CAPVT II.

DE MICROSCOPIIS HVIVS GENERIS MAGIS COMPOSITIS.

Problema I.

250.

Microſcopium huius generis ex quinque lentibus conſtruere, quae ita ſint diſpoſitae, vt prior imago realis inter lentem ſecundam et tertiam, poſterior vero inter tertiam et quartam cadat.

Solutio.

Cum igitur prior imago in interuallum ſecundum, poſterior vero in tertium cadere debeat, litterarum P, Q, R, S ſecunda et tertia Q et R debent eſſe negatiuae. Statuatur ergo $Q = -k$ et $R = -k'$, vt ſit $Pkk'S = \mathfrak{M}$, exiſtente $\mathfrak{M} = \frac{ma}{b}$. Deinde vero ſit $M = \frac{\mathfrak{q} + \mathfrak{r} + \mathfrak{s} + \mathfrak{t}}{\mathfrak{M} - 1}$, vt fiat ſpatii in obiecto conſpicui ſemidiameter $= z = Ma\xi$. Quare vt campus euadat maximus, efficiendum eſt, vt litterarum $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ tot fiant vnitati aequales, quam reliquae circumſtantiae permittunt, quod cum de omnibus ſtatui nequeat, ſaltem pro poſtremis ponamus

$$\mathfrak{s} = 1 \text{ et } \mathfrak{t} = 1, \text{ vt ſit } M = \frac{\mathfrak{q} + \mathfrak{r} + 2}{\mathfrak{M} - 1}.$$

Tum vero habebuntur ſequentes aequationes

$$1^{ma}\ \mathfrak{B}\mathfrak{q} = (P - 1)M$$

$$2^{da}\ \mathfrak{C}\mathfrak{r} = -(Pk + 1)M - \mathfrak{q}$$

$$3^{tia}\ \mathfrak{D} = (Pkk' - 1)M - \mathfrak{q} - \mathfrak{r}$$

quibus adiungatur aequatio, qua margo coloratus deſtruitur,

$$0 = \frac{\mathfrak{q}}{P} - \frac{\mathfrak{r}}{Pk} + \frac{1}{Pkk'} + \frac{1}{Pkk'S}$$

ex qua deducitur

$$k' = \frac{1 + \frac{1}{S}}{\mathfrak{r} - k\mathfrak{q}}$$

vnde patet, eſſe debere $\mathfrak{r} > k\mathfrak{q}$. Ante quam autem hinc quidquam definire valeamus, lentium interualla conſiderare debemus, quae ſunt

$$1^{mum} = Aa\left(1 - \frac{1}{P}\right)$$

$$2^{dum} = -\frac{AB}{P} \cdot a\left(1 + \frac{1}{k}\right)$$

$$3^{tium} = -\frac{ABC}{Pk} \cdot a\left(1 + \frac{1}{k'}\right)$$

$$4^{tum} = -\frac{ABCD}{Pkk'} \cdot a\left(1 - \frac{1}{S}\right)$$

quae omnia debent eſſe poſitiua; quibus adiungatur diſtantia focalis vltimae lentis

$$\mathfrak{t} = \frac{ABCD}{Pkk'S} \cdot a = \frac{ABCD}{\mathfrak{M}} \cdot a;$$

quae

quae etiam debet esse positiua, vt prodeat distantia oculi post eam, $O = \frac{tt}{M\mathfrak{M}}$, positiua; vnde ob $t = 1$ euidens est, esse debere t positiuum, ideoque $ABCD > o$. Quocirca vt quartum interuallum etiam fiat positiuum, necesse est, vt sit $1 - \frac{1}{S} < o$, siue $S < 1$. Vnde euidens est, productum Pkk' fore $> \mathfrak{M}$ ideoque numerum praemagnum; vnde tertia illa aequatio, si loco M eius valor substituatur, dabit

$$\mathfrak{D} = \frac{(Pkk'-1)(q+r+2)}{\mathfrak{M}-1} - q - r$$

vbi cum Pkk' et $\mathfrak{M}$ sint numeri praemagni et $Pkk' > \mathfrak{M}$, fiat proxime

$$\mathfrak{D} = \frac{Pkk'}{\mathfrak{M}}(q + r + 2) - q - r$$
$$= \frac{2Pkk'}{\mathfrak{M}} + (q + r)\left(\frac{Pkk'}{\mathfrak{M}} - 1\right)$$

siue ob

$Pkk' = \frac{\mathfrak{M}}{S}$, erit $\mathfrak{D} = \frac{2}{S} + (q + r)\left(\frac{1}{S} - 1\right)$;

vnde cum $q + r$ certe sit < 2, euidens est, fore $\mathfrak{D} > 1$ ideoque D negatiuum. Erit ergo $ABC < o$. hinc tertinm interuallum sponte fit positiuum. Quare cum ob secundum interuallum esse debeat $AB < o$, oportet esse $C > o$ hincque $\mathfrak{C} > o$ et $\mathfrak{C} < 1$. Vnde si fuerit $A > o$, ideoque $P > 1$ ob interuallum primum; debebit esse $B < o$, hincque vel $\mathfrak{B} < o$ vel $\mathfrak{B} > 1$; vnde sequitur fore priori casu $q < o$ altero casu $q > o$; vnde ob $q + \mathfrak{C}r < o$ fieret $r < o$ ideoque $k' < 6$ quod est absurdum. Sin autem esset

$A < o$

$\mathfrak{A} < o$ hincque $P < 1$, debent esse $\mathfrak{B} > o$ ideoque $\mathfrak{B} > o$ et $\mathfrak{B} < 1$ vnde iterum fieret $q < o$. Neque igitur etiamnum constat, vtrum ambo isti casus consistere queant. Quia vero in vtroque fit $q < o$, statuamus, vt ante, $q = -\omega$, vt fit $\omega = \frac{(1-P)M}{\mathfrak{B}}$ et ob secundam aequationem esse debet $\omega - \mathfrak{C}\,\mathfrak{r} > o$, tum vero erit

$$k' = \frac{1 + \frac{1}{S}}{\mathfrak{r} + k\,\omega};$$

vnde ob $P\,k\,k' = \frac{\mathfrak{M}}{S}$, fiet $P\,k = \frac{\mathfrak{M}(\mathfrak{r} + k\omega)}{1 + S}$.

Nunc igitur litteras ω et $\mathfrak{r}$ ex calculo eliminemus et cum fit $\omega = \frac{1-P}{\mathfrak{B}}\,M$ ponamus breuitatis gratia $\frac{1-P}{\mathfrak{B}} = \zeta$ vt fit $\omega = \zeta\,M$; deinde fit etiam breuitatis gratia

$1 + P\,k = \eta$ fietque $\mathfrak{r} = \frac{(\zeta - \eta)M}{\mathfrak{C}}$.

Ergo porro

$$\mathfrak{r} - \omega = \frac{(\zeta(1-\mathfrak{C}) - \eta)M}{\mathfrak{C}} \text{ et}$$

$$2 - \omega + \mathfrak{r} = \frac{2\mathfrak{C} + (\zeta(1-\mathfrak{C}) - \eta)M}{\mathfrak{C}}$$

at cum fit $M = \frac{2 + \mathfrak{r} - \omega}{\mathfrak{M} - 1}$ erit

$$2 + \mathfrak{r} - \omega = M(\mathfrak{M} - 1);$$

vnde concludimus fore

$$M = \frac{2\mathfrak{C}}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}$$

quo valore inuento erit

$$\omega = \frac{2\mathfrak{C}\zeta}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta} \text{ et } \mathfrak{r} = \frac{2(\zeta - \eta)}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}$$

ex

ex quibus valoribus porro conficitur

$$v + k\omega = \frac{2(\zeta - \eta) + 2k\mathfrak{C}\zeta}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}$$

hincque

$$Pk = \frac{2\mathfrak{M}(\zeta - \eta + k\mathfrak{C}\zeta)}{(1+S)(\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta)} = \eta - 1$$

ex quo definitur

$$k = \frac{(\eta-1)(1+S)\mathfrak{C}(\mathfrak{M} + \zeta - 1) - 2\mathfrak{M}(\zeta - \eta) - (\eta - 1)(1+S)(\zeta - \eta)}{2\mathfrak{M}\mathfrak{C}\zeta}$$

vnde porro inuenitur $P = \frac{\eta - 1}{k}$. Quia autem k debet esse positiuum, in eius numeratore coefficiens ipsius $\mathfrak{M}$ debet esse positiuus, vnde sequitur $\mathfrak{C} > \frac{2(\zeta - \eta)}{(\eta - 1)(1+S)}$ at vero vidimus esse $\mathfrak{C} < 1$, sicque necesse est, vt sit

$$\frac{2(\zeta - \eta)}{(\eta - 1)(1+S)} < 1 \text{ seu } 2(\zeta - \eta) < (\eta - 1)(1+S);$$

ex qua conditione concludimus

$$\zeta < \eta + \frac{(\eta - 1)(1+S)}{2}.$$

Nouimus autem esse debere $\zeta > \eta$; vnde littera ζ capi debebit intra limites η et $\eta + \frac{(\eta-1)(1+S)}{2}$, vbi manifestum est esse debere $\eta > 1$. Nostrum igitur problema sequenti modo resolui conueniet: Pro lubitu capi possunt litterae S et η, dummodo obseruetur, esse debere $S < 1$ et $\eta > 1$, quia $Pk = \eta - 1$. Deinde littera ζ capiatur intra limites η et $\eta + \frac{(\eta-1)(1+S)}{2}$ At $\mathfrak{C}$ capiatur intra hos limites

$$\mathfrak{C} < 1 \text{ simulque } \mathfrak{C} > \frac{2(\zeta - \eta)}{(\eta - 1)(1+S)}$$

vnde ſimul C definitur. Tum vero capiatur

$$k = \frac{(\eta - 1)(1 + S)\mathfrak{C}(\mathfrak{M} + \zeta - 1) - 2\mathfrak{M}(\zeta - \eta) - (\eta - 1)(1 + S)(\zeta - \eta)}{2\mathfrak{M}\mathfrak{C}\zeta}$$

ex quo habebitur

$$P = \frac{\eta - 1}{k} \text{ et } k' = \frac{\mathfrak{M}}{(\eta - 1)S}.$$

Poſtea capiatur $\mathfrak{B} = \frac{1 - P}{\zeta}$; vnde definitur B. Denique ob $P k k' = \frac{\mathfrak{M}}{S}$ formula ſupra pro $\mathfrak{D}$ inuenta dabit

$$\mathfrak{D} = (\tfrac{\mathfrak{M}}{S} - 1) M - (\mathfrak{M} - 1) M + 2$$
$$= M \mathfrak{M} (\tfrac{1}{S} - 1) + 2;$$

vbi ſubſtituto valore ipſius M prodit

$$\mathfrak{D} = \frac{\frac{2\mathfrak{M}\mathfrak{C}}{S} + 2\mathfrak{C}(\zeta - 1) - 2(\zeta - \eta)}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}$$

qui valor, cum ſit $\mathfrak{M}$ numerus praemagnus, erit proxime $\mathfrak{D} = \frac{2}{S}$; hincque adcuratius

$$\mathfrak{D} = \frac{2}{S} + \frac{\frac{2}{S}(\zeta - \eta - \mathfrak{C}(\zeta - 1)) + 2\mathfrak{C}(\zeta - 1) - 2(\zeta - \eta)}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}$$

ſiue

$$\mathfrak{D} = \frac{2}{S} + \frac{2(\frac{1}{S} - 1)(\zeta - \eta - \mathfrak{C}(\zeta - 1))}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}$$

vel

$$\mathfrak{D} = \frac{2}{S} - \frac{2(\frac{1}{S} - 1)(\mathfrak{C}(\zeta - 1) - \zeta + \eta)}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}$$

vnde ſaltem patet, certe fore $\mathfrak{D} > 1$. ideoque D ne-

gati-

gatiuum, vt supra iam notauimus. Iam prouti fuerit P siue > 1 siue < 1, capi debebit vel $A > 0$ vel $A < 0$; ita, vt nunc omnia elementa sint determinata; habebuntur enim distantiae focales

$$p = \mathfrak{A}\,a;\; q = -\frac{A\mathfrak{B}}{P}\,a;\; r = -\frac{AB\mathfrak{C}}{Pk}\cdot a;$$

$$s = -\frac{ABC\mathfrak{D}}{Pkk'}\cdot a \text{ et } t = \frac{ABCD\cdot a}{\mathfrak{m}};$$

deinde interualla

$$I^{mum} = A\,a\left(1 + \frac{1}{P}\right);$$

$$II^{dum} = -\frac{AB\cdot a}{P}\left(1 + \frac{1}{k}\right);$$

$$III^{tium} = -\frac{ABC\cdot a}{Pk}\left(1 + \frac{1}{k'}\right);$$

$$IV^{tum} = -\frac{ABCD}{Pkk'}\cdot a\left(1 - \frac{1}{S}\right);$$

tum vero erit

$$M = 2 : \left(\mathfrak{M} + \zeta - 1 - \frac{(\zeta - \eta)}{\mathfrak{C}}\right) \text{ et } O = \frac{\mathfrak{t}}{\mathfrak{M}\,M}$$

et apertura primae lentis ex aequatione notissima $\frac{1}{k^3} = \ldots$ definietur.

Coroll. I.

251. Conditio, quam inuenimus $\zeta > \eta$ hic plus non inuoluit, quam ne ζ minus, quam η, accipiatur. Nihil ergo impedit, quominus statuamus $\zeta = \eta$, etsi enim hic valor campum aliquantillum diminuit; tamen is adhuc satis prodit notabilis. Tum autem fiet $\mathfrak{t} = 0$, sicque tertia lens quam minimam requiret aperturam, ita, vt simul officium diaphragmatis angustissimi praestet

 Coroll.

Coroll. 2.

252. Quodſi vero ſtatuamus $\zeta = \eta$; ſufficit capere $\mathfrak{C}$ intra limites o et 1. vnde ſimul C fit poſitiuum. Tum vero capiatur

$$k = \frac{(\eta - 1)(1 + S)(\mathfrak{M} + \eta - 1)}{2 \mathfrak{M} \eta}$$

ex quo habebitur

$$P = \frac{2 \mathfrak{M} \eta}{(1 + S)(\mathfrak{M} + \eta - 1)};$$

ita, vt pro maioribus multiplicationibus ſit proxime

$$k = \frac{(\eta - 1)(1 + S)}{2 \eta} \text{ et } P = \frac{2 \eta}{1 + S};$$

vnde patet, eſſe $P > 1$ ideoque A poſitiuum. Tum vero erit porro $\mathfrak{B} = -\frac{(P - 1)}{\eta}$, ita, vt ſit tam $\mathfrak{B} < o$, quam $B < o$. Denique hoc caſu fiet

$$\mathfrak{D} = \frac{2 \mathfrak{M} + 2 S (\eta - 1)}{S (\mathfrak{M} + \eta - 1)} \text{ hincque } D = -\frac{2 \mathfrak{M} - 2 S (\eta - 1)}{\mathfrak{M} (2 - S) + S (\eta - 1)}$$

atque $M = \frac{2}{\mathfrak{M} + \eta - 1}$.

Scholion.

253. Praeterquam quod hic caſus $\zeta = \eta$ ad praxin inprimis eſt accommodatus, etiam hanc praerogatiuam complectitur, vt a littera $\mathfrak{C}$ reliqua elementa prorſus non pendeant, ita, vt quomodocunque $\mathfrak{C}$ accipiamus intra limites ſcilicet o et 1, reliqua elementa nullam inde mutationem patiantur. Hoc autem modo facillime euitari poterit, ne lens obiectiva nimis fiat exigua, quod vero inſuper commodius per litteram A praeſtatur, niſi forte de teleſcopiis ſit

ſermo,

fermo, vbi $A\,a = p$; fuperfluum igitur foret hic alios cafus praeter iftum $\zeta = \eta$ euoluere, atque nunc inprimis operae pretium erit aliquot valores pro η confiderare, vt inde intelligere queamus, quinam inde ad praxin maxime futuri fint idonei. Pro littera vero S, quam vnitate minorem effe debere vidimus, ftatuamus femper $S = \frac{1}{2}$, quia hinc fatis idoneum intervallum inter binas lentes poftremas oritur. Tum autem noftrae conditiones fequenti modo exprimentur:

1°. $k = \frac{3(\eta - 1)(\mathfrak{M} + \eta - 1)}{4\mathfrak{M}\eta}$

2°. $P = \frac{4\mathfrak{M}\eta}{3(\mathfrak{M} + \eta - 1)}$

3°. $\mathfrak{B} = -\frac{(4\eta - 3)\mathfrak{M} + 3(\eta - 1)}{3\eta(\mathfrak{M} + \eta - 1)}$

4°. $\mathfrak{C}$, vt iam notauimus, arbitrio noftro permittitur, dummodo fit intra 0 et 1.

