SECTIO QVARTA.

DE

MICROSCOPIIS COMPOSITIS,

IN QVIBVS DVAE IMAGINES REALES OCCVRRVNT.

CAPVT I.

DE MICROSCOPIIS SIMPLICIORIBVS HVIVS GENERIS.

Praemonitum.

Cum microſcopia ad hanc ſectionem relata iterum ſitu erecto obiecta repraeſentent, litterae q, r, s, t etc. vna cum multiplicatione *m* eadem retinent ſigna, quae in praeceptis generalibus ſunt vſurpata.

Problema I.

224. Microscopium huius generis ex tribus lentibus componere, eiusque qualitates et defectus investigare.

Solutio.

Cum hic tantum tres lentes occurrant ideoque duo interualla, in quorum vtroque imago realis existit, ambae litterae P et Q statuendae sunt negatiuae quamobrem ponamus $P = -k$ et $Q = -k'$. vt sit $k k' = \frac{m a}{b}$; distantiae vero focales lentium erunt

$$p = \mathfrak{A}\, a; \quad q = \frac{A \mathfrak{B}. a}{k}; \qquad r = \frac{A B a}{k k'} = A B \frac{b}{m}.$$

Interualla vero lentium

$$I^{mum} = A\, a \left(1 + \frac{1}{k}\right);$$

$$II^{dum} = \frac{A B. a}{k} \left(1 + \frac{1}{k'}\right);$$

ita, vt prima imago realis distet a prima lente interuallo $= A\, a$ et a secunda interuallo $= \frac{A a}{k}$; posterior vero imago realis post lentem secundam cadit interuallo $= \frac{A B a}{k}$ et ante tertiam interuallo $= \frac{A B a}{k k'}$ ac si spatii in obiecto conspicui semidiameter sit $= z$, semidiameter prioris imaginis erit $= A\, z$, quae est inuersa; posterioris vero $= A\, B\, z$, quae iterum est erecta. Hinc igitur patet, esse debere $A > 0$. et $B > 0$, vnde quoque fient $\mathfrak{A} > 0$ et $\mathfrak{B} > 0$, ita tamen, vt sit $\mathfrak{A} < 1$ et $\mathfrak{B} < 1$. Tum vero erit

$$z = \frac{\mathfrak{q}+\mathfrak{r}}{ma-b} \cdot ab\xi \text{ et } M = \frac{\mathfrak{q}+\mathfrak{r}}{ma-b} \cdot b$$

vt sit $z = Ma\xi$. vnde nanciſcimur

$$\mathfrak{B}\mathfrak{q} = -(1+k)M;$$

ex quo perſpicuum eſt, cum $\mathfrak{B}$ ſit poſitiuum, fieri $\mathfrak{q}$ negatiuum, eoque ergo campum apparentem diminui; quare, ne is penitus ad nihilum redigatur, tribui debebit litterae $\mathfrak{r}$ maximus valor, qui eſt vnitas et poſito $\mathfrak{q} = -\omega$, debet eſſe $\omega < 1$, cum ſit

$$M = \frac{1-\omega}{ma-b} \cdot b; \text{ deinde ob}$$

$$\mathfrak{B} = \frac{1+k}{\omega} M = \frac{1+k}{\omega} \cdot \frac{1-\omega}{ma-b} \cdot b$$

quia $\mathfrak{B} < 1$. debebit eſſe

$$(1+k)(1-\omega)b < \omega(ma-b),$$

quae quidem conditio facile impletur, ſi fuerit

$$\omega > \frac{(1+k)b}{ma+bk} \text{ et quia inſuper eſt } \omega < 1,$$

ad hoc requiritur, vt ſit $ma > b$, quae quidem conditio pro maioribus multiplicationibus ſponte habet locum. Quodſi vellemus aſſumere $\omega = \frac{(1+k)b}{ma+bk}$, prodiret $\mathfrak{B} = 1$ hincque $B = \infty$ et inſtrumentum fieret infinite longum; ex quo perſpicuum eſt, neceſſario capi oportere $\omega > \frac{(1+k)b}{ma+bk}$.

