

**ON THE TRUE PRINCIPLES OF HARMONY
AS PRESENTED THROUGH THE *SPECULUM MUSICUM****

By

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1 All harmony, and, what is more, all music, rests upon four or five simple consonances which novices in the art should be taught to accustom their ears to, and to produce, whether by voice or by instruments, as exactly as possible. These consonances are the following:

I. *Unison*; II. *Octave*, or *Diapason*; III. *Fifth*, or *Diapente*; IV. *Major Third*; on which four consonances music of previous ages was built up; more recently an additional consonance, usually signified by the name *seventh*, seems to have been adopted. Let us, then, investigate these five consonances with considerable care as the pillars of harmony, since many who have attempted to treat of this science have dealt with these elements too negligently.

2 Let us thus begin with the unison, which consists of perfect equality of two or more musical tones. Since indeed every sound is produced by a vibratory motion, or tremor, agitated in the air, whether that tremor be regular or irregular, no tones are admitted in music except when all vibrations are *isochronous* among themselves, that is to say, completed in identical small intervals of time. Thus we will have an adequate notion of some musical tone when we have learned how many vibrations are produced in a given time, that is, within a small span; two or more tones which in the same span give forth the same number of vibrations will be *unisons*; and because tones are customarily characterized by the number of vibrations they give forth in a given time, the nature of the unison will lie in a relationship of equality. On the other hand, those tones which do not give forth equally many vibrations in the same time are sensed as different. Those which give forth more frequent vibrations in the same time span are generally called *higher*; those, on the other hand, giving forth fewer are *deeper*.

3 We perceive tones, then, to the extent that these vibrations excited in the air are sent through the ear into the organ of hearing, and our hearing will be roused to awareness by just so many vibrations, whence when two equal **tones** are given simultaneously, our sense is affected by a certain pleasantness by this relationship of equality itself, while on the contrary if there is the slightest deviation from this relationship, it experiences a certain dissatisfaction. This perception of equal tones by all people thus seems to come from inborn nature, so that they not only recognize such equality easily, but are even able to produce it, either by voice or by instruments; truly nothing is easier than to so stretch two strings so that they give equal tones, and the smallest aberration is, as it were, intolerable to the hearing.

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4 The second principal consonance is called the *octave*, or *diapason*. It comes so close to the nature of the unison that they who are unable to attain a given tone because of its deepness or highness spontaneously give a tone an octave higher or lower, from which it is done that in music, sounds differing by one or more octaves may be regarded as similar, and are usually designated by the same signs or letters- thus if some rather low sound is signified by A, those higher than it by one or more octaves are generally indicated by a, a', a'', a''', etc.

5 Now, two such tones distant by an interval of an octave in this way affect the hearing by a most agreeable harmony, and so much seem to please, by general agreement, that they almost may be held to be one and the same tone. The cause of this most beautiful consonance lies in this: the numbers of vibrations given by these tones hold a double ratio between themselves, so that if the deeper should complete one hundred vibrations in a small time interval, the other would complete two hundred in the same time. Just as this ratio is quite easily perceived by the intellect, so also two tones holding this ratio between themselves delight the hearing with a remarkable smoothness. Indeed, even the least deviation from this ratio greatly offends the sense of hearing, from which even novices easily learn the character of this consonance. On this account, since all tones are best represented by the number of vibrations that they give in a certain time, if tone A gives n vibrations, the subsequent tones a, a', a'', a''' give 2n, 4n, 8n, 16n vibrations.

6 The third principal consonance, called the *fifth*, or *diapente*, also offers the ears a very pleasing harmony, although its character diverges greatly from the nature of an octave, and the ability of perceiving and recognizing it demands greater practice, whence novices should be strictly trained so that they should learn to recognize and accurately produce it, whether by voice or by instruments. The origin of this consonance lies in a triple ratio, which, just as it is, after the double ratio, most easily recognized, so also it displays the most pleasing harmony to the ears after the octave; but since ratio 1:3 encompasses a greater interval than one octave, if the deeper tone is A and is designated by the number of vibrations n, that tone which produces 3n vibrations in the same time will be higher than tone a, but nevertheless deeper than a', and will bear to the a the interval called *diapente*, but related to tone A itself will establish an interval composed of an octave and a fifth. From this, therefore, two tones distant by an interval of a fifth will hold ratio 2:3.

