Eulogy to Mr. Euler

By the Marquis de Condorcet

History of the Royal Academy of Sciences 1783, Paris 1786, Pages 37-68

Leonhard Euler, Director of the Mathematics Class at the Academy of Petersburg, and prior to that of Berlin, of the Royal Society of London, the Academies of Turin, Lisbon and Basel, Foreign member to all the scientific academies, was born in Basel on 15 April 1707 to Paul Euler and Marguerite Brucker.

His father who became the pastor of the village of Riehen near Basel was his first teacher and he had the good fortune to see the hopes of this glorious son with talents which were so sweet to his fatherly heart, to be brought to life under his eyes and by his care.

He had studied Mathematics under Jacob Bernoulli and we know that this great man accompanied his genius for the Sciences with a profound philosophy which although it may not always be coupled with genius, serves as an extension to it, to make it more useful. His teaching provided his students with the understanding that Geometry was not an isolated science but one that was at the very base to human understanding and that science can best observe the progression of the soul and is the one which best exercises our knowledge since it also provides us an understanding of the certainty and correctness at the same time. Finally, it is considered by the variety of its applications and by the advantage of enforcing the habit of a methodology of reasoning which could be further developed to seek truth in all its avenues and provide guidance for the conduct of our lives.

After being thoroughly introduced to the philosophy of his teacher, Paul Euler sought to teach the elements of mathematics to his son even though he had prepared him towards theological studies. When the young Euler was sent to the University of Basel, he was able to study under Johann Bernoulli. Assiduous in his studies and having a pleasant character he was able to become friends with Daniel and Nicolas Bernoulli, disciples and already rivals of their father. He even managed to gain the friendship of the irascible Johann Bernoulli who was willing to provide him with a private lesson once a week to help illuminate the problems which occurred during his lectures and his

studies. The remaining days were put to good purpose by Mr. Euler so as to profit from this unusual prerogative.

This excellent teaching method prevented his budding genius from exhausting itself against invincible obstacles or stray from the new insights that he was investigating. This method guided and seconded his efforts, but at the same time it obliged him to employ all of his forces which Bernoulli gauged by allowing for his age and the extent of his knowledge.

He was not able to long enjoy this advantage since he had barely been awarded his Master of Arts when his father, who had determined that he should succeed him, obliged him to renounce mathematics for theology. Happily this imposition was only passing; he was easily convinced that his son was born to replace Johann Bernoulli and not to be the pastor of Reihen.

M. Euler wrote a paper when he was nineteen years of age on the masting of ships proposed by the Paris Academy of Sciences for which he obtained an accessit in 1727, more than a great accomplishment since the young alpine native could not have taken advantage of any practical knowledge. He was only succeeded by Mr. Bouguer an excellent mathematician who was not only at the height of his career but also a professor of hydrology in a seafaring city.

Mr. Euler was also a candidate for a chair at the University of Basel at this time. Positions were offered by a drawing of lots conducted by the administrators, who determined these places, but fate was unfavorable and we cannot fault Mr. Euler but rather his country which lost him forever a few days later. Two year prior to this moment Daniel and Nicholas Bernoulli had been called to Russia and Mr. Euler who ruefully saw them leave managed to extract a promise that they look for the same opportunities which he so desperately sought to share, and one to which no one should be surprised. The splendor of the capital of such a great empire sparkles over all which surrounds her. She is the theater and the men who inhabit her feel that she can be the seat of their glory. How easily she seduces youth, snares the poor and obscure free citizens of a little republic. The Bernoulli brothers were true to their promise and took as much trouble to have a formidable rival at their side, as most men would have taken to keep themselves free from such circumstances.

Mr. Euler's trip was made under sad circumstances since he learned that Nicolas Bernoulli had already succumbed to the rigors of the Russian climate. On the day that he stepped onto Russian soil, Catherine 1 died and it appeared to announce the eminent dissolution of the Academy of which this Princess, true to her husband's guidance, had just completed its inauguration. Mr. Euler was now distanced from his country and not being able yet to enjoy the reputation and respect of a celebrated name such as Daniel Bernoulli, decided to enter the Russian Navy. One of Peter 1 admirals had provided a lieutenancy aboard one of his ships when the storm which threatened the Sciences, lifted and providentially saved Mathematics. Mr. Euler obtained the title of Professor and succeeded Mr. Daniel Bernoulli in 1733 when this famous man decided to return to his native Switzerland. It was the same year that he married Mlle Gsell a compatriot and daughter of a painter who Peter 1st had brought back with him to Russia after his first trip. Thus being able to use Bacon's expression, M. Euler felt that he had given hostages to fortune and that the country where he could hope to build a home that could accommodate his family had in fact become a country for him. Born within a nation where the governments conserve at least the appearance and the language of a republican government or in spite of the real distinctions which separate the first slaves and the last of his subjects from a despotic master, they have carefully maintained all forms of equality. The respect for the laws extends to the most indifferent usage, insofar as they have been consecrated by antiquity or public opinion. Mr. Euler now found himself located in a country where the prince exercises an authority without limits, a country where the most sacred law of the absolutists which determines the succession of the Empire was unequivocal and despised; where the administrators are as slaves to the Monarch who rules despotically over an enslaved people. It was at this moment that the Empire, trembled under the tyrannical Biren or Bühren an ambitious, cruel and rogue foreigner. This was of consequence to the scientists who had come to seek in the bosom of glory, fortune and the freedom and to taste in peace the sweetness of study.