Nunc etiam videamus, num margo coloratus deſtrui poſſit; quem in finem ante locus oculi examinari debet hac aequatione determinatus

$$O = \frac{\mathfrak{r}}{M\alpha} \cdot \frac{b}{m} \text{ ob } \mathfrak{r} = 1,$$

Quo-

Quoniam igitur r eſt poſitiuum, vtique erit $O > o$, vnde pro deſtructione marginis colorati habebitur iſta aequatio $o = \frac{\omega}{k} + \frac{1}{kk'}$; quod cum fieri nequeat, manifeſtum eſt, huiusmodi microſcopia inſigni vitio marginis colorati laborare; ita, vt ſuperfluum foret, in reliqua conſtructionis praecepta inquirere.

Coroll. 1.

225. Cum ob duas imagines reales pauciores, quam tres, lentes adhiberi nequeant, conſtructio in problemate contenta vtique eſt ſimpliciſſima, quae locum habere queat; quare cum eam repudiare cogamur, ad minimum quatuor lentibus vti oportebit.

Coroll. 2.

226. Quoniam formula pro deſtructione marginis colorati duabus conſtat partibus poſitiuis, iſta confuſio multo erit maior, quam in teleſcopiis et microſcopiis ex duabus tantum lentibus formatis ideoque multo minus tolerari poterit.

Scholion.

227. Cum igitur tribus lentibus hic propoſitis vnam ad minimum inſuper adiici oporteat, id tripliti modo fieri poterit, primo enim haec noua lens inter lentem obiectiuam et primam imaginem realem; ſecundo inſuper inter imaginem realem primam et ſecundam, ita, vt in hoc interuallo duae

lentes

lentes constituantur; tertio vero inter imaginem realem secundam et lentem ocularem. Verum hic tertius casus eodem vitio laborabit, quod hic est reprehensum; litterae enim P et Q eosdem retinebunt valores $-k$ et $-k'$, quippe quibus tantum tertia littera R adiungitur, sicque littera q retinebit quoque valorem negatiuum, qui sit $q = -\omega$, vnde pro margine colorato destruendo habebitur ista aequatio

$$o = \frac{\omega}{k} + \frac{r}{kk'} + \frac{s}{kk'R}$$

quae neutiquam subsistere potest, nisi vel r vel s capiatur negatiuum, quod autem cum iam q habeat valorem negatiuum, neutiquam expedit, quoniam alioquin campus nimis redderetur angustus, quocirca tantum bini casus priores nobis euoloendi relinquuntur.

Problema 2.

228. Microscopia huius generis ita ex quatuor lentibus componere, vt secunda adhuc ante priorem imaginem realem cadat; tertia vero inter ambas imagines ideoque sola ocularis post secundam imaginem, in quo id potissimum efficiatur, vt margo coloratus euanescat.

Solutio.

Hic ergo habentur tria interualla totidemque litterae P, Q et R, quarum duae posteriores debent esse negatiuae. Ponamus itaque $Q = -k$ et $R = -k'$, vt vt fit $P k k' = \frac{ma}{b}$; diftantiae porro focales harum lentium erunt:

$$p = \mathfrak{A} a; \; q = -\frac{A\mathfrak{B}}{P} . a; \; r = -\frac{AB\mathfrak{C}.a}{Pk} \text{ et}$$

$$s = -\frac{ABC}{Pkk'} a. = -A.BC.\frac{b}{m}.$$

tum vero interualla lentium

$$\text{I}^{mum} = A a (1 - \tfrac{1}{P});$$

$$\text{II}^{dum} = -\frac{ABa}{P} (1 + \tfrac{1}{k});$$

$$\text{III}^{tium} = -\frac{ABCa}{Pk} (1 + \tfrac{1}{k'});$$

vnde patet, effe debere $-AB > o$ et $C > o$, deinde notetur, primam imaginem cadere poft lentem fecundam ad interuallum $= -\frac{ABa}{P}$ et ante tertiam interuallo $= -\frac{ABa}{Pk}$; pofteriorem vero imaginem cadere poft lentem tertiam interuallo $= -\frac{ABCa}{Pk}$; et ante ocularem interuallo $= -\frac{ABCa}{Pkk'}$: praeterea vero imaginis prioris inuerfae radium effe $= AB z$; pofterioris vero erectae $= ABC. z.$, exiftente

$$z = \frac{q + r + s}{ma - b} . a b \zeta. \text{ hincque } M = \frac{q + r + s}{ma - b} . b,$$

