

REFLECTIONS ON SPACE AND TIME

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I

The principles of mechanics are already so solidly established that it would be a great error to continue to doubt their truth. Even though we would not be in condition to prove them through the general principles of metaphysics, the marvelous accord of all the conclusions that one obtains through calculation, with all the motion of bodies on earth both solid and fluid, and even with the motion of celestial bodies, shall be sufficient to put their truth beyond doubt. It is therefore an indisputable truth that a body once at rest will remain perpetually at rest, unless its state is disturbed by some outside force. It is equally certain that a body put into motion will continue to be in motion with the same speed following the same direction, provided that it does not meet any obstacles contrary to the conservation of this state.

II

As so indubitably established, these two truths must be absolutely grounded upon the nature of bodies; and as it is metaphysics which is concerned with the study of the nature and properties of bodies, the awareness of these truths will be able to serve as a guide in these thorny investigations. For we shall be right to reject in this science all reasoning and all ideas that lead to conclusions contrary to these truths, however justified they may be elsewhere; and we shall be authorized to admit only principles that are consistent with these same truths. The first ideas that we form of things that are beyond us are ordinarily so obscure and so poorly established that it is extremely dangerous to draw guaranteed results from them. It is therefore always a great advantage when one already knows from elsewhere some conclusions to which the first principles of metaphysics must lead; and it shall be upon these conclusions that the first ideas of metaphysics must be regulated and determined.

III

Far from denying these principles of truth established by mechanics, the metaphysicians try instead to deduce and prove them through their ideas. But they reproach the mathematicians for inappropriately applying these principles to ideas of space and time, which for them are merely imaginary and devoid of all reality. It is quite possible that a true principle can, without losing any of its truth, be expounded in an awkward manner so that it does not conform to the precise ideas that one must have of things. But then the metaphysician shall be obliged to

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correct this error and substitute real ideas for imaginary ones in the exposition of these principles.

IV

Therefore this shall also be the case for the principles of mechanics that are intertwined with the ideas of space and time, ideas that according to the metaphysicians are devoid of any truth. We must then see if it is possible to extract such imaginary ideas and substitute real ideas in their place. From these real ideas we have formed the imaginary one by way of abstraction, but in such a way that the sense and force of these principles are not changed. For there is no doubt that in following these principles, bodies will not conform to things that exist only in our imagination. Rather it is certainly the case that the laws, which bodies follow in the conservation of their state, refer to things that are quite real.

V

Therefore it is evident that if it were not possible to conceive of the two alleged principles of mechanics without mixing in the ideas of space and time, it would be a sure sign that these ideas are not entirely imaginary, as the metaphysicians claim. One should rather conclude that both absolute space and time, as mathematicians represent them, are real things that exist beyond our imagination, since it would be absurd to support the idea that pure imagination could serve as a foundation for real principles of mechanics.

VI

As a way to begin this study, I will start with the first principle concerning the state of rest of bodies. In mechanics space and position are considered real, and on the basis of this principle we maintain that a body at rest in a particular position will stay there perpetually unless it is chased by some outside force. Thus in this case the body will stay in the same position relative to absolute space. It would be fine with me if the ideas of space and time were only imaginary, but show me where in reality bodies behave in accordance with this law; while instead the mathematicians are content with using the imaginary notions of space and position.

VII

One will first say that position is nothing more than the relation of a body to other bodies around it. Therefore let us substitute this notion for the idea of position, and we shall be obliged to say that according to this principle a body that is in a certain relation with other bodies surrounding it shall persist in maintaining this relation. That is to say, we must claim that body A surrounded by other bodies B, C, D, E, etc., will try to preserve itself in this same configuration perpetually. And thus when the mathematician says that a body at rest stays in the same position with respect to absolute space, the metaphysician would say that this body retains the same relationship with respect to other bodies in its surroundings.

VIII

Let us see if these two ways of reasoning are equivalent, and if one can always, without falling into error, substitute the metaphysical for the mathematical expression that we are already convinced is true. Therefore, to put these two expressions in accord with one another, let us suppose that body A as well as its neighbors B, C, D, E, etc. are at rest. In this case body A stays in the same configuration with bodies B, C, D, E, etc. according to both the metaphysical rule and also the rules of mathematics. And in this case it will not be a mistake to substitute one for the other.

