



## How Euler Did It



## by Ed Sandifer

## The Euler Society

May, 2005

Every summer since 2002, twenty or so Euler fans have gathered for the three-day annual meeting of The Euler Society. The first meeting was in Rumford, Maine, and subsequent meetings have been at the Conference Center at Roger Williams University in Portsmouth, RI. This year's meeting will return to Roger Williams from the evening of Sunday, August 7 until lunchtime the following Wednesday. This should be timed so that people have about 30 hours to get to Rhode Island from MathFest in Albuquerque.

The featured talk of the conference is always The Euler Lecture. The 2004 Euler Lecture was by Rüdiger Thiele, who teaches in Leipzig and lives 20 km to the west in Halle, just two blocks from where Cantor lived. Euler has a connection to Halle. His son attended university there and Euler himself escorted his son when he went away to college, along some of the very roads Rüdiger drives to work.

Rüdiger's talk, *Leonhard Euler:* 1750 – 1760, dwelt mostly on Euler's life and times in the 1750's. His life was strongly influenced by three main factors, the European Enlightenment, his own interests in mathematics and natural sciences, and his own deep sense of Christian tradition. This particular decade had an additional, and mostly unmentioned influence: Euler's mother lived in his house near Berlin from 1750 to 1761.

Euler had immense responsibilities in the Berlin Academy. He supervised the observatory, the library, preparation of



almanacs, publications and the botanical gardens. The preparation of almanacs was particularly sensitive, since the sale of those almanacs was the principal source of revenue for the entire Academy. He also advised King Frederick on diverse technical matters regarding mines, lotteries, canals and fountains, all while essentially running the St. Petersburg Academy by mail, writing endlessly, and serving a key role in his Parish Council.

Despite all this service, Euler was unpopular in Frederick's court, in part because courtiers valued wit over wisdom, and in part because Frederick himself viewed science as a servant of the ends of the State.

Three days after The Euler Society Conference, at MathFest in Providence, the MAA presented Rüdiger with its Lester R. Ford award for his article *Hilbert's Twenty-Fourth Problem*, which appeared in the *Monthly* in January 2003.

Larry D'Antonio of Ramapo University in New Jersey, gave the first talk of The Euler Society Conference, *Euler's Christmas Present of 1751*. Larry has a variety of interests, from bioinformatics to some topics in medieval Arab mathematics, as well as being the Coordinator for Student Chapters for the New Jersey Section of the MAA. This day, though, Larry was telling the story of the *Collected Works* of Count Gulio Carlo de Toschi de Fagnano. Fagnano (1682-1766) did his best work in elliptic

integrals, though his name is also attached to "Fagnano's identity,"  $\ln \sqrt{\frac{1-i}{1+i}} = -\frac{\mathbf{p}i}{4}$ .

Two days before Christmas 1751 (a holiday Euler would have observed with more piety than celebration), Euler received a copy of Fagnano's *Collected Works*, containing, in a very modern "theorem-proof" style, a great many new results in elliptic integrals, including the so-called addition formula for elliptic arc lengths and the "trisection" of the lemniscate. Fagnano's ideas so inspired Euler that he spent the next month preparing his article [E252] "Observations on the comparison of arcs of

irrectifiable curves." He presented the article to the Academy on January 27, just five weeks after he received Fagnano's *Works*, though it wasn't published until 1761. In this paper, and enough other papers to fill two full volumes of the *Opera Omnia*, Euler presented and extended Fagnano's results and applied them to diverse subjects as the oscillation of pendulums with large amplitudes, the arithmetic of elliptic integrals, the measurement of the earth, and the three-body problem.

John Glaus of Rumford, Maine, described the long and fascinating correspondence between Euler and Johann Kaspar Wettstein (1695-1759), Chaplain to the Prince of Wales and later to his widow. Wettstein helped distribute the Berlin Academy's almanacs in England. In these letters, Euler describes his wonderful relationship with King Frederick when he first arrived in Berlin, and we can



see the gradual deterioration of their respect for each other. Euler also gives some details not found elsewhere of the role he played in the expedition of Vitus Bering that showed that Alaska was not connected to Asia and that California was not an island.

Karin Reich of the University of Hamburg spoke on *Euler and Gauss*. She studied Gauss's books and the circulation records of the library at Götthingen to document the great extent to which Gauss studied Euler. She told an anecdote from 1798, when the 21-year old student Gauss had a bit of money and, like Erasmus<sup>1</sup>, spent it to buy four books by Euler. In one of those books, Euler's great 1744 treatise on the calculus of variations, the *Methodus inveniendi*, immediately after the so called *Theoria elegantissimum*, there is a careful tracing of a portrait of Euler. Karin speculates that Gauss himself traced it, though maybe it was by one of his friends. Gauss often wrote and drew in his books.

<sup>&</sup>lt;sup>1</sup> Erasmus is often quoted as saying "If I get some money, I spend it on books. If there is any left over, I buy food, clothing and shelter." Whether or not the quote is accurate, it is a popular sentiment.

Stacy Langton from the University of San Diego spoke on Jakob Bernoulli's seminal probability book *Ars Conjectandi*. Euler Society President Rob Bradley of Adelphi spoke on *Euler*, *D'Alembert and the Vibrating String Problem*. Lee Stemkoski of Dartmouth talked about recent additions to The Euler Archive, and Vicki Hill showed her wonderful documentary film on Constantin Carathéodory. Carathéodory, in addition to being a fascinating individual and a towering 20<sup>th</sup> century mathematician, also edited the *Opera Omnia* volumes on the calculus of variations, Series I volumes 24 and 25. Your columnist suggested 23 problems for further research on Euler.

Among the unique aspects of The Euler Society Conferences are the Original Sources Workshops. As a group, those who are interested (and that's pretty much everyone) assemble after dinner with a refreshment and a copy of an original source from Euler's life or work. We read it over. Those with the necessary language skills read in the original language and translate for us, and we discuss what we've read. Usually it is not what we expected to find.

A very smart man from Berlin Said, "Constructions and curves are akin; But some, without doubt, Leave parts of curves out, While others put extra curves in!"

Last year we read Goldbach's famous letter to Euler in which he first makes Goldbach's Conjecture, that every even number, starting with 6, is the sum of two odd primes. We were delighted to read that that's not exactly how Goldbach phrased it.

We also looked at an as-yet unpublished letter of Euler to Cramer, part of the conversation on Cramer's Paradox that formed the basis of this column last August. That column was based, in part, on this letter. That workshop apparently inspired the anonymous limerick at the left that appeared the next morning before breakfast.

Perhaps some readers of this column will want to join this summer's meeting. It is a congenial group, a mixture of people who specialize in Euler and people who are just interested. Details are available (or will be soon) at the Society's website, EulerSociety.org. Maybe you'll be among the happy faces in this year's group picture.

References:

[O'C] O'Connor, J. J., and E. F. Robertson, "Guilio Carlo Fagnano dei Toschi", The MacTutor History of Mathematics archive, http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Fagnano\_Giulio.html.

[E252] Euler, Leonhard, Observationes de comparatione arcuum curvarum irrectificabilium, Novi commentarii academiae scientiarum Petropolitanae 6 (1756/57) 1761, p. 58-84, reprinted in Opera Omnia Series I vol 20 p. 80-107. Available through The Euler Archive at ,

www.EulerArchive.org

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