

	<h1>How Euler Did It</h1> <p>by Ed Sandifer</p>	
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Fallible Euler

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By now, regular readers of this column might have come to believe that, except for occasional computational errors beyond the 15th decimal place, and except for a regular and flagrant disregard of the issues of convergence when dealing with series, Euler was always right about everything. Now that 2007, the so-called “Euler year” is over and the celebrations of the 300th anniversary of his birth are winding down, perhaps we will be forgiven if we admit an uncomfortable fact: Euler was sometimes wrong. We are devoting this month’s column to a few of the things Euler was wrong about.

Lunar atmosphere

Euler thought that the moon had an atmosphere. In [E142], *Sur l’atmosphere de la Lune prouvée par la dernier eclipse annulaire du Soliel*, (On the atmosphere of the Moon, proved by the recent annular eclipse of the Sun), Euler describes the observations made in Berlin of the eclipse of July 25, 1748. Euler says that he himself took part in the observations, and this would be a rare example of Euler taking his own data. Other sources indicate that sisters Christine and Margarethe Kirsch assisted him. Christine is known for carefully keeping a diary of the weather for many years.

Euler and his assistants set up a telescope in a darkened room, making what we call a *camera oscura*. This allowed the image of the sun to be projected onto a white screen. The details of the eclipse had been calculated in advance by Johann Kies, and they had used Kies’s calculations to draw a circle on the screen in the position where the eclipse was predicted. If the calculations were accurate, the image of the eclipse at its maximum would exactly coincide with the circle at exactly the time predicted.

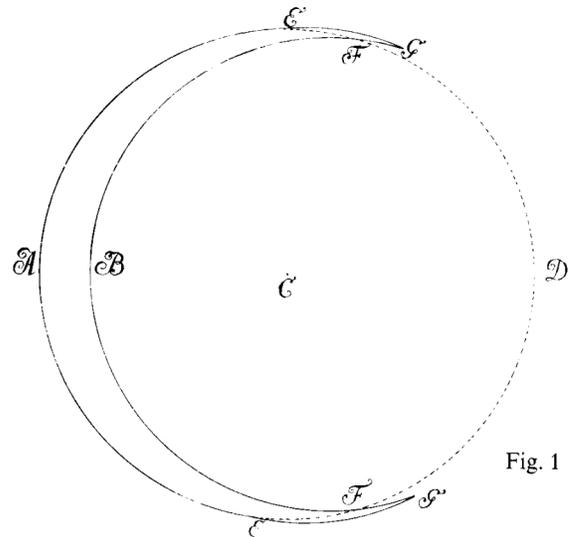
They didn’t. Though the time and position of the images were as predicted, the sizes were observed to differ in two significant ways.

First in order of occurrence, but probably second in importance, the crescent of the sun as the eclipse approached its maximum (an annular eclipse doesn’t reach *totality*) did not behave as expected. When an annular eclipse occurs, the moon is a little too far from the earth for totality, so the disk of the moon is not large enough to cover the whole disk of the sun. As a result, as the disk of the moon almost covers the sun, the horns of the crescent of the sun become very sharp as they apparently move around the disk of the sun. This is shown in Euler’s Fig. 1.

As the eclipse approached its maximum, the tips of the horns were observed to move *outside* the circle that Euler and his assistants had drawn on their screen. That circle is $DFEAEF$ in the diagram, while the disk of the moon is seen as arc $GFBFG$. The tips of the horns are seen outside the predicted disk of the sun at the two points marked G .

The second way in which the observations deviated from the predictions was that at the maximum, the apparent diameter of the disk of the sun was clearly larger than it should have been.

For this particular eclipse, Kies had calculated the apparent radius of the sun to be 952 seconds of arc, and that of the moon to be 898". Moreover, Kies calculated that at the maximum of the eclipse, the apparent centers of the sun and the moon would not quite coincide, but would be 53 seconds of arc apart. Since $952 - 898 = 54$, the annulus at the maximum should have been obviously eccentric, only 1" at its thinnest, and 107" at its thickest. Euler observed the thick part of the annulus to be 107", but at its thinnest it was 26" instead of 1".



Colin Maclaurin had made a similar observation of an annular eclipse on February 18, 1737, and had concluded that the image of the moon appeared smaller than expected, but Euler's observations, along with the carefully drawn circle on the screen, showed that it was the sun's image that was too large, not the moon's that was too small.

