NOVA METHODVS MOTVM CORPORVM RIGIDORVM DETERMINANDI.

Auctore

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§. 1.

Quanquam in tractatu meo de motu corporum rigidorum iſtam Theoriam ſatis felici ſucceſſu expediui: tamen fateri cogor, ſolutiones quas dedi non ſolum nimis eſſe intricatas, ſed etiam applicationem ad quosuis caſus particulares maxime eſſe moleſtam et plurimis difficultatibus implicatam. Poſtquam enim motum centri grauitatis determinaſſem, quod quidem nulla laborabat difficultate, ad quoduis tempus tam poſitionem axis gyrationis quam celeritatem difiniri oportebat; deinde vero imprimis ſitum ternorum axium principalium aſſignare neceſſe erat, ad quod ingens multitudo quantitatum variabilium in calculum introduci debebat.

Haec tanta incommoda etiam ſagaciſſimus Geometra *la Grange* animaduertiſſe videtur, dum hoc argumentum in Commentariis Academiae Boruſſicae pro Anno 1773 alia methodo tractandum ſuſcepit,

ſuſcepit; cuius quidem profundiſſimas meditationes maxima cum auiditate perluſtrare ſum conatus, veruntamen a me impetrare non potui, vt omnes eius calculos penetrarem. Statim enim primum Lemma ita me deterruit, vt ob defectum oculorum nullo modo ſperare potuerim, omnia artificia analytica, quibus eſt vſus, perſcrutari.

§. 2. Cum autem nuper, dum partem geometricam, cui iſta inueſtigatio innititur, accuratius euoluendam ſuſcepi, hanc inſignem proprietatem demonſtraſſem, quod, quomodocunque corpus rigidum ex ſtatu initiali in alium quemcunque ſtatum fuerit translatum, in eo ſemper talis axis aſſignari poſſit, cuius directio in vtroque ſtatu maneat inuariata: haec pulcherrima proprietas mihi ſtatim viſa eſt, eximium ſubſidium ſuppeditare, vnde omnia, quae ad motum huiusmodi corporum pertinent, multo facilius et ſine tanta farragine tot quantitatum variabilium definiri poſſent. Poſtquam enim motus centri grauitatis fuerit definitus, in ſtatu initiali ille quaeratur axis, qui in ſtatu translato etiamnunc eandem ſeruat directionem; tum vero quaeratur angulus, quo corpus interea circa hunc axem fuerit conuerſum. Hocque modo ad quoduis tempus ſitus corporis perfecte cognoſcetur, ita vt tota inueſtigatio ad determinationem iſtius axis pro quouis tempore cum angulo conuerſionis reducatur, quamobrem iſtam ideam hic accuratius ſum perſecuturus.

§. 3. Conſideremus igitur corpus in ſtatu ſuo initiali, in quo pro lubitu accipiamus punctum quodcunque, per quod ducamus ternos axes fixos inter se normales, quorum respectu situs singulorum elementorum corporis more solito per ternas coordinatas definiri queat. Illud quidem punctum in tractatu meo in ipso centro grauitatis corporis constitui; verum nunc nihil referre obseruaui, si in alio quocunque loco constituatur. Deinde etiam necesse non est, vt terni illi axes sint simul axes principales corporis, sed perinde ac ipsum illud punctum penitus arbitrio relinquuntur. Id tantum hic notasse iuuabit, calculos sequentes multo fore faciliores et concinniores, si illud punctum in ipso centro inertiae accipiatur simulque terni axes fuerint principales, pro statu scilicet corporis initialis. Tum vero ex illo puncto tanquam centro corpori Sphaera circumscribi concipiatur, cum ipso corpore firmiter cohaerens et cum ipso simul mobilis, cuius radius vnitate indicetur, vt hoc modo omnes inuestigationes ad doctrinam sphaericam reuocari queant, quandoquidem hoc pacto omnes calculi multo facilius expediri poterunt.

Tab. II. Fig. 3. §. 4. Repraesentet igitur triangulum sphaericum ABC octantem sphaerae, cuius centro adscripta intelligatur littera I, vnde rectae eductae IA, IB et IC referant ternos illos axes fixos, ita vt arcus AB, BC, CA sint quadrantes inter se angulos rectos A, B, C constituentes. Tum vero ad situm singulorum elementorum corporis cognoscendum, per quoduis eorum ductus intelligatur radius IZ; et ex puncto Z agantur terni arcus ZA, ZB, ZC, qui vocentur $ZA = \zeta$, $ZB = \eta$, et $ZC = \theta$; tum enim si distantia

ſtantia elementi corporis a centro I dicatur $= s$, ternae coordinatae axibus IA, IB, IC parallelae erunt $s \operatorname{cof.} \zeta$, $s \operatorname{cof.} \eta$, $s \operatorname{cof.} \theta$; quibus ergo ſitus cuiusque elementi in ſtatu initiali more ſolito determinabitur. Semper autem ex natura trianguli Sphaerici ABC erit

$$\operatorname{cof.} \zeta^2 + \operatorname{cof.} \eta^2 + \operatorname{cof.} \theta^2 = 1;$$

praeterea vero ex ſphaericis notentur ſequentes determinationes

$$\operatorname{cof.} BAZ = \frac{\operatorname{cof.} \eta}{\operatorname{ſin.} \zeta}; \quad \operatorname{cof.} ABZ = \frac{\operatorname{cof.} \zeta}{\operatorname{ſin.} \eta}; \quad \operatorname{cof.} BCZ = \frac{\operatorname{cof.} \eta}{\operatorname{ſin.} \theta}$$

$$\operatorname{ſin} BAZ = \frac{\operatorname{cof.} \theta}{\operatorname{ſin.} \zeta}; \quad \operatorname{ſin.} ABZ = \frac{\operatorname{cof.} \theta}{\operatorname{ſin.} \eta}; \quad \operatorname{ſin.} BCZ = \frac{\operatorname{cof.} \zeta}{\operatorname{ſin.} \theta}.$$

Porro vero erit

$$\operatorname{cof.} AZB = -\frac{\operatorname{cof.} \zeta \operatorname{cof.} \eta}{\operatorname{ſin.} \zeta \operatorname{ſin.} \eta}; \operatorname{cof.} BZC = -\frac{\operatorname{cof.} \eta \operatorname{cof.} \theta}{\operatorname{ſin.} \eta \operatorname{ſin.} \theta}; \operatorname{cof.} CZA = -\frac{\operatorname{cof.} \zeta \operatorname{cof.} \theta}{\operatorname{ſin.} \zeta \operatorname{ſin.} \theta}$$

$$\operatorname{ſin.} AZB = +\frac{\operatorname{cof.} \theta}{\operatorname{ſin.} \zeta \operatorname{ſin.} \eta}; \operatorname{ſin.} BZC = +\frac{\operatorname{cof.} \zeta}{\operatorname{ſin.} \eta \operatorname{ſin.} \theta}; \operatorname{ſin.} CZA = +\frac{\operatorname{cof.} \eta}{\operatorname{ſin.} \zeta \operatorname{ſin.} \theta}$$

§. 5. His pro ſtatu initiali definitis conſideremus ſtatum corporis, in quo elapſo tempore $= t$ reperietur; ac primo quidem in quemcunpue locum centrum I fuerit translatum, quandoquidem eius determinatio nulla difficultate laborat, eius locus cogitatione ſaltem in punctum I transferatur, ſimulque cum eo totum corpus, motu ſcilicet ſibi parallelo. Et quoniam in ſitu translato ſemper datur talis radius, cuius directio eadem eſt atque in ſtatu initiali, ſit O in ſuperficie ſphaerica id punctum, quod tam in ſtatu initiali quam in translato poſt tempus t eundem locum occupat, ita vt radius eandem directionem obtineat pro vtroque corporis ſtatu. Pro eius ergo ſitu ducantur ad angulos arcus OA, OB et OC, qui vocentur $OA = \alpha$, $OB = \beta$ et $OC = \gamma$, quos

quos ergo tanquam certas functiones temporis t spectari oportet, atque ex his nanciscemur sequentes determinationes:

$$\cos. OAB = \frac{\cos. \beta}{\sin. \alpha}; \quad \cos. ABO = \frac{\cos. \alpha}{\sin. \beta}; \quad \cos. BCO = \frac{\cos. \beta}{\sin. \gamma}$$

$$\sin. OAB = \frac{\cos. \gamma}{\sin. \alpha}; \quad \sin. ABO = \frac{\cos. \gamma}{\sin. \beta}; \quad \sin. BCO = \frac{\cos. \alpha}{\sin. \gamma}.$$

Porro vero erit

$$\cos AOB = -\frac{\cos. \alpha \cos. \beta}{\sin. \alpha \sin. \beta}; \quad \cos. BOC = -\frac{\cos. \beta \cos. \gamma}{\sin. \beta \sin. \gamma}; \quad \cos. COA = -\frac{\cos. \alpha \cos. \gamma}{\sin. \alpha \sin. \gamma}$$

$$\sin. AOB = -\frac{\cos. \gamma}{\sin. \alpha \sin. \beta}; \quad \sin. BOC = -\frac{\cos. \alpha}{\sin. \beta \sin. \gamma}; \quad \sin. COA = -\frac{\cos. \beta}{\sin. \alpha \sin. \gamma}.$$

Insuper vero erit, vt ante

$$\cos. \alpha^2 + \cos. \beta^2 + \cos. \gamma^2 = 1.$$

§. 6. Sit iam Φ ille angulus, per quem totum corpus tempore $= t$ circa radium IO fuerit conversum, ita vt angulus Φ etiam spectandus sit tanquam functio temporis t, dum arcus primum introducti ζ, η, θ, tantum a variabilitate cuiusque elementi corporis ad quod referuntur pendent. Punctum igitur sphaerae, quod initio erat in Z, nunc elapso tempore $= t$ ita translatum erit in z, vt ductis arcubus OZ et Oz futurus sit angulus Z O $z = \Phi$ existente O z = OZ. Hinc ergo si istius puncti z quaerantur distantiae ab angulis A B C, cognoscetur verus situs, in quem radius corporis IZ a situ initiali elapso tempore $= t$ erit translatus. Quamobrem pro motu corporis perfecte cognoscendo totum negotium eo reducitur, vt pro quouis tempore elapso $= t$ tam situs puncti O siue arcus α, β, γ quam angulus Z O $z = \Phi$ determinentur; quandoquidem hoc

hoc modo ad quoduis tempus verus situs singulorum corporis elementorum assignari poterit.