5°. $\mathfrak{D} = \frac{4\mathfrak{M} + 2(\eta - 1)}{\mathfrak{M} + \eta - 1}$ et $D = -\frac{4\mathfrak{M} - 2(\eta - 1)}{3\mathfrak{M} + \eta - 1}$

Hinc itaque diftantiae focales erunt

$p = \mathfrak{A}\,a$;

$q = -\frac{A\mathfrak{B}}{P}.\,a = \frac{(4\eta - 3)\mathfrak{M} - 3(\eta - 1)}{4\mathfrak{M}\eta}.\,A\,a.$

$r = -\frac{B\mathfrak{C}}{Pk}.\,A\,a = -\frac{B\mathfrak{C}}{\eta - 1}.\,A\,a$;

$s = -\frac{BC\mathfrak{D}}{2\mathfrak{M}}.\,A\,a.$ et $t = \frac{BCD}{\mathfrak{M}}.\,A\,a.$

Deinde lentium interualla

$\mathrm{I}^{mum} = A\,a\left(1 - \frac{1}{P}\right) = \frac{(4\eta - 3)\mathfrak{M} - 3(\eta - 1)}{4\mathfrak{M}\eta}.\,A\,a.$

$\mathrm{II}^{dum} = -B\left(\frac{\mathfrak{M}(7\eta - 3) + 3\eta(\eta - 2) + 3}{4\mathfrak{M}\eta(\eta - 1)}\right)A\,a.$

$III^{tium} = - B C \left(\frac{\eta}{\eta - 1} + \frac{1}{2 \mathfrak{M}} \right) A a.$

$IV^{tum} = \frac{B C D}{2 \mathfrak{M}} . A a.$

Deinde ob

$M = \frac{2}{\mathfrak{M} + \eta - 1}$, erit $Q = \frac{1}{2} t \left(1 + \frac{\eta - 1}{\mathfrak{M}} \right)$.

Pro prima lente ſemidiameter aperturae erit $= x$ ex aequatione notiſſima definiendus; pro ſecunda autem ob $\omega = \frac{2 \eta}{\mathfrak{M} + \eta - 1}$ erit ſemidiameter aperturae $= \frac{\eta}{2 (\mathfrak{M} + \eta - 1)} q + \frac{x}{P}$ et ob $r = 0$ ſemidiameter aperturae tertiae lentis $= \frac{x}{Pk} = \frac{x}{\eta - 1}$; duabus autem reliquis lentibus apertura maxima tribuitur.

Exempl. I.

254. Sumamus $\eta = 2$ eritque $k = \frac{3 (\mathfrak{M} + 1)}{8 \mathfrak{M}}$ et quia tantum de maioribus multiplicationibus agitur, ſumi poterit $k = \frac{3}{8}$ hinc $P = \frac{8 \mathfrak{M}}{3 (\mathfrak{M} + 1)} = \frac{8}{3}$ proxime. Porro

$\mathfrak{B} = - \frac{5}{6}$ et $B = - \frac{5}{11}$;

$\mathfrak{D} = 4$ et $D = - \frac{4}{3}$;

vnde diſtantiae focales erunt

$p = \mathfrak{A} a$; $q = \frac{5}{16} A a$;

$r = \frac{5}{11} . \mathfrak{C} . A a$; $s = \frac{10}{11} . \frac{C}{\mathfrak{M}} A a$.

$t = \frac{20}{33} . \frac{C}{\mathfrak{M}} . A a$.

et lentium interualla

$1^{mum} = \frac{5}{8} A a$.

2^{dum}

$2^{dum} = \frac{5}{11} \cdot A\, a.$

$3^{tium} = \frac{5}{11} \cdot C \left(1 + \frac{1}{2\mathfrak{M}}\right) A\, a.$

$4^{tum} = \frac{10}{33} \cdot \frac{C}{\mathfrak{M}} \cdot A\, a.$ et

$M = \frac{2}{\mathfrak{M}+1}$; hinc $z = \frac{1}{2} \cdot \frac{a}{\mathfrak{M}+1}$ et

$O = \frac{1}{2} t \left(1 + \frac{1}{\mathfrak{M}}\right);$

tum vero ſemidiameter aperturae lentis

$2^{dae} = \frac{q}{\mathfrak{M}+1} + \frac{5}{8} x$ et $3^{tiae} = x.$

Pro x autem inueniendo ſatisfiat huic aequationi:

$$\frac{z^3}{k^3} = \frac{\mu \mathfrak{M} x^3}{\mathfrak{A}^3 a^3} \begin{cases} \lambda + \nu \mathfrak{A} (1 - \mathfrak{A}) \\ - \frac{(1-\mathfrak{A})^3}{\mathfrak{B}^3 P} (\lambda' + \nu \mathfrak{B} (1 - \mathfrak{B})) \\ + \frac{(1-\mathfrak{A})^3}{B^3 \mathfrak{C}^3 Pk} (\lambda'' + \nu \mathfrak{C} (1 - \mathfrak{C})) \\ - \frac{(1-\mathfrak{A})^3}{B^3 C^3 \mathfrak{D}^3 Pkk'} (\lambda''' + \nu \mathfrak{D} (1 - \mathfrak{D})) \\ + \frac{(1-\mathfrak{A})^3}{B^3 C^3 D^3} \cdot \lambda'''' \end{cases}$$

quae aequatio commodius ita repraeſentabitur:

$$\frac{z^3}{k^3} = \mu \mathfrak{M} x^3 \begin{cases} \frac{\lambda + \nu \mathfrak{A} (1 - \mathfrak{A})}{p^3} \\ - \frac{1}{P^3 q^3} (\lambda' + \nu \mathfrak{B} (1 - \mathfrak{B})) \\ + \frac{1}{(Pk)^3 r^3} (\lambda'' + \nu \mathfrak{C} (1 - \mathfrak{C})) \\ - \frac{1}{(Pkk')^3 s^3} (\lambda''' + \nu \mathfrak{D} (1 - \mathfrak{D})) \\ + \frac{\lambda''''}{(Pkk'S)^3 \cdot t^3} \end{cases}$$

Coroll.

Coroll.

255. Si haec ad telescopia transferantur ponendo $\mathfrak{A}a = Aa = p$ et $\mathfrak{M} = m$, fient distantiae focales

$$q = \tfrac{5}{16} \cdot p;\ r = \tfrac{5}{11}\mathfrak{C}.\, p;$$
$$s = \tfrac{10}{11} \cdot \tfrac{C}{\mathfrak{M}} \cdot p. \text{ et } t = \tfrac{20}{11} \cdot \tfrac{C}{\mathfrak{M}} \cdot p.$$

interualla

$$1^{mum} = \tfrac{5}{8}p;$$
$$2^{dum} = \tfrac{5}{11}p;$$
$$3^{tium} = \tfrac{5}{11}C(1 + \tfrac{1}{2\mathfrak{M}})p \text{ et}$$
$$4^{tum} = \tfrac{10}{11} \cdot \tfrac{C}{\mathfrak{M}} \cdot p.$$

et semidiameter campi $\Phi = \frac{1718}{\mathfrak{M}+1}$ min. Longitudo ergo erit propemodum $= \frac{95}{88}p + \frac{5}{11}C.\,p$. Nunc autem p ex aequatione ante data definiri poterit, vbi erit $\mathfrak{A} = 0$ ob $a = \infty$.

Exemplum II.

256. Sit nunc $\eta = 3$ erit

$$k = \tfrac{\mathfrak{M}+2}{2\mathfrak{M}}, \text{ sumi ergo poterit } k = \tfrac{1}{2}$$
$$P = \tfrac{4\mathfrak{M}}{\mathfrak{M}+2} = 4$$
$$\mathfrak{B} = -\tfrac{3\mathfrak{M}+2}{3(\mathfrak{M}+2)} = -1; \text{ ergo } B = -\tfrac{1}{2}$$
$$\mathfrak{D} = \tfrac{4\mathfrak{M}+4}{\mathfrak{M}+2} = 4; \text{ ergo } D = -\tfrac{4}{3}.$$

Hinc ergo fient distantiae focales

$$p = \mathfrak{A}a;\ q = \tfrac{1}{2}Aa;\ r = \tfrac{1}{4}\mathfrak{C}.\,Aa;$$

s =

$s = \frac{C}{\mathfrak{M}} . A a.$ et $t = \frac{2}{3} . \frac{C}{\mathfrak{M}} . A a.$

et lentium interualla

$1^{mum} = \frac{3}{4} . A a.$

$2^{dum} = \frac{3}{8} A a.$

$3^{tium} = \frac{1}{4} C (1 + \frac{2}{\mathfrak{M}}) A a.$

$4^{tum} = \frac{1}{3} \frac{C}{\mathfrak{M}} . A a.$

hinc porro erit

$M = \frac{2}{\mathfrak{M} + 2}$ hinc $x = \frac{1}{2} . \frac{q}{\mathfrak{M} + 2}$ et

$O = \frac{1}{2} t (1 + \frac{2}{\mathfrak{M}})$

tum vero femidiameter aperturae lentis

$2^{dae} = \frac{3 q}{2 (\mathfrak{M} + 2)} + \frac{1}{4} x;$ et $3^{tiae} = \frac{1}{2} . x.$

Cetera fe habent, vt ante.

Coroll.

257. Facta applicatione ad telefcopia, vbi fit $A a = p$ omnia elementa facile determinantur, vt ante; tum vero longitudo inftrumenti omiffis partibus per $\mathfrak{M}$ diuifis erit $= \frac{9}{8} p + \frac{1}{4} C . p$, quae minor eft, quam cafu praecedente.

Exemplum III.

258. Statuamus $\eta = 6$, vt fiat $M = \frac{2}{\mathfrak{M} + 5}$ eritque

$k = \frac{5 (\mathfrak{M} + 5)}{8 \mathfrak{M}} = \frac{5}{8}$ proxime.

$P = 8. \quad \mathfrak{B} = \frac{-7\mathfrak{M}+5}{6(\mathfrak{M}+5)} = -\frac{7}{6}$ proxime

vnde fit

$B = -\frac{7}{13}; \; \mathfrak{D} = 4$ et $D = -\frac{4}{3}$.

vnde diſtantiae focales prodibunt

$p = \mathfrak{A}\,a; \; q = \frac{7}{48}.\, A\,a; \; r = \frac{7}{65}\,\mathfrak{C}.\, A\,a;$

$s = \frac{14}{13}.\, \frac{C}{\mathfrak{M}}.\, A\,a; \; t = \frac{28}{39}\,\frac{C}{\mathfrak{M}}.\, A\,a.$

et lentium interualla

$1^{mum} = \frac{7}{8}.\, A\,a.$

$2^{dum} = \frac{7}{40}\, A\,a.$

$3^{tium} = \frac{7}{13}\, C.\, (\frac{1}{5} + \frac{1}{2\mathfrak{M}})\, A\,a.$

$4^{tum} = \frac{14}{39}\, \frac{C}{\mathfrak{M}}.\, A\,a.$

Praeterea

$z = \frac{1}{2}.\, \frac{a}{\mathfrak{M}+5}$ et $O = \frac{1}{2}\, t\, (1 + \frac{5}{\mathfrak{M}})$.

tum vero ſemidiameter aperturae lentis

$2^{dae} = \frac{3q}{\mathfrak{M}+5} + \frac{1}{8}\, x$ et $3^{tiae} = \frac{1}{5}\, x$.

Coroll.

259. Translatione igitur ad teleſcopia facta prodiret hoc caſu eorum longitudo $= \frac{2}{25}\, p + \frac{7}{65}\, C.\, p.$ quae longitudo ſatis eſt exigua, vt etiam in aliis generibus vix minor ſperari queat.

Scho-

Scholion.

260. Etsi iste casus $\zeta = \eta$ in praxi summum vsum praestare videtur, tamen etiam considerari conueniet quempiam casum, quo $\zeta > \eta$, quandoquidem hoc modo campo quodpiam augmentum affertur. Manente autem $S = \frac{1}{2}$, alter limes pro ζ erat $\zeta < \eta + \frac{3(\eta - 1)}{4}$ siue $\zeta < \frac{7}{4}\eta - \frac{3}{4}$. Huic autem limiti ipsi aequari nequit, quia alioquin $\mathfrak{C}$ deberet esse $= 1$ hincque $C = \infty$. Sumamus igitur

$$\zeta = \eta + \tfrac{2}{4}(\eta -) = \tfrac{3}{2}\eta - \tfrac{1}{2}$$

ac reperietur $\mathfrak{C} > \frac{2}{3}$ et $\mathfrak{C} < 1$. Sumatur igitur $\mathfrak{C} = \frac{3}{4}$ vt fiat $C = 3$. hincque fiet

$$k = \frac{(\eta - 1)(2\mathfrak{M} + 15(\eta - 1))}{12\mathfrak{M}(3\eta - 1)} \text{ et } P = \frac{12\mathfrak{M}(3\eta - 1)}{2\mathfrak{M} + 15(\eta - 1)}$$

hincque porro

$$\mathfrak{B} = -\frac{4\mathfrak{M}(18\eta - 7) + 30(\eta - 1)}{(3\eta - 1)(2\mathfrak{M} + 15(\eta - 1))} \text{ et}$$

$$\mathfrak{D} = \frac{24\mathfrak{M} + 10(\eta - 1)}{6\mathfrak{M} + 5(\eta - 1)} \text{ seu proxime } \mathfrak{D} = 4.$$

Tum vero prodibit

$$M = \frac{12}{6\mathfrak{M} + 5(\eta - 1)} = \frac{2}{\mathfrak{M} + \frac{5}{6}(\eta - 1)},$$

cum antea fuisset $M = \frac{2}{\mathfrak{M} + \eta - 1}$. Quodsi iam sumamus, vt in exemplo 2^{do}, $\eta = 3$ fient haec elementa

$$k = \frac{\mathfrak{M} + 15}{24\mathfrak{M}} \text{ et } P = \frac{48\mathfrak{M}}{\mathfrak{M} + 15} \text{ hinc}$$

$$\mathfrak{B} = \frac{-47\mathfrak{M} + 15}{4(\mathfrak{M} + 15)}; \quad B = \frac{-47\mathfrak{M} + 15}{51\mathfrak{M} + 45}$$

 tum

tum

$$\mathfrak{C} = \tfrac{3}{4};\ C = 3;\ \mathfrak{D} = 4\ \text{et}\ D = -\tfrac{4}{3};$$

vnde ſingula momenta pro conſtructione definiri poſſunt. Quia vero hic tanti occurrunt numeri, quos prae $\mathfrak{M}$ negligere non amplius licet; in adplicatione ad exempla ſtatim quoque pro $\mathfrak{M}$ determinatum valorem aſſumi conueniet. Praeterea vero hic ad ſpecialiora non progredimur, quia adhuc lente obiectiua ſimplice vtimur, ita, vt confuſio aliter tolli nequeat, niſi aperturam lentis obiectiuae diminuendo; quod remedium cum praxis ſponte offerat, non opus erit, litteram x. moleſto illo calculo definire, ſiquis enim microſcopium ſecundum huiusmodi menſuras conſtruxerit, ipſe vſus aperturam declarabit; quando autem in ſequente capite per multiplicationem lentis obiectiuae omnem confuſionem ad nihilum redigemus, tum demum neceſſe erit completas determinationes pro ſingulis momentis, vti hactenus fecimus, exhibere.

Problema 2.

261. Microſcopium huius generis ex quinque lentibus conſtruere, quae ita ſint diſpoſitae, vt prior imago realis in interuallum ſecundum, poſterior vero in interuallum quartum incidat.

Solutio.