IX

To better situate our ideas, let us suppose that body A is in still water. While it remains in the same position it will also stay in the same configuration with the water particles surrounding it, and as such this body obeys the rule of mathematics as well as that of metaphysics. But let us now suppose that the water starts to flow. According to the rule of mathematics the body nevertheless remains in the same position, unless it is carried by the force of the water. And yet according to the rule of metaphysics this body should instead follow the movement of water perfectly to retain its position among the very same particles of water that surrounded it before. In this case, therefore, the rule drawn from metaphysics shall no longer conform to the truth.

X

At this point let us consult the experiment that teaches us that a body at rest in still water will be put into motion as soon as the water begins to flow. This seems to favor the rule as it is conceived metaphysically. But mechanics shows very clearly that the body does not follow the water current so much as it is struck by the water particles, and consequently an outside force puts the body in motion. Therefore without this force the body would remain at rest in flowing water as it does in still water, and thus the body in the conservation of its state of rest does not follow the bodies that immediately surround it. From this it follows that what is called position in mechanics does not allow the explanation offered by metaphysics which claims that position is nothing but the relationship of the body with respect to other bodies that surround it.

XI

To this property of bodies, by which they try to conserve in their state as much rest as movement, we shall give the name inertia. Therefore this inertia, as we have just seen, does not relate to neighboring bodies, but it is quite certain that it conforms to the idea of position that the mathematicians consider real and the metaphysicians regard as imaginary. Since we are not allowed to substitute the relation of a body to those immediately surrounding it for this idea of position, it remains for us to consider a body's relation to more distant ones in order to assess this general principle of inertia. But I strongly doubt that the metaphysicians will want to risk supporting the view that bodies by virtue of inertia are inclined to preserve the same relationship with regard to bodies which are some distance away;

for it would be easy to show the falsity of this explanation by way of reflections similar to those that I made concerning neighboring bodies.

XII

If they say that it is with regard to fixed stars that one must explain the principle of inertia, it would be quite difficult to refute them, because fixed stars, being themselves at rest, are so far away from us that the bodies that are at rest with respect to absolute space, as one regards it in mathematics, are also considered at rest with regard to fixed stars. But apart from the fact that it would be a very strange proposition and contrary to a number of other dogmas of metaphysics to say that fixed stars direct bodies in their inertia, this rule would also be found to be false if we were permitted to apply it to bodies that are close to a fixed star. These matters noted there are no more real ideas that we can substitute for the so-called imaginary ideas of space and position in the explanation of inertia.

XIII

Therefore we see that the idea of position, as mathematicians conceive it, cannot be explained through any relationship with other bodies near or far; and thus it is inappropriate to introduce those metaphysical notions which we believe to correspond to the mathematical idea of position to the explanation of the mechanical principle in question. That is to say, a body's conservation of state has to do with position as one conceives it in mathematics and not at all with respect to other bodies. Yet we cannot say that this principle of mechanics is grounded upon something that only exists in our imagination. Given this it must be concluded definitively that the mathematical idea of position is not imaginary, but rather that there is something real in the world that fulfills this idea. Therefore there is in the world, apart from the bodies that constitute it, some reality that we represent by the idea of position.

XIV

Hence the metaphysicians are wrong to want to banish space and position entirely from the world, claiming that they are only abstract and imaginary ideas. Consequently whatever sort of proof they bring forth to argue their opinion shall indeed be poorly grounded, and there will be some paralogism hidden in them. It is true that the senses are not capable of furnishing us with ideas of space and position, and it is only through reflection that we form them. From this metaphysicians conclude that these are merely abstract ideas, similar to the ideas of genus and species, which only exist in our understanding and to which no real object corresponds. But it seems to me that this conclusion is hurried; for if we think about it, we will easily realize that the manner through which one arrives at the idea of space and of time is quite different from that through which one forms the ideas of genus and species. And it would be a mistake to support the idea that nothing exists that can only be known through reflection.

XV

I agree that everything that exists is perfectly determined. But suppose we excise some determining characteristic from the notion of such an object so that out of it a generic idea arises, a notion that no longer accounts for the existing object. It is

in this manner that we form the idea of extension¹ in general, namely from excising all determining characteristics from the idea of the body save for its extension. But the idea of the position that a body occupies is not formed by excising some determination of the body; it results from removing the entire body, so that position is not a determining characteristic of the body, as it still remains after removing the entire body with all quantities involved. For it must be noted that the position that a body occupies is quite different from its extension; extension belongs to the body and is maintained by the body when moving from one position to another, while position and space are not susceptible to any type of movement.