§. 7. Cum igitur distantiae puncti z ab angulis A B C ante omnia debeant indagari, inuestigemus primo distantiam A z; quod quo facilius fieri possit, in figura nullas alias lineas repraesentemus praeter eas, quae ad hunc scopum referuntur. Ac primo quidem quaeri debebit quantitas arcus O Z ex triangulo O A Z, in quo habentur arcus O A $= \alpha$ et arcus Z A $= \zeta$. Praeterea vero definiri poterimus angulum O A Z ex angulis B A O et B A Z, cum sit O A Z = B A O − B A Z ideoque Tab. II. Fig. 4.

$$\text{sin. } OAZ = \text{sin. } BAO \text{ cos. } BAZ - \text{cos. } BAO \text{ sin. } BAZ = \frac{\text{cos. }\gamma \text{ cos. }\eta - \text{cos. }\beta \text{ cos. }\theta}{\text{sin. }\alpha \text{ sin. }\zeta} \text{ et}$$

$$\text{cos. } OAZ = \text{cos. } BAO \text{ cos. } BAZ + \text{sin. } BAO \text{ sin. } BAZ = \frac{\text{cos. }\beta \text{ cos. }\eta + \text{cos. }\gamma \text{ cos. }\theta}{\text{sin. }\alpha \text{ sin. }\zeta} \text{ et tang. } OAZ = \frac{\text{cos. }\gamma \text{ cos. }\eta - \text{cos. }\beta \text{ cos. }\theta}{\text{cos. }\beta \text{ cos. }\eta + \text{cos. }\gamma \text{ cos. }\theta}$$

quibus notatis per sphaerica fiet

$$\text{cos. } OZ = \text{cos. }\alpha \text{ cos. }\zeta + \text{cos. }\beta \text{ cos. }\eta + \text{cos. }\gamma \text{ cos. }\theta \text{ et}$$

$$\text{tang. } AOZ = \frac{\text{cos. }\gamma \text{ cos. }\eta - \text{cos. }\beta \text{ cos. }\theta}{\text{sin. }\alpha^2 \text{ cos. }\zeta - \text{cos. }\alpha (\text{cos. }\beta \text{ cos. }\eta + \text{cos. }\gamma \text{ cos. }\theta)};$$

§. 8. Ponamus autem ad abbreuiandum angulum O A Z = A, ita vt sit

$$\text{sin. } A = \frac{\text{cos. }\gamma \text{ cos. }\eta - \text{cos. }\beta \text{ cos. }\vartheta}{\text{sin. }\alpha \text{ sin. }\zeta}; \quad \text{cos. } A = \frac{\text{cos. }\beta \text{ cos. }\eta + \text{cos. }\gamma \text{ cos. }\vartheta}{\text{sin. }\alpha \text{ sin. }\zeta}$$

$$\text{et tang. } A = \frac{\text{cos. }\gamma \text{ cos. }\eta - \text{cos. }\beta \text{ cos. }\vartheta}{\text{cos. }\beta \text{ cos. }\eta + \text{cos. }\gamma \text{ cos. }\vartheta};$$

pro quaesitis autem sit arcus O Z = Z et angulus A O Z = O, atque integrale O A Z erit

$$\text{cos. } Z = \text{cos. }\alpha \text{ cos. }\zeta + \text{sin. }\alpha \text{ sin. }\zeta \text{ cos. } A \text{ et}$$

$$\text{tang. } O = \frac{\text{sin. }\zeta \text{ sin. } A}{\text{sin. }\alpha \text{ cos. }\zeta - \text{cos. }\alpha \text{ sin. }\zeta \text{ cos. } A},$$

vnde

vnde ſi loco A valor inuentos ſcribatur, prodibit vti iam inuenimus

$$\cos. Z = \cos.\alpha \cos.\zeta + \cos.\beta \cos.\eta + \cos.\gamma \cos.\vartheta$$ et

$$\text{tang.}\, O = \frac{\cos.\gamma \cos.\eta - \cos.\beta \cos.\vartheta}{\sin.\alpha^2 \cos.\zeta - \cos.\alpha(\cos.\beta \cos.\eta + \cos.\gamma \cos.\vartheta)}.$$

§. 9. His iam inuentis aggrediamur triangulum ſphaericum AOz, in quo cognoſcimus bina latera $OA = \alpha$ et $Oz = Z$, vna cum angulo intercepto $AOz = O + \Phi$, vnde colligimus

$$\cos. Az = \cos.\alpha \cos. Z + \sin.\alpha \sin. Z \cos.(O + \Phi)$$

quae ergo expreſſio reducitur ad hanc:

$$\cos. Az = \cos.\alpha \cos. Z + \sin.\alpha \sin. Z \cos. O \cos.\Phi$$
$$- \sin.\alpha \sin. Z \sin. O \sin.\Phi$$

vbi iam definiuimus $\cos. Z$. Verum tam $\sin. Z$ quam $\sin. O$ et $\cos. O$ non habentur euoluti; neque vero conducit has quantitates immediate ex inventis deriuare, quoniam perueniremus ad ſigna radicalia, quae, vtrum poſitiue an negatiue capi debeant incertum maneret. Quamobrem hanc euolutionem ſequenti modo inſtitui conueniet.

§. 10. Ex triangulo AOZ ſtatim deducimus hanc proportionem:

$\sin. A : \sin. Z = \sin. O : \sin.\zeta$, vnde colligitur

$\sin. O = \frac{\sin. A \sin.\zeta}{\sin. Z}$, qui diuiſus per tang. O praebet

$$\cos. O = \frac{\sin.\alpha \cos.\zeta - \cos.\alpha \sin.\zeta \cos. A}{\sin. Z}.$$

Ex quibus ergo formulis ſponte ſe produnt illae ipſae formulae, quibus indigemus: ſcilicet $\sin. Z \sin. O$ et $\sin. Z \cos. O$; quare, ſi pro $\cos. Z$ inuentum valorem

rem substituamus, perueniemus ad istam expressionem:

$$\begin{aligned}\cos. Az = {} & \cos.\alpha^2 \cos.\zeta + \sin.\alpha \cos.\alpha \sin.\zeta \cos. A\\ & + \sin.\alpha^2 \cos.\Phi \cos.\zeta - \sin.\alpha \cos.\alpha \cos.\Phi \sin.\zeta \cos. A\\ & - \sin.\alpha \sin.\Phi \sin.\zeta \sin. A.\end{aligned}$$

Eliso igitur angulo A ista expressio sequentem induet formam:

$$\begin{aligned}\cos. Az = {} & \cos.\alpha^2 \cos.\zeta + \cos.\alpha \cos.\beta \cos.\eta + \cos.\alpha \cos.\gamma \cos.\vartheta\\ & + \sin.\alpha^2 \cos.\Phi \cos.\zeta - \cos.\alpha \cos.\Phi \cos.\beta \cos.\eta - \cos.\alpha \cos.\Phi \cos.\gamma \cos.\vartheta\\ & - \sin.\Phi \cos.\gamma \cos.\eta + \sin.\Phi \cos.\beta \cos.\vartheta.\end{aligned}$$

§. 11. Ista expressio eo magis est notatu digna, quod non solum nullis inquinata sit fractionibus, sed etiam in singulis terminis tantum occurrant cosinus trium angulorum ζ, η et ϑ sinibus penitus exclusis. Hanc ob caussam singulos hos terminos secundum istos ternos cosinus ordinemus, sicque impetrabimus sequentem formam:

$$\begin{aligned}\cos. Az = {} & \cos.\zeta(\cos.\alpha^2 + \sin.\alpha^2 \cos.\Phi) + \cos.\eta(\cos.\alpha \cos.\beta\\ & - \cos.\alpha \cos.\beta \cos.\Phi - \cos.\gamma \sin.\Phi) + \cos.\vartheta(\cos.\alpha \cos.\gamma\\ & - \cos.\alpha \cos.\gamma \cos.\Phi + \cos.\beta \sin.\Phi)\end{aligned}$$

vbi meminisse iuuabit, angulos ζ, η, ϑ tantum ad situm initialem spectare, iisque locum cuiuslibet elementi corporis definiri, neque igitur a tempore pendere; dum contra reliqui anguli α, β, γ, Φ sunt functiones ipsius temporis t tantum.