Quatuor ergo litterarum P, Q, R, S ſecunda et quarta erunt negatiuae; vnde ponatur $Q = -k$ et

S =

$S = -k'$, vt fit $PkRk' = \frac{ma}{b} = \mathfrak{M}$, hinc erit vltimae lentis diſtantia focalis

$$t = \frac{ABCD}{PkRk'} a = \frac{ABCD}{\mathfrak{M}} \cdot a,$$

quae debet eſſe poſitiua aeque ac lentium interualla, quae ſunt

$$1^{mum} = A a (1 - \tfrac{1}{P});$$

$$2^{dum} = -\frac{AB'}{P} \cdot a (1 + \tfrac{1}{k});$$

$$3^{tium} = -\frac{ABC}{Pk} a (1 - \tfrac{1}{R});$$

$$4^{tum} = +\frac{ABCD}{PkR} a (1 + \tfrac{1}{k'});$$

ergo vt tam vltima lens, quam vltimum interuallum, fiant poſitiua, debet eſſe $ABCD > 0$. Hinc vt tertium quoque fiat poſitiuum, debet eſſe

$$-D (1 - \tfrac{1}{R}) > 0.$$

ſicque circa D nihil definitur. Ob ſecundum autem interuallum debet eſſe $-AB > 0$ et ob primum $A (1 - \frac{1}{P}) > 0$. Tum ergo debebit eſſe $CD < 0$. Statuatur nunc $M = \frac{\mathfrak{q} + \mathfrak{r} + \mathfrak{s} + \mathfrak{t}}{\mathfrak{M} - 1}$ et quia campus maximus deſideratur, ſtatim ſumi poterit $\mathfrak{s} = 1$ et $\mathfrak{t} = 1$, vt fiat $M = \frac{\mathfrak{q} + \mathfrak{r} + 2}{\mathfrak{M} - 1}$ hincque diſtantia oculi $O = \frac{t}{M \mathfrak{M} \mathfrak{r}}$ quae cum ſit poſitiua, deſtructio marginis praebet:

$$0 = \frac{\mathfrak{q}}{P} - \frac{\mathfrak{r}}{Pk} - \frac{1}{PkR} + \frac{1}{PkRk'}$$

vnde colligitur

$$\frac{1}{k'} = -\mathfrak{q} k R + \mathfrak{r} R + 1$$

 hinc

hinc erit

$$P k R = \mathfrak{M} (1 + \mathfrak{r} R - \mathfrak{q} k R)$$

sicque patet esse $1 + \mathfrak{r} R > \mathfrak{q} k R$. Praeterea vero considerare debemus sequentes aequationes:

$$1.\ \mathfrak{B} \mathfrak{q} = (P - 1) M$$

$$2.\ \mathfrak{C} \mathfrak{r} = -(1 + P k) M - \mathfrak{q}$$

$$3.\ \mathfrak{D} = -(1 + P k R) M - \mathfrak{q} - \mathfrak{r}$$

Ponatur hic, vt ante, breuitatis gratia

$$\frac{1 - P}{\mathfrak{B}} = \zeta \text{ et } 1 + P k = \eta, \text{ fietque}$$

$$\mathfrak{q} = -\zeta M \text{ et } \mathfrak{r} = \frac{(\zeta - \eta) M}{\mathfrak{C}}$$

vnde colligitur

$$2 + \mathfrak{q} + \mathfrak{r} = \frac{2\mathfrak{C} + (\zeta(1 - \mathfrak{C}) - \eta) M}{\mathfrak{C}} = (\mathfrak{M} - 1) M;$$

ex qua aequatione deducitur

$$M = \frac{2\mathfrak{C}}{\mathfrak{C}(\mathfrak{M} - 1) - \zeta(1 - \mathfrak{C}) + \eta} = \frac{2\mathfrak{C}}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}$$

ex quo valore viciffim erit

$$\mathfrak{q} = -\frac{2\zeta\mathfrak{C}}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta} \text{ et}$$

$$\mathfrak{r} = \frac{2(\zeta - \eta)}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}.$$

Nunc vt marginis colorati rationem habeamus, erit statim

$$1 + \mathfrak{r} R - \mathfrak{q} k R = \frac{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta + 2(\zeta - \eta) R + 2\zeta\mathfrak{C} k R}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}$$

Et cum ob

$$P k = \eta - 1 \text{ fit } P k R = (\eta - 1) R,$$

erit

erit

$$\mathfrak{C}(\eta-1)R(\mathfrak{M}+\zeta-1)-(\eta-1)(\zeta-\eta)R-\mathfrak{M}\mathfrak{C}(\mathfrak{M}+\zeta-1)$$
$$+\mathfrak{M}(\zeta-\eta)-2\mathfrak{M}(\zeta-\eta)R$$
$$-2\mathfrak{M}\zeta\mathfrak{C}kR=0.$$

Ante quam autem hinc vel k vel R determinemus, confiderare debemus rationem litterae $\mathfrak{D}$ ex fuperiori tertia aequatione; cum igitur PkR fit fine dubio numerus magnus inuoluens $\mathfrak{M}$, facile intelligitur litteram $\mathfrak{D}$ effe negatiuam; vnde etiam erit D negatiuum, adeoque concludimus fore $C>0$, hincque $\mathfrak{C}<1$ Ob eandem vero rationem debebit effe $R>1$; ita, vt haec littera aliquatenus tanquam nota fpectari poffit; quare ex illa aequatione colligimus

$$k=\left\{\begin{array}{l}\mathfrak{C}(\eta-1)R(\mathfrak{M}+\zeta-1)-(\eta-1)(\zeta-\eta)R\\ -\mathfrak{M}\mathfrak{C}(\mathfrak{M}+\zeta-1)+\mathfrak{M}(\zeta-\eta)\\ -2\mathfrak{M}(\zeta-\eta)R\end{array}\right\}:2\mathfrak{M}\zeta\mathfrak{C}R$$

hincque $P=\frac{(\eta-1)}{k}$; ita, vt fit $\eta>1$. Quare vt valor ipfius k fiat pofitiuus, debet effe

$$R(\mathfrak{C}(\eta-1)(\mathfrak{M}+\zeta-1)-(\eta-1)(\zeta-\eta)-2\mathfrak{M}(\zeta-\eta))$$
$$>\mathfrak{M}(\mathfrak{C}\mathfrak{M}+\mathfrak{C}(\zeta-1)-\zeta+\eta)$$

ad quod primo requiritur, vt quantitas litteram R multiplicans fit pofitiua, et quia $\mathfrak{M}$ eft numerus praemagnus, ipfius coefficiens ante omnia debet effe pofitiuus, vnde colligimus $\mathfrak{C}(\eta-)>2(\zeta-\eta)$ vnde con-

concluditur $\mathfrak{C} > \frac{2(\zeta - \eta)}{\eta - 1}$; quia igitur $\mathfrak{C} < 1$, erit

$$2(\zeta - \eta) < \eta - 1;\ \text{sicque}\ \zeta < \frac{3\eta - 1}{2}.$$

Qua conditione impleta debet esse

$$R > \frac{\mathfrak{M}(\mathfrak{C}\mathfrak{M} + \mathfrak{C}(\zeta - 1) - \zeta + \eta)}{\mathfrak{C}(\eta - 1)(\mathfrak{M} + \zeta - 1) - 2\mathfrak{M}(\zeta - \eta) - (\eta - 1)(\zeta - \eta)}$$

atque hinc retrogrediendo omnia elementa determi- nabuntur. Reliqua vero expediuntur, vt in praecedente problemate.

Coroll.

262. Hic igitur littera R denotabit numerum magnum $\mathfrak{M}$ inuoluentem; deinde conditio $\zeta < \frac{3\eta - 1}{2}$ instituto nostro maxime fauet, cum campi conditio inprimis postulet, ne ζ vltra necessitatem augeatur. Quare cum semper sit $\eta > 1$; commodissime videtur statui posse $\zeta = \eta$, vti in praecedante problemate; ita, vt tertiae lentis apertura iterum fiat minima prodeatque

$$M = \frac{2\mathfrak{C}}{\mathfrak{C}\mathfrak{M} + \mathfrak{C}\eta - \mathfrak{C}} = \frac{2}{\mathfrak{M} + \eta - 1}.$$

Coroll. 2.

263. Sumto autem $\zeta = \eta$, pro $\mathfrak{C}$ limites erunt $\mathfrak{C} < 1$ et $\mathfrak{C} > 0$. Porro capi debet $R > \frac{\mathfrak{M}}{\eta - 1}$; indeque fiet

$$k = \frac{((\eta - 1)R - \mathfrak{M})(\mathfrak{M} + \eta - 1)}{2\mathfrak{M}\eta R} \quad \text{et} \quad P = \frac{2\mathfrak{M}\eta(\eta - 1)R}{((\eta - 1)R - \mathfrak{M})(\mathfrak{M} + \eta - 1)}$$

Prae-

Praeterea vero erit

$$\mathfrak{B} = -\frac{(\eta-1)R((2\eta-1)\mathfrak{M}-\eta+1)-\mathfrak{M}(\mathfrak{M}+\eta-1)}{\eta((\eta-1)R-\mathfrak{M})(\mathfrak{M}+\eta-1)}$$

Denique vero reperietur

$$\mathfrak{D} = -\frac{2(1+(\eta-1)R-\eta)}{\mathfrak{M}+\eta-1}$$

siue cum $\mathfrak{M}$ et R sint numeri praemagni, erit proxime $\mathfrak{D} = -\frac{2(\eta-1)R}{\mathfrak{M}}$, qui valor certo est negatiuus, vt supra iam posuimus.

Coroll. 3.

264. Quin etiam statui poterit $\zeta = 0$; vnde pro $\mathfrak{C}$ limites erunt $\mathfrak{C} < 1$ et $\mathfrak{C} > -\frac{2\eta}{\eta-1}$; cui satisfit, dummodo $\mathfrak{C}$ intra limites 1 et 0 contineatur. Tum vero sumi debebit

$$R > \frac{\mathfrak{M}(\mathfrak{C}\mathfrak{M}-\mathfrak{C}+\eta)}{\mathfrak{C}(\eta-1)(\mathfrak{M}-1)+2\mathfrak{M}\eta+\eta(\eta-1)}$$

siue ob $\mathfrak{M}$ numerum praemagnum $R > \frac{\mathfrak{C}\mathfrak{M}}{\mathfrak{C}(\eta-1)+2\eta}$ Verum hinc sequitur porro $k = \infty$ et $P = 0$. ita, vt sit $Pk = \eta - 1$. Praeterea vero prodit $\mathfrak{B} = \infty$ et $B = -1$. Denique vero ob

$$\mathfrak{q} = 0 \text{ et } \mathfrak{r} = -\frac{\eta M}{\mathfrak{C}} \text{ erit}$$

$$\mathfrak{D} = -(1+(\eta-1)R-\frac{\eta}{\mathfrak{C}})M$$

ideoque ob $M = \frac{2\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}-1)+\eta}$, qui valor ipſius M aliquanto minor eſt, quam caſu praecedente, fiet $\mathfrak{D} = -\frac{2(\eta-1)R}{\mathfrak{M}}$.

Scholion.

265. Quantumuis hic caſus paradoxus videatur, cum ob $\mathfrak{B} = \infty$ tum vero ob $P = 0$, tamen reuera eſt realis et ad caſum in praecedente capite expoſitum reducitur, quia enim $\mathfrak{B} = \infty$; ſecundae lentis diſtantia focalis eſt infinita, ſicque res eodem redit ac ſi ſecunda lens plane abeſſet, ita, vt non amplius quaeſtio ſit de eius loco, quare etſi primum interuallum prodeat $= Aa(1 - \frac{1}{P}) = -\infty$, et ſecundum

$$Aa\left(\frac{1}{P} + \frac{1}{\eta-1}\right) = +\infty,$$

tamen horum ſumma, quae ſola nunc eſt ſpectanda, fit finita

$$= Aa\left(1 + \frac{1}{\eta-1}\right) = \frac{\eta}{\eta-1} \cdot Aa.$$

Cum igitur tantum quatuor lentes hic habeantur, hic caſus ad praecedens caput eſt referendus. Interim tamen hinc incommodum naſci debebit, quando ζ prope ad 0 accedit, quia tum P etiam erit vnitate minus, ita, vt A debeat eſſe numerus negatiuus et $B > 0$. Cum autem ſit $\mathfrak{B} = \frac{1-P}{\zeta}$, erit quidem $\mathfrak{B} > 0$, verum inſuper neceſſe eſt, vt ſit $1 - P < \zeta$, vel $P > 1 - \zeta$, ſiue P contineri debebit intra limites 1 et $1 - \zeta$ ſeu eſſe debet $k < \frac{\eta-1}{1-\zeta}$, quare cum $\mathfrak{M}$

et

et R ſint numeri praemagni, debebit eſſe

$$R\left(\frac{\mathfrak{C}(\eta-1)(1-3\zeta)}{1-\zeta}-2\right)<\mathfrak{C}\mathfrak{M}$$

quod ſponte euenit, ſi fuerit

$$\frac{\mathfrak{C}(\eta-1)(1-3\zeta)}{1-\zeta}<2 \text{ ſiue } \mathfrak{C}<\frac{2(1-\zeta)}{(\eta-1)(1-3\zeta)}.$$

Sin autem ſit

$$\frac{\mathfrak{C}(\eta-1)(1-3\zeta)}{1-\zeta}>2 \text{ debet eſſe } R<\frac{(1-\zeta)\mathfrak{C}\mathfrak{M}}{\mathfrak{C}(\eta-1)(1-3\zeta)-2(1-\zeta)}$$

quibus obſeruatis aliquot caſus fuſius euoluamus.

CASVS I.

quo $\zeta=\eta$.

266. Hoc caſu iam vidimus, lentem tertiam noſtro arbitrio relinqui, dummodo pro ea capiatur $\mathfrak{C}<1$ et $\mathfrak{C}>0$, vt C fiat numerus poſitiuus, vnde ſi circumſtantiae poſtulent, vt C ſit numerus ſatis magnus, tum $\mathfrak{C}$ parum ab vnitate deficere debebit; deinde etiam notauimus, capi debere $R>\frac{\mathfrak{M}}{\eta-1}$, vnde cum ſemper ſit $\eta>1$, ſi etiam fuerit >2, tunc commode ſumi poterit $R=\mathfrak{M}$. Notetur autem, litteram η non nimis magnam ſumi conuenire, quia pro campo ſit $M=\frac{2}{\mathfrak{M}+\eta-1}$. Deinde vero prodit

$$k=\frac{((\eta-1)R-\mathfrak{M})(\mathfrak{M}+\eta-1)}{2\mathfrak{M}\eta R}$$

quare pro maioribus multiplicationibus tuto ſumi poterit $k=\frac{(\eta-1)R-\mathfrak{M}}{2\eta R}$ vnde patet, litteram k eo fore minorem, quo minus R limitem praeſcriptum $\frac{\mathfrak{M}}{\eta-1}$

ſuperet; vnde fit

$$P = \frac{\eta - 1}{k} = \frac{2\eta(\eta - 1)k}{(\eta - 1)R - \mathfrak{M}}.$$

Pro reliquis elementis primo prodit

$$\mathfrak{B} = -\frac{(\eta - 1)R((2\eta - 1)\mathfrak{M} - \eta + 1) - \mathfrak{M}(\mathfrak{M} + \eta - 1)}{\eta((\eta - 1)R - \mathfrak{M})(\mathfrak{M} + \eta - 1)}$$

hincque proxime

$$\mathfrak{B} = -\frac{(\eta - 1)(2\eta - 1)R - \mathfrak{M}}{\eta(\eta - 1)R - \eta\mathfrak{M}}.$$

qui valor manifeſto eſt negatiuus, ex quo etiam B erit negatiuum. Deinde cum ſit $P > 1$; ob eandem rationem littera A debebit eſſe poſitiua; ex quo productum A B ob

$$B = -\frac{(\eta - 1)(2\eta - 1)R - \mathfrak{M}}{(\eta - 1)(3\eta - 1)R - (\eta - 1)\mathfrak{M}}$$

ſponte fit negatiuum, prorſus, vti conditiones praeſcriptae poſtulant. Denique vero reperitur

$$\mathfrak{D} = -\frac{2(1 + (\eta - 1)R - \eta)}{\mathfrak{M} + \eta - 1}$$

ideoque proxime

$$\mathfrak{D} = -\frac{2(\eta - 1)R}{\mathfrak{M}}; \text{ vnde fit } D = -\frac{2(\eta - 1)R}{\mathfrak{M} + 2(\eta - 1)R}.$$

His definitis erunt primo diſtantiae focales

$$p = \mathfrak{A}a.\ q = -\frac{A\mathfrak{B}}{P}a = \frac{(\eta - 1)(2\eta - 1)R + \mathfrak{M}}{2\eta^2(\eta - 1)R}.\ A\,a.$$

$$r = -\frac{AB\mathfrak{C}}{\eta - 1}.\ a;\ s = +\frac{ABC\mathfrak{D}}{(\eta - 1)R}.\ a.$$

$$\text{et } t = \frac{ABCD}{\mathfrak{M}}.\ a.$$

Deinde

Deinde vero interualla

$$I^{mum} = A\,a\left(1 - \frac{1}{P}\right);$$

$$II^{dum} = -A\,B.\,a\left(\frac{1}{P} + \frac{1}{\eta - 1}\right);$$

$$III^{tium} = -\frac{A\,B\,C\,a}{\eta - 1}\left(1 - \frac{1}{R}\right);$$

$$IV^{tum} = A\,B\,C\,D.\,a\left(\frac{1}{(\eta - 1)R} + \frac{1}{\mathfrak{M}}\right).$$

Distantia vero oculi fiet

$$O = \frac{t}{M\mathfrak{M}} = \frac{t}{2}.\frac{\mathfrak{M} + \eta - 1}{\mathfrak{M}} = \frac{1}{2}t\left(1 + \frac{\eta - 1}{\mathfrak{M}}\right) = \frac{1}{2}t \text{ proxime.}$$

Pro aperturis vero lentium primae quidem apertura tutissime per experientiam definitur; vnde reperitur littera x, ex eaque mensura claritatis $= \frac{20\,x}{\mathfrak{M}}$, si scilicet x in digitis exprimatur.