At this moment one senses everything that tried Euler's spirit, how this stay was bound by chains that could not be broken, perhaps it

is necessary, so as to understand this period of his life that his obtuseness for work which by now was a habit, and which became his sole resource in a Capital where one found minions or ministerial enemies, some occupied to enforce his suspicions and others to cast them aside. This impression was so powerful to Mr. Euler that he still felt its full measure when, in 1741 the year after Biren's fall from power that the tyrannical period was replaced by a more moderate and humane government. Euler left Saint Petersburg to go to Berlin where the Prussian king had called him.

When he was presented to the Prussian Queen Mother, who so enjoyed the conversation of enlightened men whom she gathered to her with the same noble familiarity which is part of the princely mien and independent of their titles and whose familiarity had become part of this august family. However the Queen Mother could not elicit anything but monosyllables from Euler. She reproached him for being so shy and the embarrassment that she did not feel that she inspired. "Why will you not speak to me?" She asked. "Madame, he replied," because I have come from a country where one can be hanged for what one says."

Having arrived at the moment to provide an overview of the immense works of Mr. Euler, I have personally experienced the impossibility to follow the details and to provide the knowledge of the astonishing amount of discoveries, new methods, ingenious views covering more than thirty works published outside and the more than seven hundred Mémoires of which two hundred were deposited before his death and are destined to enrich the collection that the Academy publishes.

However, a particular characteristic seems to distinguish him from other illustrious men who have followed similar careers and obtained the glory over which his work does not appear to be shadowed. He has been embraced by the universality of the mathematical sciences and has perfected its different parts, to have enriched them by his different important discoveries and to have provided the revolutionary environment in which to deal with them. I have therefore found it advisable to construct a method to chart the different scientific branches which define the progress that was made in each and the beneficial changes that occurred thanks to Mr. Euler. Then should I

still have the strength to continue I will have at least provided a better idea of this famous man, who by so many extraordinary qualities was, so to say, a phenomenon for which the History of Science has only just provided us with an example.

Algebra had been for a considerable period of time, a very limited Science. This method was used to consider the idea of dimension as the distillation of abstraction which the human mind can attain only by the rigorous application with which one separates this notion by occupying the imagination which otherwise might benefit from some assistance or some rest to one's intelligence. Finally, the over usage of notations that this Science employs rendering it in certain ways too foreign to our nature, too far from our pedestrian concepts, so that the human spirit might easily enjoy itself and acquire some ease in its practice. Even the direction of algebraic methods rebuffed those who meditated on such things and if the point were complicated, it forced them to forget it entirely or to think only of the formulas. The road which we follow is sure, however the goal where we wish to go and the point from which we left disappears in the eyes of the Geometer. It certainly took a great deal of courage to lose sight of the earthly trappings and so be exposed to an entirely new science. As we cast our looks, towards the works of the great mathematicians of the last century, these very same ones to which algebra owes its greatest discoveries, we will see how little they knew and how best to employ these very same methods that they perfected. At the same time one will not be able to deny the very revolutionary aspect of Euler's transformation of algebraic analysis into a shinning, universal method applicable in all its aspects and easy to use.

After having provided the steps to the roots of algebraic equations, and their general solvability, numerous new theories and some ingenious and insightful views, Mr. Euler's research was directed to the calculation of transcendental quantities. Leibniz and the two Bernoulli each share the glory for having introduced exponential and logarithmic functions into algebraic analysis. Cotes had already provided the way in which to represent the roots of certain algebraic equations by sine and cosine.

These discoveries led Euler to an important discovery by observing the unique characteristics of exponential and logarithmic

quantities born within the circle and following methods by which the solutions make the problems disappear, the terms of the imaginaries which would then be present and which would have complicated the calculation, even though the are known to collapse, reduced the formulas to simpler and more convenient expressions. He was able to provide an entirely new understanding to the part of analysis which concerns itself with the questions of Astronomy and Physics. This process has been adopted by all mathematicians and has become a useful and basic tool and has produced in this section of mathematics about the same revolutionary effect as the discovery of logarithms had into ordinary calculations.

It has been known that after certain periods of great efforts, the mathematical sciences appeared to have exhausted human capabilities and to have reached their limits. When all of a sudden new ways to calculate arrived at the very moment that it seemed that they have reached the limit of their progress; a new method was introduced into the Sciences and provided them with new impetus. They are quickly enriched by the solutions to a great number of problems that the Mathematicians dared not deal with because of the difficulty and the physical impossibility to conduct their calculations to a satisfactory conclusion. Does one think that justice should be reserved to the one who knew to introduce these methods and make them useful or that a portion of the glory should go to all those who use them with success will at least have the recognition of priority so that they might quibble without being ungrateful.

At every turn in Euler's life, series analysis always occupied a special place. It is the part of his oeuvre where we see the sparkle of his brilliance, the wisdom and the variety of methods at his disposal and the resources that characterize his approach.

Continued fractions invented by Viscount Brouckner had nearly been forgotten until Mr. Euler came and perfected their theory, multiplied their applications and elicited their importance.

His novel research into the series of indefinite products provided the necessary resources into solutions to a great many useful and curious questions. It was above all by imagining the new series forms and by employing them not only to approximations, to which we are so often forced to take, but also into the discovery of absolute and

rigorous proofs that Mr. Euler has been able to expand this branch of analysis, which has grown so large as opposed to a time when before Euler it was limited to a small number of methods and applications.

Mr. Euler's oeuvre changed the face of integral calculus as the ripest discovery which man has ever possessed. He perfected, extended and simplified all of the known or proposed methods prior to him. He is responsible for the general solution of linear equations which are so varied and useful as well as the first of all formulas for approximations. There are a great number of particular methods based on different principles which are spread throughout his works and brought together in his Treatise of Integral Calculus. There we are able to observe by the propitious method of substitutions or by using an already known method to solve obstinate equations or by reductions to first differentials of first order equations and then by considering the integrals' forms; he deduced the differential equations conditions to which they may be satisfied, sometimes by the thorough examination of the factors which provides for a complete differential and other times lead him to conclude the formulation of a general class of integral equations. There is a particularity that he noticed in an equation which provided him with the opportunity to separate the indiscriminant which appeared confusing, otherwise if in an equation, where they are separate slips through the known methods, it is by mixing the indiscriminant that he was able to recognize the integral.

