



# How Euler Did It



## by Ed Sandifer

### Euler and the Pirates

April 2009

We sometimes celebrate the first of April with a column on the lighter side of Euler scholarship. We continue that occasional tradition with some stories intended to help perpetuate the idea that no matter where we look, we can find a connection with Euler.

Once is amusing. Twice is a coincidence. Three times is worth remarking about.

We've recently come across a third, and maybe a fourth, person with connections both to Euler and to piracy or privateering. For our collective amusement, we thought we'd share them with you.

#### Maupertuis

The first, of course, was Maupertuis (1698–1759), President of the Berlin Academy for much of the time Euler was there. According to Mary Terrall's award winning biography, *The Man Who Flattened the Earth*, [Terrall 2002] his family fortune was built on piracy. His father, René Moreau, had been a merchant and ship owner working out of the French port of San Malo. When he got a license from the King and agreed to prey mainly on British ships, he was quite successful and made a fortune in the 1690s by attacking English shipping. He married well enough to be a candidate for the aristocracy, and after he retired from seafaring, he was awarded the hereditary title "sieur de Maupertuys" for Services to the Crown.

As a scientist, Maupertuis is best known for leading an expedition to Lapland to take measurements on the shape of the earth, and then combining those measurements with data from other expeditions that he had organized to determine that the Earth was not a perfect sphere, but instead it bulges at the equator, as Newton had predicted.



Maupertuis is also credited with the discovery of the so-called Principle of Least Action, which he stated as, "In all the changes that take place in the universe, the sum of the products of each body

multiplied by the distance it moves and by the speed with which it moves is the least possible." Though Maupertuis was not the clearest of writers, the Principle of Least Action is a great and basic principle of physics. It leads to several important laws of conservation and explains why so many problems in physics involve maximization and minimization.

Euler used the Principle of Least Action to great advantage, and it provided much of the impetus for his work in the calculus of variations. Indeed, some people suspect that Euler had formulated the Principle of Least Action before Maupertuis did, but ceded priority to avoid disputes at the Academy. They might argue that René Moreau had not been the only pirate in the family.

### Henry Watson

The second privateer was Henry Watson (1737? –1786), the Englishman living in India who translated and published Euler's *Théorie complette* [E426] from its original French in 1776, just three years after Euler published it. Watson's friends published a second edition of the translation in 1790, to which they added a "Sketch of the life and character of the late Col. Henry Watson." There we learn that "though *Holland* may boast a *Coehorn*, and *France* a *Vauban*, yet *England* can boast their superiors in a *Robins* and a *Watson*." Menno Coehorn (1641–1704) and the Marquis de Vauban (1633–1707) were important military engineers, and Benjamin Robins (1707–1751) wrote the book on artillery that Euler subsequently translated from English into German. [E77, Sandifer December 2006] All three were more famous in 1790 than they are today.



The Coll Hen WAT SON. Chief Engineer.

Watson's friends also tell us that "as early as 1753, he cut a conspicuous figure as a mathematician in the Ladies Diary," a popular forum in England at the time for exchanging recreations and problems in

mathematics, and whose readership was not confined to women. [Peterson 2009] "He gave signal proofs of his superior abilities as an engineer; particularly at the siege of Belleisle in 1761." That siege was the subject of a Mother Goose nursery rhyme. In 1762, during the siege of the Morro Castle in Havana, he was carried to his tent, thought dead, but he revived and returned to the field to see the fort captured.

Watson lost a fortune reported to be £100,000 (worth about \$25 million today) to the British East India Company when they let his project to modernize the docks in Calcutta go bankrupt, then bought the nearly-completed project for pennies on the pound. He angrily went back to England to try to sue the BEIC to recover his fortune, but died of a fever before he arrived.

Shortly before that, he'd borrowed money to build three ships based on Euler's innovations, completing only two of them. THEN he got around to applying to the Crown for a privateer's license, intending to advance England's interests near the Philippines. He didn't get the license and instead used them as merchant ships. Some sources say he used them to transport opium, but others say that he put them into the tea trade where, his (very sympathetic) eulogists tell us that they were among the swiftest and best-handling ships on the seas.