Pro secunda vero lente cum sit

$$q = -\eta\,M = -\frac{2\eta}{\mathfrak{M} + \eta - 1} = -\frac{2\eta}{\mathfrak{M}},$$

erit eius aperturae semidiameter

$$= \frac{1}{4}q\,q + \frac{x}{P} = -\frac{1}{2}.\frac{\eta}{\mathfrak{M}}.q + \frac{x}{P}.$$

Pro tertia vero lente ob $r = 0$ sufficit aperturae semidiameter $= \frac{x}{\eta - 1}$; reliquas vero lentes vtrinque aeque conuexas confici conuenit.

Coroll.

267. Si sumatur $R = \frac{\mathfrak{M}}{\eta - 1}$, vt fiat $k = 0$ et $P = \infty$, existente $P\,k = \eta - 1$; tum fiet $\mathfrak{B} = -\frac{P}{\eta} = \infty$ et

et $B = -1$. Tum igitur secunda lens in ipsam imaginem realem priorem incidet ob primum intervallum $= Aa = \alpha$, eiusque distantia focalis erit $q = \frac{Aa}{\eta}$. At vero secundum interuallum hoc casu euadet $= -\frac{ABa}{\eta-1} = \frac{Aa}{\eta-1}$. Reliqua vero definiuntur, vt in genere, si modo notetur, fore $\mathfrak{D} = -2$. et $D = -\frac{2}{3}$.

Exempl. I.

Ponamus $\eta = 2$ et capi debebit $R > \mathfrak{M}$. Nihil vero impedit, quominus secundum praecedens Coroll. sumamus $R = \mathfrak{M}$ ita, vt tum fiat $k = 0$ et $P = \infty$ quare primo distantiae focales ita exprimentur

$$p = \mathfrak{A}a;\ q = \frac{Aa}{2};$$

$$r = A\,\mathfrak{C}.\,a;\ s = \frac{2\,A\,C.}{\mathfrak{M}}.\,a;$$

$$\text{et } t = \tfrac{2}{3}.\,\frac{C}{\mathfrak{M}}.\,A\,a. = \tfrac{1}{3}\,s.$$

Deinde interualla ita se habebunt:

$$1^{mum} = A\,a;\ 2^{dum} = A\,a;$$

$$3^{tium} = C.\,(1 - \tfrac{1}{\mathfrak{M}})\,A\,a;$$

$$4^{tum} = \tfrac{4}{3}\,\frac{C}{\mathfrak{M}}.\,A\,a.$$

distantia vero oculi $= \frac{1}{2}\,t\,(1 + \frac{1}{\mathfrak{M}})$.

Pro valore ipsius x siue per experientiam siue per formulam notam definito, fiat secundae lentis semidiameter aperturae $= \frac{q}{\mathfrak{M}}. = \frac{1}{2}.\,\frac{Aa}{\mathfrak{M}}$ et tertiae lentis $= x$.

$= x.$ ſemidiameter ſpatii conſpicui erit $z = \frac{\alpha}{2\mathfrak{M}}$ et menſura claritatis $= \frac{20 \cdot x}{\mathfrak{M}}$.

Corollarium.

268. Hae formulae quoque ad teleſcopia accommodari poterunt, ſumendo $Aa = p$ et $\mathfrak{M} = m$. Tum vero ſumi debebit ipſa diſtantia focalis

$$p = m \sqrt[3]{\mu m \left(\lambda * + \frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right)}$$

omiſſis terminis ſequentibus per $\mathfrak{M}$ diuiſis.

Exempl. II.

269. Sit nunc $\eta = 3$, vt eſſe debeat $R > \frac{\mathfrak{M}}{2}$ ſumaturque $R = \mathfrak{M}$, vnde fiet $k = \frac{1}{6}$ et $P = 12$; quare reliqua elementa fient $\mathfrak{B} = -\frac{11}{3}$; hincque $B = -\frac{11}{14}$ et $\mathfrak{D} = -4$ et $D = -\frac{4}{5}$ vnde diſtantiae focales euadent

$$p = \mathfrak{A}\, a;\ q = \frac{11}{36} A a.$$
$$r = \frac{11}{28} \mathfrak{C}. A a.\ s = \frac{11}{7}. \frac{C}{\mathfrak{M}}. A a. \text{ et}$$
$$t = \frac{22}{35} C. A a. = \frac{2}{5}. s$$

atque interualla lentium

$$1^{mum} = \frac{11}{12} A a;$$
$$2^{dum} = \frac{11}{84} A a;$$
$$3^{tium} = \frac{11}{28} C \left(1 - \frac{1}{\mathfrak{M}}\right) A a.$$
$$4^{tum} = \frac{11}{35}. \frac{C}{\mathfrak{M}}. A a.$$

oculi-

oculique diſtantia $O = \frac{1}{2} t (1 + \frac{2}{\mathfrak{M}})$, ſpatii vero in obiecto conſpicui erit ſemidiameter $z = \frac{a}{2.(\mathfrak{M}+2)}$. Definito x ſiue per experientiam ſiue per formulam notam erit ſemidiameter aperturae

ſecundae lentis $= \frac{3}{2} . \frac{q}{\mathfrak{M}} + \frac{x}{12}$ et tertiae $= \frac{x}{8}$.

et gradus claritatis $= \frac{20 x}{\mathfrak{M}}$.

Coroll.

270. Hae formulae etiam ad teleſcopia accommodari poſſunt; erit enim $A a = p$ et $\mathfrak{M} = m$. Tum vero lentis obiectiuae diſtantia focalis definitur per hanc formulam

$$p = m \sqrt[3]{\mu m \left(\lambda + \frac{1}{12}\left(\frac{3^3}{11^3} \lambda' - \frac{3.14\nu}{11^2}\right) + \frac{14^3}{11^3.2}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right)\right)}$$

Longitudo huius teleſcopii erit circiter $= (\frac{11}{8} + \frac{11}{28} C) p$.

Exempl. III.

271. Sit nunc $\eta = 6$, vt eſſe debeat $R > \frac{\mathfrak{M}}{5}$ et ſumatur $R = \frac{\mathfrak{M}}{2}$ ac reperitur $k = \frac{1}{4}$ et $P = 20$. Hinc porro fiet $\mathfrak{B} = -\frac{19}{6}$ et $B = -\frac{19}{25}$. Tum vero $\mathfrak{D} = -5$. et $D = -\frac{5}{6}$. Diſtantiae ergo focales ita ſe habebunt:

$p = \mathfrak{A} a$; $q = \frac{19}{120} A a$;

$r = \frac{19}{125} \mathfrak{C}. A a$; $s = \frac{38}{25} \frac{C}{\mathfrak{M}} A a$. et

$t = \frac{19}{30} . \frac{C}{\mathfrak{M}} . A a$ ſeu $t = \frac{5}{12} . s$.

et

et interualla lentium

$1^{mum} = \frac{19}{20}. A\,a;$

$2^{dum} = \frac{19}{100} A\,a;$

$3^{tium} \frac{11}{125} C(1 - \frac{2}{\mathfrak{M}}) A\,a.$ et

$4^{tum} = \frac{133}{150}. \frac{C}{\mathfrak{M}}. A\,a.$

oculique diſtantia $O = \frac{1}{2} t (1 + \frac{5}{\mathfrak{M}})$.

Spatii conſpicui ſemidiameter $z = \frac{a}{2(\mathfrak{M}+5)}$.

Definito denique x, vt ante, erit ſemidiameter aperturae lentis

ſecundae $= \frac{5}{\mathfrak{M}} q + \frac{x}{20}$ et tertiae $= \frac{1}{5} x$;

gradus autem claritatis manet $= 20. \frac{x}{\mathfrak{M}}$.

Exempl. IV.

272. Sit vt ante $\eta = 6$, ſumatur vero $R = \mathfrak{M}$, ac reperitur $k = \frac{1}{5}$ et $P = 15$. Hinc porro fit

$\mathfrak{B} = -\frac{7}{3}$ et $B = -\frac{7}{10}$, at

$\mathfrak{D} = -10$ et $D = -\frac{10}{11}$.

Diſtantiae ergo focales erunt

$p = \mathfrak{A}\,a;\ q = \frac{7}{45} A\,a;$

$r = \frac{7}{50} \mathfrak{C}. A\,a;\ s = \frac{7}{5}. \frac{C}{\mathfrak{M}}. A\,a;$

$t = \frac{7}{11}. \frac{C}{\mathfrak{M}}. A\,a$ ſeu $t = \frac{5}{11} s$.

et lentium interualla

$1^{mum} = \frac{14}{15} \cdot \mathrm{A}\, a;$

$2^{dum} = \frac{14}{75} \cdot \mathrm{A}\, a;$

$3^{tium} = \frac{7}{50} \cdot \mathrm{C} \cdot (1 - \frac{1}{\mathfrak{M}}) \mathrm{A}\, a;$

$4^{tum} = \frac{42}{55} \cdot \frac{\mathrm{C}}{\mathfrak{M}} \cdot \mathrm{A}\, a.$

et diſtantia Oculi $= \frac{1}{2} t (1 + \frac{5}{\mathfrak{M}})$ pariter, ac reliqua momenta, ſe habet, vt ante.

Corollarium.

273. Si haec ad teleſcopia transferantur, ponendo $\mathrm{A}.a = p$ et $\mathfrak{M} = m$, caſus hic poſterior antecedenti praeferendus videtur, ſiquidem praebet longitudinem parumper minorem, quippe quae neglectis terminis per $\mathfrak{M}$ diuiſis erit $= (1 \frac{3}{25} + \frac{7}{50} \cdot \mathrm{C}) p$. cum ex exemplo praecedente fuerit $= (1 \frac{7}{50} + \frac{19}{125} \mathrm{C}) p$ Caſu ergo vltimi exempli lentis obiectiuae diſtantia focalis ita definietur, vt ſit

$$p = m \sqrt[3]{\mu m \left(\lambda + \frac{1}{15}\left(\frac{5^3}{7^3}\lambda' - \frac{30 \nu}{7^2}\right) - \frac{200}{343}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{\mathrm{C}\mathfrak{C}}\right)\right)}$$

CASVS II.

quo $\zeta = 1$.

274. Cum ſit $\zeta = 1$, limites pro $\mathfrak{C}$ erunt $\mathfrak{C} < 1$ et $\mathfrak{C} > -2$, ita, vt $\mathfrak{C}$ aeque arbitrio noſtro permittatur, ac ante. Tum vero eſſe debebit

$$\mathrm{R} > \frac{\mathfrak{M}(\mathfrak{C}\mathfrak{M} + \eta - 1)}{(\eta - 1)(\mathfrak{C}\mathfrak{M} + 2\mathfrak{M} + \eta - 1)}$$

ſeu

seu neglectis minoribus partibus $R > \frac{\mathfrak{C}\mathfrak{M}}{(\eta-1)(2+\mathfrak{C})}$. Statuatur ergo $R = i\mathfrak{M}$, sumendo scilicet $i > \frac{\mathfrak{C}}{(\eta-1)(2+\mathfrak{C})}$ Tum ergo fiet

$$k = \frac{\mathfrak{C}(\eta i - i - 1) + 2i(\eta-1)}{2i\mathfrak{C}} \text{ et } P = \frac{2i(\eta-1)\mathfrak{C}}{\mathfrak{C}((\eta-1)i-1)+2i(\eta-1)}$$

Hinc ergo fiet

$$P - 1 = \frac{\mathfrak{C}(i(\eta-1)+1) - 2i(\eta-1)}{\mathfrak{C}((\eta-1)i-1)+2i(\eta-1)}$$

qui valor erit positiuus, seu $P > 1$, si fuerit

$$i < \frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})}.$$

Hoc ergo casu prodit

$$\mathfrak{B} = \frac{-\mathfrak{C}(i(\eta-1)+1)+2i(\eta-1)}{\mathfrak{C}(i(\eta-1)-1)+2i(\eta-1)}$$

qui valor cum sit negatiuus, etiam B erit negatiuum et ob $P > 1$ debebit A esse positiuum, vt superiores conditiones postulant; sin autem esset

$$i > \frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})};$$

tum foret $P < 1$ sumique deberet A negatiue, ac prodiret $\mathfrak{B} > 0$. vnde vt etiam B fiat positiuum, insuper necesse est, vt sit $\mathfrak{B} < 1$, quod manifestum est, cum sit $\mathfrak{B} = 1 - P$. Postea vero pro $\mathfrak{D}$ inueniendo notetur esse

$$M = \frac{2\mathfrak{C}}{\mathfrak{C}\mathfrak{M}+\eta-1} \text{ et}$$

$$\mathfrak{q} = -M \text{ et } \mathfrak{r} = -\frac{(\eta-1)M}{\mathfrak{C}};$$

ex quibus prodit

$$\mathfrak{D} = -i(\eta-1)M\mathfrak{M} = -2i(\eta-1).$$

 illi

illi vero valores abibunt in hos:

$$\mathfrak{q} = -\frac{2}{\mathfrak{M}}; \text{ et } \mathfrak{r} = -\frac{2(\eta-1)}{\mathfrak{C}\mathfrak{M}}.$$

His valoribus inuentis considerentur primo diſtantiae focales, quae ſunt:

$$p = \mathfrak{A}a; \quad q = -\frac{A\mathfrak{B}}{P}.a;$$

$$r = -\frac{AB\mathfrak{C}}{\eta-1}.a; \quad s = \frac{ABC\mathfrak{D}}{i(\eta-1)\mathfrak{M}},$$

$$\text{et } t = \frac{ABCD}{\mathfrak{M}}.a.$$

Tum vero interualla

$$1^{mum} = A\,a\left(1-\frac{1}{P}\right);$$

$$2^{dum} = -A\,B.\,a\left(\frac{1}{P}+\frac{1}{\eta-1}\right);$$

$$3^{tium} = -\frac{ABC.a}{\eta-1}\left(1-\frac{1}{i\mathfrak{M}}\right);$$

$$4^{tum} = \frac{ABCD.a}{\mathfrak{M}}\left(\frac{1}{i(\eta-1)}+1\right);$$

diſtantia vero oculi

$$O = \frac{t}{\mathfrak{M}M} = \frac{1}{2}t\left(1+\frac{\eta-1}{\mathfrak{C}\mathfrak{M}}\right).$$

Deinde ſimili modo, vt iam ante notauimus, littera x ſiue per experientiam ſiue ex formula nota definiri poterit. Tum vero erit lentis ſecundae ſemidiameter aperturae

$$= \frac{1}{4}\mathfrak{q}q + \frac{x}{P} = \frac{1}{2}.\frac{q}{\mathfrak{M}} + \frac{x}{P}.$$

tertiae vero lentis

$$= \frac{1}{4}.\mathfrak{r}r + \frac{x}{\eta-1} = \frac{1}{2}.\frac{(\eta-1)r}{\mathfrak{C}\mathfrak{M}} + \frac{x}{\eta-1}.$$

reliquae vero lentes quia ſunt vtrinque aeque convexae, maximam aperturam admittunt. Pro ſpatio deni-

denique conſpicuo erit

$$z = \frac{1}{2} \cdot \frac{\mathfrak{C} \cdot a}{\mathfrak{C}\mathfrak{M} + \eta - 1} = \frac{a}{2\mathfrak{M}} \text{ proxime}$$

et menſura claritatis $= 20 \cdot \frac{x}{\mathfrak{M}}$.

Coroll. 1.

275. Si littera i contineatur intra hos limites, ſcilicet

$$i > \frac{\mathfrak{C}}{(\eta - 1)(2 + \mathfrak{C})} \text{ et } i < \frac{\mathfrak{C}}{(\eta - 1)(2 - \mathfrak{C})}$$

tum fit $P > 1$ et litterae $\mathfrak{B}$ et B negatiuae, littera vero A ſumi debet poſitiue; vnde omnia elementa eiusdem ſunt naturae, vti in caſu praecedente.

Coroll. 2.

276. Sin autem adeo fuerit $i > \frac{\mathfrak{C}}{(\eta - 1)(2 - \mathfrak{C})}$; tum littera P vnitate fit minor, hincque tam littera $\mathfrak{B}$, quam B fiunt poſitiuae; at vero littera A eſſe debebit negatiua, id quod duplici modo euenire poteſt, altero quo $\mathfrak{A} > 1$; altero vero, quo $\mathfrak{A} < 0$. quo poſteriore caſu lens prima euaderet concaua et inſtrumentum multis incommodis foret obnoxium.

Coroll. 3.