At first glance it may appear that the choice and success of these methods might belong to chance. However, such frequent and confident successes obligate one to consider another cause, since it is not always possible to follow the thread which has guided genius. If for example one considers the form of the substitutions employed by Mr. Euler, one will soon discover what made him presume that the operation would produce the given effect that he envisaged, and if one examines the form of one of his best methods, he expects the factors of a second degree equation and one will note that he has stopped at one of those which is particular to this order of equations. In reality this flow of ideas which the analyst conducts, is less a method of the conduct of the flow of ideas than it is a sort of particular instinct of which it is very difficult to be aware. Often he preferred not to reveal the process of his thinking rather than to be exposed to the suspicion

of a slight of hand and that he arrived at the solution only after the fact.

Mr. Euler arrived at the conclusion that differential equations are susceptible to particular solutions which are not included in general solutions in which Mr. Clairaut made the a similar statement. However, it is Mr. Euler who has shown why these particular integrals are excluded from the general solution and he is the first to have devoted some time to this theory, which has since been perfected by other celebrated geometers and the mémoire in which Mr. de la Grange has left nothing unknown concerning the nature of these integrals and their use in the solution of problems.

Here is an opportunity to mention another body of calculus which belongs almost in its entirety to Mr. Euler. This occurs when one is seeking particular integrals for a certain determined value of the unknowns which are contained within the equation. This theory is all the more important to us since the general integral is hidden from our search and that when there are problems where the approximate value of these particular integrals are not within the parameters that we expect, the knowledge of these particular integrals can overcome these deficiencies. In effect one then has, at least for certain points, a rigorous value and this knowledge appears with the general approximate value as sufficient for nearly all analytical needs.

No one has made a more extended and better use of the methods which provide for the approximate value of a determined quantity through the use of differential equations for which one has obtained the first values. He has also provided a direct method to immediately deduce the same equation with a value very close to the real one, through which the powers are removed from their difference and can be discarded; a method by which the approximations used by the mathematicians can not be extended though the equations for which the observations or the particular considerations do not provide the first value for these known methods.

That which has been said is sufficient to show to which point Mr. Euler has plumbed the nature of differential equations, the fountainhead of difficulties which oppose integration and the way in which to elucidate or conquer them. His great work on this subject is not only a valuable collection of new and extended methods, it is

more like a mine of fertile veins, so that any man born with some intelligence would not travel across its pages without bringing out some rich spoils. It can be said of these works by Mr. Euler, as with his many others, that the methods that have been preserved will provide, long after he has gone, the ability to resolve important and difficult questions, and that his works will produce more than one discovery and more than one reputation.

A nearly unknown but intelligent work on finite differences by Mr. Taylor was made into an important branch of integral calculus by assigning a simple and workable notation which was found to apply successfully to the theory of series. This was done by searching for the sums or the expression of their general terms and to those of the roots or determinant equations, by which to obtain with a simple calculation the approximate value of the products or the indefinite sums of certain numbers.

The discovery of partial differentials, in reality belongs to Mr. d'Alembert, since it is due to him that there is an understanding of the general form of their integrals. However, in Mr. d'Alembert's works we often saw the results of the calculations rather than the calculations themselves. It is to Mr. Euler that we have the notation. He knew how to best present it, through the deep understanding of the theory which provided him the way in which to solve a great number of these equations, to distinguish the forms of the orders of integrals, for the different number of variables, to reduce these equations when they attain a certain form and become ordinary integrations, to provide for a way in which to remember these forms, through substitutions after which they vanish; in one word, to discover within the nature of these partial differential equations most of these singular properties which render the general theory so difficult and thorny, qualities which are nearly inseparable in Geometry where the degree of difficulty is so often the measurement of interest that one takes in a question and the honor that one attaches to a discovery. The influence of new truth in Science itself or to some application is the sole advantage that can balance the scale against the odds of such a difficult victory within the body of men for whom it is a pleasure to perceive a truth and one which is always proportional to the efforts that have been required. Mr. Euler did not neglect to look at any part of Analysis; he provided

proofs to some of Fermat's theories concerning indeterminate analysis and found some not less interesting and not less difficult to discover. The Knight's move in chess, and other situational problem not only pricked his curiosity but exercised his genius, he mixed his most important research and these apparent recreations sometimes more difficult but seemingly useless even to the progress of the Sciences until now. Mr. Euler was much too intelligent to not sense the inconvenience of spending too much time on what might appear to be merely curiosities, but at the same time his intelligence was too broad not see that their uselessness was only momentary and that the best way to end the search was to seek to deepen and generalize them.

Since Descartes, the application of Algebra to Geometry had occupied nearly all of the mathematicians of the past century; however Mr. Euler found that they had not come even close to exhausting the subject. It is to him that we are indebted for the new research concerning the number of points which determine a curved line of which the degree is known and also of the intersections of lines at different degrees. Equally, for the developments of general equations of curves of the second and third degrees, in other words the development of any random order which are similar to the generating curve; a remarkable equation due to its extreme simplicity.

The general theory of curved surfaces was little known and Mr. Euler is the first to have developed it into an elementary work to which he added that of oscillating radii of these surfaces; he arrived at the singular conclusion that the elemental curve of a surface is determined by two of the oscillating radii of the curves formed by the intersection of the surface and that of the plane which passes through a given perpendicular point. These rays are either the greatest or smallest of all those that belong to the series of curves formed in this way and that finally they always find themselves in planes perpendicular to one another.