7 Since the tone in lowest position in the musical scale is usually designated by the letter C, and its octaves by letters c, c', c'', c''', the tone above C itself by a fifth is designated by letter G, and its subsequent octaves by g, g', g'', g''', etc. For if we first represent tone C by some number n, all of these subsequent tones will be shown by the subscripted numbers:

C,	G,	c,	g,	c',	g',	c'',	g'',	c''',	g''''
n	3n/2,	2n,	3n,	4n,	6n,	8n,	12n,	16n,	24n

Since, however, the ratio 1:3 is doubtless simpler and more easily perceived than the ratio 2:3, it will be easier in music to produce tone g than tone G, in relation to a given tone C, and so it will be easier for the ears to recognize the interval of tones C:g, and the least deviation from the true ratio 1:3, than it would be for C:G, the interval of a fifth itself. From this, if a musical instrument is to be tuned properly by relaxation or tightening of strings, the tone g may be formed immediately from an established tone C, and from there, by descent through one octave, the tone G is reached. Yet meanwhile, a little exercise will be enough for novices accurately to form this interval C:G of one fifth without mediation, and because tone G is distant by an interval of a fourth from tone c, we justly require that novices also learn thoroughly this interval that is made up of ratio 3:4, and that they become accustomed to judging its character by ear.

8 The fourth principal consonance, called the *major third*, presents a certain singular type of smoothness to the hearing. It will be proper for novices to be trained with exceptional care for recognizing and producing it, by voice or by instruments, as accurately as possible. Now, this consonance is made up of ratio 4:5, which, as it is less simple than the preceding, should be worked out with greater care, so that the sense of hearing may become accustomed to recognizing and judging it. In the usual scale of tones, the tone above the fundamental C by such an interval is usually designated by the letter E; from which, if the number n is assigned to tone C, then $5n/4$ is ascribed to this E. This, therefore, let us attach, along with its octaves of higher order:

C	E	G	c	e	g	c'	e'	g'	c''	e''	g''	c'''	e'''	g'''
n	$(5/4)n$	$(3/2)n$	2n	$(5/2)n$	3n	4n	5n	6n	8n	10n	12n	16n	20n	24n

9 Because the ratio 4:5 is not so easily perceived by the intellect as the ratio 2:5 or even 1:5, so in a similar way in music, for a given tone C the tone e is more easily produced than E, and perhaps easier yet is tone e', which relates to C as 5:1. Whether we make tone e or tone e', the rest, whether deeper or higher, will be displayed readily.

10 And these are the four principal consonances upon which all music was once built. More recently, however, they have introduced an additional, fifth, principal consonance, which may be called the minor seventh, even though it does not occur in the system of tones in which musical instruments are generally set up. Now, this new consonance is comprised of the ratio 4:7, which, since it differs too little from the ratio 5:9 or 9:16, one of these is likely to be used incorrectly in place of that 4:7. Meanwhile it will nevertheless be useful above all for novices to exercise in recognition and judgment of this ratio 4:7, as to whether tones hold the ratio precisely, or not. On this account, it will be necessary to produce tones of this sort, holding ratio 4:7, on a monochord, and to accustom the ears to them, which will thereupon experience a not small kind of pleasure.

11 With the principal consonances upon which all music is built established, let us see how it is suitable that these tones be set up in musical instruments, because the variation by which this art most pleases requires many different tones made firm by the true principals of harmony. And indeed, having taken as we wish some tone F, from which, generally, the rest of the tones are seen derived in musical instruments, which tone let us designate by number n, which indicates how many vibrations are put forth in a small time interval, from which by an ascent through octaves, we reach the following tones, designated by their numbers

$$f = 2n; f' = 4n; f'' = 8n; f''' = 16n; \text{ etc.}$$

And if one may descend to yet deeper tones, they may be represented in this way:

$$F, = n/2; F,, = n/4; F,,, = n/8; \text{ etc.}$$

Then to each of these tones let us attach a fifth, holding ratio 2:3, and from F will arise a tone expressed by the number $(3/2)n$, which musicians usually designate by letter c, from which the tone C, deeper by an octave, is expressed by the number $(3n)/4$, and in this way we arrive at the series of tones:

$$C = (3n)/4; c = (3n/2); c' = 3n; c'' = 6n; c''' = 12n; \text{ etc.}$$

Clearly, we rise in this fashion from the fundamental tone F by the interval of a fifth. If we should now ascend from these tones in turn by an interval of a fifth, we will obtain the following new tones

$$G = (9/8)n; g = (9/4)n, g' = (9/2)n, g'' = 9n; g''' = 18n; \text{ etc.}$$

From here let us ascend once again through such an interval of a fifth, and it will produce the following series of new tones:

$$D = (27/32)n; d = (27/16)/n; d' = (27/8)/n; d'' = (27/4)n; d''' = (27/2)n; \text{ etc.}$$