XVI

I do not want to enter into a discussion of the objections made against the reality of space and position. For having shown that this reality can no longer be revoked, it inevitably follows that all these objections must not be very sound. Even so we will not be in a position to respond to them. If we believe it to be absurd that all different positions or parts of space are similar to one another, contrary to the principle of indiscernibles, then I do not know whether this principle is as general as we think. Perhaps it is only applicable to bodies and spirits, a generality with which we could be quite content. But as space and position are things so essentially different from spirits and bodies, we cannot judge them by the same principles.

XVII

The reality of space shall still be established by the other principle of mechanics, which concerns the conservation of uniform motion following the same direction. For if space and position are only the relation among coexisting bodies, what would we mean by the idea of 'same direction'? We would be burdened with furnishing an idea, by the sole mutual relation of coexisting bodies, without incorporating that of immobile space. For no matter how bodies move and change position among themselves, it does not prevent us from preserving a clear enough idea of a fixed direction that the bodies try to follow in their motion, despite all the changes that other bodies undergo. From this it is evident that the identity of direction, which is a truly essential circumstance in the general principles of motion, could not be explained absolutely by the relation or the order of the coexisting bodies. Therefore, there must still be some other real thing apart from a body to which the idea of 'same direction' relates, and there is no doubt that it is to space, which we have just established to be real.

XVIII

The ideas of space and of time have almost always had the same fate, so that those who have denied the reality of one have also denied the reality of the other. As we have established the reality of space, we shall not be surprised to recognize time as something that is also real. It does not only exist in our mind, but truly flows, serving as measurement for the duration of things. We have a very clear idea of time, and I claim that we form it out of the succession of changes that we perceive.

¹Translators' note: In this section Euler is describing the idea of volume, for which Euler uses the French word 'étendue', here translated to English as 'extension'. In *Recherches physiques sur la nature des moindres parties de la matière* (E91) Euler uses the term 'volume' alongside 'étendue' to discuss the idea of the space occupied by matter.

With this view I agree that the idea of time only exists in our imagination. But we have reason to ask whether or not the idea of time and time itself are different from one another. And it seems to me that the metaphysicians, in doing away with the reality of time, have confused time with the notion of it.

XIX

The principle of motion of bodies, according to which a body set into motion must continue to be in motion with the same speed and following the same direction, this principle, I say, furnishes us with new proofs, not only for the reality of space, but also for that of time. For since uniform motion describes equal [amounts of]² space in equal [amounts of]³ time, I first ask, what are equal amounts of space according to those who deny the reality of space? I highly doubt that the metaphysicians will risk saying that the equality of spaces must be judged by the equality of the number of monads which fill them, for they would maintain that monads are equally dispersed throughout the body. Even so, they would like to hold on to this explanation, even though it would be overturned as soon as we consider those bodies in motion, in relation to which we would seek to determine the equality of spaces. For we maintain, and the principle of motion teaches us, that when a body travels through equal amounts of space, these equal amounts do not depend at all on the other bodies that surround it, and it remains the same, no matter to what sort of changes the other bodies are exposed.

XX

It is much the same for the equality of time, for if time, as the metaphysicians hold, is nothing but the order of successions, how will one render the equality of time as intelligible? We profess that each being on earth is subject to continual changes, and that it is the succession of these changes that causes time. Following this explanation two periods of time must be equal when the same number of successions occurs for both. But if one considers a body that travels through equal amounts of space in equal times, based on which changes, or which body, must we judge the equality of those two times? Or should all bodies be subject to equally frequent changes, in which case we arrive at the same issue, that is, for which body would we like to choose to measure the equality of time in relation to the number of changes that happens to it. But I am certain that if we weigh this explanation at all, we shall find in it so many shortcomings that we shall easily be obliged to abandon it.

XXI

It is not a question of our regard for the equality of times, which no doubt depends on the state of our soul. It is a question of the equality of times, during which a body, moving uniformly, travels through equal spaces. As this equality could not be explained through the order of succession any more than the equality of space could be explained through the order of coexisting things, and as it enters essentially into the principle of motion, we cannot say that the body, as it moves, conforms to something that only exists in our imagination. We shall therefore be obliged to

²Brackets are translators' insertion.

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admit, as we have done for space, that time is something that exists beyond our minds or that it is as real as space. I speak here to those metaphysicians who still recognize some reality in bodies and in motion. As for those who completely deny this reality and who only grant them the status of phenomena, because they consider both motion itself and the laws of motion as chimera, I do not presume that these reflections should make the least impression on them.