Armed with these observations, and with the theory of refraction of light, Euler concluded that the image of the sun was being magnified by the refraction effects of an atmosphere on the moon.

Euler continued his calculations and found that to produce the observed refraction, the atmosphere of the moon was about 1/200 as dense as the atmosphere of the earth.

He is not entirely confident of his "discovery" of a lunar atmosphere, though, for he writes that "this celebrated question has agitated astronomers for a long time: *whether or not the moon has an atmosphere*, has not yet been decided.

Euler proposed further research on the lunar atmosphere. He suggested that people could measure the apparent angles between stars as the moon occulted one of them. I do not know whether these experiments were ever performed.

Now that men have walked on the moon, we are sure that Euler was wrong; the moon has no atmosphere. The phenomena Euler observed are optical effects of light passing close to a sharp edge, and not the refraction of a lunar atmosphere.

Aether and planetary friction

From our 21st century point of view, it is easy for us to respect Euler's error about the atmosphere of the moon. He followed good scientific methodology in making his observations. He

formulated a plausible theory consistent with well-established scientific facts, and he proposed experiments to attempt to test his theories.

It is a little more difficult for us to set aside our modern prejudices to accept the 18th century ideas of so-called *aether*. Not to be confused with the soporific solvent “ether”, the aether was a kind of subtle fluid that filled the spaces between material substances and provided a medium for transmitting phenomena like gravity, light, magnetism and electric fields.

The 18th century view of the world resisted the idea of “action at a distance”. Natural philosophers of the time thought that for things to affect one another, there must be some kind of connection between them to carry the effects. Sound behaves like light in many ways, and vacuum experiments in the 17th century had shown that sound does not travel in a vacuum. It must be carried by some kind of physical medium, usually air. It stood to reason that the other physical phenomena required a physical medium to transmit them. The theory of aether provided that medium.

The physical properties of aether were hard to pin down. It had to be light, flexible and porous. It had to be strong enough to hold things apart, but thin enough to allow things to move. It had pores that allowed light to move in any direction in a way that one beam of light never interfered with another beam of light. It had to be so subtle that it never got in the way of any physical phenomenon, but so powerful and omnipresent that it was always there when it was needed.

Scientists, philosophers and theologians of the time needed aether, for without it, phenomena like magnetism, electrical fields and gravity would be “action at a distance”, and they could not accept that. Action at a distance was an occult phenomenon, in both senses of the word “occult”, “hidden” and “ungodly”. They generally agreed with Leibniz that the world was a beautiful and righteous creation, and at the same time agreed with Newton that the workings of the universe could be studied by analysis and experimentation. The existence of occult phenomena was at odds with both outlooks. Hence, they needed aether to avoid action at a distance.

It requires a deep understanding both of modern science and of 18th century natural philosophy to understand whether this is really different from some science today, when people spend careers and fortunes in a search that transcends galaxies and dimensions, looking for gravity waves and gravity particles.

Having stirred up that hornets’ nest, we’ll quickly move on.

Euler, like most scientists of his day, believed in the aether and he would appeal to its properties when he tried to explain natural phenomena. He, and others, hypothesized that the aether might exert friction on the planets and comets in their orbits. This phenomenon might be too subtle to observe on earth, perhaps because of the interference of other matter, or perhaps because the planets move so much more quickly than anything moves on earth.

Euler wrote one paper [E89] and three published letters [E183, E184, E218], and his son Johann Albrecht wrote two versions of one paper [A8, A8²] on the resistance the aether exerted on the motion of planets and comets.

In the late 1740s and early 1750s, astronomers and mathematicians were struggling to explain minute discrepancies in orbits of the moon and planets and in the earth's rotation that had been observed using the improved and more accurate scientific instruments that had been developed. New instruments and new mathematics combined, for example, to help Euler understand how the fact that the orbit of Saturn is slightly inclined to the orbit of Jupiter made their influences on each other's orbits different than the simpler co-planar models predicted. [W] Similar progress enabled Clairaut to explain irregularities in the moon's orbit around the earth. [K]

When 18th century observations suggested that the length of the year might be shorter than it had been in the three previous centuries, Euler noted also that the orbital period of Halley's comet had also been decreasing, and he set out to offer an explanation for these apparently related observations.

It may seem counterintuitive, especially to those who have not studied physics, but a shorter orbital period means that the body is slowing down. A slower body moves into a lower orbit, hence has a shorter distance to travel each time it orbits. The shorter distance more than compensates for the slower speed, and the time it takes for the slower body to orbit is actually *less* than the time it takes for a faster body.