However, after we have ascended by an interval of a fifth three times, here further progression should be stopped. If indeed we wish to ascend by a fifth above D, we reach a tone expressed by $81/64$, which differs too little from tone A, expressed by the number $5/4$, for both to be introduced simultaneously into music and to be distinguishable from one another; truly moreover this tone A = $(5/4)n$, which stands at an interval of a major third from the fundamental, unavoidably occupies an important position in music, since otherwise this excellent consonance would be banished. On this account let us ascend from the individual tones already established above by an interval of a major third, from which will arise the following tones:

From F		A = $(5/4)n$;	a = $(5/2)n$;	a' = $5n$;	a'' = $10n$;	etc.
		C E = $(15/16)n$;	e = $(15/8)n$;	e' = $(15/4)n$;	e'' = $(15/2)n$;	etc.
		G H = $(45/32)n$;	h = $(45/16)n$;	h' = $(45/8)n$;	h'' = $(45/4)n$;	etc.
		D F# = $(135/128)n$;	f# = $(135/64)n$;	f#' = $(135/32)n$	f#'' = $(135/16)n$;	etc.

12 So in this fashion, led by the principles of harmony themselves, we arrive at the **genus** of music which is commonly called Diatonic, except that the tone F#, which older musicians omitted, has come in; but it nonetheless enters into this type of necessity. Let us set forth, then, those tones constituting the diatonic **genus**, with their numbers ordered for display.

Let us take the number 128, which is convenient for use, so that tone F, to which we have assigned number n, gives forth precisely 128 vibrations in some small time interval, just as experiments arranged with strings have taught. In this way, all the numbers displayed in the following table will show, simultaneously, of how many vibrations given out in a small time interval any tone is made up:

C = 96		c = 192		c' = 384		c'' = 786		c''' = 1536
D = 108		d = 216		d' = 432		d'' = 864		d''' = 1728
E = 120		e = 240		e' = 480		e'' = 960		e''' = 1920
F = 128		f = 256		f' = 512		f'' = 1024		f''' = 2048
F# = 135		f# = 270		f#' = 540		f#'' = 1080		f#''' = 2160
G = 144		g = 288		g' = 576		g'' = 1152		g''' = 2304
A = 160		a = 320		a' = 640		a'' = 1280		a''' = 2560
H = 180		h = 360		h' = 720		h'' = 1440		h''' = 2880
c = 192		c' = 384		c'' = 768		c''' = 1536		c'''' = 3072

And from this are taken the terms:

I Octaves, since, with the omission of the tone F#, there are eight tones from C to c, **II** fifths, since there are five tones from C to G, **III** fourths, since there are four tones from C to F, **IV** thirds, since there are three tones from C to E.

13 Just as here we ascend from the four previously established tones F, C, G, D by an interval of a third, so if we duplicate this leap anew, we reach four new tones, by the addition of which a musical **genus** still in use results, which is generally called Diatonic-Chromatic, the source of which may be seen from this scheme:

Ascending		Ascending by a fifth					
by a major		F	C	G	D		
		A	E	B	F#		
third							
		C#	G#	D#	Bb		
		100	150	112½	168¾		

Here, obviously, we have written the appropriate numbers beneath the four new tones.

14 From this table it is clearly seen how musical instruments might most easily be adjusted to this system of tones. For, with fundamental tone F having been established, we may twice ascend from it by major thirds to tones A and C#, and then from any one of these three tones we may ascend three times by fifths, and in this way all twelve tones of one octave will be obtained. From this, all remaining octaves will be furnished with their tones very easily, and so the entire instrument will be best tempered to true harmony.