ita, vt fit $z = M a \xi$, quae quantitas per hypothefin debet effe pofitiua; ex hoc autem valore deductae funt fequentes formulae:

$$\mathfrak{B} q = (P - 1) M;$$

$$\mathfrak{C} r = -(P k + 1) M - q.$$

Ob conditionem $C > o$ autem modo allatam debet effe $\mathfrak{C} > o$ et $\mathfrak{C} < 1$. ex quo perfpicuum eft, vel q

vel r

vel $\mathfrak{r}$ esse debere negatiuum. Vtrum igitur locum habeat, conueniet $\mathfrak{s}$ sumi positiue atque adeo poni $\mathfrak{s} = 1$. vt sit $M = \frac{1 + \mathfrak{q} + \mathfrak{r}}{ma - b} . b$. Hinc autem oculi distantia post lentem ocularem prodibit $O = \frac{\mathfrak{s} s}{M} . \frac{b}{ma}$; quia igitur $s > 0$; haec distantia fiet positiua ideoque margo coloratus destructur ope huius aequationis:

$$0 = \frac{\mathfrak{q}}{P} - \frac{\mathfrak{r}}{Pk} + \frac{1}{Pkk'}.$$

quae neutiquam subsistere posset, si esset $\mathfrak{r} < 0$, vnde necesse est, vt sit $\mathfrak{q} < 0$. Statuatur ergo $\mathfrak{q} = - \omega$, eritque $\frac{1}{k'} = k\omega + \mathfrak{r}$, atque nunc nouimus esse debere

$$\mathfrak{B} = \frac{1 - P}{\omega} . M \text{ et } \mathfrak{C} = - \frac{(Pk + 1) M + \omega}{\mathfrak{r}}$$

qui valor cum esse debeat positiuus, erit

$$(Pk + 1) M < \omega, \text{ hincque } \omega > \frac{(Pk + 1)(1 + \mathfrak{r}) b}{ma + Pkb}.$$

Cum autem sit

$$\frac{ma}{b} = Pkk' = \frac{Pk}{k\omega + \mathfrak{r}}$$

orietur haec aequatio

$$Pk^2 \omega^2 + \omega (1 + \mathfrak{r})(P - Pk - 1) k$$
$$- (Pk + 1)(1 + \mathfrak{r}) \mathfrak{r} > 0$$

quae aequatio conditionem continet, secundum quam littera ω debet definiri. Definitis autem conuenienter litteris ω et $\mathfrak{r}$ indeque deductis valoribus $\mathfrak{B}$ et $\mathfrak{C}$ saltim quam proxime, reliqua elementa innotescunt; tum vero nihil aliud superest, nisi vt apertura lentis obiectiuae ex aequatione pro semidiametro confusionis determinetur.

Coroll. 1.

229. Ponamus breuitatis gratia

$$\frac{ma}{b} = \mathfrak{M}, \text{ vt fit } M = \frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1},$$

et habebimus:

$$Pk = \mathfrak{M}(k\omega + 1) \text{ et } \mathfrak{B}\omega = (1-P)\left(\frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1}\right)$$

$$\text{et } (Pk+1)\frac{(1-\omega+\mathfrak{r})}{\mathfrak{M}-1} = \frac{(\mathfrak{M}(k\omega+\mathfrak{r})+1)(1-\omega+\mathfrak{r})}{\mathfrak{M}-1}$$

$= \omega - \mathfrak{C}\mathfrak{r}$; ex quo patet, fore $\omega > \mathfrak{C}\mathfrak{r}$, vbi constat, esse $\mathfrak{C} > 0$ et $\mathfrak{C} < 1$.

Coroll. 2.

230. Cum igitur ω notabiliter maius esse debeat, quam $\mathfrak{C}\mathfrak{r}$; videamus, an fieri possit $\omega = \mathfrak{r}$; quem in finem ponamus $\omega = \mathfrak{r}$ et vltima aequatio fiet

$$\mathfrak{r}(1-\mathfrak{C}) = \frac{(\mathfrak{M}(1+k)\mathfrak{r}+1)}{\mathfrak{M}-1};$$

vnde concluditur

$$\mathfrak{r} = \frac{-1}{\mathfrak{M}(\mathfrak{C}+k)+1-\mathfrak{C}}$$

quod, cum esse debeat $\mathfrak{r} > 0$, fieri nequit, sicque etiam certum est, esse debere $\omega > \mathfrak{r}$, ita, vt campus ne ad valorem eius simplicem quidem $z = \frac{ab\xi}{ma-b}$ augeri possit ob $1 - \omega + \mathfrak{r} < 1$.

