



How Euler Discovered America

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This month some people celebrate Columbus Day, the anniversary of that day in 1492 when the Italian navigator Christopher Columbus, sailing on behalf of Spain, first spotted the lands of the New World, thinking they were part of China or India. His confusion about where he really was lingers in the name he gave to the inhabitants of the islands he saw, “Indians.”

Whether or not Columbus deserves credit for “discovering” America has become a familiar topic of controversy. There are those who point out that Columbus died still thinking that he’d found a shorter route to India and China, not realizing that the Americas were whole continents blocking the way to East Asia. And he probably never set foot on the mainland itself. If he so misunderstood what he had discovered, does he deserve credit for the discovery?

Further, Columbus was probably not the first European to visit America. Norse visitors apparently left remains of their settlements in eastern Canada, a place they called Vinland, and people make similar claims for Irish fishermen, Irish monks, and even for wayward Roman and Egyptian mariners.

The Original Peoples of the Americas, of course, have a considerably different point of view. They had known about the Americas since the beginning of time, so Columbus’ discovery was old news to them.

On the other hand, visits by the Norse stopped after a few decades, and other pre-Columbian visitors don’t seem to have established a permanent presence. Columbus opened the doors, and Europeans from many countries followed. The Columbus voyages were the ones that changed America. The Norse, the Irish, the Romans, and so forth, had virtually no impact in America or in Europe.

In this column, we will make the (slightly silly) case that Euler deserves some credit for the European discovery of America. The main purpose of the column is to tell that story, but we will also make the small point that this whole “Discovery of America” story can be viewed as a metaphor for mathematical discovery as well.

Let us try to set up some working criteria for a “discovery.”¹

1. You have to find something you didn't know about before.
2. You have to identify what you have found.
3. You have to announce it, and people have to remember that you found it.
4. You have to demonstrate that what you have found is something new and previously unknown, at least to the people around you.

Credit for a discovery can be shared, either if people do things together or if some people do some things and other people do others. There is room for debate, discussion and exceptions. These aren't axioms.

By these criteria, Leif Erikson's Norse settlement wasn't a discovery of America because it didn't have a sufficiently lasting effect. People abandoned his settlements and forgot that they had been there.

Columbus deserves only partial credit because he so egregiously misunderstood what he had found. By 1520, though, when Cortez conquered Mexico City, people had figured out that New Spain wasn't India, Japan or China, but a new place altogether. So, it is evident that by 1520, the Spanish had satisfied criteria 1, 2, 3 and part of criterion 4, for a European discovery of America.

What, the reader asks, did the Spaniards miss about Criterion 4? Despite sending expeditions clear around South America and up the Pacific coast to Northern California, they were unable to establish that North America was not attached to Asia. If America were really part of Asia, then India and China could be the Far East, and America would just be Farther East, and not a New World. (Also, Columbus would be right; he'd discovered part of Asia.)

Establishing that America was not Asia fell to the Russians. In the mid-1600's, a Russian Czar sent an expedition to establish the limits of the Russian Empire. They came back a few years later with a report that showed that Siberia ended at the sea, but they did not try to find what, if anything, was beyond that sea. They did, though, establish that Siberia wasn't connected to America. But by the time they returned to Moscow, the Czar who had sent them had died and the new Czar didn't care. They filed the report in the archives, where it rested forgotten for over 200 years.

The issue of the extent of Russia resurfaced shortly after the establishment of the St. Petersburg Academy, Euler's employer from 1728 to 1741 and from 1766 to his death in 1783. Peter the Great, not knowing about the expedition in the 1600's, sent the First Kamchatka Expedition, led by Vitus Bering, to explore the far reaches of Siberia. [Anon 1, Anon 2] Bering didn't answer all of Czar Peter's questions, though, so Peter's successors sent him to lead the Second Kamchatka Expedition as well, which lasted from 1733 to 1741. Bering himself died on the trip, but survivors reported back in 1743 with, among other things, the news of the Bering Strait that separated Asia from North America