277. Sin autem fuerit $i = \frac{\mathfrak{C}}{(\eta -)(2 - \mathfrak{C})}$; tum fiet $P = 1$. hincque $\mathfrak{B} = 0$ et $B = 0$. Tum igitur ne fiat $q = 0$, neceſſe erit ſumi $A = \infty$, ita, tamen, vt ſit $AB = A\mathfrak{B} = -\frac{q}{a}$. Vnde cum ſit $1 - P = \mathfrak{B}$

 erit

erit primum interuallum

$$= A\,a\left(1 - \frac{1}{p}\right) = -A\,\mathfrak{B}.\,a = q$$

quare ob $\mathfrak{A} = 1$. fient distantiae focales

$$p = a;\; q = q;\; r = \frac{\mathfrak{C}q}{\eta - 1};$$

$$s = -\frac{C\mathfrak{D}.}{i(\eta - 1)\mathfrak{M}}\,q \text{ seu } s = \frac{2C}{\mathfrak{M}}.\,q \text{ et}$$

$$t = -\frac{CD}{\mathfrak{M}}.\,q \text{ seu } t = \frac{2i(\eta - 1)C}{2i(\eta - 1) + 1}.\,\frac{q}{\mathfrak{M}}.$$

Interualla vero lentium

$$1^{mum} = q;$$

$$2^{dum} = \frac{\eta}{\eta - 1}.\,q;$$

$$3^{tium} = \frac{C}{\eta - 1}\left(1 - \frac{1}{i\mathfrak{M}}\right)q;$$

$$4^{tum} = -\frac{CD}{\mathfrak{M}}\left(1 + \frac{1}{i(\eta - 1)}\right)q.$$

$$= \frac{2i(\eta - 1) + 2}{2i(\eta - 1) + 1}.\,\frac{C}{\mathfrak{M}}.\,q.$$

In reliquis vero momentis nihil mutandum occurrit.

Scholion.

278. Probe autem hic est notandum, casus in his duobus postremis corollariis contentos neutiquam ad telescopia transferri posse. Pro telescopiis enim ob $a = \infty$ necessario sumi debet $\mathfrak{A} = 0$ et $A = 0$, cum in his casibus debeat esse $\mathfrak{A}$ vel infinitum vel negatiuum.

Exempl. I.

279. Sumamus $\eta = 2$ et quia C in iis tantum terminis occurrit, qui per $\mathfrak{M}$ sunt diuisi ideo-

que

que ſemper numerum praemagnum ſignificare debet, pro $\mathfrak{C}$ recte vnitatem aſſumere poterimus, hinc ergo pro littera R primus limes erit $i > \frac{1}{3}$, vt noſtra inſtrumenta ad caſum corollarii primi pertineant, ſumi quoque debet $i < 1$ hinc ergo capiatur $i = \frac{1}{2}$, vt fit $R = \frac{1}{2}\mathfrak{M}$; vnde colligemus $k = \frac{1}{2}$ et $P = 2$. Deinde $\mathfrak{B} = -1$ et $B = -\frac{1}{2}$ et $\mathfrak{D} = -1$. hinc $D = -\frac{1}{2}$. Hinc diſtantiae focales erunt

$$p = \mathfrak{A}\, a;\; q = \tfrac{1}{2} A\, a;$$
$$r = \tfrac{\mathfrak{C}}{2} . A\, a;\; s = \tfrac{CA a}{\mathfrak{M}} \text{ et}$$
$$t = \tfrac{1}{4} . \tfrac{C}{\mathfrak{M}}\; A\, a. \text{ ſeu } t = \tfrac{1}{4} s.$$

Interualla vero lentium erunt

$$1^{mum} = \tfrac{1}{2} A\, a;$$
$$2^{dum} = \tfrac{3}{4} A\, a.;$$
$$3^{tium} = \tfrac{C}{2}\left(1 - \tfrac{2}{\mathfrak{M}}\right) A\, a;$$
$$4^{tum} = \tfrac{3}{4} . \tfrac{C}{\mathfrak{M}} A\, a.$$

ac diſtantia oculi $O = \frac{1}{2} t \left(1 + \frac{1}{\mathfrak{M}}\right)$.

Semidiametri denique aperturarum lentis

$$1^{mae} = x;$$
$$2^{dae} = \tfrac{1}{2} \tfrac{q}{\mathfrak{M}} + \tfrac{1}{2} x;$$
$$3^{tiae} = \tfrac{1}{2} \tfrac{r}{\mathfrak{M}} + x.$$

Exempl.

Exemplum 2.

280. Maneat $\eta = 2$, ſumatur vero $i = 1$, ſiue $R = \mathfrak{M}$, atque erit $k = 1$ et $P = 1$, tum vero $\mathfrak{B} = 0 = B$ et $\mathfrak{D} = -2$, et $D = -\frac{2}{3}$; vnde ex corollario tertio nanciſcimur

$$p = a;\; q = q;\; r = q;$$

$$s = \frac{2C}{\mathfrak{M}} q;\; t = \frac{2}{3} \cdot \frac{C}{\mathfrak{M}} \cdot q. \text{ ſeu } t = \frac{1}{3} s.$$

Interualla vero lentium erunt

$1^{mum} = q;$

$2^{dum} = 2q;$

$3^{tium} = C(1 - \frac{1}{\mathfrak{M}}) q;$

$4^{tum} = \frac{4}{3} \cdot \frac{C}{\mathfrak{M}} q;$

reliqua vero momenta perinde ac in praecedente exemplo ſe habebunt.

Exempl. 3.

281. Maneat $\eta = 2$, et ſumatur $i = 2$, vt ſit $R = 2\mathfrak{M}$; erit ergo $k = \frac{5}{4}$ et $P = \frac{4}{5}$, vnde $\mathfrak{B} = \frac{1}{5}$, hinc $B = \frac{1}{4}$; tum vero $\mathfrak{D} = -4$ et $D = -\frac{4}{5}$. Hinc cum A debeat eſſe negatiuum, ſtatuatur

$$A = -\alpha, \text{ vt ſit } \mathfrak{A} = -\frac{\alpha}{1-\alpha} = \frac{\alpha}{\alpha - 1};$$

ex quo diſtantiae focales erunt

$$p = \frac{\alpha}{\alpha - 1} \cdot a;\; q = \frac{1}{4} \alpha a;$$

$$r = \frac{1}{4} \mathfrak{C} \alpha a;\; s = \frac{1}{2} \cdot \frac{}{\mathfrak{M}} \alpha a$$

et

et $t = \frac{1}{5} \frac{C}{\mathfrak{M}} \alpha a$ fiue $t = \frac{2}{5} s$.

Interualla vero erunt

$1^{mum} = \frac{1}{4} \alpha a$;

$2^{dum} = \frac{1}{10} \alpha a$.

$3^{tium} = \frac{1}{4} C \left(1 - \frac{1}{2\mathfrak{M}}\right) \alpha a$.

$4^{tum} = \frac{3}{10} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha a$.

Reliqua vero momenta funt iterum, vt in exemplo primo. Hic autem probe notandum eft, fi capiatur $\alpha = 1$, prodire $p = \infty$ ideoque primam lentem plane reiici poffe, ita, vt microfcopium ex folis lentibus pofterioribus componatur. Quia autem tum confufio prodiret enormis, cum in formulis capitis praecedentis fumi deberet $\mathfrak{A} = \frac{1}{2}$, hoc inftrumentum merito reiicimus multoque magis ea, quae prodirent, fi effet $\alpha < 1$. Ac fi α vnitatem haud parum fuperet, haec inftrumenta in praxi vfum habere poffe videntur.

Exempl. 4.

282. Sit nunc $\eta = 4$ et cum primus limes fit $i > \frac{1}{9}$, pro cafu corollarii primi fumamus $i < \frac{1}{3}$; fit igitur $i = \frac{1}{6}$ fumto $\mathfrak{C} = 1$, eritque $k = \frac{1}{2}$; $P = 2$; hinc $\mathfrak{B} = -1$; $B = -\frac{1}{2}$; $\mathfrak{D} = -1$; $D = -\frac{1}{2}$; vnde diftantiae focales erunt

$p = \mathfrak{A} a$; $q = \frac{1}{2} A a$;

$r = \frac{1}{6} . Aa$; $s = \frac{C}{\mathfrak{M}} . Aa$; et

$t = \frac{1}{4} . \frac{C}{\mathfrak{M}} . Aa$. ſeu $t = \frac{1}{4} s$.

Interualla vero lentium

$1^{mum} = \frac{1}{2} Aa$;

$2^{dum} = \frac{5}{12} Aa$;

$3^{tium} = \frac{C}{6}\left(1 - \frac{6}{\mathfrak{M}}\right) Aa$;

$4^{tum} = \frac{3}{4} \frac{C}{\mathfrak{M}} . Aa$.

Oculi diſtantia $O = \frac{1}{2} t \left(1 + \frac{3}{\mathfrak{M}}\right)$.

Porro $z = \frac{a}{2(\mathfrak{M} + 1)}$.

Semidiametri porro aperturarum erunt lentis

primae $= x$;

ſecundae $= \frac{1}{2} \frac{q}{\mathfrak{M}} + \frac{1}{2} x$;

tertiae $= \frac{3}{2} . \frac{r}{\mathfrak{M}} + \frac{1}{3} x$.

Exempl. 5.

283. Maneat $\eta = 4$, at ſumatur $i = \frac{1}{3}$ ſecundum Coroll. tertium; eritque $k = 1$ et $P = 1$; vnde colligantur diſtantiae focales

$p = a$; $q = q$; $r = \frac{1}{3} q$;

$s = \frac{2C}{\mathfrak{M}} q$. et $t = \frac{2}{3} . \frac{C}{\mathfrak{M}} . q = \frac{1}{3} s$.

et lentium interualla

$1^{mum} = q$; $2^{dum} = \frac{4}{3} q$;

3^{tium}

$3^{tium} = \frac{C}{3}(1 - \frac{3}{\mathfrak{M}})q;$

$4^{tum} = \frac{4}{3} \cdot \frac{C}{\mathfrak{M}} q. = 2t.$

Diſtantia oculi $O = \frac{1}{2} t (1 + \frac{3}{\mathfrak{M}})$ et reliqua momenta omnia ſunt, vt ante.

Exempl. 6.

284. Manente $\eta = 4$, ſumatur $i = 1$. eritque $k = 4$ et $P = \frac{3}{4}$; tum vero $\mathfrak{B} = \frac{1}{4}$ et $B = \frac{1}{3}$ et $\mathfrak{D} = -6$; $D = -\frac{6}{7}$. Cum igitur B ſit poſitiuum, littera A negatiua eſſe debet. Sit igitur $A = -\alpha$ fietque $\mathfrak{A} = \frac{\alpha}{\alpha - 1}$, vnde prodibunt diſtantiae focales

$$p = \frac{\alpha}{\alpha - 1} \cdot a; \; q = \frac{1}{3} \alpha a;$$

$$r = \frac{1}{9} \mathfrak{C}. \alpha a; \; s = \frac{2}{3} \cdot \frac{C}{\mathfrak{M}} \alpha a. \text{ et}$$

$$t = \frac{2}{7} \cdot \frac{C}{\mathfrak{M}} \alpha a; \; t = \frac{3}{7} s.$$

et interualla lentium

$1^{mum} = \frac{1}{3} \alpha a.$

$2^{dum} = \frac{5}{9} \alpha a.$

$3^{tium} = \frac{1}{9} C. (1 - \frac{3}{\mathfrak{M}}) \alpha a.$

$4^{tum} = \frac{8}{21} \cdot \frac{C}{\mathfrak{M}} \alpha a.$

Diſtantia oculi O cum reliquis momentis eadem, ac ante, manet.

Problema 3.

285. Microſcopium ex ſex lentibus conſtruere, quae ita ſint diſpoſitae, vt prior imago realis in inter-

teruallum fecundum, pofterior vero in quartum incidat.

Solutio.

Quinque igitur litterarum P, Q, R, S, T fecunda et quarta debent effe negatiuae; quare ponatur $Q = -k$ et $S = -k'$, vt fit $P k R k' T = \mathfrak{M} = \frac{ma}{b}$. Hinc erit vltimae lentis diftantia focalis

$$u = -\frac{ABCDE}{\mathfrak{M}} . a,$$

quae debet effe pofitiua aeque ac lentium interualla, quae funt

$$1^{mum} = A\, a\left(1 - \frac{1}{P}\right);$$
$$2^{dum} = -\frac{AB}{P}\, a\left(1 + \frac{1}{k}\right);$$
$$3^{tium} = -\frac{ABC}{Pk}\, a\left(1 - \frac{1}{R}\right);$$
$$4^{tum} = \frac{ABCD}{PkK}\, a\left(1 + \frac{1}{k'}\right);$$
$$5^{tum} = \frac{ABCDE}{PkRk'}\, a\left(1 - \frac{1}{T}\right).$$

Ob quintum ergo debet effe $T < 1$. ob quartum vero $E < 0$. hincque $ABCD > 0$. Ob fecundum vero effe debet $AB < 0$. hincque etiam CD negatiuum. Statuatur nunc $M = \frac{\mathfrak{q} + \mathfrak{r} + \mathfrak{s} + \mathfrak{t} + \mathfrak{u}}{\mathfrak{M} - 1}$ et quia campus maximus defideratur, fumi poterit $\mathfrak{s} = 1$; $\mathfrak{t} = 1$ et $\mathfrak{u} = 1$, vt fiat

$$M = \frac{\mathfrak{q} + \mathfrak{r} + 3}{\mathfrak{M} - 1} \text{ hincque } z = M a \xi = \tfrac{1}{4}. M a.$$

et diftantia oculi $O = \frac{\mathfrak{u}}{\mathfrak{M} M}$ quae cum fit pofitiua, deftructio marginis colorati praebet

$O =$

$$o = \frac{\mathfrak{q}}{P} - \frac{\mathfrak{r}}{Pk} - \frac{1}{PkR} + \frac{1}{PkRk'} + \frac{1}{PkRk'T}$$

vnde colligitur

$$\frac{1}{k'}\left(1 + \frac{1}{T}\right) = -\mathfrak{q}\,k\,R + \mathfrak{r}\,R + 1$$

et quia conſtat, eſſe $T < 1$, ſtatuatur breuitatis gratia $1 + \frac{1}{T} = \mathfrak{D}$; vt ſit $\mathfrak{D} > 2$, hincque $T = \frac{1}{\theta - 1}$; ex quo habebitur

$$\frac{1}{k'} = -\frac{\mathfrak{q}kR + \mathfrak{r}R + 1}{\theta};\ \text{ergo ob}\ \frac{PkRk'}{\theta - 1} = \mathfrak{M}$$

fiet ſtatim

$$P\,k\,R = \frac{(\theta - 1)\mathfrak{M}}{k'} = \frac{\mathfrak{M}(\theta - 1)(1 + \mathfrak{r}R - \mathfrak{q}kR)}{\theta}.$$

Praeterea iam conſiderandae ſunt aequationes ſequentes:

1°. $\mathfrak{B}\,\mathfrak{q} = (P - 1)\,M$;

2°. $\mathfrak{C}\,\mathfrak{r} = -(1 + P\,k)\,M - \mathfrak{q}$;

3°. $\mathfrak{D} = -(1 + P\,k\,R)\,M - \mathfrak{q} - \mathfrak{r}$;

4°. $\mathfrak{E} = (P\,k\,R\,k' - 1)\,M - \mathfrak{q} - \mathfrak{r} - 1$;

pro quarum reſolutione ſtatuamus breuitatis gratia

$\frac{1 - P}{\mathfrak{B}} = \zeta$ et $1 + P\,k = \eta$, vt fiat

$\mathfrak{q} = -\zeta M$ et $\mathfrak{r} = \frac{(\zeta - \eta)M}{\mathfrak{C}}$ ergo

$$3 + \mathfrak{q} + \mathfrak{r} = \frac{3\mathfrak{C} + (\zeta(1 - \mathfrak{C}) - \eta)M}{\mathfrak{C}} = M(\mathfrak{M} - 1);$$

vnde concluditur

$$M = \frac{3\mathfrak{C}}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta};\ \text{vnde viciſſim}$$

$$\mathfrak{q} = -\frac{3\zeta\mathfrak{C}}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}\ \text{et}\ \mathfrak{r} = \frac{3(\zeta - \eta)}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}$$