He additionally provided for a method to determine surfaces which can be developed in a plane and a theory for spherical geographic projections. These two works contain a link to the application of partial differential equations to geometrical problems that may be employed in many applications which Mr. Euler was the first to introduce.

His research into curves which are inscribed onto a sphere and are algebraically rectifiable curved surfaces for which the corresponding parts are parts of a given plan and are equal to themselves, moved Euler to a new type of analysis for which he coined the term indeterminate infinitesimal analysis. As is the case in ordinary indeterminate analysis the quantities which remain arbitrary are subject to certain conditions and in the same way that indeterminate analysis could sometimes help in the improvement of algebra, Mr. Euler regarded his new analysis as a science which one day would be useful in the progress of integral calculus.

These particular questions which are not part of the body of methodology of the mathematical sciences and are not considered as applications that can be useful must not be looked upon as a way in which to exercise the genius of the geometricians who are nearly all in the sciences. One begins by separately cultivating some isolated parts and as the successive discoveries become apparent, and more often than not, it is the light which shines from these conjunctions from which the greatest discoveries become momentous in the history of the human spirit.

The question to determine the curves or the surfaces onto which certain indefinite functions are greater or lesser for all the others had perplexed the most famous geometricians during the past century. The solutions to the problems of the least resistant solid, and the curve of quickest descent and the problem of largest isoperimetric areas were much celebrated in Europe. The general method for solving the problem was hidden in these solutions, especially in those of Jakob Bernoulli who had found the answer to the isoperimetric question which provided him with an advantage over his brother since so many masterpieces subsequently fathered by Johann Bernoulli. However, to develop this method it was necessary to reduce it into a general formula, and this is what Mr. Euler accomplished in 1744 which became one of his most important works. To find these formulas it was necessary to consider the use of curved lines; and fifteen years after the young Geometer M. de la Grange, who within the content of his first Essays established himself as a worthy successor to Mr. Euler by successfully resolving the identical problem by purely analytical means. Mr. Euler was the first to appreciate the novel

efforts in the art of calculating and busied himself to expose this new method and offered the new principles, which were drawn with such clarity that the elegance sparkles in his work. Never has nor never will genius provide for such gracious generosity and never did he show himself to be superior by having to share a little glory which passion so violently manifests in ordinary men.

We will end this exposition of Mr. Euler's works into pure analysis with the observation that it would be unfair to limit his influence only to the field of Mathematics, and not to the innumerable discoveries which fill his works. The lines of communications that he opened between all the parts of such a vast science, the general objectives that he often refrains from mentioning but which hardly escape his attentive gaze, the paths which he not only opened but removed the initial obstacles have so benefited a science which will be enriched and which posterity will enjoy even at the point of forgetting the hands from which it received them.

The treatise on Mechanics that Mr. Euler produced in 1736 is the first great work in which analysis was applied to the science of motion. The number of novelties or those topics presented in such a new form which make their appearance in this treatise might have surprised other Geometricians had Mr. Euler not already published the majority of the work in a separate publication.

In a number of works concerning the same subject, Euler remained steadfast to Analysis and the employment of this method provided it with the necessary acceptance to make it the most widely used method of all.

The solution to the problem that is sought for the motion of an object which is launched into space and is attracted towards two points has become famous by Euler's ability of make the necessary substitutions thought a reduction to quadratic equations so that their complexity and form might have made them appear to be insoluble.

To the motion of a solid body of a given figure he applied analysis to motion which provided him with the beautiful theory which had already been discovered by Segner; that a body of undetermined shape can turn freely in a uniform motion around three perpendicular axes between themselves, to a body of knowledge with various singular

properties of the three principal axis, irrespective of their shape and the laws of the accelerating forces which act onto their parts.

The vibrating string problem and all those that belong to the theory of sound or the laws of oscillations in air had been subjected to analysis by these new methods which in turn enriched the calculus of partial differentials equations. A theory for the motion of fluids, applied with the same calculations is astonishing by the clarity he has brought to so many thorny issues and the ease with which he knew to impart to those methods founded on such a profound analysis.

All of the problems in astronomical physics which were treated during this century were resolved by Euler's particular analytical methods. His calculations concerning terrestrial perturbations and especially his theory of the moon are models of simplicity with which precision can be brought to bear with these methods. By reading this last work, one is no less astonished to see the lengths to which a great man of genius, animated exclusively by the desire to leave nothing to chance concerning important issues, can push the limits of patience and the obstinacy to work.

In the past Astronomy employed only geometrical methods, and Mr. Euler felt that it could do well if rescued by analysis and he provided proof by examples which, imitated by many famous scientists which one day would provide any entirely new platform for this science. He embraced naval science in a large work of which a truly knowledgeable analysis served as a basis and one where the most difficult questions were subjected to useful and general methods which he knew how to create so well. Shortly after he published an abridged version of this same work, in which he encapsulated in an identical and simpler form whatever is most practical and what those who dedicate themselves to naval service need to know. Even though this opus was destined exclusively to the schools of the Russian empire, he received compensation from the King who believed that this work to be useful to all mankind and should have the acknowledgement of all Sovereigns who wished to demonstrate that from even the far reaches of Europe such rare talent could not escape his view nor his rewards. Mr. Euler was particularly taken by this mark of recognition by such a powerful king. He was also recognized by the hand that had helped him, it was that of Mr. Turgot, Minister

respected throughout Europe as much by his dazzling brilliance as by his virtues, a man made to command opinion rather than to obey it and whose opinion was always dictated by the truth and never to attract onto himself public approval, could flatter a wise man too accustomed to glory to still remain sensitive to the murmur of his renown.