Watson was an important and mostly-successful man of his times, and though he was never actually a pirate, he'd wanted to be one.

#### Kenelm Digby

A third privateer has more tenuous connections to Euler. Euler read Fermat rather systematically, and used Fermat's problems as a source of inspiration, especially in number theory, but also in geometry. One interesting problem in geometry [Sandifer December 2008] was from a letter from Fermat to Kenelm Digby (1603–1665). [Fermat 1658, Fermat 1894] The original copy of that letter is now lost, but as was common in that era before photocopying machines, Digby transcribed the entire letter and sent it on to John Wallis, and that transcription survives among Wallis's correspondence.

That problem involved the figure below, in which *AMB* is a semicircle and *ABFE* is a rectangle with sides in the ratio  $\sqrt{2}$ :1. Then for any point *M* on the circumference of the semicircle, if *M* is joined to the points *E* and *F* and if the line segments *ME* and *MF* cut the diameter at *R* and *S* respectively, then the segments *AS*, *SB* and *AB* satisfy the relation  $AS^2 + RB^2 = AB^2$ . It suggests a Pythagorean theorem, but none is evident. Euler solved the problem in [E135].



A second connection might be made through a substance called the "powder of sympathy." This alchemical concoction was made from copper sulfate and rainwater. According to Digby, he could cure a wound by rubbing the powder of sympathy on the weapon that had inflicted the wound or even by immersing bandages that had covered the wound in a solution containing the powerful material. Digby explained that it worked by "attraction and by the small material particles given off by all objects." Moreover, and this is a key detail, Digby observed that his patients would swoon and cry out as he applied his cure to the weapons or bandages.

A hundred years later, in Euler's time, the greatest technological problem of the era was to find a way of calculating longitude at sea. Untold thousands of lives and, perhaps worse in some eyes, valuable treasures had been lost when ships were lost at sea for want of more accurate navigational tools. In the 17th century, the Paris Academy had offered a generous prize for a means of finding longitude at sea, but had squandered the prize on a fairly simple water anemometer that didn't work if there were any currents involved and turned out to be useless. In 1714, the British Parliament offered an even more lavish prize, up to  $\pounds 20,000$  (worth about \$5 million today.) That prize was eventually awarded for a clock invented by John Harrison, but the Longitude Board was so slow in making its decision that Harrison had died before collecting the full prize. The Board also awarded a  $\pounds 300$  consolation prize to Euler for work he had done on accurate calculations of the orbits of the moon and

planets that were, in themselves, very useful and *could* have led to a different solution to the problem. Therein lies the Euler connection.

One of many unsuccessful solutions, a method that Dava Sobel describes as "canine vivisection," [Sobel] would have used Digby's powder of sympathy. The plan was that before a ship was to set out, they would wound a dog and then bandage the wound. [Brown 1949] As the ship was about to set sail, they would remove the bandage. Then, every hour on the hour, someone back home would immerse the dog's bandage in a solution containing the powder of sympathy. The dog was supposed to swoon and cry out, and the mariner at sea would know what time it was back in his homeport. By comparing the time back home to the local time, observed by measuring the angle of the sun or some other reliable astronomical object, and using the conversion factor 1 hour = 15 degrees, the mariner could calculate his longitude. The solution wasn't that much different from Harrison's scheme, but it involved dogs and the powder of sympathy instead of a clock. Also, unlike Harrison's clock, canine vivisection didn't work.



Those are the two Euler connections. What about Digby as a pirate?

Kenelm Digby was everything. His father had been executed following the Gunpowder Plot. Digby distinguished himself as an alchemist and was one of the founders of the Royal Society.

In 1627, Digby led an expedition of privateers to the Mediterranean, hoping to gain wealth and fame by capturing and plundering French merchant ships. All the way east across the Mediterranean, he and his crews sighted nothing but fishing boats and neutral shipping. But when they got to the far northeastern corner of the Sea, off a Turkish port named Scanderoon, he found and attacked a group of French and Venetian merchant ships and managed to fill his two ships with loot and return to England.