 Nunc

Nunc ergo habebimus $PkR = (\eta - 1)R$, ſeu

$$(\eta - 1)R = \mathfrak{M}.\frac{\theta - 1}{\theta}\left(1 + \frac{3R(\zeta + \zeta\mathfrak{C}k - \eta)}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}\right)$$

vnde ob rationes ante allegatas litteram k definire conuenit; quem in finem notaſſe iuuabit litteras ζ et η vna cum $\mathfrak{C}$ ſemper prae multiplicatione $\mathfrak{M}$ fore valde exiguas; alioquin enim campus praeter neceſſitatem diminueretur; contra vero R etiam fore numerum praemagnum; vnde ſuperior illa aequatio induet hanc formam

$$(\eta - 1)R = \frac{\theta - 1}{\theta}\left(\frac{\mathfrak{C}\mathfrak{M} + 3R(\zeta - \eta) + 3R\zeta\mathfrak{C}k}{\mathfrak{C}}\right)$$

ex qua ſequitur

$$k = \frac{(\eta - 1)\mathfrak{C}\theta R - (\theta - 1)\mathfrak{C}\mathfrak{M} - 3(\theta - 1)(\zeta - \eta)R}{3(\theta - 1)\zeta\mathfrak{C}R}$$

qui valor cum debeat eſſe poſitiuus, debebit eſſe

$$R > \frac{(\theta - 1)\mathfrak{C}\mathfrak{M}}{(\eta - 1)\mathfrak{C}\theta - 3(\theta - 1)(\zeta - \eta)}$$

exiſtente

$(\eta - 1)\mathfrak{C}\theta > 3(\theta - 1)(\zeta - \eta)$, ſeu $\mathfrak{C} > \frac{3(\theta - 1)(\zeta - \eta)}{(\eta - 1)\theta}$. Cum vero vt in problemate praecedente eſſe debeat $\mathfrak{C} < 1$.; numerator illius limitis minor eſſe debet ſuo denominatore, hincque $\zeta < \frac{\theta(4\eta - 1) - 3\eta}{3(\theta - 1)}$. His ergo conditionibus obſeruatis, ponamus breuitatis gratia iterum, vt ante, $R = i\mathfrak{M}$, ita, vt eſſe debeat

$$i > \frac{(\theta - 1)\mathfrak{C}}{(\eta - 1)\mathfrak{C}\theta - 3(\theta - 1)(\zeta - \eta)}$$

habebimus inde

$$k = \frac{i(\eta - 1)\mathfrak{C}\theta - (\theta - 1)\mathfrak{C} - 3i(\theta - 1)(\zeta - \eta)}{3i(\theta - 1)\zeta\mathfrak{C}} \text{ et } P = \frac{\eta - 1}{k};$$

pro quo valore duos caſus conſiderari conuenit. Si $P > 1$; tum debet eſſe $A > 0$. ideoque $B < 0$. quod quidem ſponte euenit, cum prodeat $\mathfrak{B} < 0$. Hoc ergo euenit, quando $k < \eta - 1$; ex quo concluditur

$$i < \frac{(\theta - 1)\mathfrak{C}}{(\eta - 1)\mathfrak{C}\theta - 3(\theta - 1)((\eta - 1)\zeta\mathfrak{C} + \zeta - \eta)}$$

qui limes manifeſto maior eſt ſuperiore. Sin autem littera i adeo hunc limitem ſuperet, tunc fiet $P < 1$. ideoque A negatiue ſumi debebit et quia $\mathfrak{B}$ prodit poſitiuum, B eatenus tantum erit poſitiuum, vti requiritur, quatenus fit $\mathfrak{B} < 1$. Fit vero ſemper $\mathfrak{B} < 1$, niſi fuerit $\zeta < 1$. atque ſi etiam fuerit $\zeta < 1 - P$ caſus erit impoſſibilis. Deinde cum ſit

$$P k R = (\eta - 1) i \mathfrak{M},$$

neglectis terminis prae $\mathfrak{M}$ valde paruis ob $\mathfrak{M} M = 3$ proxime, erit

$$\mathfrak{D} = -3i(\eta - 1), \text{ hinc } D = -\frac{3i(\eta - 1)}{3i(\eta - 1) + 1}.$$

Porro cum ſit

$$P k R k' = \frac{\mathfrak{M}}{T} = \mathfrak{M}(\theta - 1),$$

fiet eodem modo

$$\mathfrak{E} = 3\mathfrak{D} - 4 \text{ et } E = \frac{3\theta - 4}{5 - \theta} \text{ ſeu } E = -\frac{(3\theta - 4)}{3\theta - 5}.$$

Hinc ergo diſtantiae focales ita ſe habebunt:

$$p = \mathfrak{A} a; \; q = -\frac{A\mathfrak{B}}{P}. a; \; r = -\frac{AB\mathfrak{C}}{k - 1}. a;$$

$s =$

$$s = + \frac{ABC.3i(\eta-1)}{(\eta-1)i.\mathfrak{M}} . a = - \frac{3.ABC}{\mathfrak{M}} . a;$$
$$t = - \frac{3i(\eta-1)(2\theta-1)}{(3i(\eta-1)+1)(\theta-1)} . \frac{ABC}{\mathfrak{M}} . a;$$
$$u = - \frac{3i(\eta-1)(3\theta-4)}{(3i(\eta-1)+1)(3\theta-5)} . \frac{ABC}{\mathfrak{M}} . a;$$

vbi notetur eſſe quoque $q = \frac{1}{\zeta}(1 - \frac{1}{P}) A a$, ita, vt ſit q ad primum interuallum, vt $1 : \zeta$. Interualla autem erunt

$$1^{mum} = A a (1 - \frac{1}{P}) = \zeta q;$$
$$2^{dum} = - A B. a (\frac{1}{P} + \frac{1}{\eta-1});$$
$$3^{tium} = - A B C. a (\frac{1}{\eta-1} - \frac{1}{i(\eta-1)\mathfrak{M}})$$
$$= - \frac{ABC}{\eta-1} . a (1 - \frac{1}{i\mathfrak{M}});$$
$$4^{tum} = \frac{ABCD.a}{\mathfrak{M}} (\frac{1}{i(\eta-1)} + \frac{1}{\theta-1})$$
$$= - \frac{3(i(\eta-1)+\theta-1)}{(3i(\eta-1)+1)(\theta-1)} . \frac{ABC}{\mathfrak{M}} . a;$$
$$5^{tum} = - \frac{3i(\eta-1)(3\theta-4)(\theta-2)}{(3i(\eta-1)+1)(\theta-1)(3\theta-5)} . \frac{ABC}{\mathfrak{M}} . a.$$

Diſtantia vero oculi erit

$$O = \frac{u}{\mathfrak{M}M} = \frac{1}{3} u. \text{ proxime.}$$

Spatii vero conſpicui ſemidiameter erit

$$z = \frac{1}{4} a M = \frac{3.a}{4.\mathfrak{M}}.$$

Tum vero ſemidiameter aperturae lentis primae eſt $= x$ ſiue per formulam notam ſiue per experientiam definiendus. Lentis vero

$$2^{dae} = \frac{1}{4} \mathfrak{q} q + \frac{x}{P} = \frac{1}{4} . \frac{\zeta}{\mathfrak{M}} q + \frac{x}{P};$$
$$3^{tiae} = \frac{1}{4} \mathfrak{r} r + \frac{x}{Pk} = \frac{1}{4} . \frac{\zeta - \eta}{\mathfrak{C}\mathfrak{M}} r + \frac{x}{\eta-1};$$

reliquarum vero lentium, quae debent eſſe vtrinque aeque

aeque conuexae, semidiametri aperturarum erunt respectiue $\frac{1}{4}s$; $\frac{1}{4}t$ et $\frac{1}{4}u$. Denique autem mensura claritatis fiet $= 20. \frac{x}{\mathfrak{M}}$.

Coroll.

286. Si statuatur $\zeta = 0$, pro secunda lente erit $q = \infty$, qui casus eodem redit, ac si haec lens plane abesset; tum autem erit $k = \infty$ et $P = 0$, $\mathfrak{B} = \infty$ et $B = -1$, vnde etsi primum interuallum fit $= -\infty$, ob secundam lentem deficientem interuallum primae et tertiae lentis fiet nihilominus finitum $= \frac{\eta}{\eta - 1}$. A a. Deinde vero capi debebit

$$i > \frac{(\theta - 1)\mathfrak{C}}{(\eta - 1)\mathfrak{C}\theta + \eta(\theta - 1)}.$$

Pro $\mathfrak{C}$ vero sufficit, vt capiatur intra limites 1 et 0, quandoquidem C debet esse numerus positiuus ob $D < 0$. Reliquae vero determinationes manent, vt ante, si modo notetur, esse $B = -1$.

Coroll. 2.

287. Quia hoc casu lens secunda tollitur, hoc modo solutio habebitur problematis, quo microscopium ex quinque lentibus constructum quaeritur, quae ita sint dispositae, vt prima imago realis in primum interuallum, posterior vero in tertium incidat; cuius ergo problematis solutio etiam suppeditat campum triplicatum.

Coroll. 3.

288. Quia in genere ob rationes ante allegatas littera C semper designare debet numerum satis magnum, ne scilicet lentes posteriores fiant nimis exiguae, satis prope erit $\mathfrak{C} = 1$ atque adeo in praxi tuto statuere licebit $\mathfrak{C} = 1$. Tum igitur sumi debebit

$$i > \frac{\theta - 1}{(\eta - 1)\theta - 3(\theta - 1)(\zeta - \eta)} \text{ deinde vero}$$

$$k = \frac{i(\eta - 1)\theta - 3i(\theta - 1)(\zeta - \eta) - \theta + 1}{3i(\theta - 1)\zeta}.$$

CASVS I.

quo $i = \infty$ et $\vartheta = 3$.

289. Hoc ergo casu esse debet $\zeta < \frac{3\eta - 1}{2}$ tum vero erit

$$k = \frac{3\eta - 2\zeta - 1}{2\zeta} \text{ et } P = \frac{2(\eta - 1)\zeta}{3\eta - 2\zeta - 1}.$$

Vt igitur fiat $P > 1$, debebit esse $\zeta > \frac{3\eta - 1}{2\eta}$. Quare fiet $P > 1$, si capiatur ζ intra limites $\frac{3\eta - 1}{2}$ et $\frac{3\eta - 1}{2\eta}$; quo ergo casu A sumi debet positiue et quia reperitur $\mathfrak{B} < 0$, sponte fit $B < 0$. Sin autem sit $\zeta < \frac{3\eta - 1}{2\eta}$, tunc erit $P < 1$ hincque $A < 0$, ita, vt debeat esse $B > 0$, hincque $\mathfrak{B} < 1$, erit vero $\mathfrak{B} < 1$, si $1 - P < \zeta$, siue $P > 1 - \zeta$ quod quidem semper euenit, nisi sit $\zeta < 1$. Superest ergo examinare casum $\zeta < 1$. et quia tum esse debet $P > 1 - \zeta$ oritur inde haec relatio $\zeta(5\eta - 1) - 2\zeta^2 > 3\eta - 1$. vnde patet, esse debere

$\zeta =$

$$\zeta = \frac{5\eta - 1}{4} - \frac{1}{4}\sqrt{25\eta^2 - 34\eta + 9 - \alpha}$$

denotante α numerum quempiam positiuum, siue

$$\zeta > \frac{5\eta - 1}{4} - \frac{1}{4}\sqrt{25\eta^2 - 34\eta + 9}$$

qui ergo limes pro ζ pendet ab η, ita, vt sumto $\eta = 2$ debeat esse $\zeta > \frac{9 - \sqrt{41}}{4}$ seu $\zeta > \frac{5}{8}$ sin autem fuerit $\eta = 4$, debet esse $\zeta > \frac{19 - \sqrt{272}}{4}$ seu proxime $\zeta > \frac{5}{8}$. At si sit $\eta = 6$. prodit $\zeta > \frac{29 - \sqrt{785}}{4}$ seu proxime $\zeta > \frac{5}{8}$ vt ante, sicque patet ζ nunquam infra $\frac{5}{8}$ accipi posse. Nunc igitur pronuntiare poterimus, limites, intra quos ζ capi debeat, esse $\frac{5}{8}$ et $\frac{5\eta - 1}{8}$. His notatis distantiae focales erunt

$$p = \mathfrak{A}a;\ q = -\frac{A\mathfrak{B}}{P}a = \frac{1}{\zeta}\left(1 - \frac{\zeta}{P}\right)Aa.$$

$$r = -\frac{AB\mathfrak{C}}{\eta - 1}.a;\ s = -3\frac{ABC}{\mathfrak{M}}a.$$

$$t = -\frac{5}{2}.\frac{ABC}{\mathfrak{M}}a;\ u = -\frac{5}{4}.\frac{ABC}{\mathfrak{M}}.a.$$

et lentium interualla

$1^{mum} = Aa\left(1 - \frac{\zeta}{P}\right);$

$2^{dum} = -AB\left(\frac{\zeta}{P} + \frac{1}{\eta - 1}\right)a;$

$3^{tium} = -\frac{ABC}{\eta - 1}.a;$

$4^{tum} = -\frac{1}{2}.\frac{ABC}{\mathfrak{M}}.a;$

$5^{tum} = -\frac{5}{8}.\frac{ABC}{\mathfrak{M}}.a.$

Reliqua momenta se habent vti in problemate, quippe quae ab u non pendent.

Scholion.

290. Mirum hic videbitur, quod hoc casu tam P maius vnitate, quam minus vnitate fieri possit, cum in solutione problematis ostenderimus, tum solum P fieri > 1, quando littera i contineatur intra limites

$$\frac{(\theta - 1)\mathfrak{C}}{(\eta - 1)\mathfrak{C}\theta - 3(\theta - 1)(\zeta - \eta)} \text{ et } \frac{(\theta - 1)\mathfrak{C}}{(\eta - 1)\mathfrak{C}\theta - 3(\theta - 1)(\zeta - \eta + (\eta - 1)\zeta\mathfrak{C})}$$

quando vero i etiam posteriorem limitem superauerit; tum semper fore $P < 1$. Quare cum hic adeo sumserimus $i = \infty$, hinc vtique sequi videtur semper esse debere $P < 1$, quod tamen, vt vidimus, secus euenit. Ad quod dubium soluendum natura posterioris limitis accuratius perpendi debet, si enim is ipse iam fieret infinitus, tum certe mirari desinemus, si etiam sumto $i = \infty$ reperiatur $P > 1$. Sin autem in hoc limite posteriore denominator non solum euanescat, sed adeo negatiuus euadat; tum ipse limes non tam negatiuus, quam infinito maior spectari debebit, ita, vt positio $i = \infty$ adhuc inter illos limites contineri sit censenda. Nunc igitur manifestum est, limitem posteriorem fieri $= \infty$, si sumto $\mathfrak{C} = 1$ fuerit

$$\zeta = \frac{(4\eta - 1)\theta - 3\eta}{3\eta(\theta - 1)},$$

sumtoque $\mathfrak{D} = 3$, vti fecimus, si fuerit $\zeta = \frac{3\eta - 1}{2\eta}$. Sin autem sit $\zeta > \frac{3\eta - 1}{2\eta}$ (semper autem esse debet $\zeta < \frac{3\eta - 1}{2}$); tum ille limes fit quasi infinito maior, hincque $i = \infty$ ipso minor; vnde necessario fieri de-

bebit

bebit $P > 1$. Sin autem fit $\zeta < \frac{2\eta - 1}{2\eta}$; tum ille limes adhuc erit finitus ideoque valor $i = \infty$ illo erit fine dubio maior; vnde etiam his tantum cafibus fiet $P < 1$. Hoc notato iftum cafum aliquot exemplis illuftremus.

Exempl. I.

291. Sumamus $\eta = 2$ et cum pro ζ prior limes fit $\zeta < \frac{5}{2}$; pofterior vero limes $\frac{5}{4}$; fumamus $\zeta = 2$, vt cadat intra hos limites; vnde fiet $k = \frac{1}{4}$ et $P = 4$, hinc $\mathfrak{B} = -\frac{5}{3}$ et $B = -\frac{5}{8}$ vnde diftantiae focales erunt

$$p = \mathfrak{A}\, a;\ q = -\tfrac{1}{4}\mathfrak{B}.\ A\, a = \tfrac{5}{8} A\, a;$$
$$r = \tfrac{5}{3} A\, a;\ s = \tfrac{9}{5}\cdot\tfrac{C}{\mathfrak{M}}\cdot A\, a;$$
$$t = \tfrac{3}{2}\,\tfrac{C}{\mathfrak{M}}\cdot A\, a,\ u = \tfrac{3}{4}\cdot\tfrac{C}{\mathfrak{M}}\cdot A\, a.$$

et lentium interualla

$$1^{mum} = \tfrac{3}{4} A\, a;\ 2^{dum} = \tfrac{5}{4} A\, a;$$
$$3^{tium} = \tfrac{5}{5} C.\ A\, a;\ 4^{tum} = \tfrac{3}{10}\cdot\tfrac{C}{\mathfrak{M}}\cdot A\, a.$$
$$\text{et }\ 5^{tum} = \tfrac{3}{8}\,\tfrac{C}{\mathfrak{M}}\cdot A\, a.$$

Diftantia oculi $O = \frac{1}{3} u$ proxime; $z = \frac{3\alpha}{4\mathfrak{M}}$; femidiameter aperturae lentis primae $= x$; fecundae $= \frac{3}{2\mathfrak{M}} q + \frac{x}{4}$; tertiae $= x$. ac denique menfura claritatis $= 20.\frac{x}{\mathfrak{M}}$, vti femper.

Exemplum 2.