Within the content of men of superior genius, the extreme simplicity of character can be allied with spiritual characteristics which may or may not appear to announce cleverness or refinement. Mr. Euler despite the simplicity which never left him always knew how to wisely distinguish and always indulge the appreciation of enlightened admiration from those whose vanity leeches onto great men and assures them at least of being noticed for their enthusiasm.

His works concerning Dioptics are based on a less deep analysis and we are tempted to be grateful to him as one would for a trophy. The different rays from which one solar ray is formed, is subjected to different refractions; thus separated from its neighbors they appear solitary or less mixed and each provide its own unique color; this refrangibility varies within each media for each ray and follows a law which is not the same as that of average refraction for these environments. This observation gave reason to believe that two prisms both unequal and comprised of different substances combined together might divert a ray from its course without decomposing it or rather by replacing, by a triple refraction, the elementary rays in a parallel direction.

The proof of this conjecture might have depended in the glass and whether the elimination of iridescence of colors which project through the glass lenses. Mr. Euler was convinced of the possibility of success based on the meta-physical idea that if the eye is composed of different parts, it is uniquely so with the intention of eliminating the distortionary effects of refrangibility. Therefore it was only a matter of finding the way to imitate Nature's way and he proposed a method based on a theory that he had formulated. His first experiments excited the Physicists to re-evaluate an object that they had apparently neglected since their experiments did not agree with Mr. Euler's theory, but they confirmed the opinions that he had concerning the improvements to glasses. His insight provided the laws of dispersion

in different medias. He abandoned his first ideas and submitted new ones to proof by experiments and enriched Dioptics with analytical formulas which were simple, useful, general and applicable for every instrument that could be built.

Also from Mr. Euler are some Essays on the general theory of Light, whose phenomenon he had hoped to match with those of fluid oscillations in a liquid since the hypothesis of light rays being emitted in a straight lines presented insurmountable difficulties. Magnetic theory, the propagation of fire, the laws of body adhesion and that of friction provided the opportunity for lengthy calculations applied to hypothesis which unfortunately should have been based on experimentation.

Probability and political arithmetic were also part of his indefatigable undertakings. We will only mention his research in the tables of mortality and the ways of improving actuarial science. The improvements that he made in deducing phenomenon with more precision, his methodology to increase precision in error theory, the necessary reasoning for savings institutions with the goal to assure that widows and children receive either a fixed sum or lump sum payment, payable subsequent to the death of the husband or a father as a ingenious alternative to life annuities and to make available immediately to families the small savings that the head of the family could make on his daily wages, or by appointments of either a commission or a sinecure.

We have seen in the Eulogy to Mr. Bernoulli, that he was the only other person to share the distinction of having won the prize of the Academy of Sciences thirteen times. They often worked on the same topics and the honor to beat out ones competitor also had to be shared without ever having this rivalry interfere with the reciprocity of their mutual feelings for one another or cool their friendship. In the examination of the subjects in which one or the other gained their victory, one can see that the success depended especially on the character of their talent. If the question demanded some special disposition in the presentation or a useful way to employ the outcome of an experiment or new and ingenious breakthroughs in Physics then the advantage was to Mr. Bernoulli. If the matter were to conquer great difficulties in calculation and was it necessary to create new and

improved methods in analysis, it would then be Mr. Euler who would be designated. If one necessitated the foolishness to choose between them, it would not be between the two men that one might have to select but rather between two different types of character and two ways to employ genius.

We might only have provided for a very imperfect idea of Mr. Euler's extraordinary output, had we not added to this short sketch on his works that there were many important subjects through which he retraced his steps even to the extent of remastering his work many times. At times he substituted a direct and analytical work with an indirect method, sometimes extending his work to include cases which had escaped his initial intentions, adding new examples which he knew to choose with singular ability amongst those which provided some useful application or some interesting remark. His only intention was to provide his work with a more methodical rigor and to insist on greater clarity, to add a new degree of perfection. When he published a mémoire on a new subject, he exposed with the greatest simplicity the path that he chose to follow, he brought to light the difficulties or detours and after having his readers scrupulously follow the cadence of his walk during the initial steps, he showed them how it became possible to find a simpler road to follow. It can be seen that he much preferred the education of his students than the small satisfaction derived from astonishment; he never believed that he had truly done enough for Science if he did not feel that that he had added new truths to enrich it and the exposure of the simplicity of the idea which lead him there.

This way of embracing all of Mathematics was to always have, so to say, a closeness within himself all the questions and theories that was for Mr. Euler a source of discovery closed to nearly everyone else and available to him alone. In such a way that in the course of his work there sometimes appeared a unique method to integrate a differential equation or sometimes a remark concerning a question in Analysis or Mechanics lead him to a solution to a very complicated differential equation which did not lend itself to direct methods. At other times it would be a problem that appeared insurmountable that he resolved in an instant by a very simple method or an elementary problem with a very difficult solution that could only be overcome with the greatest

efforts. At other times simple numbers, or a new series presented questions novel by their uniqueness which took him to unexpected proofs. When Mr. Euler made mention that it was to chance that he owed such discovery, it was not as an attempt to diminish the importance, since one could easily see that chance favors the prepared mind of a person who possesses a vast body of knowledge coupled with a rare wisdom. In fact we should praise him for his candor, even though it cost him some glory. Men of genius rarely display the underhandedness of vanity which only serves to belittle in the eyes of an enlightened jury that which it enlarges in the opinion of the crowd, and that is that the man of genius feels that he will never be larger that in showing himself as he is, as self opinion has no hold on him but that it so well exercises with such power over other men.