§. 12. Facta hac euolutione pro arcu Az superfluum plane foret, si similem inuestigationem pro duobus reliquis arcubus Bz et Cz instituere vellemus. Cum enim tria puncta A, B, C sint inter

se

ſe permutabilia, tantum opus erit, vt tam anguli ζ, η, ϑ, quam α, β, γ ſecundum ordinem promoveantur, dum interea angulus Φ idem conſeruatur. Hoc igitur modo reperiemus ſequentes expreſſiones

$$\text{coſ.}\,Bz = \text{coſ.}\,\eta(\text{coſ.}\,\beta^2 + \text{ſin.}\,\beta^2\,\text{coſ.}\,\Phi) + \text{coſ.}\,\vartheta(\text{coſ.}\,\beta\,\text{coſ.}\,\gamma - \text{coſ.}\,\beta\,\text{coſ.}\,\gamma\,\text{coſ.}\,\Phi - \text{coſ.}\,\alpha\,\text{ſin.}\,\Phi) + \text{coſ.}\,\zeta(\text{coſ.}\,\beta\,\text{coſ.}\,\alpha - \text{coſ.}\,\beta\,\text{coſ.}\,\alpha\,\text{coſ.}\,\Phi + \text{coſ.}\,\gamma\,\text{ſin.}\,\Phi)$$

$$\text{coſ.}\,Cz = \text{coſ.}\,\vartheta(\text{coſ.}\,\gamma^2 + \text{ſin.}\,\gamma^2\,\text{coſ.}\,\Phi) + \text{coſ.}\,\zeta(\text{coſ.}\,\gamma\,\text{coſ.}\,\alpha - \text{coſ.}\,\gamma\,\text{coſ.}\,\alpha\,\text{coſ.}\,\Phi - \text{coſ.}\,\beta\,\text{ſin.}\,\Phi) + \text{coſ.}\,\eta(\text{coſ.}\,\gamma\,\text{coſ.}\,\beta - \text{coſ.}\,\gamma\,\text{coſ.}\,\beta\,\text{coſ.}\,\Phi + \text{coſ.}\,\alpha\,\text{ſin.}\,\Phi).$$

§. 13. Statuamus nunc breuitatis gratia pro ſitu translato puncti Z

$$Az = \zeta', \quad Bz = \eta', \quad Cz = \vartheta',$$

ex quorum valoribus coordinatae pro iſto puncto deinceps formabuntur. Coſinus ergo horum trium arcuum hic iunctim aſpectui exponamus

$$\text{coſ.}\,\zeta' = \text{coſ.}\,\zeta(\text{coſ.}\,\alpha^2 + \text{ſin.}\,\alpha^2\,\text{coſ.}\,\Phi) + \text{coſ.}\,\eta(\text{coſ.}\,\alpha\,\text{coſ.}\,\beta(1 - \text{coſ.}\,\Phi) - \text{coſ.}\,\gamma\,\text{ſin.}\,\Phi) + \text{coſ.}\,\vartheta(\text{coſ.}\,\alpha\,\text{coſ.}\,\gamma(1 - \text{coſ.}\,\Phi) + \text{coſ.}\,\beta\,\text{ſin.}\,\Phi)$$

$$\text{coſ.}\,\eta' = \text{coſ.}\,\eta(\text{coſ.}\,\beta^2 + \text{ſin.}\,\beta^2\,\text{coſ.}\,\Phi) + \text{coſ.}\,\vartheta(\text{coſ.}\,\beta\,\text{coſ.}\,\gamma(1 - \text{coſ.}\,\Phi) - \text{coſ.}\,\alpha\,\text{ſin.}\,\Phi) + \text{coſ.}\,\zeta(\text{coſ.}\,\alpha\,\text{coſ.}\,\beta(1 - \text{coſ.}\,\Phi) + \text{coſ.}\,\gamma\,\text{ſin.}\,\Phi)$$

$$\text{coſ.}\,\vartheta' = \text{coſ.}\,\vartheta(\text{coſ.}\,\gamma^2 + \text{ſin.}\,\gamma^2\,\text{coſ.}\,\Phi) + \text{coſ.}\,\zeta(\text{coſ.}\,\alpha\,\text{coſ.}\,\gamma(1 - \text{coſ.}\,\Phi) - \text{coſ.}\,\beta\,\text{ſin.}\,\Phi) + \text{coſ.}\,\eta(\text{coſ.}\,\beta\,\text{coſ.}\,\gamma(1 - \text{coſ.}\,\Phi) + \text{coſ.}\,\alpha\,\text{ſin.}\,\Phi).$$

§. 14. Operae pretium hic erit nonnullos ſitus principales puncti Z contemplari. Sumamus igitur primo punctum Z in ipſo puncto A, ita vt ſit $\zeta = 0$ et $\eta = \vartheta = 90°$; vnde pro ſitu puncti z erit

coſ.

$$\cos.\zeta' = \cos.\alpha^2 + \sin.\alpha^2 \cos.\Phi = 1 - \sin.\alpha^2 (1 - \cos.\Phi)$$
$$\cos.\eta' = \cos.\alpha \cos.\beta (1 - \cos.\Phi) + \cos.\gamma \sin.\Phi$$
$$\cos.\vartheta' = \cos.\alpha \cos.\gamma (1 - \cos.\Phi) - \cos.\beta \sin.\Phi.$$

Simili modo ſi punctum Z capiatur in puncto B, ita vt ſit $\zeta = \vartheta = 90°$ et $\eta = 0$, tum erit pro ſitu puncti z

$$\cos.\zeta' = \cos.\alpha \cos.\beta (1 - \cos.\Phi) - \cos.\gamma \sin.\Phi$$
$$\cos.\eta' = \cos.\beta^2 + \sin.\beta^2 \cos.\Phi = 1 - \sin.\beta^2 (1 - \cos.\Phi)$$
$$\cos.\vartheta' = \cos.\beta \cos.\gamma (1 - \cos.\Phi) + \cos.\alpha \sin.\Phi.$$

Sin autem denique punctum Z capiatur in puncto C tum locus puncti z ita determinabitur, vt ſit

$$\cos.\zeta' = \cos.\alpha \cos.\gamma (1 - \cos.\Phi) + \cos.\beta \sin.\Phi$$
$$\cos.\eta' = \cos.\beta \cos.\gamma (1 - \cos.\Phi) - \cos.\alpha \sin.\Phi$$
$$\cos.\vartheta' = 1 - \sin.\gamma^2 (1 - \cos.\Phi).$$

§. 15. Iſtae nouem formulae ideo omni attentione ſunt dignae, quod ex iis formulae generales pro $\cos.\zeta'$, $\cos.\eta'$, $\cos.\vartheta'$ ex illis tam pulchre componuntur. Manifeſtum enim eſt, formulam generalem pro $\cos.\zeta'$ compoſitam eſſe ex tribus formulis ſpecialibus pro $\cos.\zeta'$ modo inuentis. Simili modo formula generalis pro $\cos.\eta'$ componitur ex ternis formulis ſpecialibus eiusdem nominis. Tandem etiam formula generalis $\cos.\vartheta'$ componitur ex ternis ſpecialibus pro eodem angulo.

§. 16. Haec inſignis proprietas, quae inter translationes punctorum A, B, C et alius cuiusuis puncti Z intercedit, maxime eſt memoratu digna,

 atque

atque omnino meretur, vt diligentius perpendatur; quam ob rem eam in sequenti Theoremate generalissimo complecti conueniet, cuius veritas ex calculis, quos modo absoluimus satis elucet, dum alias profundissimas indagationes postularet.

Theorema.

§. 17. Si sphaera circa centrum suum fixum vtcunque de situ suo deturbetur, vt terna eius puncta A, B, C in eius superficie quadrantibus a se inuicem distantia transferantur in puncta a, b, c; tum aliud punctum quodcunque Z ita transferetur in z vt sit

Tab. II. Fig. 5.

$$\text{I. } \cos zA = \cos ZA \cos aA + \cos ZB \cos aB + \cos ZC \cos aC$$
$$\text{II. } \cos zB = \cos ZB \cos bB + \cos ZC \cos bC + \cos ZA \cos bA$$
$$\text{III. } \cos zC = \cos ZC \cos cC + \cos ZA \cos cA + \cos ZB \cos cB.$$

§. 18. Quoniam puncta litteris maiusculis et minusculis designata inter se permutare licet, ita vt minusculae pertineant ad statum initialem, maiusculae vero ad statum mutatum: aequationes etiam sequentes veritati erunt consentaneae

$$\text{I. } \cos Za = \cos za \cos Aa + \cos zb \cos Ab + \cos zc \cos Ac$$
$$\text{II. } \cos Zb = \cos zb \cos Bb + \cos zc \cos Bc + \cos za \cos Ba$$
$$\text{III. } \cos Zc = \cos zc \cos Cc + \cos za \cos Ca + \cos zb \cos Cb.$$

Applicatio horum symptomatum ad coordinatas orthogonales.

§. 19. Applicemus nunc omnes formulas, quas hactenus inuenimus, ad ternas coordinatas, directionibus

bus fixis I A, I B, I C parallelas. Ac primo quidem pro ſtatu initiali conſideremus elementum corporis quodcunque in Z, pro cuius loco ſtatuamus coordinatas $IX = X$, $XY = Y$ et $YZ = Z$; vnde ſi diſtantia iſtius puncti Z a centro I vocetur $= s$, euidens eſt, ſi iſtud punctum Z in eo ſphaerae radio accipiatur, qui ante per punctum Z tranſibat, fore $X = s \operatorname{cof.} \zeta$, $Y = s \operatorname{cof.} \eta$, $Z = s \operatorname{cof.} \vartheta$. Hinc igitur erit $s^2 = XX + YY + ZZ$. Simili modo pro radio illo I O, cuius directio in ſtatu translato non mutatur, ternae coordinatae inter ſe erunt vt coſinus angulorum α, β, γ, quas autem in figura exprimere non eſt opus. Tab. II. Fig. 6.