292. Manente $\eta = 2$, aequetur ζ ipſi alteri limiti, ſcilicet $\zeta = \frac{5}{4}$ fietque $k = 1$ et $P = 1$, hinc ergo prodit $\mathfrak{B} = 0$ et $B = 0$. Quare ne tam prima lens, quam interualla euaneſcant, ſumi debet $A = \infty$ ideoque $\mathfrak{A} = 1$, ita, vt ſit $A \mathfrak{B}$ ſiue $A B = -\frac{q}{a}$ et cum ſit $\mathfrak{B} = \frac{4}{5}(1 - P)$, erit reuera $1 - P = \frac{5}{4}\mathfrak{B}$ hincque $A a (1 - \frac{1}{P}) = \frac{5}{4} q$. Quare diſtantiae focales erunt

$$p = a;\; q = q;\; r = \mathfrak{C} q;$$

$$s = 3 \cdot \frac{C}{\mathfrak{M}} \cdot q;\; t = \frac{5}{2} \cdot \frac{C}{\mathfrak{M}} \cdot q;\; u = \frac{5}{4} \cdot \frac{C}{\mathfrak{M}} \cdot q;$$

vbi ſecunda q arbitrio noſtro relinquitur. Tum vero lentium interualla

$$1^{mum} = \frac{5}{4} q;\; 2^{dum} = 2 q;$$

$$3^{tium} = C q;\; 4^{tum} = \frac{1}{2} \cdot \frac{C}{\mathfrak{M}} \cdot q;\; 5^{tum} = \frac{5}{8} \cdot \frac{C}{\mathfrak{M}} \cdot q.$$

Valores O et z erunt, vt ante; at ſemidiameter aperturae lentis

$$2^{dae} = \frac{15}{16 \mathfrak{M}} q + x \text{ et } 3^{tiae} = \frac{9}{16 \mathfrak{M}} r + x.$$

Exempl. 3.

293. Manente $\eta = 2$, ſumatur $\zeta < \frac{5}{4}$ et cum eſſe debeat $\zeta > \frac{5}{8}$ vti oſtendimus, ſumatur $\zeta = \frac{3}{4}$ eritque $k = \frac{7}{5}$ et $P = \frac{3}{7}$; vnde fit $\mathfrak{B} = \frac{16}{21}$, hincque $B = \frac{16}{5}$, qui valor cum ſit poſitiuus, littera A negatiue capi debet, vti etiam primum interuallum poſtulat ob

$P < 1.$

$P < 1$. Sit igitur $A = -\alpha$ ideoque $\mathfrak{A} = \frac{\alpha}{\alpha - 1}$ eruntque distantiae focales

$$p = \frac{\alpha a}{\alpha - 1};\ q = \tfrac{16}{9}\,\alpha a;$$
$$r = \tfrac{16}{5}\,\mathfrak{C}\,\alpha a;\ s = \tfrac{48}{5}.\,\frac{C}{\mathfrak{M}}\,\alpha a;$$
$$t = 8.\,\frac{C}{\mathfrak{M}}\,\alpha a;\ u = 4.\,\frac{C}{\mathfrak{M}}\,\alpha a.$$

Interualla lentium

$$1^{mum} = \tfrac{4}{3}\,\alpha a;\ 2^{dum} = \tfrac{32}{5}\,\alpha a;$$
$$3^{tium} = \tfrac{16}{5}\,C.\,\alpha a;\ 4^{tum} = \tfrac{8}{5}.\,\frac{C}{\mathfrak{M}}\,\alpha a;$$
$$5^{tum} = 2.\,\frac{C}{\mathfrak{M}}.\,\alpha a.$$

Reliqua se habebunt, vt ante ac siqua differentia in aperturis deprehenditur, ea in praxi attendi non meretur; interim tamen semidiameter secundae lentis

$$= \tfrac{9}{16}\,\frac{q}{\mathfrak{M}} + \tfrac{7}{3}\,x \text{ et } 3^{tiae} = \tfrac{3}{16}\,\frac{r}{\mathfrak{M}} + x.$$

Exempl. 4.

294. Statuatur nunc $\eta = 4$ et cum esse debeat $\zeta < \frac{11}{3}$, posterior vero limes sit $\frac{11}{3}$, quo scilicet casus $P > 1$ et $P < 1$ distinguntur. Sumatur $\zeta = 3$ et erit $k = \frac{5}{6}$ et $P = \frac{48}{5}$. Vnde fit $\mathfrak{B} = -\frac{13}{15}$ et $B = -\frac{13}{28}$. Distantiae ergo focales lentium erunt

$$p = \mathfrak{A}\,a;\ q = \tfrac{13}{34}\,A\,a;$$
$$r = \tfrac{13}{84}\,\mathfrak{C}.\,A\,a;\ s = \tfrac{39}{28}.\,\frac{C}{\mathfrak{M}}.\,A\,a;$$
$$t = \tfrac{65}{56}.\,\frac{C}{\mathfrak{M}}.\,A\,a;\ u = \tfrac{65}{112}.\,\frac{C}{\mathfrak{M}}.\,A\,a = \tfrac{1}{2}\,t.$$

Inter-

Interualla vero lentium

$$1^{mum} = \tfrac{13}{18}\,\mathrm{A}\,a;\quad 2^{dum} = \tfrac{143}{504}\,\mathrm{A}\,a;$$
$$3^{tium} = \tfrac{13}{54}\cdot \mathrm{C}.\ \mathrm{A}\,a;\quad 4^{tum} = \tfrac{13}{56}\cdot \tfrac{\mathrm{C}}{\mathfrak{M}}\cdot \mathrm{A}\,a;$$
$$5^{tum} = \tfrac{65}{224}\cdot \tfrac{\mathrm{C}}{\mathfrak{M}}\cdot \mathrm{A}\,a.$$

Denique ſemidiameter aperturae lentis

$$2^{dae} = \tfrac{9}{4\mathfrak{M}}\cdot q + \tfrac{5}{18}\,x.\ \text{et}\ 3^{tiae} = \tfrac{1}{4\mathfrak{M}}\,r + \tfrac{1}{3}\,x.$$

Exempl. 5.

295. Manente $\eta = 4$, fit $\zeta = \frac{11}{8}$ ac erit $k = 3$ et $\mathrm{P} = 1$ vnde $\mathfrak{B} = \frac{1-\mathrm{P}}{\zeta} = \frac{8}{11}(1-\mathrm{P}) = 0$ et $\mathrm{B} = 0$. Vnde aſſumi debet $\mathrm{A} = \infty$, ita, vt fiat $\mathfrak{A} = 1$; tum igitur introducto q in calculum fiet $\mathrm{A}\mathfrak{B} = \mathrm{A}\mathrm{B} = -\frac{q}{a}$; vnde fit $\mathrm{A}\,a\frac{(1-\mathrm{P})}{\mathrm{P}} = \frac{11}{8}\,q$ ſicque diſtantiae focales erunt

$$p = a;\ q = q;$$
$$r = \tfrac{1}{3}\,\mathfrak{C}\,q;\ s = 3\cdot \tfrac{\mathrm{C}}{\mathfrak{M}}\,q;$$
$$t = \tfrac{5}{2}\cdot \tfrac{\mathrm{C}}{\mathfrak{M}}\,q\ \text{et}\ u = \tfrac{5}{4}\cdot \tfrac{\mathrm{C}}{\mathfrak{M}}\,q.$$

et lentium interualla

$$1^{mum} = \tfrac{11}{8}\,q;\ 2^{dum} = \tfrac{4}{3}\,q;$$
$$3^{tium} = \tfrac{1}{3}\,\mathrm{C}\,q;\ 4^{tum} = \tfrac{1}{2}\cdot \tfrac{\mathrm{C}}{\mathfrak{M}}\,q$$
$$\text{et}\ 5^{tum} = \tfrac{5}{8}\cdot \tfrac{\mathrm{C}}{\mathfrak{M}}\,q.$$

Semidiameter aperturae lentis

$$2^{dae} = \tfrac{33}{32}\cdot \tfrac{q}{\mathfrak{M}} + x;\ 3^{tiae} = \tfrac{63}{32}\cdot \tfrac{r}{\mathfrak{M}} + \tfrac{1}{3}\,x.$$

Exempl.

Exempl. 6.

296. Manente adhuc $\eta = 4$, fit $\zeta = \frac{2}{3}$, ac reperitur $k = \frac{29}{4}$ et $P = \frac{12}{29}$ vnde fit $\mathfrak{B} = \frac{51}{58}$ et $B = \frac{51}{7}$ ex quo tanto valore iam perſpicuum eſt, huiusmodi microſcopiis in praxi locum concedi non poſſe.

CASVS II.

quo $\eta = 4$ et $\vartheta = 3$.

297. Quoniam debet eſſe $\eta > 1$ eiusque valor nimis paruus quibusdam incommodis eſt obnoxius; nimis magnus vero campo nocet, mediocri ſemper valore vti conueniet, cuiusmodi eſt $\eta = 4$; tum vero valor $\vartheta = 3$ ſeu $T = \frac{1}{2}$ idoneum interuallum inter vltimas lentes praebet; littera autem $\mathfrak{C}$ tam parum ab vnitate deficere debet, vt in noſtris formulis liceat ſumere $\mathfrak{C} = 1$. His praemiſſis pro ζ limes erit $\zeta < \frac{11}{2}$. Pro $\mathfrak{C}$ vero habebimus

$$\mathfrak{C} < 1 \text{ et } \mathfrak{C} > \tfrac{2}{3}(\zeta - 4);$$

vnde patet, etiamſi ſit $\zeta = \frac{11}{2}$, tamen fore $\mathfrak{C} = 1$, ita, vt certe ſumi poſſit $\mathfrak{C} = 1$. Porro pro littera i fiet $i > \frac{2}{33 - 6\zeta}$. Deinde obtinebimus

$$k = \frac{(33 - 6\zeta)i - 2}{6\zeta i} \text{ vnde fit } P = \frac{18\zeta i}{(33 - 6\zeta)i - 2}.$$

Hinc oritur $\mathfrak{B} = \frac{(33 - 24\zeta)i - 2}{((33 - 6\zeta)i - 2)\zeta}$.

Hic duos caſus diſtingui oportet, prior eſt, quo fit $P > 1$ hincque A valorem poſitiuum habere debet, quod euenit, si $\mathfrak{B}$ fiat < 0 ideoque $i < \frac{2}{33 - 24\zeta}$, siue quando i continetur intra limites $\frac{2}{33 - 6\zeta}$ et $\frac{2}{33 - 24}$ atque hoc casu etiam B fit negatiuum, ita, vt sit $AB < 0$. Alter casus, quo $P < 1$; locum habet, si fuerit $i > \frac{2}{33 - 24\zeta}$, quo casu A valorem habebit negatiuum; quo igitur B nanciscatur valorem positiuum, debet esse $\mathfrak{B} < 1$. hincque $i < \frac{2(1 - \zeta)}{6\zeta^2 - 57\zeta + 33}$. Cum igitur sit $i > \frac{2}{33 - 24\zeta}$, id euenit, si fuerit

$$\frac{1 - \zeta}{6\zeta^2 - 57\zeta + 33} > \frac{1}{33 - 24\zeta}$$

vnde sequeretur $\zeta > 0$. His igitur notatis distantiae focales erunt

$$p = \mathfrak{A} a;\; q = -\frac{A\mathfrak{B}}{P} . a;$$

$$r = -\frac{1}{3} A B \mathfrak{C} a;\; s = -3 . \frac{ABC}{\mathfrak{M}} . a;$$

$$t = -\frac{45 i}{38 i + 2} . \frac{ABC}{\mathfrak{M}} . a. \text{ et}$$

$$u = -\frac{45 i}{36 i + 4} . \frac{ABC}{\mathfrak{M}} . a.$$

siue etiam erit $q = +\frac{1}{\zeta}(1 - \frac{1}{P}) A a$ et interualla lentium sunt

$$1^{mum} = A a (1 - \frac{1}{P})$$

$$2^{dum} = -A B a (\frac{1}{P} + \frac{1}{3})$$

$$3^{tium} = -\frac{ABC}{3} . a (1 - \frac{1}{i\mathfrak{M}})$$

$$4^{tum} = -\frac{3(3i + 2)}{2(9i + 1)} . \frac{ABC}{\mathfrak{M}} . a$$

$$5^{tum} = -\frac{45 i}{4(9i + 1)} . \frac{ABC}{\mathfrak{M}} . a.$$

Deinde

Deinde cum ſit

$$M = \frac{3}{\mathfrak{M}+3} \text{ fiet } z = \frac{1}{4} \cdot \frac{3a}{\mathfrak{M}+3}$$

et diſtantia oculi

$$O = \frac{1}{3} u \left(1 + \frac{2}{\mathfrak{M}}\right).$$

Semidiameter porro aperturae lentis ſecundae erit

$$= \frac{3}{4} \frac{\zeta}{\mathfrak{M}} q + \frac{x}{P}, \text{ et } 3^{tiae} = \frac{3}{4} \frac{(\zeta-4)r}{\mathfrak{M}} + \frac{1}{3} x.$$

Denique definito x erit menſura claritatis $= \frac{20 \cdot x}{\mathfrak{M}}$.

Exempl. I.

298. Sit $\zeta = 0$ et cum eſſe debeat $i > \frac{2}{37}$, praeterea vero pro $P > 1$ ſit $i < \frac{2}{37}$ ſtatuamus $i = \frac{2}{37}$ eritque $P = \frac{0}{0}$. ſiue P non determinatur, modo non ſit minus vnitate; adeoque $\mathfrak{B} = -\frac{(P-1)}{\zeta}$, ita, vt $\mathfrak{B}$ ſemper ſit ∞, niſi capiatur $P = 1$. Primo igitur non ſit $P = 1$. erit $\mathfrak{B} = \infty$ et $B = -1$, ita, vt A ſit maius nihilo; hinc igitur diſtantiae focales erunt $p = \mathfrak{A} a$; $q = \infty$; ſeu quod idem eſt ſecunda lens tollitur;

$$r = \frac{1}{3} A \mathfrak{C} . a; \; s = \frac{3AC}{\mathfrak{M}} . a;$$

$$t = \frac{15}{17} . \frac{AC}{\mathfrak{M}} . a \text{ et } u = \frac{15}{34} . \frac{AC}{\mathfrak{M}} . a.$$

et interualla lentium

$$1^{mum} + 2^{dum} = \frac{4}{3} A a;$$

$$3^{tium} = \frac{AC}{3}\left(1 - \frac{33}{2\mathfrak{M}}\right); \; 4^{tum} = \frac{75}{34} . \frac{AC}{\mathfrak{M}} . a;$$

$$\text{et } 5^{tum} = \frac{15}{136} . \frac{AC}{\mathfrak{M}} . a.$$

Reliqua manent, nisi quod sit semidiameter aperturae lentis tertiae $= 3 \cdot r + \frac{1}{3} x$.

Sin autem caperetur $P = 1$, vtcunque calculus instituatur, primum interuallum semper euanesceret; verum superfluum est ad hunc casum attendere, cum in prioribus formulis littera P plane ex calculo euanuerit, ita, vt illae formulae subsistant, quicunque valor ipsi P tribuatur atque adeo non solum si ponatur $P = 1$, sed etiam si P vnitate minus sumeretur, quod etsi nostrae hypothesi repugnat, tamen ob lentem secundam prorsus deficientem haec anomalia admitti debet.

Exempl. 2.

299. Maneat $\zeta = 0$, sed capiatur $i > \frac{2}{33}$ si fieri queat, quo casu fiet $P < 1$, quia autem hoc ipso casu iterum esse debet $i < \frac{2}{33}$, hic casus ad praecedentem redigitur; quem quidem iam notauimus aeque ad valores $P < 1$, quam ad $P > 1$ patere. Interim tamen cum secunda lens plane deficiat, posterior limes $i < \frac{2}{33}$ sponte cessat, ita, vt nunc liceat assumere $i > \frac{2}{33}$, vti iam obseruauimus in Coroll. 1. problemati subnexo. Tum igitur erit

$$p = \mathfrak{A}\, a;\ q = \infty;\ \text{seu lens secunda deest;}$$

$$r = \frac{A\mathfrak{C}}{3} \cdot a;\ s = \frac{2\,AC}{\mathfrak{M}} \cdot a;$$

$$t = \frac{45 \cdot i}{18\, i + 2} \cdot \frac{AC}{\mathfrak{M}} \cdot a;\ u = \frac{45 \cdot i}{36\, i + 4} \cdot \frac{AC}{\mathfrak{M}} \cdot a.$$

Inter-

Interualla vero lentium

$$1^{mum} + 2^{dum} = \tfrac{4}{3} \cdot A\, a;$$

$$3^{tium} = \tfrac{1}{3} A\, C\, a \left(1 - \tfrac{1}{i\mathfrak{M}}\right);$$

$$4^{tum} = \tfrac{3(3i+2)}{2(9i+1)} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a;$$

$$5^{tum} = \tfrac{45 i}{8(9i+1)} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a.$$

Quod autem ad litteram i attinet, quoniam k non amplius in calculum iugreditur, ex aequatione, vnde k definiuimus, iam i definiatur eritque $i = \frac{2}{33 - 6\zeta} = \frac{2}{33}$. Proprie autem hae formulae continent ſolutionem problematis, quo quinque tantum lentes poſtulantur, ita disponendae, vt ambae imagines reales in primum et tertium interuallum incidant.