Depending on whose side you're on, this was either a daring attack on enemy shipping or it was senseless aggression on what had previously been a friendly port. Either way, Digby became wealthy and he was knighted for his success on behalf of the English King.

#### John Paul Jones

We might make an equally tenuous connection with John Paul Jones, ex-slave trader and "Father of the American Navy." Jones served in the navy of Catherine the Great for a few months in 1788. He was stationed in St. Petersburg, but apparently never actually sailed under the Russian flag. Euler had died there five years earlier. During the American Revolution, John Paul Jones had been charged with piracy in the Netherlands for attacking ships "under an unknown flag," the flag of the new United States of America. When the American flag was duly entered into the appropriate Dutch records, the charges were dropped.



PAUL JONES OF PIRATE

JPJ's months in St. Petersburg were tarnished with scandal. The account of this episode on Wikipedia is highly sanitized. A letter from one of Euler's sons in St. Petersburg to relatives in Berlin survives in the archives at Yale, and the "gossip" there seems to be true.

So, does this connection between Euler and JP Jones count? Does Jones count as a pirate, since the charges were soon dropped?

### Conclusions

Euler's name does not loom large in the history of piracy and privateering. Indeed, our four "pirates" are all better known for something other than piracy, if, indeed, they are known at all.

In recent years, it has become popular in historical analysis to emphasize how things are connected to other things, what Lovejoy calls the "essentiality of relations." [Lovejoy 1950, p. 10] Indeed, in the Internet Age, connections are a modern way to understand things, including history. But sometimes, as with Euler and the Pirates, there are connections, but they don't mean anything. They're just fun. We hope this was fun.

#### **References:**

- [Brown 1949] Brown, Lloyd A., *The Story of Maps*, Little, Brown & Co., reprinted by Dover, New York, 1980. Relevant parts reprinted as "The Longitude" in *The World of Mathematics*, vol. 2, pp. 780-819, James R. Newman, ed., Simon and Schuster, NY, 1956.
- [E77] Euler, Leonhard, *Neue Grundsätze der Artillerie aus dem Englischen des Herrn Benjamin Robins übersetzt und mit fielen Anmerkungen versehen*, Berlin, 1745. Reprinted in *Opera Omnia* Series II vol 14 p. 3-409.
- [E135] Euler, Leonhard, Variae demonstrations geometriae, Novi commentarii academiae scientiarum Petropolitanae 1 (1747/48) 1750, pp. 38-38, 49-66. Reprinted in Opera omnia, Series I vol. 26, pp. 15-32. Also available online, along with an English translation by Adam Glover, at EulerArchive.org.
- [E426] Euler, Leonhard, *Théorie complette de la construction et de la manoeuvre des vaisseaux*, St. Petersburg, 1773. English translations by Henry Watson, 1776 and 1790.
- [Fermat 1658] Fermat, Pierre de, Commercium epistolicum de quaestionibus quibusdam mathematicis etc., Oxford, 1658.
- [Fermat 1894] Fermat, Pierre de, *Oeuvres de Fermat*, vol. 2, Correspondance, Tannery and Henry, eds., Gauthier-Villars, Paris, 1894. Available at Google Books.
- [Lovejoy 1950] Lovejoy, Arthur O., *The Great Chain of Being: A Study of the History of an Idea*, Harvard University Press, Cambridge, 1950.
- [Peterson 2009] Peterson, Ivars, The Ladies' Diary, The Mathematical Tourist, MAA Online,
- [Sandifer December 2006] Sandifer, Ed, Cannonball curves, *How Euler Did It*, MAA Online, December 2006. March 26, 2009.
- [Sandifer December 2008] Sandifer, Ed, A Forgotten Fermat Problem, How Euler Did It, MAA Online, December 2008.
- [Sobel] Sobel, Dava, The Longitude Problem, http://www.themodernword.com/eco/eco\_longitude.html. Link valid as of March 22, 2009.
- [Terrall 2002] Terrall, Mary, *The Man Who Flattened the Earth: Maupertuis and the Sciences in the Enlightenment*, U. Chicago Press, Chicago, 2002.

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