

FACILLIMA METHODUS  
PLURIMOS PRIMOS PRAEMAGNOS  
INVENIENDI.

AUCTORE  
L. E U L E R O.

Convenerunt exhibita die 16. Mart. 1778.

1. Sufficiet hanc methodum unico exemplo, numerorum in hac formula  $2^2aa + 1$  contentorum, explicasse, ubi omnes valores ipsius  $a$  investigab, quibus haec formula numeros compositos producit; his enim exclusis. omnes reliqui numeri pro  $a$  assumi numero. primos supeditabunt.

2. At si formula  $23^2aa + 1$  praebeat numerum compositum, tum semper talis aequatio locum habebit:

$23^2aa + 1 = 23^2xx + yy^2$ ; propterea quod  $23^2 = 8.29$  est numerus idoneus  $\gamma$ ). Sic igitur erit  $23$  ( $aa - xx$ )  $= yy - 1$ . Jam ponatur  $\gamma = 1 + 5^2z$ , ac facta divisione per  $23^2$  prodibit ista aequatio:  $aa - xx = \frac{1}{2}z. 29z + 1$ , five, quia unitatem tam positive quam negative accipere licet, erit

$$aa - xx = \frac{1}{2}z(29z + 1).$$

3. Tribuantur nunc ipsi  $z$  ordine omnes valores 1, 2, 3. etc. et quia formula  $\frac{1}{2}z. 29z + 1$  cente duos factores, facpiusque plures, involvet, ea semel quoque, vel pluribus modis, per productam  $rs$  representari poterit, cum cum aequalis esse debeat

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for-

<sup>\*)</sup> Const. Nov. Act. T. III. p. 5. 53.

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debet, ut N — xz dividi queat per numerum  $\gamma$ . Hoc mo-  
do ad novae formulae novae aequationis

divisionem per 8 admittat. Hinc factum ponatur  $a = 8b$

formula  $aa - xx = (a+x)(a-x)$ , factuatur  $a+x = r$  et  $a-x = s$ , unde fit  $a = \frac{r+s}{2}$ , hocque modo referentur valores pro  $a$  excludendi. Hic igitur evidens est, ambos factores  $r$  et  $s$  simul vel pares vel imparis esse debere.

4. His jam notatis famamus  $z = 1$ , eritque duplici modo pro ambiguitate signi, vel  $rs = 14$  vel  $sr = 15$ , ubi prior valor, utpote impariter par, ad nostrum institutum est ineptus; posterior vero  $rs = 15$  praebet duas exclusiones, vel enim erit  $r = 15$  &  $s = 1$ , vel  $r = 5$  et  $s = 3$ , unde valores pro  $a$  excludendi erunt 8 et 4.

5. Sit nunc  $z = 3$ , eritque  $rs = 57$ , vel  $rs = 59$ . Prior admittit duas resolutiones:  $r = 57, r = 19$   
 $s = 1, s = 3$   
posterior unicam:  $r = 59$  et  $s = 1$ , unde oriuntur hae tres exclusiones:  $a = 29, a = 11, a = 3c$ .

6. Sit nunc  $z = 3$ , eritque vel  $rs = 3.43$ , vel  $rs = 3.44$ , quorum valorum resolutiones ita referentur:

$$r = 129, r = 43, r = 66, r = 22$$
$$s = 1, s = 3, s = 2, s = 6$$

unde quatuor exclusiones procedunt

$$a = 65, a = 23, a = 34, a = 14$$

7. Sit  $z = 4$ , erit vel  $rs = 250$ , vel  $rs = 234$ , qui numeri, utpote impariter pares, nullas dant exclusiones.

8. Sit nunc  $z = 5$ , erit  $rs = 5.73$ , vel  $rs = 5.73$ , unde ex priore sequentes procedunt resolutiones:

$$r = 180, r = 9c, r = 6c, r = 36, r = 3c, r = 20$$
$$s = 2, s = 4, s = 6, s = 10, s = 15, s = 18$$

Hinc ergo valores ipsius  $a$  excludendi erunt 9, 14, 17, 33, 33, 21, 19  
Alter

prietatis absolute necessaria est ad omnes numeros primos hac ratione explorandos Nisi enim formula  $xx - yy$  hac proprietate gaudeat, de numeris, qui unico modo in ea

Alter valor dat  $r = 73, s = 5$ , vel  $r = 365, s = 1$ , unde exclusiones hinc natae sunt 39 et 183.

8. Sit  $z = 6$ , erit vel  $rs = 3.173$ , vel  $rs = 3.175$ , unde sequentes resolutiones:

$$r = 173, 519, r = 525, 175, 105, 75, 35, 25$$
$$s = 3, 15 = 1, 3, 5, 7, 15, 21.$$

Exclusiones ergo nascuntur istae:

$$88, 260, 263, 89, 55, 41, 25, 23$$

9. Sit nunc  $x = 7$ , erit vel  $rs = 7.101$ , vel  $rs = 7.102$ . Hic solus prior casus dat has resolutiones:

$$r = 707, r = 101$$
$$s = 1, s = 7, \text{ hincque exclusiones erunt } 354 \text{ et } 54.$$

Ut autem nobis terminum praescribamus, in posterum omnes exclusiones majores quam 300 praetermittamus.