When one reads the life of a great man, one becomes convinced of the imperfection that is attached to humanity, either because of the righteousness of which we are capable does not allow us to attain a just recognition in our fellow men of a superiority from which nothing else can console us, or either that the idea of perfection in another wounds or humiliates more than that of greatness. It then would appear that we have need to find a weak point, we seek some fault that can raise us in our own eyes, and that we are involuntarily transported to defy the sincerity of the writer if he fails to reveal this soft underbelly or if he fails to lift the unfortunate veil which hides these faults.

At times Mr. Euler appeared only to enjoy the pleasures of calculation; and to look at the point in Mechanics or Physics which he examined only as an occasion to exercise his genius and to surrender to his dominant passion. Other scientists have reproached him for having at times used his mental forces to prove physical hypotheses or even metaphysical principles for which he had neither sufficiently examined the truth or its solidness. He was criticized for having relied too heavily on his mathematical abilities and that he neglected those questions which if he might have examined them in closer detail might very well have given him the answers that he was attempting to resolve. We will concede that the first reproach is not without foundation and we admit that Mr. Euler the Metaphysician or even the Physicist was not as great as the Geometer, it is regrettable

without a doubt that in some parts of his works, for example of those concerning Naval science and Artillery were practically entirely useful towards the progress of mathematics, however we feel that the second criticism is much less justified since everywhere in Mr. Euler's works we see him occupied by adding to the richness of analysis, and extending and multiplying its applications at the same time that it appears to be his only instrument we realize that he has only wished to make it a universal tool. The natural progress of the mathematical sciences should have lead this revolution, but he saw it, so to say, develop right in front of his eyes, and it is to his genius that we know it as the prize of his efforts and his discoveries. Therefore, when he appeared at times to extend the useful resources of his art, and to extract every secret to resolve what might be solved with some foreign thought to mathematics might very well provide the perfectly correct answer, he only wished to exhibit the power of his art. And we must forgive him if he appeared to be occupied with another science, but it was still and only to the progress and propagation of Analysis to which these works were dedicated since the revolution which is the fruit is only of the first rights which recognizes man and one of the greatest entitlements to glory.

I did not think it necessary to interrupt the details of Mr. Euler's work due to the telling of the very simple and unvaried events of his life.

He established himself in Berlin in 1741 and he remained until 1766.

The Princess of Anhalt-Dessau, the King of Prussia's niece wished to receive some lessons in Physics; these lessons have been published under the title: Letters to a Princess of Germany. A very valuable work due to the singular clarity with which he has exposed the most important truths concerning Mechanics, Astronomical physics, Optics, Theory of sounds and with less philosophy but more ingenious ways and much more wisdom than Fontenelle's Plurality of the Worlds could show vortices. Euler's name, so highly regarded in the Sciences and the imposing way in which his insights reveal the most thorny and abstract ideas, reveals in these simple and easily readable Letters and unique charm and those who have not studied Mathematics, are astonished and flattered to be able to understand a

work by Euler and are grateful that his message has been placed within their grasp. These elementary notions in Science, acquire grandeur due to the closeness that we achieve by approaching the glory and genius of the famous man who penned them.

The King of Prussia employed Mr. Euler for the calculation of the currency, for the water pumps at Sans Souci, for the assessment of several navigation canals; this Prince who had not been born with blinders believed that great talents and a profound knowledge were neither dangerous nor superfluous qualities and that happiness was reserved by Nature as a value above ignorance and mediocrity.

In 1750, Mr. Euler travelled to Frankfurt to meet his mother who then a widow and returned with him to live in Berlin. He had the good fortune to keep her until 1761. During those eleven years she enjoyed her son's reputation as only the heart of a mother can and was perhaps made even more joyful through his tender and attentive care which his glory made priceless.

It was during his stay in Berlin that Mr. Euler, bound by gratitude to Mr. de Maupertuis, felt an obligation to defend the principle of least action onto which the president of the Prussian Academy had place his hope of a great reputation. But the way that Mr. Euler chose could only be used by him, since it was to resolve this Principle through some very difficult problems in Mechanics, as it was in legendary times that the Gods deigned to make for their favorite warriors, impenetrable armor to withstand the blows of their adversaries. We only wish that the recognition of Mr. Euler was limited to the exercise of a more noble and worthy protection, but it cannot he hidden that he exhibited too strongly worded condemnations of Koenig. It is painful that we are obliged to count a great man amongst the enemies of a great persecuted scientist. Happily Euler's life places him above more serious suspicion. Without this simplicity and indifference to fame of which he was noted, one might have thought that the pleasantries of that illustrious companion of Koenig's (pleasantries that M. Voltaire has himself condemned and for good reason) might have altered the character of this wise and peaceful Geometer whose only fault was an excess of gratitude, which was a selfless sentiment and for which he was wrong for the only time in his life.

The Russians penetrated through the Brandenburg Marshes in 1760 and pillaged Euler's farm near Charlottenburg, but General Tottleben had not come to make war on the Sciences and when informed of Euler's losses, he hastened to pay for reparations above and beyond the actual value of the damages, and having realized the effect of this involuntary as a lack of respect to her Imperial Majesty Elizabeth who also added four thousand florins as an additional indemnification which in itself was more than sufficient. This apparent concern was unknown in Europe, and we quote with enthusiasm similar actions to the same effect that our forefathers long ago had transmitted through the ages. That this difference in our judgment is it not proof that this happy progress of the human race, which some writers still deny for no only reason then that they may be accused of having been accomplices.

The Russian government had never treated Mr. Euler as a foreigner, and a portion of his salary was always paid to him in spite of his absence and when the Empress recalled him in 1766 he consented to return to Saint Petersburg.

In 1735 the efforts that he had expended towards an astronomical calculation for which other mathematicians had required months to do and which he accomplished in just a few days, caused him an illness which then caused the loss of an eye. He had reason to fear for complete blindness if he exposed himself once again in such an inclement environment. The interest of his children overcame this fear, and if we are to believe that Euler's studies were an exclusive passion, we will undoubtedly arrive at the conclusion that the examples of paternal love have better proved that it is the most effective and the most tender of our affections.