§. 20. Iam elapſo tempore $= t$, quod perpetuo in minutis ſecundis exprimi aſſumimus translatum ſit centrum corporis I in punctum i, pro quo vocemus coordinatas $If = f$, $fi = g$, $ig = h$. Punctum autem, quod ante fuerat in Z nunc reperiatur in z, pro quo vocemus coordinatas $Ix = x$, $xy = y$, $yz = z$, et quoniam iſtud punctum z perinde ſe habet reſpectu puncti i, vt in praecedentibus figuris punctum Z ad centrum I, ita vt ſit diſtantia $iz = s$: vt ante manifeſtum eſt eſſe debere

$$x - f = s \operatorname{cof.} \zeta', \quad y - g = s \operatorname{cof.} \eta', \quad z - h = s \operatorname{cof.} \vartheta'.$$

§. 21. Quod ſi iam loco horum coſinuum, valores ſupra inuentos ſubſtituamus, ob $s \operatorname{cof.} \zeta = X$, $s \operatorname{cof.} \eta = Y$, $s \operatorname{cof.} \vartheta = Z$ nanciſcemur ſequentes valores:

$$x=f+X(\operatorname{cos}.\alpha^2+\operatorname{sin}.\alpha^2\operatorname{cos}.\Phi)+Y(\operatorname{cos}.\alpha\operatorname{cos}.\beta(1-\operatorname{cos}.\Phi)-\operatorname{cos}.\gamma\operatorname{sin}.\Phi)$$
$$+Z(\operatorname{cos}.\alpha\operatorname{cos}.\gamma(1-\operatorname{cos}.\Phi)+\operatorname{cos}.\beta\operatorname{sin}.\Phi)$$
$$y=g+Y(\operatorname{cos}.\beta^2+\operatorname{sin}.\beta^2\operatorname{cos}.\Phi)+Z(\operatorname{cos}.\beta\operatorname{cos}.\gamma(1-\operatorname{cos}.\Phi)-\operatorname{cos}.\alpha\operatorname{sin}.\Phi)$$
$$+X(\operatorname{cos}.\alpha\operatorname{cos}.\beta(1-\operatorname{cos}.\Phi)+\operatorname{cos}.\gamma\operatorname{sin}.\Phi)$$
$$z=h+Z(\operatorname{cos}.\gamma^2+\operatorname{sin}.\gamma^2\operatorname{cos}.\Phi)+X(\operatorname{cos}.\alpha\operatorname{cos}.\gamma(1-\operatorname{cos}.\Phi)-\operatorname{cos}.\beta\operatorname{sin}.\Phi)$$
$$+Y(\operatorname{cos}.\beta\operatorname{cos}.\gamma(1-\operatorname{cos}.\Phi)+\operatorname{cos}.\alpha\operatorname{sin}.\Phi).$$

§. 22. Quoniam autem istae expressiones nimis sunt complicatae, ponamus breuitatis gratia

$$x=f+FX+F'Y+F''Z$$
$$y=g+GX+G'Y+G''Z$$
$$z=h+HX+H'Y+H''Z$$

ita vt sit

$$F=\operatorname{cos}.\alpha^2+\operatorname{sin}.\alpha^2\operatorname{cos}.\Phi$$
$$G=\operatorname{cos}.\alpha\operatorname{cos}.\beta(1-\operatorname{cos}.\Phi)+\operatorname{cos}.\gamma\operatorname{sin}.\Phi$$
$$H=\operatorname{cos}.\alpha\operatorname{cos}.\gamma(1-\operatorname{cos}.\Phi)-\operatorname{cos}.\beta\operatorname{sin}.\Phi$$

$$F'=\operatorname{cos}.\alpha\operatorname{cos}.\beta(1-\operatorname{cos}.\Phi)-\operatorname{cos}.\gamma\operatorname{sin}.\Phi$$
$$G'=\operatorname{cos}.\beta^2+\operatorname{sin}.\beta^2\operatorname{cos}.\Phi$$
$$H'=\operatorname{cos}.\beta\operatorname{cos}.\gamma(1-\operatorname{cos}.\Phi)+\operatorname{cos}.\alpha\operatorname{sin}.\Phi$$

$$F''=\operatorname{cos}.\alpha\operatorname{cos}.\gamma(1-\operatorname{cos}.\Phi)+\operatorname{cos}.\beta\operatorname{sin}.\Phi$$
$$G''=\operatorname{cos}.\beta\operatorname{cos}.\gamma(1-\operatorname{cos}.\Phi)-\operatorname{cos}.\alpha\operatorname{sin}.\Phi$$
$$H''=\operatorname{cos}.\gamma^2+\operatorname{sin}.\gamma^2\operatorname{cos}.\Phi.$$

§. 23. Cum iam distantia puncti iz etiamnunc aequalis esse debeat distantiae IZ, ideoque

$$(x-f)^2+(y-g)^2+(z-h)^2=X^2+Y^2+Z^2,$$

calculum instituendo comperietur reuera fore

$$FF+GG+HH=1 \quad FF'+GG'+HH'=0$$
$$F'F'+G'G'+H'H'=1 \quad F'F''+G'G''+H'H''=0$$
$$F''F''+G''G''+H''H''=1 \quad FF''+GG''+HH''=0.$$

Atque

Atque de hoc ſatis ſumus certi, etiamſi calculus non parum fieret moleſtus.

Formulae generales pro motu corporum rigidorum a viribus quibuscunque ſollicitatorum.

§. 24. Conſideremus igitur corpus quodcunque rigidum, inquo pro lubitu certum punctum pro centro I fuerit aſſumtum, et cuius ternae directiones fixae IA, IB, IC in ſitu initiali, ſe inuicem normaliter decuſſent, quorum reſpectu ſingulorum corporis elementorum loca per ternas coordinatas X, Y et Z determinentur. Deſignetur autem elementum corporis in puncto Z conſtituti charactere dM ita vt littera M, denotet maſſam totius corporis; vbi ergo probe notetur, has tres variabiles X, Y, Z tantum ad ſtatum corporis initialem pertinere, neque vllo modo a tempore t pendere.

§. 25. His igitur pro ſtatu initiali conſtitutis, vbi corpori motus quicunque impreſſus fuiſſe eſt concipiendus. Hunc in finem ponamus elapſo tempore t centrum corporis I perueniſſe in i, elementum vero dM, quod in puncto Z conſiderauimus translatum fuiſſe in punctum z, cuius locus per coordinatas x, y, z ſupra aſſignatas definiatur; locum autem centri i per coordinatas f, g, h exhiberi. Hic igitur non ſolum tres quantitates f, g, h certae erunt functiones temporis, tam ex motu corpori impreſſo quam ex viribus ſollicitantibus determinandae, ſed etiam litterae F, G, H cum ſuis deriuatis tanquam tales functio-

 nes

nes erunt ſpectandae; quandoquidem per angulos α, β, γ et Φ, qui ſunt functiones temporis, exprimuntur. Totum negotium enim eo redit, vt tam ex motu corpori initio impreſſo, quam viribus ſollicitantibus ad quoduis tempus tam valores litterarum f, g, h quam angulorum α, β, γ et Φ eliciantur, quippe quibus inuentis motus corporis perfecte cognoſcetur.

§. 26. Cum nunc elementum corporis quodcunque in genere conſideratum verſetur in puncto z, cuius locus per dupplicis generis quantitates variabiles exprimitur, quarum tres X, Y, Z referuntur ad locum Z, quem in ſtatu initiali obtinuit: reliquae vero ſunt functiones temporis. Ex harum poſteriorum igitur variabilitate tam motus huius elementi quam eius acceleratio determinari poterit, id quod commodiſſime fiet, ſi eius motum ſecundum ternas directiones fixus I A, I B et I C reſoluamus, vnde primo ternas celeritates eiusdem elementi, hincque porro etiam accelerationes ſecundum easdem directiones definire licebit. Sumto igitur ſolo tempore t pro variabili, formulae $(\frac{dx}{dt})$, $(\frac{dy}{dt})$, $(\frac{dz}{dt})$ dabunt illas ternas celeritates, differentialia autem ſecunda $(\frac{ddx}{dt^2})$, $(\frac{ddy}{dt^2})$, $(\frac{ddz}{dt^2})$ accelerationes.

§. 27. Quod ſi nunc ſimili modo omnes vires, quibus corpus hoc tempore ſollicitatur etiam ſecundum iſtas ternas directiones reſoluantur, atque ex omnibus coniunctis pro directionibus IA, IB, IC vires oriantur P, Q et R, per principia motus neceſſe

cesse est, vt istae vires aequentur summis omnium virium acceleratricium, quae ex omnibus corporis elementis dM iunctim sumtis nascuntur. Scilicet si g denotet altitudinem lapsus grauium vno minuto secundo, loco $2g$ autem scribamus litteram i, quoniam littera g iam tanquam functio temporis in calculum ingreditur impetrabimus tres aequationes sequentes:

$$\int dM \left(\frac{ddx}{dt^2}\right) = iP$$
$$\int dM \left(\frac{ddy}{dt^2}\right) = iQ$$
$$\int dM \left(\frac{ddz}{dt^2}\right) = iR$$

vbi signum integrationis solas variabiles X, Y et Z respicit, ita vt in his integrationibus tempus t tanquam constans spectari debeat, etiamsi in formulis differentialibus id solum pro variabili est habitum. Praeterea vero etiam omnia momenta virium acceleratricium respectu ternorum axium fixorum simul sumta aequari debent momentis, quae ex omnibus viribus sollicitantibus respectu eorundem axium deducuntur; quamobrem designemus ista momenta, quae ex omnibus viribus sollicitantibus pro ternis axibus IA, IB, IC nascuntur, litteris S, T, V, ita vt his quantitatibus per i multiplicatis summae omnium momentorum elementarium, quas singulae vires acceleratrices suppeditant aequari debeant.