Coroll. 3.

231. Cum igitur sit $\omega - \mathfrak{r} > 0$; plurimum interest, nosse, quomodo isti formulae minimus valor con-

conciliertur; quem in finem litteris ω et $\mathfrak{r}$ vt variabilibus ſpectatis hoc eueniet, ſi ſit $d\omega = d\mathfrak{r}$, cui regulae conuenienter differentietur noſtra aequatio:

$$(\omega - \mathfrak{C}\mathfrak{r})(\mathfrak{M} - 1) = (\mathfrak{M}(k\omega + \mathfrak{r}) + 1)(1 - \omega + \mathfrak{r})$$

ac prodibit

$$(1 - \mathfrak{C})(\mathfrak{M} - 1) = \mathfrak{M}(1 - \omega + \mathfrak{r})(k + 1)$$

vnde colligimus

$$1 - \omega + \mathfrak{r} = \frac{(1 - \mathfrak{C})(\mathfrak{M} - 1)}{\mathfrak{M}(k + 1)}$$

ita, vt ſit $M = \frac{1 - \mathfrak{C}}{\mathfrak{M}(k + 1)}$, quae formula praebet maximum campum, quem quidem obtinere licet. Hic autem campus maximus obtinebitur capiendo

$$\omega = \mathfrak{r} + \frac{\mathfrak{M}(\mathfrak{C} + k) + 1 - \mathfrak{C}}{\mathfrak{M}(k + 1)}$$

qui valor in noſtra aequatione ſubſtitutus dabit

$$\mathfrak{r}(1 - \mathfrak{C})(\mathfrak{M} - 1) + \frac{(\mathfrak{M} - 1)^2\mathfrak{C} + (\mathfrak{M} - 1)(\mathfrak{M}k + 1)}{\mathfrak{M}(k + 1)}$$

$$= \frac{(1 - \mathfrak{C})(\mathfrak{M} - 1)}{(k + 1)}\left(\mathfrak{r}(k + 1) + \frac{(\mathfrak{M} - 1)k\mathfrak{C} + \mathfrak{M}k^2 + k}{\mathfrak{M}(k + 1)}\right)$$

vbi membra litteram $\mathfrak{r}$ continentia ſe mutuo tollunt, relinquitur haec aequatio

$$(1 - \mathfrak{C})(1 + \mathfrak{C}k) + \mathfrak{M}(\mathfrak{C} + k)(\mathfrak{C}k + 1) = 0$$

quae reducitur ad hanc

$$\mathfrak{M}(\mathfrak{C} + k) + 1 - \mathfrak{C} = 0;$$

quae cum ſit impoſſibilis, ſequitur, hunc campum maximum ne quidem obtineri poſſe.

Scholion.

232. Parum vero refert, vtrum campum illum maximum obtinere queamus nec ne, cum etiam hic non desint remedia, campum pro lubitu amplificandi; quare relicta hac inuestigatione aliquot casus euoluamus, qui ad praxin inprimis accommodati videntur ac primo quidem apparet litteram P vnitati non nimis vicinam assumi posse, quia tum secunda lens primae tam esset propinqua, vt ambae tanquam vna spectari possent: ex quo casus praecedente problemate tractatus resultaret, quem locum habere non posse vidimus. Quamobrem pro P numerum satis magnum accipi conueniet, deinde etiam cum semper sit $\omega > 1$, e re erit 1 quam minimum accipere, denique etiam vt ad campum maximum quantum fieri licet appropinquemus, conueniet litteras k et $\mathfrak{C}$ quam minimas assumi.

CASVS I.

quo $P = \infty$.