15 Let us set into view all the tones of this diatonic-quadratic **type**, together with their appropriate numbers, and to avoid fractions, let us quadruple the preceding numbers, and then indeed let us express the same numbers by prime factors, by which the ratios they hold among each other may more easily be seen. Now, it will be enough to develop a single octave in that way:

Tone		Numbers		Expressed
Names		Numbers		by Factors
C		384		$2^7 \cdot 3$
C#		400		$2^4 \cdot 5^2$
D		432		$2^4 \cdot 3^3$
D#		450		$2 \cdot 3^2 \cdot 5^2$
E		480		$2^5 \cdot 3 \cdot 5$
F		512		2^9
F#		540		$2^2 \cdot 3^3 \cdot 5$
G		576		$2^6 \cdot 3^2$
G#		600		$2^3 \cdot 3 \cdot 5^2$
A		640		$2^7 \cdot 5$
Bb		675		$3^3 \cdot 5^2$
B		720		$2^4 \cdot 3^2 \cdot 5$
c		768		$2^8 \cdot 3$

16 With these tones established, then, all music is reduced to this: that by the various tones of this kind linked with each other, pleasing harmony may be offered to the hearing, the nature and characteristic of which harmony is to be sought in the perception of the principal consonances explained above, since from ears accommodated to music, no more is required than that they become well acquainted with those principal consonances, and that they be able to judge whether they be accurate, or not. Once they have become adept at this ability by frequent practice, they will come to feel, by its very nature, a certain special pleasure. However, in the beginning we justly pass over that fifth principal consonance consisting of ratio 4:7, since those tones suitable for producing it have not yet been introduced into music, but in their place, musicians would be apt to misuse other tones differing, truly, too little from them, but since in this way the purity of harmony is neglected, one may truly doubt whether music would be taken to a higher degree of perfection in this way. For all that, I have explained more broadly the use of these new consonances, to the extent that they are usually employed by composers, in the Proceedings of the Royal Prussian Academy.

17 It will not be permitted in this **genus** of music to leap directly from any given tone to other tones, unless they are remote from it by an interval of an octave, of a fourth or fifth, or of a major third; which leaps, for that reason, it may be permitted to call *simple*. In this itself the main principle of composition should be understood to be contained. Now, we have noted above that the ratios 1:3 and 1:5 are more easily perceived than the ratios 2:3 and 4:5 by which the true intervals of a fifth and a third are expressed; for a leap through these intervals be able to be undertaken, it will help to have demonstrated that rather fully.

If it is wanted, thus, to ascend from tone f through a fifth to c', it will be done more easily by interpolation of tone F, or tone c'', in the following way:

$$\begin{array}{rcl}
 f : F : c' & & f : c'' : c' \\
 2 : 1 & \text{or} & 1 : 3 \\
 1 : 3 & & 2 : 1 \quad .
 \end{array}$$

But if it is wanted to go from tone c through a fourth to tone f, it will most easily be able to be accomplished thus:

$$\begin{array}{rcl}
 c : c' : F : f & & \\
 1 : 2 & & \\
 3 : 1 & & \\
 1 : 2 \quad . & &
 \end{array}$$

And finally, if it is wanted to leap from tone f through a major third to a, it will be done most conveniently thus:

$$\begin{array}{rcl}
 f : F : a' : a & & \\
 2 : 1 & & \\
 1 : 5 & & \\
 2 : 1 \quad . & &
 \end{array}$$

18 Rightly, then, we assume the sense of hearing to be so refined already that it is able immediately to grasp and to recognize intervals of a fifth, fourth, and a major third, so that we are able to regard these leaps as simple ones. In the musical **genus** diatonic-chromatic, however, it is not permitted to pass from all tones through these intervals, because the tones which would be reached do not occur in our scale. It is not permitted, thus, to ascend from the three tones D, F#, and B through an interval of a fifth, nor indeed to ascend by an interval of a fourth from tones F, A, and C#; nor yet to ascend by an interval of a major third from the four tones C#, G#, D#, nor, finally, to descend through the same interval from tones F, C, G, and D. From all others except those just mentioned, such leaps succeed.