§. 28. Cum igitur elemento dM, quod in puncto z concipimus, primo applicata sit vis $= dM\left(\frac{ddx}{dt^2}\right)$ secundum directionem IA agens, ex

ea

ea nullum nascitur momentum pro hoc axe; pro axe autem I B nascetur momentum $= z\, d\, \mathrm{M}\, (\frac{d\, d\, x}{d\, t^2})$ et pro axe I C momentum $= y\, d\, \mathrm{M}\, (\frac{d\, d\, x}{d\, t^3})$. Simili modo ex vi acceleratrice secundum directionem I B, quae est $d\, \mathrm{M}\, (\frac{d\, d\, y}{d\, t^2})$ nascitur momentum pro axe I A $= z\, d\, \mathrm{M}\, (\frac{d\, d\, y}{d\, t^2})$, at pro axe I C momentum $= x\, d\, \mathrm{M}\, (\frac{d\, d\, y}{d\, t^2})$. Denique ex vi acceleratrice secundum I C, quae est $d\, \mathrm{M}\, (\frac{d\, d\, z}{d\, t^2})$ nascitur momentum pro axe I A $= y\, d\, \mathrm{M}\, (\frac{d\, d\, z}{d\, t^2})$ et pro axe I B momentum $= x\, d\, \mathrm{M}\, (\frac{d\, d\, z}{d\, t^2})$. Hinc igitur pro quolibet axe habemus bina momenta elementaria, quae in partes contrarias vergunt; vnde pro axe I A summa omnium momentorum elementarium erit

$$+\int z\, d\, \mathrm{M}\, (\frac{d\, d\, y}{d\, t^2}) - \int y\, d\, \mathrm{M}\, (\frac{d\, d\, z}{d\, t^2}) = i\, \mathrm{S}.$$

Eodem modo pro axe I B obtinebimus hanc aequationem:

$$\int x\, d\, \mathrm{M}\, (\frac{d\, d\, z}{d\, t^2}) - \int z\, d\, \mathrm{M}\, (\frac{d\, d\, x}{d\, t^2}) = i\, \mathrm{T}.$$

Tertia vero aequatio erit pro axe I C

$$\int y\, d\, \mathrm{M}\, (\frac{d\, d\, x}{d\, t^2}) - \int z\, d\, \mathrm{M}\, (\frac{d\, d\, y}{d\, t^2}) = i\, \mathrm{U}.$$

§. 29. Hac igitur ratione sex nacti sumus aequationes, quas hic coniunctim conspectui exponamus

I. $\int d\, \mathrm{M}\, (\frac{d\, d\, x}{d\, t^2}) = i\, \mathrm{P}$ IV. $\int z\, d\, \mathrm{M}\, (\frac{d\, d\, y}{d\, t^2}) - \int y\, d\, \mathrm{M}\, (\frac{d\, d\, z}{d\, t^2}) = i\, \mathrm{S}$

II. $\int d\, \mathrm{M}\, (\frac{d\, d\, y}{d\, t^2}) = i\, \mathrm{Q}$ V. $\int x\, d\, \mathrm{M}\, (\frac{d\, d\, z}{d\, t^2}) - \int z\, d\, \mathrm{M}\, (\frac{d\, d\, x}{d\, t^2}) = i\, \mathrm{T}$

III. $\int d\, \mathrm{M}\, (\frac{d\, d\, z}{d\, t^2}) = i\, \mathrm{R}$ VI. $\int y\, d\, \mathrm{M}\, (\frac{d\, d\, x}{d\, t^2}) - \int z\, d\, \mathrm{M}\, (\frac{d\, d\, y}{d\, t^2}) = i\, \mathrm{U}.$

In

In quibus formulis integralibus folae litterae maiusculae X, Y et Z pro variabilibus habentur, dum contra in formulis differentialibus folae quantitates a tempore t pendentes tanquam variabiles fpectantur.

§. 30. Vt nunc iftas formulas rite euoluamus, pro ternis coordinatis x, y, z valores contractos §. 22. datos adhibeamus, et quoniam in formulis differentialibus quantitates X, Y, Z vt conftantes fpectantur habebimus

$$\frac{d\,d\,x}{d\,t^2} = \left(\frac{d\,d\,f}{d\,t^2}\right) + X\left(\frac{d\,d\,F}{d\,t^2}\right) + Y\left(\frac{d\,d\,F'}{d\,t^2}\right) + Z\left(\frac{d\,d\,F''}{d\,t^2}\right)$$
$$\frac{d\,d\,y}{d\,t^2} = \left(\frac{d\,d\,g}{d\,t^2}\right) + X\left(\frac{d\,d\,G}{d\,t^2}\right) + Y\left(\frac{d\,d\,G'}{d\,t^2}\right) + Z\left(\frac{d\,d\,G''}{d\,t^2}\right)$$
$$\frac{d\,d\,z}{d\,t^2} = \left(\frac{d\,d\,h}{d\,t^2}\right) + X\left(\frac{d\,d\,H}{d\,t^2}\right) + Y\left(\frac{d\,d\,H'}{d\,t^2}\right) + Z\left(\frac{d\,d\,H''}{d\,t^2}\right)$$

Facile autem intelligitur, fi iftos valores in fuperioribus fex aequationibus fubftituere vellemus, formas maxime diffufas effe prodituras, quas tamen operae pretium erit hic adhibuiffe.

§. 31. Subftituamus ergo actu loco litterarum x, y, z et formularum $\left(\frac{d\,d\,x}{d\,t^2}\right)$, $\left(\frac{d\,d\,y}{d\,t^2}\right)$ et $\left(\frac{d\,d\,z}{d\,t^2}\right)$ valores ante euolutos, et fex aequationes noftrae fequenti modo exprimentur:

$$\text{I.}\quad i\,P = \left(\frac{d\,d\,f}{d\,t^2}\right)\int d\,M + \left(\frac{d\,d\,F}{d\,t^2}\right)\int X\,d\,M + \left(\frac{d\,d\,F'}{d\,t^2}\right)\int Y\,d\,M + \left(\frac{d\,d\,F''}{d\,t^2}\right)\int Z\,d\,M$$

$$\text{II.}\quad i\,Q = \left(\frac{d\,d\,g}{d\,t^2}\right)\int d\,M + \left(\frac{d\,d\,G}{d\,t^2}\right)\int X\,d\,M + \left(\frac{d\,d\,G'}{d\,t^2}\right)\int Y\,d\,M + \left(\frac{d\,d\,G''}{d\,t^2}\right)\int Z\,d\,M$$

$$\text{III.}\quad i\,R = \left(\frac{d\,d\,h}{d\,t^2}\right)\int d\,M + \left(\frac{d\,d\,H}{d\,t^2}\right)\int X\,d\,M + \left(\frac{d\,d\,H'}{d\,t^2}\right)\int Y\,d\,M + \left(\frac{d\,d\,H''}{d\,t^2}\right)\int Z\,d\,M.$$

$$\text{IV. } i\,S = \left(\frac{h\,ddg - g\,ddh}{dt^2}\right)\int dM$$
$$+ \left(\frac{h\,ddG - G\,ddh}{dt^2}\right)\int X\,dM + \left(\frac{H\,ddg - g\,ddH}{dt^2}\right)\int X\,dM$$
$$+ \left(\frac{h\,ddG' - G'\,ddh}{dt^2}\right)\int Y\,dM + \left(\frac{H'\,ddg - g\,ddH'}{dt^2}\right)\int Y\,dM$$
$$+ \left(\frac{h\,ddG'' - G''\,ddh}{dt^2}\right)\int Z\,dM + \left(\frac{H''\,ddg - g\,ddH''}{dt^2}\right)\int Z\,dM$$
$$+ \left(\frac{H\,ddG - G\,ddH}{dt^2}\right)\int X^2\,dM + \left(\frac{H'\,ddG' - G'\,ddH'}{dt^2}\right)\int Y^2\,dM$$
$$+ \left(\frac{H''\,ddG'' - G''\,ddH''}{dt^2}\right)\int Z^2\,dM$$
$$+ \left(\frac{H'\,ddG' - G'\,ddH'}{dt^2}\right)\int XY\,dM + \left(\frac{H'\,ddG - G'\,ddH}{dt^2}\right)\int XY\,dM$$
$$+ \left(\frac{H'\,ddG'' - G''\,ddH'}{dt^2}\right)\int YZ\,dM + \left(\frac{H''\,ddG' - G'\,ddH''}{dt^2}\right)\int YZ\,dM$$
$$+ \left(\frac{H\,ddG'' - G''\,ddH}{dt^2}\right)\int XZ\,dM + \left(\frac{H''\,ddG - G\,ddH}{dt^2}\right)\int XZ\,dM$$

$$\text{V. } i\,T = \left(\frac{f\,ddh - h\,ddf}{dt^2}\right)\int dM$$
$$+ \left(\frac{f\,ddH - H\,ddf}{dt^2}\right)\int X\,dM + \left(\frac{F\,ddh - h\,ddF}{dt^2}\right)\int X\,dM$$
$$+ \left(\frac{f\,ddH' - H'\,ddf}{dt^2}\right)\int Y\,dM + \left(\frac{F'\,ddh - h\,ddF'}{dt^2}\right)\int Y\,dM$$
$$+ \left(\frac{f\,ddH'' - H''\,ddf}{dt^2}\right)\int Z\,dM + \left(\frac{F''\,ddh - h\,ddF''}{dt^2}\right)\int Z\,dM$$
$$+ \left(\frac{F\,ddH - H\,ddF}{dt^2}\right)\int X^2\,dM + \left(\frac{F'\,ddH' - H'\,ddF'}{dt^2}\right)\int Y^2\,dM$$
$$+ \left(\frac{F''\,ddH'' - H''\,ddF''}{dt^2}\right)\int Z^2\,dM$$
$$+ \left(\frac{F\,ddH' - H'\,ddF}{dt^2}\right)\int XY\,dM + \left(\frac{F'\,ddH - H\,ddF'}{dt^2}\right)\int XY\,dM$$
$$+ \left(\frac{F'\,ddH'' - H''\,ddF'}{dt^2}\right)\int YZ\,dM + \left(\frac{F''\,ddH' - H'\,ddF''}{dt^2}\right)\int YZ\,dM$$
$$+ \left(\frac{F\,ddH'' - H''\,ddF}{dt^2}\right)\int XZ\,dM + \left(\frac{F''\,ddH - H\,ddF''}{dt^2}\right)\int XZ\,dM$$

$$\text{VI. } i\,U = \left(\frac{g\,ddf - f\,ddg}{dt^2}\right)\int dM$$
$$+ \left(\frac{g\,ddF - F\,ddg}{dt^2}\right)\int X\,dM + \left(\frac{G\,ddf - f\,ddG}{dt^2}\right)\int X\,dM$$
$$+ \left(\frac{g\,ddF' - F'\,ddg}{dt^2}\right)\int Y\,dM + \left(\frac{G'\,ddf - f\,ddG'}{dt^2}\right)\int Y\,dM$$
$$+ \left(\frac{g\,ddF'' - F''\,ddg}{dt^2}\right)\int Z\,dM + \left(\frac{G''\,ddf - f\,ddG''}{dt^2}\right)\int Z\,dM$$