Hoc ergo casu fit interuallum primum $= A\alpha$, ideoque $A > 0$ et $\mathfrak{A} < 1$. ac secunda lens cadet in ipsam imaginem primam, cuius distantia focalis ne fiat $= 0$. debet esse $\mathfrak{B} = \infty$ ita, vt fit

$$\frac{P}{\mathfrak{B}} = -\zeta \text{ hincque } q = \frac{A\alpha}{\zeta}.$$

Deinde cum sit $r = -\frac{AB\mathfrak{C}\alpha}{Pk}$, ob $\mathfrak{B} = \infty$ fit $B = -1$

et

et ob $\mathfrak{C} < 1$ manifeſtum eſt, eſſe debere $k = 0$, vt fieri poſſit Pk quantitas finita, at quia $k = 0$ erit $\frac{1}{k^{\prime}} = \mathfrak{r}$, hincque $P.k = \mathfrak{M}\,\mathfrak{r}$ exiſtente $\mathfrak{M} = \frac{ma}{b}$; ex quo erit

$$r = \frac{A\,\mathfrak{C}\,a}{\mathfrak{M}\,\mathfrak{r}}, \text{ et } s = A\,C.\,\frac{b}{m};$$

vnde pro magnis multiplicationibus eſſe debet C numerus praemagnus hincque $\mathfrak{C}$ ab vnitate parum deficere. Reliqua vero interualla erunt

$$II^{dum} = \frac{Aa}{\mathfrak{M}\,\mathfrak{r}} = \frac{r}{\mathfrak{C}} \text{ et}$$

$$III^{tium} = \frac{AC.a}{\mathfrak{M}\,\mathfrak{r}}(1 + \mathfrak{r}) = (1 + C)\,r + s.$$

Praeterea vero diſtantia oculi erit $O = \frac{s}{M\,\mathfrak{M}}$.

Nunc autem cum ſit $M = \frac{1 - \omega + \mathfrak{r}}{\mathfrak{M} - 1}$; hunc valorem in binis formulis $\mathfrak{B}\,\omega$ et $\mathfrak{C}\,\mathfrak{r}$ non ſubſtituamus, ſed in iis litteram M retineamus, quo eam facilius deinceps definire queamus; tum autem ob $\frac{P}{\mathfrak{B}} = -\zeta$ ex priore inuenimus $\omega = \zeta M$ ex poſteriore vero

$$\mathfrak{C}\,\mathfrak{r} = -M - \mathfrak{M}\,\mathfrak{r}\,M + \zeta M \text{ ſiue}$$

$$\mathfrak{r}(\mathfrak{C} + M\,\mathfrak{M}) = (\zeta - 1)\,M \text{ hincque}$$

$$\mathfrak{r} = \frac{(\zeta - 1)M}{\mathfrak{C} + M\,\mathfrak{M}}, \text{ hinc ergo colligimus}$$

$$\omega - \mathfrak{r} = \frac{(\mathfrak{C} + M\,\mathfrak{M} - 1)\,\zeta M + M}{\mathfrak{C} + M\,\mathfrak{M}} \text{ vel}$$

$$\omega - \mathfrak{r} = \frac{(\mathfrak{M}\,M\,\zeta - (1 - \mathfrak{C})\,\zeta + 1)\,M}{\mathfrak{M}\,M + \mathfrak{C}} \text{ et}$$

$$1 - \omega + \mathfrak{r} = \frac{\mathfrak{M}\,M + \mathfrak{C} - \mathfrak{M}\,M^2\zeta + (1 - \mathfrak{C})\,\zeta M - M}{\mathfrak{M}\,M + \mathfrak{C}}$$

quae

quae expressio aequalis esse debet huic $(\mathfrak{M}-1)M$; vnde nascitur haec aequatio:

$$M^2\mathfrak{M}(\mathfrak{M}-1+\zeta)-M(1-\mathfrak{C})(\mathfrak{M}-1+\zeta)-\mathfrak{C}=0$$

ex qua, cum sit proxime $\mathfrak{C}=1$, colligimus etiam proxime $M=\frac{1}{\sqrt{\mathfrak{M}(\mathfrak{M}-1+\zeta)}}$; adcuratius vero erit

$$M=\frac{1-\mathfrak{C}}{2\mathfrak{M}}+\sqrt{\frac{\mathfrak{C}}{\mathfrak{M}.(\mathfrak{M}-1+\zeta)}}$$

revera autem

$$M=\frac{1-\mathfrak{C}}{2\mathfrak{M}}+\sqrt{\left(\frac{\mathfrak{C}}{\mathfrak{M}.(\mathfrak{M}-1+\zeta)}+\frac{(1-\mathfrak{C})^2}{4\mathfrak{M}^2}\right)}$$