19 When, therefore, it is necessary either to ascend or descend from some tone through any other interval, it is by no means permissible to proceed by a single leap, but rather it will be necessary for a transit through two or more simple leaps to be arranged. We may use the following convenient symbols, by which we may put such composite leaps more clearly before the eyes:

Let us denote an ascent through an interval of a fifth by +V, and a descent, naturally, by -V; and similarly the symbol +III denotes an ascent through an interval of a major third and -III a descent through the same interval, and by these symbols we will be able to represent succinctly all leaps from any tone of our scale to any other. So, then, let us set forth in order whether these transits are to be carried out by two, three, or more leaps, and whether by fifths or by thirds.

I. Transit through +V +V, that is,
through interval 8:9

20 This interval 8:9 is usually called the *major tone*, and the following such occur in our scale:

F:G , C:D , A:B , E:F# , C#:D# , G#:Bb.

The leaps by which these intervals may be produced are thus:

$F:G = (F:C)(C:G); \quad C:D = (C:G)(G:D);$
 $A:B = (A:E)(E:B); \quad E:F\# = (E:B)(B:F\#);$
 $C\#:D\# = (C\#:G\#)(G\#:D\#); \quad G\#:Bb = (G\#:D\#)(D\#:Bb).$

In this way, evidently, such intervals are completed by a pair of simpler leaps. In musical practice, to be sure, it is not always necessary, in the act, to interpolate these middle tones, for if there should be concerted singing by several voices, it is sufficient that a different voice give out the interpolating tone, which is commonly to be observed by practitioners. The transition -V -V, holding ratio 9:8, would now follow, but it is clear that the preceding transitions taken in reverse order may be referred to, whence it would be superfluous to develop it in turn; which is also to be understood concerning the sections that follow.

II. Transit through +V +III, that is,
through interval 16:15

21 This interval is usually called a *major semitone*, and occurs between the following tones in our scale:

F:E; C:B; G:F#; A:G#; E:D#; B:Bb.

Individually, now, these intervals may be resolved in a twofold way, according to which double leap is taken, either +V +III, or in reverse order:

	+V +III		+III +V
F:E	= (F:C) (C:E)	=	(F:A) (A:E)
C:B	= (C:G) (G:B)	=	(C:E) (E:B)
G:F#	= (G:D) (D:F#)	=	(G:B) (B:F#)
A:G#	= (A:E) (E:G#)	=	(A:C#) (C#:G#)
E:D#	= (E:B) (B:D#)	=	(E:G#) (G#:D#)
B:Bb	= (B:F#) (F#:Bb)	=	(B:D#) (D#:Bb)

If on the other hand we should wish to descend by a major semitone, it is only necessary to arrange the tones here exhibited in inverse order; since, then, these transits are of a double type, in singing in ensemble these major semitones may be employed in a double way, obviously provided that the tones interpolated here are expressed by different voices, and this differing usage is even deemed to relate to different musical **genera**; harmony itself takes on differing aspects according to whether one or the other interpolation is applied.

III. Transit through +V -III, that is,
through interval 5:6 or 5:3

22 The interval 5:6 is called the minor third, and the other one, 5:3, the major sixth. Such intervals are found in our musical scale:

A:C; E:G; B:D; C#:E; G#:B; D#:F#.

The twofold transit is given here, to wit

	+V -III		-III +V
A:C	= (A:E) (E:C)	=	(A:F) (F:C)
E:G	= (E:B) (B:G)	=	(E:C) (C:G)
B:D	= (B:F#) (F#:D)	=	(B:G) (G:D)
C#:E	= (C#:G#) (G#:E)	=	(C#:A) (A:E)
G#:B	= (G#:D#) (D#:B)	=	(G#:E) (E:B)
D#:F#	= (D#:B) (B:F#)	=	(D#:B) (B:F#)

Clearly, this twofold transit to the minor third is commonly to be assigned to different genera by musicians.

IV. Transit through +III +III, that is,
through interval 16:25

23 This interval, little customary in music, usually falls under the name of *augmented fifth*. In our scale, four such intervals occur in all:

F:C#; C:G#; G:D#; D:Bb

each of which is resolved in one way only

F:C# = (F:A) (A:C#); C:G# = (C:E) (E:G#)
G:D# = (G:B) (B:D#) D:Bb = (D:F#) (F#:Bb).

If the first of the tones is raised by an octave, so that the interval becomes 32:25, it is usually called in music an *augmented third* or *diminished fourth*; moreover, the term in this matter is plainly of no importance. If these tones are inverted pairwise, the formulas need only be read in reverse order.