Thus, a shorter orbital period would mean that the earth and Halley's comet, and presumably other bodies orbiting the sun as well, were slowing down.

On the other hand, comparisons with ancient observations made by Ptolmey about 150 CE seemed to suggest that the earth's year was instead getting longer, not shorter. This was consistent with the theory of vortices, an alternative to the theory of aether and the point of view favored by Descartes. I confess that I find vortex theory even more confusing than the theory of the aether. It seems that according to Descartes, all of space is filled with fine particles, moving in complex, swirling paths. In their larger and longer swirling paths, the vortices carry the planets and comets in their orbits, while in on a smaller scale they help manage trajectories, currents and other phenomena.

The property of vortices that we care about here is that sometimes a moving body could build up vortices behind it that help to push the body along. Thus, vortices could provide what some have called "negative resistance."

The theory of vortices was not entirely inconsistent with the theory of aether, but they were rival theories on many points. In particular, if the year were getting longer, as suggested by Ptolmey's data, then that would tend to support the vortex theories, but if were getting shorter, as suggested by Halley's comet and other 18th century data, then it tended to support planetary resistance due to the aether.

In 1746, Euler wrote *De relaxatione motus planetarum* (On the enlarging of the motion of the planets) [E89] and also exchanged some letters with the astronomer Joseph Nicolas Delisle, (1688-1768) [R517]¹ to make some speculations, backed with calculations, about these theories and observations.

¹ Over 3000 items of Euler's correspondence are cataloged and indexed in Series IV-A volume 1 of his *Opera omnia*, edited by Juskevic, Smirnov and Habicht and published by Birkhäuser in 1975. They assigned each item an "R-number," similar to the way Eneström assigned index numbers, now called E-numbers, to Euler's published works. As we shall see later in this column, some of Euler's letters have been published individually, and some of those items have both R-numbers and E-numbers. We will give both.

In his letter to Delisle, Euler proposed that a day had been lost in counting the days since Ptolmey's observations, so that rather than lengthening, the length of the year had decreased from 365 days 5 hours 55 minutes in Ptolmey's time to 365 days 5 hours 48 minutes in 1700, and that "this diminution of the year is the effect of the resistance of the aether, which I have explained in a piece that will appear." So, with Euler's proposed correction Ptolmey's data supported a shortening year rather than a lengthening one.

Of course, E89 was the paper that Euler said would appear. In that paper, Euler explained how the resistance of the aether would shorten the year, rather than lengthening it as intuition would suggest. In addition, Euler knew a lot about friction on spherical objects as they move through resisting media because of the work he'd done on ballistics and the trajectories of cannonballs. [S Dec06, S Jan07] He reversed those same calculations to try to calculate how dense the aether must be in order to provide the resistance necessary to shorten the year enough to fit the observations. He concluded that air must be about 4×10^8 times as dense as aether, and that that density would also account for the observed shortening of the period of Halley's comet.

Later, in 1749, Euler wrote two letters to Caspar Wettstein, Chaplain to the Princess of Wales and a friend of Euler from their days in St. Petersburg, asking first of Wettstein could help Euler locate some Arabic astronomical observations to corroborate those of Ptolmey [E183=R2763], and the second letter [E184=R2765] to warn that, even if ancient observations didn't confirm that the number of days in a year was decreasing, then it was also possible that the friction of the aether was also changing the length of the day, and the length of the year could be decreasing without our being able to detect it.