+

$$+\left(\frac{G\,ddF - F\,ddG}{dt^2}\right)\int X^2\,dM + \left(\frac{G'\,ddF' - F'\,ddG'}{dt^2}\right)\int Y^2\,dM$$
$$+\left(\frac{G''\,ddF'' - F''\,ddG''}{dt^2}\right)\int Z^2\,dM$$
$$+\left(\frac{G\,ddF' - F'\,ddG}{dt^2}\right)\int XY\,dM + \left(\frac{G'\,ddF - F\,ddG'}{dt^2}\right)\int XY\,dM$$
$$+\left(\frac{G'\,ddF'' - F''\,ddG'}{dt^2}\right)\int YZ\,dM + \left(\frac{G''\,ddF' - F'\,ddG''}{dt^2}\right)\int YZ\,dM$$
$$+\left(\frac{G\,ddF'' - F''\,ddG}{dt^2}\right)\int XZ\,dM + \left(\frac{G''\,ddF - F\,ddG''}{dt^2}\right)\int XZ\,dM.$$

§. 32. In his ſex aequationibus formulae integrales, quae tantum ad ſtatum corporis initialem referuntur ante omnia per totam corporis maſſam ſunt extendendae; vnde earum loco in calculum ingredientur certae quantitates conſtantes, quas ex figura et indole cuiusque corporis definiri oportet. Veluti pro primis membris ſtatim prodit $\int dM = M$ vbi M maſſam totius corporis denotat. His igitur conſtantibus loco formularum integralium in noſtras aequationes introductis, aliae variabiles non amplius inerunt, niſi quae a ſolo tempore t pendent: ſemper enim aſſumi poteſt, omnes vires, quibus corpus ſollicitari ponimus, ad quoduis tempus eſſe datas. Quoniam vero hae ſex aequationes tantopere ſunt complicatae, praecipue ſi loco litterarum F, G, H etc. ſuos valores ſubſtituere vellemus, in genere nihil praeterea hinc concludere licebit.

§. 33. Ratio autem, cur iſtae aequationes tam prolixae euaſerunt manifeſto in eo eſt ſita, quod tam punctum I quam ternos axes I A, I B, I C in ſtatu initiali prorſus pro lubitu aſſumſimus. Si enim punctum I in ipſo centro grauitatis ſeu potius

tius inertiae collocamus, ob naturam huius centri statim istae tres formulae integrales euanescunt

$$\int X\,dM = 0,\ \int Y\,dM = 0,\ \int Z\,dM = 0.$$

Si deinde praeterea ternos axes I A, I B, I C in ipsis axibus corporis principalibus constituamus, tum etiam tres sequentes formulae integrales in nihilum abeunt

$$\int XY\,dM = 0,\ \int XZ\,dM = 0,\ \int YZ\,dM = 0$$

quibus igitur omnibus membris deletis nostrae aequationes iam mirifice contrahentur.

§. 34. Remanebunt autem tantum istae tres formulae integrales

$$\int XX\,dM,\ \int YY\,dM \text{ et } \int ZZ\,dM,$$

quorum valores per totum corpus extensos, si ponamus

$$\int XX\,dM = A,\ \int YY\,dM = B,\ \int ZZ\,dM = C$$

sex nostrae aequationes ad sequentes formas reducentur

$$\text{I. } iP = M\left(\frac{ddf}{dt^2}\right).\quad \text{II. } iQ = M\left(\frac{ddg}{dt^2}\right).\quad \text{III. } iR = M\left(\frac{ddh}{dt^2}\right)$$

$$\text{IV. } iS = M\left(\frac{h\,ddg - g\,ddh}{dt^2}\right) + A\left(\frac{H\,ddG - G\,ddH}{dt^2}\right) + B\left(\frac{H'\,ddG' - G'\,ddH'}{dt^2}\right) + C\left(\frac{H''\,ddG'' - G''\,ddH''}{dt^2}\right)$$

$$\text{V. } iT = M\left(\frac{f\,ddh - h\,ddf}{dt^2}\right) + A\left(\frac{F\,ddH - H\,ddF}{dt^2}\right) + B\left(\frac{F'\,ddH' - H\,ddF'}{dt^2}\right) + C\left(\frac{F''\,ddH'' - H''\,ddF''}{dt^2}\right)$$

$$\text{VI. } iU = M\left(\frac{g\,ddf - f\,ddg}{dt^2}\right) + A\left(\frac{G\,ddF - F\,ddG}{dt^2}\right) + B\left(\frac{G'\,ddF' - F'\,ddG'}{dt^2}\right) + C\left(\frac{G''\,ddF'' - F''\,ddG''}{dt^2}\right).$$

Plus autem hinc in genere pro viribus sollicitantibus quibuscunque concludere non licet.

Appli-

Applicatio harum Formularum ad casum quo corpus nullis plane viribus sollicitatur.

§. 35. Constituto igitur puncto I in ipso corporis centro grauitatis, ternae autem rectae I A, I B, I C simul corporis axes principales referant, dum scilicet in statu initiali versabatur; atque elapso tempore $= t$ ob omnes vires P, Q, R et S, T, U euanescentes habebimus primo istas tres aequationes:

$$\text{I. } M\left(\frac{ddf}{dt^2}\right)=0, \quad \text{II. } M\left(\frac{ddg}{dt^2}\right)=0, \quad \text{III. } M\left(\frac{ddh}{dt^2}\right)=0$$

quae semel integratae praebent

$$\frac{df}{dt}=\mathfrak{a}, \quad \frac{dg}{dt}=\mathfrak{b} \quad \text{et} \quad \frac{dh}{dt}=\mathfrak{c}$$

ex quibus formulis cognoscitur, motum centri grauitatis esse aequabilem, ideoque illi aequalem, qui ipsi initio fuerit impressus. Hinc autem porro integrando colligitur

$$f=\mathfrak{a}t, \quad g=\mathfrak{b}t \quad \text{et} \quad h=\mathfrak{c}t$$

sicque via a centro grauitatis descripta erit linea recta I i.

§. 36. Tres autem reliquae aequationes ita se habebunt

$$\text{IV. } 0=A\frac{(HddG-GddH)}{dt^2}+B\frac{(H'ddG'-G'ddH')}{dt^2}+C\frac{(H''ddG''-G''ddH'')}{dt^2}$$

$$\text{V. } 0=A\frac{(FddH-HddF)}{dt^2}+B\frac{(F'ddH'-H'ddF')}{dt^2}+C\frac{(F''ddH''-H''ddF'')}{dt^2}$$

$$\text{VI. } 0=A\frac{(GddF-FddG)}{dt^2}+B\frac{(G'ddF'-F'ddG')}{dt^2}+C\frac{(G''ddF''-F''ddG'')}{dt^2}.$$

Vnde per integrationem statim deducimus sequentes

$$\mathfrak{A}dt=A(HdG-GdH)+B(H'dG'-G'dH')+C(H''dG''-G''dH'')$$

$$\mathfrak{B}dt=A(FdH-HdF)+B(F'dH'-H'dF')+C(F''dH''-H''dF'')$$

$$\mathfrak{C}dt=A(GdF-FdG)+B(G'dF'-F'dG')+C(G''dF''-F''dG'')$$

 vbi

vbi conſtantes $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}$ inuoluunt motum, qui corpori initio praeter motum centri grauitatis fuerit impreſſus.

§. 37. Tota igitur ſolutio perducta eſt ad iſtas tres aequationes differentiales primi gradus, quae cum praeter tempus t tantum continent tres variabiles: ſcilicet angulum Φ cum angulis α, β, γ, qui vti iam obſeruauimus duobus tantum aequiualent, quoniam eſt

$$\text{coſ.}\,\alpha^2 + \text{coſ.}\,\beta^2 + \text{coſ.}\,\gamma^2 = 1,$$

vnde patet, problema eſſe determinatum. Quod ſi enim integrationes ſucceſſerint, ad quod vis tempus iſtos angulos aſſignare licebit. Cognoſcetur enim per angulos α, β, γ poſitio illius axis corporis qui poſt tempus t eandem habet directionem, quam initio habuerat. Deinde vero angulus Φ oſtendet, quantam conuerſionem totum corpus circa iſtum axem IO ſubierit, quemadmodum in prima parte fuſius explicauimus.