V. Transit through +V +V +V, that is,
through interval 32:27 or even 16:27

24 The interval 32:27 also gets the name major third, **since** it falls short of the previous one by one comma. Three such intervals are found:

F:D; A:F#; C#:Bb

each of which is given by a single transit:

F:D = (F:C) (C:G) (G:D)
A:F# = (A:E) (E:B) (B:F#)
C#:Bb = (C#:G#) (G#:D#) (D#:Bb)

VI. Transit through +V +V +III, that is,
through interval 32:45 or 45:64

25 The first interval, 32:45, is called the *augmented fourth*, the other, 45:64, the *diminished fifth*; of such types of intervals in the scale there occur the following, through which triple transitions are given

F:B; C:F#; A:D#; E:Bb

	+V +V +III	+V +III +V	+III +V +V
F:B	(F:C) (C:G) (G:B)	(F:C) (C:E) (E:B)	(F:A) (A:E) (E:B)
C:F#	(C:G) (G:D) (D:F#)	(C:G) (G:B) (B:F#)	(C:E) (E:B) (B:F#)
A:D#	(A:E) (E:B) (B:D#)	(A:E) (E:G#) (G#:D#)	(A:C#) (C#:G#) (G#:D#)
E:Bb	(E:B) (B:F#) (F#:Bb)	(E:B) (B:D#) (D#:Bb)	(E:G#) (G#:D#) (D#:Bb) .

VII. Transit through +V +V -III, that is,
through interval 5:9 or 10:9

26 The interval 5:9 is called the minor seventh, as is also 9:16, and indeed the interval 10:9 bears the name *minor tone*; such intervals are

A:G; E:D; C#:B; G#:F#

through each of which a triple transition is given:

	+V	+V	-III		+V	-III	+V		-III	+V	+V	
A:G		(A:E)	(E:B)	(B:G)		(A:E)	(EC)	(C:G)		(A:F)	(F:C)	(C:G)
E:D		(E:B)	(B:F#)	(F#:D)		(E:B)	(B:G)	(G:D)		(E:C)	(C:G)	(G:D)
C#:B		(C#:G#)	(G#:D#)	(D#:B)		(C#:G#)	(G#:E)	(E:B)		(C#:A)	(A:E)	(E:B)
G#:F#		(G#:D#)	(D#:Bb)	(Bb:F#)		(G#:D#)	(D#:B)	(B:F#)		(G#:E)	(E:B)	(B:F#),

which transits are obviously to be applied to three different modes.

VIII. Transit through +III +III +V, that is,
through interval 64:75

27 This interval is also called a major third, even though it falls short by about two commas from the true ratio 5:6. There are three such intervals,

F:G#; C:D#; G:Bb,

and the transits may be set up in a triple way as follows:

	+III	+III	+V		+III	+V	+III		+V	+III	+III	
F:G#		(F:A)	(A:C#)	(C#:G#)		(F:A)	(A:E)	(E:G#)		(F:C)	(C:E)	(E:G#)
C:D#		(C:E)	(E:G#)	(G#:D#)		(C:E)	(E:B)	(B:D#)		(C:G)	(G:B)	(B:D#)
G:Bb		(G:B)	(B:D#)	(D#:Bb)		(G:B)	(B:F#)	(F#:Bb)		(G:D)	(D:F#)	(F#:Bb) .

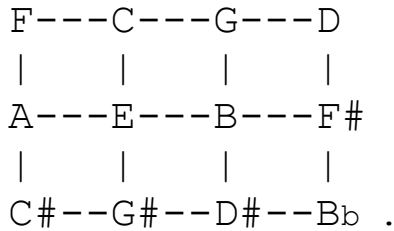
IX. Transit through +III +III -V, that is,
through interval 24:25

28 This interval is less than a semitone and is commonly called a *lesser limma*, of which kind are the three following:

C:C#; G:G#; D:D#; any one of which involves triple leaps:

	+III +III -V	+III -V +III	-V +III +III
C:C#	(C:E) (E:G#) (G#:C#)	(C:E) (E:A) (A:C#)	(F:C) (C:E) (E:G#)
G:G#	(G:B) (B:D#) (D#:G#)	(G:B) (B:E) (E:G#)	(C:G) (G:B) (B:D#)
D:D#	(D:F#) (F#:Bb) (Bb:D#)	(D:F#) (F#:B) (B:D#)	(D:G) (G:B) (B:D#) .