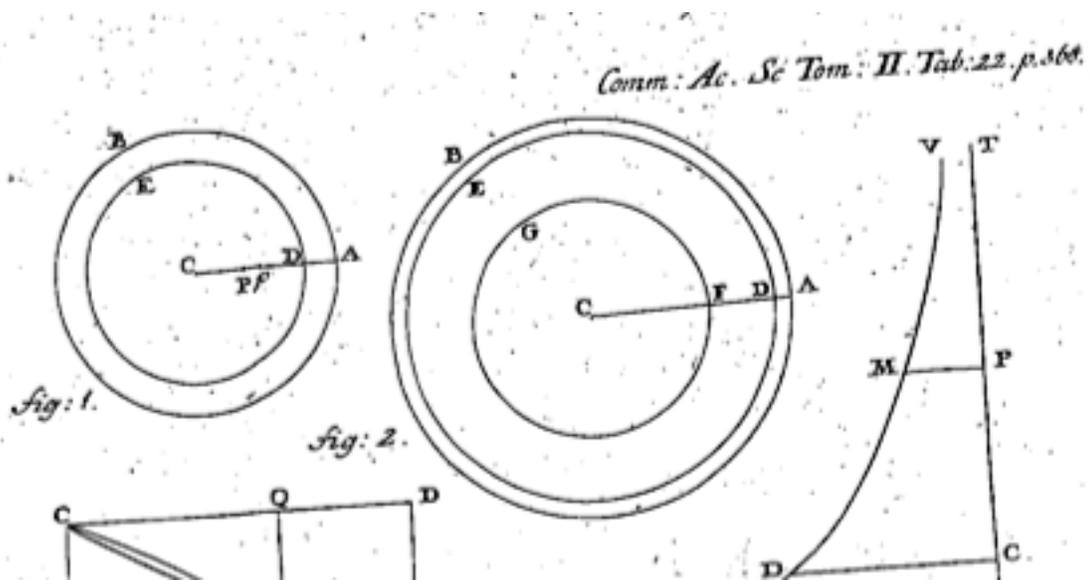
In 1754, Euler also wrote a letter to the Norwegian philosopher Erich Pontoppidan [E218=R2021] in which Euler agreed that there could be no life as we know it on a planet as distant as Saturn, hence the date of Creation had to be more recent than when the Earth's orbit was as far from the sun as that of Saturn is today.

Modern theory holds that the experiments of Michelson and Morley, performed in 1887, proved that there was no aether, that the period of the earth's year has not changed measurably, and that the period of Halley's comet has shortened because of gravitational effects of the outer planets. Euler was chasing a false theory. On the other hand, most of us agree that people don't live on Saturn.

The nature of the atmosphere

Air is a tricky thing. It is hard to see and hard to hold on to, but for some reason the air around the earth doesn't just float away into space. Euler lived in a time after Robert Boyle showed that the pressure of a gas is inversely proportional to its volume in 1660 and before Émile Clapeyron unified many properties of gasses into the Combined Gas Law in 1834. Even so, this progress in thermodynamics only described the properties of air, and not the nature of air and why it had these properties. Euler sought to provide a model for the mechanics of air that could explain its observed properties. He first made his speculations in one of his very early works, *Tentamen explicationis phaenomenorum aeris*, (Tentative explanations of the phenomena of air) [E7] and he added to his ideas late in his life in *Conjectura circa naturam aëris, pro explicandis phaenomenis in atmosphaera observatis* (Conjectures about the nature of air, for explaining phenomena observed in the atmosphere) [E527], written in 1780 and published in 1782.

Euler proposed that in its most compressed state, air consisted of bubbles of water filled with aether, as shown in Euler's fig. 1 below, taken from the 1727 volume of the journal of the St. Petersburg Academy.



This shows the bubble centered at C, with inner radius CD and outer radius CA.

Euler's fig. 2 shows what happens when air gets warmer. The aether inside the bubble begins to spin, and the centrifugal force opens a vortex inside the aether, shown as the circle with radius CF. This allows the bubble to expand, thus providing a mechanism for air to expand when it gets warmer.

The vortex also gives a way for pressure to compress the bubble, providing a means for Boyle's Law to apply.

Finally, the surface of the bubble is made of water. When water evaporates, it simply thickens the surface of the bubble, simultaneously explaining where water goes when it evaporates and why moist air is heavier than dry air.

Other scientists of the era apparently did not find Euler's speculations very convincing, and, like most ideas in the scientific ritual of trial and error, these ideas of Euler were relegated to the "error" group.

Other Euler errors

Euler had a number of other misconceptions that we don't have room to discuss here. He thought that the tails of comets had the same origins as the lights of the aurora borealis, and they were both related to a fascinating but relatively unknown phenomenon called "zodiacal light". In fact, they are three entirely different phenomena, and Euler was completely wrong about their cause. He also used hypothesized properties of the porosity of the aether to explain the phenomena of heat transfer, magnetism and electricity, and briefly flirted with the idea that Newton's inverse square law was only an approximation, and that there had to be another term, inversely proportional to r^4 , introduced by the pressure of the aether.

As we look at these errors of Euler, we see that they are all rooted in the scientific worldview of his times, and that we should probably forgive him for not knowing things that would not be discovered for a century or more after he died. It serves to remind us that Euler was not a 21st century scientist trapped in an 18th century timeline, but rather he was a citizen of his own times, some of whose accomplishments are still interesting and relevant today.

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