10. Sit nunc  $z = 8$ , erit vel  $rs = 4.5.7.11$ , vel  $rs = 4.23$ . Resolutiones hinc nascuntur sequentes:

$$r = 462, 154, 66, 42, r = 466$$
$$s = 2, 6, 14, 22, s = 2$$

unde concludimus sequentis exclusiones:

$$a = 232, 5, 47, 37, 234.$$

11. Sit  $z = 9$ , erit vel  $rs = 9.13c$ , vel  $rs = 9.131$ ,

$$r = 395, 131$$
$$s = 3, 3, \text{ excluditur igitur } a = 198, 70.$$

12. Sit  $z = 10$ , erit vel  $rs = 5.17.17$ , vel  $rs = 3.5.97$ ,

$$r = 495, 291, 97, r = 289, 85$$
$$s = 3, 5, 15, s = 5, 17, \text{ unde excluditur}$$
$$a = 147, 244, 51, 148, 56.$$

13. Sit

$z$  omnes fracesive tribuantur valores ad productum  $ab$  primi, sive si ipsius  $z$  valores excludantur, qui cum hoc producto communem habent divisorem; tam vero sufficit hos

13. Sit  $x = 11$ , erit vel  $rs = 3.11.53$ , vel  $rs = 32.5.11$ ; hinc

$r = 583, 159, 53; 440, 220, 176, 88, 80, 110, 44$ , hinc  
 $s = 3, 11, 33; 4, 8, 10, 20, 22, 10, 40$ , hinc  
 $a = 393, 85, 43, 222, 114, 93, 54, 51, 63, 42$ .

14. Sit  $x = 12$ . erit vel  $rs = 6.347$ , vel  $rs = 6.349$ , quorum neuter dat exclusionem.

15. Sit  $x = 13$ , erit vel  $rs = 13.4.57$ , vel  $rs = 27.13.7$ , hinc

$r = 94, 551, 273, 189, 117, 91, 63$ , excluduntur ergo  
 $s = 26, 7, 9, 13, 21, 27, 39$ ,  
 $6c, 179, 141, 101, 69, 59, 51$ .

16. Hoc modo calculum ulterius prosequi licet, quousque visum fuerit. Exclusiones ex singulis valoribus ipsius  $x$  ortas sequenti tabula repraesentemus:

$x$	Exclusiones
1	+
2	11, 29, 30
3	14, 23, 34, 65
4	-
5	19, 21, 23, 33, 39, 47, 91, 183
6	23, 25, 41, 55, 88, 89, 260, 263
7	54
8	32, 40, 80, 232, 234
9	70, 198,
10	11, 50, 147, 148, 244,
11	43, 43, 51, 54, 63, 85, 93, 114, 222, 293,
12	5, 6c, 69, 121, 141, 179
13	4, 53, 66, 78, 1-3, 15, 163, 266, 267,

Exclusiones

15	64, 68, 88, 116, 236,
16	61, 77, 103, 161, 191,
17	120, 132, 168,
18	265, 266,
19	75, 147, 243, 80, 88,
21	80, 85, 83, 97, 103, 109, 128, 145, 163, 221, 235, 272,
22	84, 96, 123, 276,
23	90, 122, 178,
24	92, 140, 154
25	98, 110, 154, 194
26	108, 145,
27	103, 105, 111, 129, 125, 147, 203, 209, 241.
29	122, 134, 166, 218, 225,
30	122, 131, 133, 146, 187, 214,
31	240,
32	139, 224,
33	256,
34	130, 141, 190,
35	137, 143, 162, 169, 247, 271,
37	141, 145, 171, 287,
38	212,
39	154, 202
40	156, 174, 178, 242,
41	162, 186,
42	160, 220, 268
43	164, 195, 199, 211, 284,
45	208,
46	191, 217, 257,
47	184,
48	185, 253, 241,

Exclusiones

49	260
50	191, 193
51	196, 236, 292
53	202, 223, 245, 260,
54	224
56	216, 240
58	224, 236, 296
59	225, 231, 233, 272
61	239, 241, 282
62	237,
63	240,
64	246, 282
65	252
66	274
67	282
69	263, 265, 295, 263, 265
71	278
73	286
75	286, 290
78	298

17. Inter hos numeros excludendos pro littera *a* nonnulli bis vel ter occurrunt, quod evenit, quando numerus  $232a + 1$  tres pluresve habet factores; ita videmus, numerum 265 primo a valore  $z = 18$ , deinde bis a valore  $z = 69$  excludi. Hoc autem casu revera numerus  $232.265^2 + 1$  constat his tribus factoribus:  $59 \cdot 461 \cdot 599$ , atque adeo prae-ter istam formam tres sequentes recipit:

$$\begin{aligned} 1^\circ & 232.256^2 + 1043^2 \\ 2^\circ & 232.34^2 + 4003^2 \\ 3^\circ & 232.35^2 + 4001^2 \end{aligned}$$

18.

