EXERCITATIO ANALYTICA.

Auctore

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§. 1.

Consideranti productum infinitum cosinum cuiusque anguli exprimens, quod est

$$\cos.\frac{\pi}{2n} = \left(1 - \frac{1}{nn}\right)\left(1 - \frac{1}{9nn}\right)\left(1 - \frac{1}{25nn}\right)\left(1 - \frac{1}{49nn}\right) \text{ etc.}$$

in mentem venit methodum inuestigare, cuius ope vicissim ex indole istius producti eius valor, quem nouimus esse $= \cos.\frac{\pi}{2n}$, erui queat, in quo negotio plura se obtulere artificia, quorum explicationem Geometris haud ingratam fore confido.

§. 2. Pono igitur

$$S = \left(1 - \frac{1}{nn}\right)\left(1 - \frac{1}{9nn}\right)\left(1 - \frac{1}{25nn}\right) \text{ etc.}$$

et sumtis logarithmis prodit mihi:

$$lS = l\left(1 - \frac{1}{nn}\right) + l\left(1 - \frac{1}{9nn}\right) + l\left(1 - \frac{1}{25nn}\right) + \text{etc.}$$

et cum sit

$$l\left(1 - \frac{1}{x}\right) = -\frac{1}{x} - \frac{1}{2xx} - \frac{1}{3x^3} - \frac{1}{4x^4} - \text{etc.}$$

erit his seriebus ordine dispositis signisque mutatis:

 $-lS$

$$-lS = \frac{1}{nn} + \frac{1}{2n^4} + \frac{1}{3n^6} + \frac{1}{4n^8} + \text{etc.}$$
$$+ \frac{1}{9nn} + \frac{1}{2.9^2n^4} + \frac{1}{3.9^3n^6} + \frac{1}{4.9^4n^8} + \text{etc.}$$
$$+ \frac{1}{25.nn} + \frac{1}{2.25^2.n^4} + \frac{1}{3.25^3.n^6} + \frac{1}{4.25^4.n^8} + \text{etc.}$$
$$+ \frac{1}{49.nn} + \frac{1}{2.49^2.n^4} + \frac{1}{3.49^3.n^6} + \frac{1}{4.49^4.n^8} + \text{etc.}$$
etc.

§. 3. Quodſi iam ſingulas columnas verticales in ordinem diſponamus, ſequentes ſeries pro $-lS$ obtinebimus:

$$-lS = \frac{1}{nn}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \text{etc.}\right)$$
$$+ \frac{1}{2.n^4}\left(1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \frac{1}{9^4} + \text{etc.}\right)$$
$$+ \frac{1}{3.n^6}\left(1 + \frac{1}{3^6} + \frac{1}{5^6} + \frac{1}{7^6} + \frac{1}{9^6} + \text{etc.}\right)$$
$$+ \frac{1}{4.n^8}\left(1 + \frac{1}{3^8} + \frac{1}{5^8} + \frac{1}{7^8} + \frac{1}{9^8} + \text{etc.}\right)$$
etc.

Sicque negotium perductum eſt ad ſummationem ſerierum poteſtatum parium progreſſionis harmonicae $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}$, etc.

§. 4. Oſtendi autem olim, poſito breuitatis gratia $\frac{\pi}{2} = \varrho$, ſi harum poteſtatum ſummae repraeſententur ſequenti modo:

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \text{etc.} = A\varrho^2,$$
$$1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \text{etc.} = B\varrho^4,$$
$$1 + \frac{1}{3^6} + \frac{1}{5^6} + \frac{1}{7^6} + \text{etc.} = C\varrho^6,$$
etc.

primo eſſe $A = \frac{1}{2}$, tum vero litteras reliquas ſequenti modo per praecedentes determinari:

$$B = \tfrac{2}{3}A^2,\ C = \tfrac{2}{5}.\,2AB,\ D = \tfrac{2}{7}(2AC + BB),$$
$$E = \tfrac{2}{9}(2AD + 2BC),\ F = \tfrac{2}{11}(2AE + 2BD + CC), \text{ etc.}$$

cuius

cuius veritas ſimul ex pulcherrimo conſenſu huius Analyſeos elucebit.