29 In similar fashion, the more complex transitions, which are +V +V +V ±III; +V +V ±III ±III; +V +V +V ±III ±III, would have been easy to set forth, but truly, all transitions of this sort may be represented to the gaze much more clearly and concisely by the scheme from section 13 above, which, adapted to this purpose, therefore may be called the *Speculum Musicum* because of its exceptional utility:



Now, on examination of this *Speculum*, it is clear which leaps may lead from any tone to any other, and at the same time in how many ways any given transit may be carried out, all according to the path of links, whether horizontal or vertical, to be followed, where the horizontal indicate a leap through a fifth and the vertical through a third. Thus, if it is wanted to pass from tone F to tone Bb, it is easily seen that it may be done in ten different ways, which are

- I. F:C:G:D:F#:Bb
- II. F:C:G:B:F#:Bb
- III. F:C:G:B:D#:Bb
- IV. F:C:E:B:F#:Bb
- V. F:C:E:B:D#:Bb
- VI. F:C:E:G#:D#:Bb
- VII. F:A:E:B:F#:Bb
- VIII. F:A:E:B:D#:Bb
- IX. F:A:E:G#:D#:Bb
- X. F:A:C#:G#:D#:Bb.

30 By the power of this speculum, even a rather complicated inquiry in music may be resolved, obviously to the extent that all twelve tones of the musical scale may be run through by simple leaps of a fifth and a major third, so that by such individual steps taken once each, a return be made to the first tone, from which the course was begun. Such a progression returning on itself may be set up in two ways:

I. Circulation F:C:G:D:F#:Bb:D#:B:E:G#:C#:A:F

II. Circulation F:C:E:B:G:D:F#:Bb:D#:G#:C#:A:F.

And from the same speculum, it is immediately clear through which tones of the musical scale harmonic triads may be given, whether major or minor. Now, triples of tones indicated by the sign |⁻⁻⁻ will constitute triads of that first type; those, however, indicated by the sign __| constitute a minor triad.

Behold then the following major triads:

F, A, C; C, E, G; G, B, D; A, C#,E; E, G#,B; B, D#,F#.

The minor triads, on the other hand, will be

A, C, E; E, G, B; B, D, F#; C#,E, G#; G#,B, D#; D#,F#,Bb.

In both of these modes, there are given three triads of pure harmony.

31 While, on the one hand, we do not admit other simple consonances than the octave, fifth, and third, we by no means reject more complex consonances, nor dissonances, as they are called by musicians, but rather we have shown here their final resolution into simple consonances, so that it might be plain in what way such consonances or even dissonances might be able to be called into use, and perceived, and judged. Indeed, a place in music is granted to just such more complex and dissonant consonances in so far as it is allowed to resolve them into simple consonances. And whoever wishes to use the rules given here must above all take care that a musical instrument be tuned exactly to the tones that harmony demands, and how those tones are represented in our *Speculum Musicum*.

32 With all reason, then, we are seen to assume all consonances discussed here to be displayed in musical instruments so exactly that it should not be possible for even the least aberration to be sensed. Therefore, those musicians have greatly withdrawn from this rule, which is so greatly necessary for the production of harmony, who have thought that an interval of one octave should be divided into twelve equal parts, because in such a way musical ensemble may be transposed into all keys. Because, however, in this way no pure fifth may be given in the entire musical scale, and all major thirds deviate considerably from the true ratio, this opinion has now been rejected by the majority, to be sure, of musicians, who have easily recognized that to be removed from the true principles of harmony for the sake of transposition is not at all fitting. Finally, the usual way of instructing children in music is greatly alien from the principles of harmony, in the manner in which it may be demanded that beginners learn to chant tones *do re mi fa sol la*; since in this progression *do:re* makes a major tone, next *re:mi* a minor tone, then indeed *mi:fa* a major semitone, which interval not even the most experienced musicians are able to produce, unless they have been aided by instruments or by resolution into simple leaps. All the more reason, then, that novices be taught with all zeal, right from the start, that they should learn to carry out the simple consonances, namely, the octave, fifth, and major third; thus, then, they will hone the discernment of their hearing by this exercise itself, and will become more and more accustomed to the pleasure of perceiving these consonances.

Finis