Exempl. 3.

300. Sit $\zeta = \frac{1}{2}$, ſumique debebit $i > \frac{1}{15}$ fietque $P > 1$, ſi fuerit $i < \frac{2}{21}$; ſin autem ſit $i > \frac{2}{21}$ ſimul fiet $P < 1$. Hic vero ſumamus $i = \frac{1}{12}$; fietque $P = \frac{3}{2}$, hinc $\mathfrak{B} = -1$, et $B = -\frac{1}{2}$, vnde diſtantiae focales erunt

$$p = \mathfrak{A}\, a; \; q = \tfrac{2}{3} A\, a;$$

$$r = \tfrac{1}{8} A\, \mathfrak{C}\, a; \; s = \tfrac{3}{2} \tfrac{AC}{\mathfrak{M}} \cdot a;$$

$$t = \tfrac{45}{64} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a; \; u = \tfrac{15}{56} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a.$$

et lentium interualla

$$1^{mum} = \tfrac{1}{3} A\, a; \; 2^{dum} = \tfrac{1}{2} A\, a;$$

 3^{tium}

$3^{tium} = \frac{1}{8}\,A\,C.\,a\,(1 - \frac{12}{\mathfrak{M}})$;

$4^{tum} = \frac{27}{28} \cdot \frac{AC}{\mathfrak{M}} \cdot a$; $5^{tum} = \frac{15}{112} \cdot \frac{AC}{\mathfrak{M}} \cdot a$.

Reliqua momenta sunt, vt ante, nisi quod sit semidiameter aperturae

2^{dae} lentis $= \frac{5}{8} \cdot \frac{q}{\mathfrak{M}} + \frac{2}{3}\,x$;

et $3^{tiae} = \frac{21}{8} \cdot \frac{r}{\mathfrak{M}} + \frac{1}{3}\,x$.

Has formulas commode ad telescopia adplicare licet, quia posito $A\,a = p$ longitudo tantum fit $= \frac{5}{6}p + \frac{1}{6}C.p$, ita, vt ea non multum superet p, etiamsi pro C numerus satis magnus capiatur.

Exempl. 4.

301. Maneat $\zeta = \frac{1}{2}$, sed sumatur $i > \frac{2}{21}$ et quia hinc fit $P = \frac{9i}{30i-2}$ ideoque $\mathfrak{B} = \frac{21i-2}{15i-1}$, vt fiat $\mathfrak{B} < 1$. debet esse $21.\,i - 2 < 15\,i - 1$ siue $i < \frac{1}{6}$. Capiatur ergo $i = \frac{1}{8}$ fietque $P = \frac{9}{14}$ et $\mathfrak{B} = \frac{5}{7}$, hinc $B = \frac{5}{2}$; ergo A debet esse negatiuum. Statuatur ergo $A = -\alpha$ et distantiae focales erunt

$p = \frac{\alpha}{1-\alpha}\,a$; $q = \frac{10}{9}\,\alpha\,a$.

$r = \frac{5}{6}\,\mathfrak{C}.\,\alpha\,a$. $s = \frac{15}{2} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha\,a$.

$t = \frac{225}{68} \cdot \frac{C}{\mathfrak{M}}\,\alpha.\,a$. et $u = \frac{225}{196} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha\,a$.

et lentium interualla

$1^{mum} = \frac{5}{9} \cdot \alpha\,a$. $2^{dum} = \frac{85}{18} \cdot \alpha\,a$.

3^{tium}

$$3^{tium} = \tfrac{5}{a}.\, C.\, \alpha\, a \left(1 - \tfrac{8}{\mathfrak{M}}\right).$$

$$4^{tum} = \tfrac{57}{34}.\, \tfrac{C}{\mathfrak{M}}\, \alpha\, a.$$

$$5^{tum} = \tfrac{225}{272}.\, \tfrac{C}{\mathfrak{M}}.\, \alpha\, a.$$

Tum vero ſemidiameter aperturae lentis ſecundae et tertiae

$$= \tfrac{3}{8}.\, \tfrac{q}{\mathfrak{M}} + \tfrac{14}{9} x \text{ et } \tfrac{21}{8} r + \tfrac{1}{3} x.$$

Has autem formulas ad teleſcopia adplicare non licet, quia A erat negatiuum.

Exempl. 5.

302. Sit $\zeta = 1.$ et cum ſumi debeat $i > \frac{2}{27}$ atque vt prodeat $P > 1$, $i < \frac{2}{9}$ capiatur $i = \frac{1}{10}$ fietque $P = \frac{18}{7}$ et $\mathfrak{B} = -\frac{11}{7}$ et $B = -\frac{11}{18}$; vnde diſtantiae focales erunt

$$p = \mathfrak{A}\, a;\; q = \tfrac{11}{18}\, A\, a;$$

$$r = \tfrac{11}{54}.\, \mathfrak{C}.\, A\, a;\; s = \tfrac{11}{6}.\, \tfrac{AC}{\mathfrak{M}}.\, a;$$

$$t = \tfrac{55}{76}.\, \tfrac{C}{\mathfrak{M}}.\, A\, a;\; u = \tfrac{55}{152}.\, \tfrac{C}{\mathfrak{M}}.\, A\, a.$$

et lentium interualla

$$1^{mum} = \tfrac{11}{18}\, A\, a;\; 2^{dum} = \tfrac{145}{324}.\, A\, a;$$

$$3^{tium} = \tfrac{11}{54}.\, A\, C.\, a \left(1 - \tfrac{10}{\mathfrak{M}}\right);$$

$$4^{tum} = \tfrac{253}{228}.\, \tfrac{AC}{\mathfrak{M}}.\, a.\;\; 5^{tum} = \tfrac{55}{304}.\, \tfrac{AC}{\mathfrak{M}}.\, a.$$

et ſemidiameter aperturae lentis

$$2^{dae} = \tfrac{3}{4}.\, \tfrac{q}{\mathfrak{M}} + \tfrac{7}{18} x.\;\; 3^{tiae} = \tfrac{9}{4}.\, \tfrac{r}{\mathfrak{M}} + \tfrac{1}{3} x.$$

quas

quas formulas etiam commode ad telescopia transferre licet.

Exempl. 6.

303. Maneat $\zeta = 1$. sed sumatur $i = \frac{2}{9}$ fietque $P = 1$. et $\mathfrak{B} = 0$. hinc $B = 0$ hinc A capi debet $= \infty$, ideoque $\mathfrak{A} = 1$. tum autem esse debet

$$A\mathfrak{B} = AB = -\frac{q}{a} \text{ vnde fit}$$

$$Aa\left(1 - \frac{1}{P}\right) = Aa(P-1) = -A\mathfrak{B}a = q.$$

hinc distantiae focales erunt

$$p = a;\ q = q;$$

$$r = \tfrac{1}{3}\mathfrak{C}.\, q;\ s = 3.\, \tfrac{C}{\mathfrak{M}}\, q;$$

$$t = \tfrac{5}{3}.\, \tfrac{C}{\mathfrak{M}}\, q;\ u = \tfrac{5}{1}.\, \tfrac{C}{\mathfrak{M}}.\, q.$$

et lentium interualla

$$1^{mum} = q;\ 2^{dum} = \tfrac{4}{3}.\, q;$$

$$3^{tium} = \tfrac{1}{3} C\left(1 - \tfrac{9}{2\mathfrak{M}}\right) q;\ 4^{tum} = \tfrac{4}{3}.\, \tfrac{C}{\mathfrak{M}}\, q;$$

$$5^{tum} = \tfrac{5}{12}.\, \tfrac{C}{\mathfrak{M}}\, q.$$

et semidiametri aperturae lentis

$$2^{dae} = \tfrac{3}{4}.\, \tfrac{q}{\mathfrak{M}} + x;$$

$$3^{tiae} = \tfrac{9}{4}.\, \tfrac{r}{\mathfrak{M}} + \tfrac{1}{3} x.$$

Exempl. 7.

304. Maneat $\zeta = 1$. sed capiatur $i > \frac{2}{9}$ et cum sit $P = \frac{18i}{27i-2}$, hincque $\mathfrak{B} = \frac{9i-2}{27i-2}$, quae fractio iam sponte

ſponte vnitate eſt minor, ſumamus ergo $i = 1$. erit $P = \frac{18}{25}$ et $\mathfrak{B} = \frac{7}{25}$, $B = \frac{7}{18}$ vnde A debet eſſe negatiuum, ſtatuatur ergo $= -\alpha$; eruntque diſtantiae focales

$$p = \frac{\alpha}{\alpha - 1} \cdot a; \quad q = \frac{7}{18} \cdot \alpha a;$$

$$r = \frac{7}{54} \mathfrak{C} \cdot \alpha a; \quad s = \frac{7}{6} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha a;$$

$$t = \frac{35}{40} \cdot \frac{C}{\mathfrak{M}} \alpha a; \quad u = \frac{35}{60} \cdot \frac{C}{\mathfrak{M}} \alpha a.$$

et interualla

$$1^{mum} = \frac{72}{18} \alpha a;$$

$$2^{dum} = \frac{217}{324} \alpha a;$$

$$3^{tium} = \frac{7}{54} \cdot C \cdot \alpha a \left(1 - \frac{5}{\mathfrak{M}}\right);$$

$$4^{tum} = \frac{7}{54} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha a;$$

$$5^{tum} = \frac{7}{32} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha a.$$

et ſemidiametri aperturae lentis

$$2^{dae} = \frac{3}{4} \cdot \frac{q}{\mathfrak{M}} + \frac{25}{18} x.$$

$$3^{tiae} = \frac{9}{4} \cdot \frac{r}{\mathfrak{M}} + \frac{1}{3} x.$$

Exempl. 8.

305. Sit nunc $\zeta = 4 = \eta$ et cum eſſe debeat $i > \frac{2}{9}$; vt autem fiat $P > 1$, $i < -\frac{2}{33}$ qui limes vt infinito maior ſpectari debet, ſi enim ſumſiſſemus ζ minus, ſcilicet $\zeta = \frac{11}{8}$; tum prodiiſſet $i < \frac{2}{6}$, quod indicio fuiſſet, i quantumuis magnum accipi poſſe, ſemperque fore $P < 1$. Quod autem de valore $\zeta = \frac{11}{8}$

valet, multo magis de maioribus valet. Sit ergo $i = \frac{1}{3}$ eritque $P = 24$ et $\mathfrak{B} = -\frac{23}{4}$ et $B = -\frac{23}{27}$; vnde colliguntur diſtantiae focales:

$$p = \mathfrak{A}a;\quad q = \frac{23}{96}.\,Aa;$$

$$r = \frac{23}{81}\,A\,\mathfrak{C}a;\quad s = \frac{23}{9}.\frac{AC}{\mathfrak{M}}.\,a;$$

$$t = \frac{115}{72}.\frac{AC}{\mathfrak{M}}.\,a;\quad u = \frac{115}{144}.\frac{AC}{\mathfrak{M}}.\,a.$$

et interualla lentium

$$1^{mum} = \frac{23}{24}.\,Aa;$$

$$2^{dum} = \frac{23}{72}.\,Aa;$$

$$3^{tium} = \frac{23}{81}.\,CAa\left(1 - \frac{3}{\mathfrak{M}}\right);$$

$$4^{tum} = \frac{23}{24}.\frac{AC}{\mathfrak{M}}.\,a;$$

$$5^{tum} = \frac{115}{288}.\frac{AC}{\mathfrak{M}}.\,a.$$

et ſemidiameter aperturae lentis

$$2^{dae} = \frac{57}{\mathfrak{M}} + \frac{5}{24}x. \text{ et } 3^{tiae} = \frac{1}{3}x.$$

Exempl. 9.

306. Maneat $\zeta = 4$ et ſit $i = 1$. fiet $P = \frac{72}{7}$; ergo $\mathfrak{B} = -\frac{65}{28}$; hinc $B = -\frac{65}{93}$; vnde colliguntur diſtantiae focales

$$p = \mathfrak{A}a;\quad q = \frac{65}{288}.\,Aa;$$

$$r = \frac{65}{288}\,A\,\mathfrak{C}.\,a;\quad s = \frac{65}{31}.\frac{AC}{\mathfrak{M}}.\,a;$$

$$t = \frac{195}{334}.\frac{AC}{\mathfrak{M}}.\,a;\quad u = \frac{195}{348}.\frac{AC}{\mathfrak{M}}.\,a = \frac{1}{2}t.$$

et

et interualla lentium

$1^{mum} = \frac{65}{72} . \mathrm{A}\, a;$

$2^{dum} = \frac{65}{216} . \mathrm{A}.\, a;$

$3^{tium} = \frac{65}{279}\, \mathrm{A\,C}.\, a\left(1 - \frac{1}{\mathfrak{M}}\right);$

$4^{tum} = \frac{65}{124} . \frac{\mathrm{A\,C}}{\mathfrak{M}} . a;$

$5^{tum} = \frac{195}{496} . \frac{\mathrm{A\,C}}{\mathfrak{M}} . a.$

et ſemidiameter aperturae lentis

$2^{dae} = \frac{3q}{\mathfrak{M}} + \frac{7}{72} x$ et $3^{tiae} = \frac{1}{3} x.$

Scholion.

307. Horum exemplorum ea inprimis ſunt notatu digna, in quibus fiebat $\mathrm{P} = 1.$ quia tum littera A abibat in infinitum eratque $\mathfrak{A} = 1.$ In caſu igitur ſecundo, quem hactenus ſumus contemplati, ſtatuamus in genere $\mathrm{P} = 1.$ ac ſumi debebit $i = \frac{2}{33 - 24\zeta}.$ Tum ergo erit $\mathfrak{B} = 0$, ſimulque $\mathrm{B} = 0.$ vnde ſumto $\mathfrak{A} = 1$ et $\mathrm{A} = \infty$ fiet $q = -\mathrm{A}\,\mathfrak{B}\, a$, hinc viciſſim $\mathrm{A}\,\mathfrak{B} = \mathrm{A\,B} = -\frac{q}{a}$, ita, vt nunc diſtantia focalis ſecundae lentis q arbitrio noſtro penitus relinquatur; tum autem ad primum interuallum inueniendum ob $\mathfrak{B} = \frac{1 - \mathrm{P}}{\zeta}$, erit $\mathrm{A}\, a . \frac{\mathrm{P} - \mathrm{P}}{\mathrm{P}} = \mathrm{A}\, a . \mathrm{P} - 1 = -\mathrm{A}\,\mathfrak{B}.\zeta a = \zeta q,$ atque hinc in genere diſtantiae focales ita ſe habebunt:

$p = a;\ q = q;\ r = \frac{1}{3}\,\mathfrak{C}\, q;$

$s = \frac{3\mathrm{C}}{\mathfrak{M}} q;\ t = + \frac{15}{17 - 8\zeta} . \frac{\mathrm{C}}{\mathfrak{M}} q;$

$u = \frac{15}{34 - 16\zeta} . \frac{\mathrm{C}}{\mathfrak{M}} . q$ ſeu $u = \frac{1}{2} t.$

Interualla vero lentium erunt

$$1^{mum} = \zeta q; \quad 2^{dum} = \tfrac{4}{3} \cdot q;$$

$$3^{tium} = \tfrac{1}{3} q \left(1 - \frac{33 - 24\zeta}{2\mathfrak{M}}\right) = \tfrac{1}{3} q \left(1 - \frac{5(11 - 8\zeta)}{2\mathfrak{M}}\right);$$

$$4^{tum} = \frac{12(1 - 2\zeta)}{17 - 8\zeta} \cdot \frac{C}{\mathfrak{M}} q; \quad 5^{tum} = \frac{15}{4(17 - 8\zeta)} \cdot \frac{C}{\mathfrak{M}} \cdot q.$$

vbi ergo manifeſtum eſt, neceſſario ſumi debere $\zeta < \tfrac{3}{2}$. Tum vero erit ſemidiameter aperturae lentis

$$2^{dae} = \tfrac{3}{4} \cdot \frac{\zeta}{\mathfrak{M}} \cdot q + x; \text{ et } 3^{tiae} = \tfrac{1}{4} \frac{(\zeta - 4)}{\mathfrak{M}} \cdot r + \tfrac{1}{3} x.$$

Ceterum hic manifeſtum eſt, iſtas formulas ad teleſcopia accommodari neutiquam poſſe. Cum igitur omnes caſus, qui quidem in praxi locum habere poſſunt, adeo pro ſex lentibus euoluerimus iisque tantum campum conciliauerimus, quo maior deſiderari vix queat; huic capiti finem imponimus ad ſequens idque vltimum progreſſuri, in quo oſtendimus, quemadmodum loco lentis obiectiuae duas pluresue lentes ſiue ex eodem ſiue ex diuerſo vitri genere factas ſubſtituendo omnis plane confuſio tolli poſſit, vt hoc modo microſcopia omnibus numeris abſoluta nanciſcamur.