§. 38. Quo autem facilius ipſos hos angulos in tres noſtras aequationes introducere queamus, quoniam omnia per coſinus angulorum α, β, γ commode exprimi poſſunt, ſtatuamus breuitatis gratia

$\text{coſ.}\,\alpha = p$, $\text{coſ.}\,\beta = q$ et $\text{coſ.}\,\gamma = r$, ita vt ſit
$pp + qq + rr = 1$ ideoque
$pdp + qdq + rdr = 0.$

Hinc igitur ſequentes habebimus determinationes pro litteris maiuſculis F, G, H etc. in noſtras aequationes ingredientibus

$$F = p$$

$$F = pp(1-\text{cof.}\,\Phi)+\text{cof.}\,\Phi\,;\quad F' = pq(1-\text{cof.}\,\Phi)-r\,\text{fin.}\,\Phi\,;\quad F'' = pr(1-\text{cof.}\,\Phi)+q\,\text{fin.}\,\Phi$$

$$G = pq(1-\text{cof.}\,\Phi)+r\,\text{fin.}\,\Phi\,;\quad G' = qq(1-\text{cof.}\,\Phi)+\text{cof.}\,\Phi\,;\quad G'' = qr(1-\text{cof.}\,\Phi)-p\,\text{fin.}\,\Phi$$

$$H = pr(1-\text{cof.}\,\Phi)-q\,\text{fin.}\,\Phi\,;\quad H' = qr(1-\text{cof.}\,\Phi)+p\,\text{fin.}\,\Phi\,;\quad H'' = rr(1-\text{cof.}\,\Phi)+\text{cof.}\,\Phi.$$

§. 39. Videamus igitur, quomodo hos valores commodiſſime in noſtris aequationibus ſubſtituere queamus; vbi ſtatim intelligitur, formulam primam $H\,dG - G\,dH$ oriri ex differentiatione fractionis $\frac{G}{H}$ omiſſa diuiſione per quadratum denominatoris. Cum igitur ſit

$$\frac{G}{H} = \frac{pq(1-\text{cof.}\,\Phi)+r\,\text{fin.}\,\Phi}{pr(1-\text{cof.}\,\Phi)-q\,\text{fin.}\,\Phi}$$

differentiationem more ſolito inſtituendo prodibit ſequens forma:

$$pp(1-\text{cof.}\,\Phi)^2(rdq-qdr)-prrd\Phi(1-\text{cof.}\,\Phi)-qqd\,\text{fin.}\,\Phi(1-\text{cof.}\,\Phi)$$
$$-pqqd\Phi(1-\text{cof.}\,\Phi)-\text{fin.}\,\Phi^2(qdr-rdq)-rrdp\,\text{fin.}\,\Phi(1-\text{cof.}\,\Phi)$$

quae contrahitur in hanc formam:

$$(rdq-qdr)(pp(1-\text{cof.}\,\Phi)^2+\text{fin.}\,\Phi^2)-dp(1-pp)\,\text{fin.}\,\Phi(1-\text{cof.}\,\Phi)$$
$$-p(1-pp)d\Phi(1-\text{cof.}\,\Phi)$$

quae expreſſio ergo eſt valor formulae $H\,dG - G\,dH$.

§. 40. Secunda formula $H'dG' - G'dH'$, deducitur ex differentiatione fractionis

$$\frac{G'}{H'} = \frac{qq(1-\text{cof.}\,\Phi)+\text{cof.}\,\Phi}{qr(1-\text{cof.}\,\Phi)+p\,\text{fin.}\,\Phi}$$

vnde oritur ſequens expreſſio:

$$qq(1-\text{cof.}\,\Phi)^2(rdq-qdr)-\text{cof.}\,\Phi(1-\text{cof.}\,\Phi)(qdr+rdq)$$
$$-qrd\Phi\,\text{fin.}\,\Phi+\text{fin.}\,\Phi(1-\text{cof.}\,\Phi)(2pqdq-qqdp)$$
$$+pqqd\Phi(1-\text{cof.}\,\Phi)-pd\Phi-dp\,\text{fin.}\,\Phi\,\text{cof.}\,\Phi$$

quae

quae reducitur ad ſequentem formam:

$$-dp\sin\Phi(qq(1-\cos\Phi)+\cos\Phi)+dq(1-\cos\Phi)(rqq-r\cos\Phi(1+qq)+2pq\sin\Phi)-qdr(1-\cos\Phi)(qq(1-\cos\Phi)+\cos\Phi)-d\Phi(qr\sin\Phi-pqq(1-\cos\Phi)+p).$$

§. 41. Tertia denique formula $H''ddG''-G''ddH''$ formabitur ex fractione

$$\frac{G''}{H''}=\frac{qr(1-\cos\Phi)-p\sin\Phi}{rr(1-\cos\Phi)+\cos\Phi}$$

Sufficiet enim has tres formulas euoluiſſe, quoniam ſequentes per analogiam inde deducere licebit; at differentiatione inſtituta prodit iſta forma:

$$rr(1-\cos\Phi)^2(rdq-qdr)-prrd\Phi\cos\Phi(1-\cos\Phi)+prrd\Phi\sin\Phi^2+(qdr+rdq)\cos\Phi(1-\cos\Phi)+qrd\Phi\sin\Phi\cos\Phi+qrd\Phi\sin\Phi(1-\cos\Phi)-dp\sin\Phi\cos\Phi-pd\Phi\cos\Phi^2-pd\Phi\sin\Phi^2+2prdr\sin\Phi(1-\cos\Phi)-rrdp\sin\Phi(1-\cos\Phi)$$

quae contrahitur in iſtam

$$-dp\sin\Phi(rr(1-\cos\Phi)+\cos\Phi)+rdq(1-\cos\Phi)(rr(1-\cos\Phi)+\cos\Phi)+dr(1-\cos\Phi)(2pr\sin\Phi-qrr+q\cos\Phi(1+rr))+d\Phi(qr\sin\Phi+prr(1-\cos\Phi)-p).$$

§. 42. Subſtituantur nunc hi valores in aequatione $\mathfrak{A}dt$, ac prodibit ſequens aequatio:

$$\mathfrak{A}dt=-Adp(1-pp)\sin\Phi(1-\cos\Phi)+Ardqpp(1-\cos\Phi)^2+\sin\Phi)^2-Aqdr(pp(1-\cos\Phi)^2+\sin\Phi)^2-Ad\Phi(p(1-pp)(1-\cos\Phi)-Bdp\sin\Phi(qq(1-\cos\Phi)+\cos\Phi)+Bdq(1-\cos\Phi)(rqq-r\cos\Phi(1+qq)+2pq\sin\Phi)-Bqdr(1-\cos\Phi)(qq(1-\cos\Phi)+\cos\Phi)-Bd\Phi(qr\sin\Phi-pqq(1-\cos\Phi)+p)-Cdp\sin\Phi(rr(1-\cos\Phi)+\cos\Phi)+Crdq(1-\cos\Phi)(rr(1-\cos\Phi)+\cos\Phi)+Cdr(1-\cos\Phi)(2pr\sin\Phi-qrr+q\cos\Phi(1+rr))+Cd\Phi(qr\sin\Phi+prr(1-\cos)-p).$$

Ex

Ex hac autem aequatione binae reliquae formabuntur, si litterae

A, B, C, p, q, r promouentur primo in
B, C, A, q, r, p; secundo in
C, A, B, r, p, q.

Forma autem illa etiam sequenti modo commodius exhiberi potest

$$\begin{aligned}\mathfrak{A}dt = &+dp \sin.\Phi(1-\cos.\Phi)(App-Bqq-Crr)\\ &+dp\sin\Phi\cos\Phi(A-B-C)-Adp\sin.\Phi+rdq(1-\cos\Phi)^2(App\\ &+Bqq+Crr)+rdq\cos.\Phi(1-\cos.\Phi)(A-B+C)\\ &+Ardq(1-\cos.\Phi)+2Bpqdq\sin.\Phi(1-\cos.\Phi)\\ &-qdr(1-\cos.\Phi)^2(App+Bqq+Crr)\\ &-qdr\cos.\Phi(1-\cos.\Phi)(A+B-B)-Aqdr(1-\cos.\Phi)\\ &+2Cprdr\sin.\Phi(1-\cos.\Phi)+pd\Phi(1-\cos\Phi)(App+Bqq\\ &+Crr)-qrd\Phi\sin.\Phi(B-C)-pd\Phi(A+B+C)+Apd\Phi\cos.\Phi.\end{aligned}$$

§ 43. Quia autem in his aequationibus iam per se valde prolixis variabiles p, q, r et Φ nimis inter se sunt permixtae, quam vt resolutio generalis suscipi queat, euoluamus ante omnia casum, quo corpus circa axem fixum gyrari potest; qui cum conuenire debeat cum axe supra considerato IO, anguli α, β, γ ideoque etiam litterae p, q, r pro constantibus sunt habendae, vnde deletis membris, quae differentialia dp, dq, dr implicant, ternae aequationes ad hunc casum erunt

$$\begin{aligned}\mathfrak{A}dt = &pd\Phi(1-\cos.\Phi)(App+Bqq+Crr)-qrd\Phi(B-C)\\ &-pd\Phi(A+B+C)+Apd\Phi\cos.\Phi\end{aligned}$$

$$\begin{aligned}\mathfrak{B}dt = &qd\Phi(1-\cos.\Phi)(App+Bqq+Crr)-prd\Phi(C-A)\\ &-qd\Phi(A+B+C)+Bqd\Phi\cos.\Phi\end{aligned}$$

$$\mathfrak{C}dt = rd\Phi(1-\text{cof.}\Phi)(App+Bqq+Crr)-pqd\Phi\,\text{fin.}\Phi(A-B)$$
$$-rd\Phi(A+B+C)+Crd\Phi\,\text{cof.}\Phi.$$

§. 44. Cum igitur corpus tali motu gyrari possit, si axis gyrationis incidat in aliquem axem principalem, ponamus eum incidere in axem IA ita, vt sit $p = 1$, $q = 0$, et $r = 0$, vnde tres nostrae aequationes euadent

$$\mathfrak{A}dt = Ad\Phi(1-\text{cof.}\Phi)-d\Phi(A+B+C)+Ad\Phi\,\text{cof.}\Phi$$

siue

$$\mathfrak{A}dt = -d\Phi(B+C)$$
$$\mathfrak{B}dt = 0$$
$$\mathfrak{C}dt = 0.$$

Hinc igitur statim patet, hunc casum vtique locum habere posse, sumtis constantibus $\mathfrak{B} = 0$ et $\mathfrak{C} = 0$; tum vero $d\Phi = \frac{-\mathfrak{A}dt}{B+C}$, quamobrem angulus Φ tempori erit proportionalis, seu motus erit vniformis, vti per se est manifestum.