18. Referamus autem in tabulam omnes hos numeros excludendos, secundum ordinem diffusos, atque eos, qui plus quam semel occurrunt, alienico notemus

0	4, 8
1	11, 14, 19
2	21, 23*, 25
3	3, 32, 33, 34, 39
4	40, 41, 42, 43, 47
5	51*, 54*, 55, 56, 58, 59
6	60, 61, 63, 64, 65, 66, 68, 69
7	72, 75, 77, 78
8	80*, 83, 84, 85*, 88*, 89
9	90, 91, 92, 93, 96, 97, 98
10	101, 102, 103, 105, 108, 109
11	113, 111, 114, 116
12	120, 122, 123, 125, 128, 129
13	130, 131, 132, 133, 134, 135, 137, 139
14	140, 141, 143, 145, 146, 147, 148
15	154, 156
16	160, 161, 162, 163, 164, 166, 165, 169
17	171, 174, 178, 179
18	183, 184, 185, 186, 187
19	190, 191, 193, 194, 195, 196, 198, 199
20	202, 203, 206, 207, 208, 209
21	211, 212, 214, 216, 217, 218
22	220, 221, 222, 223, 224, 225
23	231, 232, 233, 234, 235, 236, 237, 239
24	240, 241, 242, 243, 244, 245, 246, 247
25	252, 253, 256, 257,
26	260, 263, 26, 266, 268
27	271, 272, 274, 276, 278

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28|282, 284, 286, 287  
29|290, 292, 293, 295, 296, 298

His igitur numeris exclusis reliqui omnes loco a scripti  
in formula  $3^2ac - 1$  producant numeros primos. Hos igitur  
valores ipsius a sequens exhibet tabula:

1, 2, 3, 5, 6, 7, 9, 1, 12, 13, 15, 16, 17, 18,  
20, 22, 24, 26, 27, 28, 29, 31, 35, 36, 37, 38,  
44, 45, 46, 48, 49, 50, 5, 53, 57, 62, 67, 71,  
72, 73, 74, 76, 79, 81, 82, 86, 87, 94, 95, 99, 100,  
104, 106, 107, 112, 113, 11, 117, 118, 219, 121,  
124, 126, 127, 136, 138, 142, 144, 149, 150, 151,  
152, 153, 155, 157, 158, 159, 165, 167, 170, 172,  
173, 175, 176, 177, 18, 181, 182, 185, 189, 192,  
190, 197, 200, 201, 204, 205, 210, 213, 215, 215,  
226, 227, 228, 229, 230, 238, 248, 249, 250, 251,  
254, 255, 258, 259, 261, 262, 264, 267, 269, 272,  
273, 275, 277, 279, 280, 281, 283, 285, 288, 289,  
291, 294, 297, 299

horum numerorum ultimi fere ad 20 milliones exsurgunt.

ME.

### METHODUS GENERALIOR NUMEROS QUOSVIS SATIS GRANDES PERSCRUTANDI UTRUM SINT PRIMI NEC NE?

AUCTORE  
L. EULERO.

Conveneri exhibita die 16 Martii 1778.

#### §. I.

Sit  $N$  numerus propositus, et non difficile erit eum reducere ad huiusmodi formam:  $N = aa + \lambda bb$ ; tum vero inquiratur, utrum adhuc alio modo ad similem formam reduci queat. Si enim quoque fuerit  $N = xc + \lambda yy$ , ita ut sit  $aa + \lambda bb = xc + \lambda yy$ , tum certe numerus  $N$  non erit primus, sed eius factores hoc modo assignari poterunt. Quoniam hinc est  $aa - xc = \lambda (yy - bb)$ , erit  $\frac{a+x}{y-b} = \lambda \cdot \frac{y-b}{a-x}$ , quae fractiones ad minimos terminos reducti sint  $\lambda \cdot \frac{p}{q}$ . Ponatur igitur  $a+x = m p$  et  $y-b = n p$ , erit  $y+b = m q$  et  $a-x = n q$ , et hinc reperietur  $a = \frac{\lambda m p + n q}{2}$  et  $b = \frac{m q - n p}{2}$ , ex quibus valoribus, erit  $N = \frac{1}{4} (\lambda m m + n n)$  ( $\lambda p p + q q$ ); unde patet, formulam  $\lambda p p + q q$  vel ipsam, vel eius semissem, vel quadrantem esse factorem numeri propositi  $N$ .

§. II. Quando autem ad talem formam  $N = aa + \lambda bb$  pervenimus, tum statuamus  $N = xc + \lambda yy$ , ubi statim facile patebit, utrum hi numeri  $x$  et  $y$  sint pares, an vero impares, quo reperto statuat vel  $N = xc + \lambda yy$ , vel  $N = \lambda yy = xc$ ; ac priori quidem casu numerus  $x$  ita accipi debet

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