A few years later he narrowly escaped from the unfortunate situation again that he had forewarned, but he managed to preserve for himself and Science the ability to distinguish large letters traced onto a blackboard with chalk. His sons and students copied his calculations and wrote by dictation the remaining Mémoires. If we are to judge by the numbers alone, then we might think that the elimination of all distraction and the new energy that this reassessed force onto the remaining faculties allowed him to gain what the loss of his eyesight could not make him lose in desire or the ability to work.

Furthermore due to the nature of his genius, Mr. Euler had even involuntarily prepared his extraordinary resources by examining these great analytical formulas, so uncommon before his time, yet so frequent in his works, for which the combination and development so aptly placed together is such simplicity and elegance, whose form pleases the eyes as much as the spirit and one can see that these are not the fruit of calculations that have been written onto paper, but rather they are the produced entirely in his head and were created by an powerful and active imagination. There exists in Analysis (and Mr. Euler has greatly increased their numbers) formulas with a common and near daily application and he had them at his fingertips and used them in conversation. When Mr. d'Alembert saw Euler in Berlin he was astonished by the prodigious memory that Mr. Euler possessed and finally the ease with which he calculated in his head. He was fined tuned to an unbelievable acuity and we would be surprised were it not for the fact that Euler constantly produced extraordinary and prodigious results. Euler had the intention of exercising his grandson's memory to extract roots and to tabulate the first six powers of all numbers from 1 to 100 and to keep them firmly within the memory. Once there were two of his students who had calculated a convergent series to the 17^{th} term which was certainly complicated and needed to be written on paper but when the results were compared a discrepancy appeared by one number when the students asked the Master who was correct, Euler did the entire calculation in his head and his answer proved to be correct.

Since his lost eyesight there were more important things to do than to play with magnets and provide mathematics lessons to his grandson, who showed a propensity to the sciences.

He still went to the Academy principally only when difficult circumstances existed and where he felt that his presence could bring the greatest good in maintaining the freedom so necessary and so we can sense how a perpetual President, appointed by the court can trouble the rest of an Academy and all that she must fear by not having appointed the president from the ranks of a scientist. He did not even feel what their votes might bring to his reputation, how should men so uniquely qualified in their pleasant work and not know how to speak the language of science, could they defend themselves especially

if these foreigners, isolated and distanced from their native countries and who receive everything from a government from whom they request a vote of no-confidence the head of the government which has provided for them.

However there exists a state of glory where one can be above fear and that is when all of Europe would rise against a personal affront made against a great man, who can without risk , deploy his renown against unjust authority and to elevate in favor the voice of science, a voice which cannot be ignored. Mr. Euler's simple modesty felt his force and on more than on occasion used it to good purpose.

In 1771 Saint Petersburg was struck by a terrible fire the flames of which reached all the way to Euler's house. A Baseler by the name of Peter Grimm(whose name should be preserved for posterity) learned of the imminent danger to his compatriot who was blind and suffering and who dashed through the flames went right to where he was in the flaming house threw him onto his shoulders and saved him by risking his own life. The library and furniture were consumed by the flames; however Count Orloff saved the manuscripts and this fact, in the middle of the horrors of this great catastrophe, one of the great unselfish and flattering acknowledgements that the public rendered to the sciences, was that Euler's house which had been a gift of the Empress was quickly rebuilt.

Euler had thirteen children from his first wife of whom eight died in infancy, his three sons survived him but unfortunately he lost his two daughters during the last year of his life. Of the thirty-eight grandchildren, twenty-six survived him. In 1776 he remarried after the death of his first wife, Mlle Gsell her sister. He had kept all the traditions that had come from his father's house and as long as he had his eyesight he gathered his family for communal prayer and oftentimes read form the scriptures.

He was extremely religious and we have from Euler a new proof of the existence of God and the spirituality of the soul; the latter has been adopted into the studies at certain theological seminaries. He conserved his country's religion which was a conservative Calvinism and he does not appear to have adopted the example of other Protestants any personal opinions or to have formed his own religious system.

His knowledge was extremely extensive especially in the History of Mathematics; it has been presumed that he had brought his curiosity to the point of teaching himself the rules and procedures of Astrology and that he had even made some practical applications. Furthermore in 1740 he was given instructions to prepare Prince Ivan's horoscope, he explained that the function belonged to Mr. Krafft who as the court Astronomer was obliged to do it instead. Incredulously as it may seem to find such things in the Court of Russia was commonplace in the courts of Europe one century prior, in fact the courts of Asia have yet to cast of this yolk and it must be admitted that except for the most basic moral precepts, that there has never been until now any truth that can be mentioned as having fit into the general scheme of things with more general acceptance, for as long with such errors, ridiculousness and sadness.

Mr. Euler had studied nearly everything available in the branches of Physics, Anatomy, Chemistry and Botany; however he flourished in the field of Mathematics which prohibited him to consider the very least importance in his knowledge of other things, even though extensive for a man who might be susceptible to the pettiness of self importance, he aspired to a greater universality.

The study of ancient literature and languages were an original part of his education for which he preserved a taste for leaning all his life and his memory did not permit him to forget anything, but he had neither the time nor the desire to add to his original studies, he did not read any of the modern poets, but did know the Aeneid by heart. Furthermore Mr. Euler never lost sight of Mathematics even when reciting verses from Virgil. Everything was available to remind him of the object which filled his thoughts. One of his mémoires in Mechanics was sparked by a verse form the Aeneid and remains in the work.