§. 45. Porro vero etiam notum est, corpus circa omnes axes libere gyrari posse, si omnia momenta fuerint aequalia, hoc est $A = B = C$, vnde hae produnt aequationes

$$\frac{\mathfrak{A}dt}{A} = -2pd\Phi;\ \frac{\mathfrak{B}dt}{A} = -2qd\Phi;\ \frac{\mathfrak{C}dt}{A} = -2rd\Phi$$

vnde manifestum est fore $\frac{d\Phi}{dt}$ quantitatem constantem, ideoque motum gyratorium aequabilem. Quod si ergo statuatur $\frac{d\Phi}{dt} = \Delta$, hinc reperiemus ipsas quantitates p, q, r; erit enim

$$p = -\frac{\mathfrak{A}}{2A\Delta};\ q = -\frac{\mathfrak{B}}{2A\Delta};\ r = -\frac{\mathfrak{C}}{2A\Delta},$$

quare

quare cum ſit $pp + qq + rr = 1$ erit

$$\Delta = \frac{1}{2A} \sqrt{\mathfrak{A}^2 + \mathfrak{B}^2 + \mathfrak{C}^2},$$

ſicque omnia per ternas conſtantes $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}$ ſunt determinata.

§. 46. Iidem valores prodire debent ex tribus noſtris aequationibus, ſi etiam quantitates p, q, r vt variabiles ſpectantur, pro caſu $A = B = C$; tum autem aequatio prima erit

$$\frac{\mathfrak{A}dt}{A} = +2dp\,\text{ſin.}\,\Phi(pp(1-\text{coſ.}\,\Phi)-1) + 2dq(1-\text{coſ.}\,\Phi)(r+pq\,\text{ſin}\,\Phi)$$
$$-2dr(1-\text{coſ.}\,\Phi)(q-pr\,\text{ſin.}\,\Phi) - 2pd\Phi$$

quae ob $pdp + qdq + rdr = 0$ reducitur ad hanc formam:

I. $\frac{\mathfrak{A}dt}{A} = -2dp\,\text{ſin.}\,\Phi + 2(1-\text{coſ.}\,\Phi)(rdq - qdr) - 2pd\Phi$

vnde duae reliquae per analogiam erunt

II. $\frac{\mathfrak{B}dt}{A} = -2dq\,\text{ſin.}\,\Phi + 2(1-\text{coſ.}\,\Phi)(pdr - rdp) - 2qd\Phi$

III. $\frac{\mathfrak{C}dt}{A} = -2dr\,\text{ſin.}\,\Phi + 2(1-\text{coſ.}\,\Phi)(qdp - pdq) - 2rd\Phi$

atque iam certi eſſe poſſumus, his aequationibus aliter ſatisfieri non poſſe niſi modo ante expoſito, quo p, q, r ſint quantitates conſtantes, angulus vero Φ tempori proportionalis.

§. 47. Ad hoc oſtendendum eliminemus primo elementum $d\Phi$, ac I. q — II. p praebet

$$\frac{\mathfrak{A}qdt - \mathfrak{B}pdt}{A} = -2\,\text{ſin.}\,\Phi(qdp - pdq) - 2dr(1-\text{coſ.}\,\Phi)$$

Similimodo II. r — III. q dat

$$\frac{\mathfrak{B}rdt - \mathfrak{C}qdt}{A} = -2\,\text{ſin.}\,\Phi(rdq - qdr) - 2dp(1-\text{coſ.}\,\Phi)$$

 quibus

quibus adiungi potest haec combinatio: III. p — I. r, quae dat

$$\frac{\mathfrak{C}pdt - \mathfrak{A}rdt}{\mathrm{A}} = -2 \sin.\Phi(pdr - rdp) - 2dq(1 - \cos\Phi)$$

Atque nunc quidem euidens est, his aequationibus satisfieri, statuendo quantitates p, q, r constantes. Interim tamen hinc non liquet, quod nulla alia solutio locum habere possit. Quam ob rem hinc enascitur insigne Problema analyticum, quomodo haec solutio ex istis formulis deriuari debeat, id quod Geometris imprimis commendari meretur.

§. 48. Ob has igitur difficultates, quae in casu facillimo, vbi A = B = C, moram faceffunt, multo minus euolutionem generalem tentare licebit; quare, cum aliunde certi simus, etiam in genere succedere debere, quandoquidem eam ope prioris methodi, qua olim sum vsus ad finem perducere contigit, nullum est dubium, quin certa dentur artificia analytica nobis adhuc incognita, quae ad hunc scopum perducere valeant. Quia vero talia artificia non solum maximam sagacitatem, sed etiam aciem oculorum postulant, hanc inuestigationem aliis relinquere sum coactus; quandoquidem hoc argumentum summa attentione Geometrarum dignum est iudicandum.

§. 49. Fortasse etiam non parum ad felicem successum conferre poterit, si, antequam loco litterarum primo introductarum F, G, H valores suos substituamus, earum relationes mutuas accuratius perpenpendamus: forſitan enim hinc iam via commodior ſeſe offeret hoc negotium conficiendi. Cum igitur illarum litterarum valores inuenti ſint iſti

$$F = pp(1-\text{cof.}\Phi)+\text{cof.}\Phi;\quad F' = pq(1-\text{cof.}\Phi)-r\,\text{ſin.}\Phi;$$
$$F'' = pr(1-\text{cof.}\Phi)+q\,\text{ſin.}\Phi$$
$$G = pq(1-\text{cof.}\Phi)+r\,\text{ſin.}\Phi;\quad G' = qq(1-\text{cof.}\Phi)+\text{cof.}\Phi;$$
$$G'' = qr(1-\text{cof.}\Phi)-p\,\text{ſin.}\Phi$$
$$H = pr(1-\text{ſin.}\Phi)-q\,\text{cof.}\Phi;\quad H' = qr(1-\text{cof.}\Phi)+p\,\text{ſin.}\Phi;$$
$$H'' = rr(1-\text{cof.}\Phi)+\text{cof.}\Phi$$

ob $pp+qq+rr) = 1$ erit $F+G'+H'' = 1+2\,\text{cof.}\Phi$.
Tum vero inter reliquas ſex harum litterarum binae egregiae inter ſe conueniunt, vnde ſequuntur ſequentes relationes

$$G+F' = 2pq(1-\text{cof.}\Phi)$$
$$G-F' = 2r\,\text{ſin.}\Phi$$
$$H+F'' = 2pr(1-\text{cof.}\Phi)$$
$$-H+F'' = 2q\,\text{ſin.}\Phi)$$
$$H'+G'' = 2qr(1-\text{cof.}\Phi)$$
$$H'-G'' = 2p\,\text{ſin.}\Phi$$

hinc igitur porro erit

$$GG-F'F' = F''F''-HH = H'H'-G''G'' = 4pqr\,\text{ſin.}\Phi(1-\text{cof.}\Phi).$$

Praeterea vero hinc eiusmodi combinationes formari poterunt, in quibus ſolus angulus Φ inſit, cuiusmodi ſunt

$$\frac{(F''-H)(H'-G'')}{G+F'} = 2(1+\text{cof.}\Phi)$$
$$\frac{(G-F')(H'-G'')}{H+F''} = 2(1+\text{cof.}\Phi)$$
$$\frac{(G-F')(F''-H)}{H'+G''} = 2(1+\text{cof.}\Phi).$$

 Caete-

Caeterum hoc argumentum dignissimum videtur, quod omni studio excolatur; cum inde non solum in Mechanica sed etiam in Analysi egregia incrementa expectari queant.

§. 50. Antequam hoc argumentum penitus deseram fortasse haud abs re erit annotasse, ternas aequationes §. 46. satis commode ad duas reuocari posse, dum variabilis t, cuius differentiale tantum ingreditur penitus eliminatur, id quod facillime effici poterit primam aequationem per p, secundam per q et tertiam per r multiplicando; tum enim prodibit sequens aequatio satis concinna:

$$\frac{\mathfrak{A}p + \mathfrak{B}q + \mathfrak{C}r}{A} dt + 2\, d\Phi = 0,$$

vnde statim fit

$$dt = -\frac{2\, A\, d\Phi}{\mathfrak{A}p + \mathfrak{B}q + \mathfrak{C}r},$$

qui valor in duabus tantum aequationibus substitutus suppeditabit duas nouas aequationes, in quibus tantum tres variabiles insunt, quarum bina per tertiam determinari oporteret: interim tamen nulla via patet, quomodo hoc commode praestari possit; praecipue hac ratione solutio illa simplex, quae iam a priori constat, prorsus excluderetur. Quia enim quantitates p, q, r sunt constantes, angulus vero Φ tempori proportionalis, neutiquam fieri potest, vt iste angulus per quantitates p, q, r, exprimatur. Hanc igitur ob caussam ista inuestigatio omnino deserenda videtur, praecipue cum huius quaestionis ope methodi olim vsitatae iam completam solutionem elicuerim, qua igitur acquiescere debemus.

THEORE-