Again, while they were gone, times had changed. Czars and Czarinas had died, and, in the wake of xenophobic riots in St. Petersburg, Euler had left for Berlin and the St. Petersburg Academy had fallen into disarray. Bering's discoveries made Russia the largest country in the world, and at the same

¹ Watch closely here, as this is where I'm trying to “pull the wool over your eyes.” I will carefully manipulate the definition of “discovery” to carve room to give Euler credit. I do a similar thing when people ask (as they often do) “Who published more, Euler or Erdos?” I ask them who they want to win, and then I devise a scoring method to allow their favorite candidate to win. (Of course, Christian Wolff published more than both of them put together.)

time completed the discovery of America by establishing that America wasn't part of Russia. With Russia in chaos, there was nobody in Russia to make the announcement.

Making the announcement fell to Leonhard Euler, still the preeminent member of the St. Petersburg Academy, and really the only member who was still taking his responsibilities seriously. Euler chose to make his announcement in the form of a letter dated December 10, 1746 to his friend Caspar Wetstein, Chaplain and Secretary to the Prince of Wales and member of the Royal Society in London. The following February, Wetstein, read the announcement before the Society, and he placed an extract [E107] of the letter in the Society's *Philosophical Transactions* of 1748. The "extract" begins:

XIV. Extract of a Letter from Mr. Leonard Euler, Prof. Mathem. and Member of the Imperial Society at Petersburgh, to the Rev. Mr. Cha. Wetstein, Chaplain and Secretary to His Royal Highness the Prince of Wales, concerning the Discoveries of the Russians on the North-East Coast of Asia.

Berlin, Dec. 10. 1746.

Read Feb. 5. — **A**S you are desirous to hear something more particular concerning the *Russian* Expeditions to the North and North-East of *Asia*, I will here give you an Account of all that has come to my Knowledge relating to the fame.

This is one of Euler's only publications in English. The full text is in the *Opera Omnia*, Series III, volume 2, and an image of the original (from which this image is taken) is available on line at The Euler Archive.

Thus, Euler deserves a share of the credit for the discovery of America, for his role in satisfying our Criterion 4 for a discovery.

Is the reader convinced? Must we demand that American history textbooks be rewritten to take out Christopher Columbus and Henry Cabot, to be replaced by Vitus Bering and Leonhard Euler?

Of course not, but it does illustrate how "discovery" is more complicated than we expect.² Let us consider mathematical discovery, starting with $V - E + F = 2$.

Some people call the Euler formula, $V - E + F = 2$, relating the vertices, edges and faces of a simple polyhedron, the "Euler-Descartes formula," [S, E230, E231] because Descartes discovered, but did not communicate, a fact about polyhedra that modern analysis can show to imply the Euler formula. Euler's contributions fully meet the criteria for a discovery. Descartes' falls far short.

² Moreover, our choice of the word "discovery" instead of "invention" embeds some assumptions about the nature of mathematics. If we "discover" it, then we are taking a philosophical stance that mathematics exists anyway, whether or not there is a human mind to know it. If we "invent" it, then mathematics is a creation of mankind, and doesn't exist without humans. This is yet another opportunity for debate. Personally, I take both sides at once. I tell people that beautiful mathematics is discovered; ugly mathematics is invented. Euler discovered.

On the other hand, in 1777 Euler wrote a paper [E703] in which he expanded a formula describing planetary orbits, $\frac{b}{1 + e \cos j}$, as a cosine series that he wrote as

$$\Gamma : j = A + B \cos j + C \cos 2j + \text{etc.}$$

This is clearly a Fourier series and the coefficients A , B , C , etc. are Fourier coefficients.

Still, Euler thought that his result was one about planetary orbits, and not about periodic functions. That was left to Fourier, whose name, indeed, belongs on the series.

And so it goes; there are a great many more examples in the history of mathematics in general and the discoveries of Euler in particular. They show that most things in mathematics are correctly, or at least reasonably named, and the exceptions make good material for interesting talks and articles on the history of mathematics.

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