It is said that for men of great talent, the pleasure to work is a sweeter reward than glory; if this truth requires proof of examples then Mr. Euler should be proof enough.

Never within the confines of discussions with other famous scientists did there every escape some shred of evidence that might have thrown suspicions onto the fact that he was concerned with his own self-importance. He never boasted about any of his discoveries

and if someone claimed an issue with one of his works, he rushed to fix some an error without ever examining the issue of equity and whether the repair demanded a total abandonment of the issue at hand. Had some error been found, if the mention was unsubstantiated, he simply forgot about it; if it was worthy he corrected it and thought nothing more, as often the only merit of those who claimed to have found his mistakes simply consisted of applications which he himself had taught them and to theories that he had simplified from the greatest difficulties.

Nearly always the most mediocre of men wish to have themselves appreciated only by a referee proportional to the high esteem that they wish to give to their own knowledge or genius, exhausting to all those who are greater than them, they shown no compassion to the less fortunate, one might say that they have a perverse streak which advises them of the need to deflate those around them. On the contrary to Mr. Euler who wished to celebrate the talents from the moment or that he noticed some fruitful mémoires and certainly not in expectation of the public having to garnish his acknowledgement. One sees Euler using his time to remake and illuminate his works, and to resolve problems that were already resolved to which remained only the necessity of continued elegance and improved method which he did with the same passion, the same constancy which he would have put to use in the pursuit of a new truth to whose discovery would be added to his just reputation. If however, there had existed some need to satisfy the demands for glory in his heart, the very frankness of his character would not have permitted him to dissimulate its hiding. However, the glory that he never sought came and found him. The singular abundance of his genius struck even those who were not capable of understanding his work, for even though his work is uniquely scientific, his reputation extended to those who were unfamiliar with Science; and he was to Europe not only a great scientist but also a great man. It was customary in Russia to bestow military titles onto those men who were very foreign to military service; it provided a sense of validation to those who believed that it was the only noble profession and thereby acknowledging that it was baseless and there were some scientists who obtained the rank of Major-General. Euler wanted none of it but then what title would be

an appropriate to honor Euler's name. In respect for the natural rights of men to do as they pleased it is necessary to provide for a wise indifference to these tendencies that exist in human vanity which are so childish yet so dangerous.

For most of the Northern aristocracy to whom he was personally known, they had already provided him with marks of their esteem, or more like veneration that one can hardly deny when one sees the uniting of such simple virtues to such vast heightened genius. When the Royal Prussian Prince went to Saint Petersburg, he stopped to pay his respect to Mr. Euler and there he passed a number of hours at the bedside of this illustrious old man, hands locked together with one of his grandchildren on his knee and one whose precocity for mathematics had made him the particular object of grand paternal tenderness.

Of all the famous mathematicians who exist today, most are his students and there is not one who has not been formed by reading his works and who has not learned the formulas and methods that he now uses, which in his discoveries has not been guided by Euler's genius. They owe this honor to the revolution that he produced within the Mathematical Science by submitting them all to analysis and to his work ethic which permitted him to embrace the entirety of the Science and to the categorization that he knew to give to his work; to the simplicity and elegance of his formulas, to the clarity of his methods and the proofs which have increased the choice of his examples. Neither Newton nor even Descartes, whose influence today is so strongly felt, has attained this glory among the mathematicians that Mr. Euler possesses, alone.

However as a professor who educated the students which belong to him, there is one among those and we will mention his eldest son that the Academy of Sciences selected to replace him without the fear that such a such an honorable succession awarded to the Euler name, as that of Bernoulli, might set a dangerous precedent. There is a second son who today studies Medicine but who in his youth won from this Academy a prize concerning the average mean movement of the Planets. There is also Mr. Lexell whose premature death has taken him from the Sciences and finally Mr. Fuss the youngest of his disciples and his last working collaborator who was sent from Basel to

Mr. Euler by Mr. Daniel Bernoulli, who has shown himself worthy of his works and justified Bernoulli's choice and learned from Euler's lessons and who after having delivered the public eulogy at the Saint Petersburg Academy had just married one of his grand-daughters.

Of the sixteen professors attached to the Saint Petersburg Academy eight were trained under him and all are known through their works and have been awarded various academic distinctions and are proud to add the title of Euler's disciples.

He had full possession of his faculties and apparently all of his strength and absolutely no change announced itself that the Sciences were in fear of losing him. On 7 September 1783, after having enjoyed some calculations on his blackboard concerning the laws of ascending motion for aerostatic machines for which the recent discovery was the rage of Europe, he dined with Mr. Lexell and his family, spoke of Herschel's planet and the mathematics concerning its orbit and a little while later he had his grandson come and play with him and took a few cups of tea, when all of a sudden the pipe that he was smoking slipped from his hand and he ceased to calculate and live.

Such was the end of one of the greatest and most extraordinary men that Nature ever produced whose genius was equally capable of great effort and continuous work which multiplied his productive work span to beyond what one dares to expect from a human. And each of which was original, during which his mind was always occupied and his soul always calm. He had finally achieved what he had always deserved, a happiness that was cloudless coupled to a glory that has never been questioned.

His death was seen as a loss to the Public even in the country where he lived. The Saint Petersburg Academy solemnly mourned his passing and will install a marble bust at their expense to be placed in the assembly hall which had already provided an already singular honor. Designed within an allegorical setting, Geometry is leaning on a board filled with calculations, and these are the new formulas for lunar theory that the Academy gave orders to have inscribed. Thus for a country, which at the beginning of this century, we regarded as still barbarian, has taught the most enlightened nations of Europe how to honor the lives of great men in recent history. It has provided to

these nations an example that many among themselves might very well be embarrassed that they knew not how to anticipate or imitate.

Translated by
John S.D.Glaus
The Euler Society
March 2005