§. 5. His igitur valoribus ſubſtitutis nanciſcimur hanc ſeriem:

$$-lS = \frac{A\varrho^2}{nn} + \frac{1}{2}\cdot\frac{B\varrho^4}{n^4} + \frac{1}{3}\cdot\frac{C\varrho^6}{n^6} + \frac{1}{4}\cdot\frac{D\varrho^8}{n^8} + \text{etc.}$$

Quod ſi igitur ponamus $\frac{\varrho}{n} = x$, vt ſit $x = \frac{\pi}{2n}$, iſta ſeries hanc induet formam:

$$-lS = Axx + \tfrac{1}{2}Bx^4 + \tfrac{1}{3}Cx^6 + \tfrac{1}{4}Dx^8 + \text{etc.}$$

Vt fractiones $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, etc. abigamus, differentiemus, ac facta diuiſione per $2\partial x$ conſequemur.

$$-\frac{\partial S}{2S\partial x} = Ax + Bx^3 + Cx^5 + Dx^7 + \text{etc.}$$

§. 6. Statuamus hic breuitatis gratia $-\frac{\partial S}{2S\partial x} = t$, vt habeamus;

$$t = Ax + Bx^3 + Cx^5 + Dx^7 + \text{etc.}$$

vnde ſumtis quadratis orietur haec ſeries:

$$\begin{aligned} tt = A^2xx + 2ABx^4 + 2ACx^6 &+ 2ADx^8 + 2AEx^{10} + \text{etc.}\\ + BB \quad &+ 2BC \quad + 2BD \quad + \text{etc.}\\ &\qquad\quad + CC \quad\;\; + \text{etc.}\end{aligned}$$

ſicque iam pro quauis poteſtate ipſius x eas nacti ſumus formulas, quibus determinatio litterarum A, B, C, D, continetur: deſunt tantum coëfficientes illi $\frac{2}{3}, \frac{2}{5}, \frac{2}{7}$, etc.

§. 7. Hos autem coëfficientes introducemus integrando, poſtquam per $2\partial x$ multiplicauerimus. Reperietur enim

$$2\int tt\partial x = \tfrac{2}{3}A^2x^3 + \tfrac{2}{5}\cdot 2ABx^5 + \tfrac{2}{7}(2AC + BB)x^7$$
$$+ \tfrac{2}{9}(2AD + 2BC)x^9 + \tfrac{2}{11}(2AE + 2BD + CC)x^{11} + \text{etc.}$$

Cum nunc ſit

$$\tfrac{2}{3}A^2 = B,\ \tfrac{2}{5}\cdot 2AB = C,\ \tfrac{2}{7}(2AC + BB) = D,\ \text{etc.}$$

his

his valoribus restitutis perueniemus ad hanc seriem:

$$2\int tt\,\partial x = \mathrm{B}x^3 + \mathrm{C}x^5 + \mathrm{D}x^7 + \mathrm{E}x^9 + \text{etc.}$$

§. 8. Cum igitur ante habuissemus hanc seriem:

$$t = \mathrm{A}x + \mathrm{B}x^3 + \mathrm{C}x^5 + \mathrm{D}x^7 + \text{etc.}$$

hinc manifesto fluit ista aequatio:

$$t = \mathrm{A}x + 2\int tt\,\partial x,$$

quae differentiata dat

$$\partial t = \mathrm{A}\partial x + 2tt\partial x = \tfrac{1}{2}\partial x + 2tt\partial x, \text{ ob } \mathrm{A} = \tfrac{1}{2}.$$

Hinc ergo habebimus $2\partial t = \partial x(1+4tt)$, vnde fit $\partial x = \frac{2\partial t}{1+4tt}$, cuius integrale in promptu est, scilicet $x = \mathrm{A}\ \text{tang.}\ 2t$, vbi constantis adiectione non est opus, quandoquidem posito $x = 0$ t iam sponte euanescit. Hac ergo aequatione inuenta, si quantitas x vt angulus spectetur, vicissim erit $2t = \text{tang.}\ x$. Erat vero $t = -\frac{\partial S}{2S\partial x}$, vnde colligitur haec aequatio:

$$-\frac{\partial S}{S\partial x} = \text{tang.}\ x, \text{ ideoque } -\frac{\partial S}{S} = \frac{\partial x\,\text{sin.}\,x}{\text{cos.}\,x}.$$

§. 9. Cum igitur sit $\partial x\ \text{sin.}\ x = -\partial.\text{cos.}\ x$, erit $\frac{\partial S}{S} = \frac{\partial.\,\text{cos.}\,x}{\text{cos.}\,x}$, hincque integrando $l\,S = l\,\text{cos.}\,x + C$, quae constans inde debet definiri, vt posito $x = 0$ fiat $l\,S = 0$. Hinc ergo erit $C = 0$, ita vt sit $l\,S = l\,\text{cos.}\,x$, ideoque ad numeros progrediendo fiet $S = \text{cos.}\ x$.

§. 10. Posueramus autem $x = \frac{\pi}{2n}$, vnde manifesto valor quaesitus S prodit $S = \text{cos.}\,\frac{\pi}{2n}$, prorsus vti iam ante constabat. Haec igitur Analysis egregie confirmat illam relationem inter litteras A, B, C, D, quam aliunde in calculum introduxi.

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