quo valore inuento simul innotescunt litterae ω et r, vnde reliqua omnia determinabuntur. Denique pro apertura lentis obiectiuae determinanda, quia nulla ratio vitri diuersitatem suadet, satisfieri debet huic aequationi:

$$\frac{1}{k^3}=\frac{\mathfrak{M}\mu x^3}{a^3}\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}-*+\frac{r\lambda^4}{A^4 a}+*\right)$$

vbi terminus secundus sponte euanuit, tertius vero ob C numerum praemagnum tuto reiici potest; vnde si hic factor posterior ponatur $=\Lambda$, reperitur

$$x=\frac{a}{k}\sqrt[3]{\frac{1}{\mathfrak{M}\mu\Lambda}}.$$

Coroll. I.

233. Quia $\mathfrak{M}$ est numerus praemagnus, loco factoris $\mathfrak{M}-1+\zeta$ scribere licebit $\mathfrak{M}$, siquidem ζ non fuerit numerus valde magnus; nulla autem ratio suadet pro ζ tantum numerum adhibere; sufficit enim

enim, vt capiatur $\zeta > 1$, ne $\mathfrak{r}$ vel euanefcat vel adeo negatiuum euadat. Tum igitur erit

$$M = \frac{1-\mathfrak{C}}{2\mathfrak{M}} + \sqrt{\left(\frac{\mathfrak{C}}{\mathfrak{M}^2} + \frac{(1-\mathfrak{C})^2}{4\mathfrak{M}^2}\right)}$$
$$= \frac{1-\mathfrak{C}}{2\mathfrak{M}} + \frac{1+\mathfrak{C}}{2\mathfrak{M}} = \frac{1}{\mathfrak{M}}$$

vnde viciffim colligitur

$$\omega = \frac{\zeta}{M} \text{ et } \mathfrak{r} = \frac{\zeta - 1}{\mathfrak{M}(1+\mathfrak{C})}.$$

Coroll. 2.

234. Hinc ergo fequentes adipifcimur determinationes pro ipfa microfcopii conftructione:

1°. diftantiae focales lentium erunt

$$p = \mathfrak{A}a;\ q = \frac{Aa}{\zeta};$$
$$r = \frac{A\mathfrak{C}(1+\mathfrak{C})a}{\zeta - 1} \text{ et } s = A\,C.\,\frac{b}{m} = \frac{AC.a}{\mathfrak{M}}.$$

2°. lentium interualla

$1^{mum} = A\,a;$

$2^{dum}\ \frac{r}{\mathfrak{C}} = \frac{A(1+\mathfrak{C})a}{\zeta - 1}$ et

$3^{tium} = (1 + C)\,r + s = \frac{AC(1+\mathfrak{C})a}{\zeta - 1} + \frac{ACa}{\mathfrak{M}}$

tum vero diftantia oculi erit $O = s$.

3°. pro apertura inuenienda erit

$$\Lambda = \frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}} - * + \frac{(1+\mathfrak{C})}{A^3(\zeta - 1)}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right) + \frac{\lambda'''}{A^3C^2\mathfrak{M}}$$

vbi membrum vltimum manifefto omitti poteft.

Scholion.

235. Iam innuimus, nullam rationem ſuadere, cur pro ζ numerum ſatis notabilem accipere velimus, interim tamen tertium interuallum, quod eſt

$$\frac{AC(1+\mathfrak{C})\alpha}{\zeta-1}+\frac{AC.\alpha}{\mathfrak{M}}$$

fieri videtur nimis magnum, niſi ζ vnitatem multum ſuperet, quoniam pro C numerum ſatis magnum aſſumi conuenit atque etiam A numero ſatis notabili aequari debet. Interim tamen ſemper praeſtabit, maiorem inſtrumenti longitudinem tolerare, quam campum reſtringere. Verum etiamſi ζ maius acciperemus, vt menſurae prodeant ad praxin magis accommodatae; nullum aliud incommodum inde eſſet metuendum, niſi quod campus minor eſſet reuera futurus, quam intendimus; quem vero defectum aliquot inſuper lentibus adiungendis facile ſupplere licebit. At vero plurimum refert, vt numerus A ſatis notabilis accipiatur, vt $\mathfrak{A}$ ſatis prope ad vnitatem reducatur, id quod neceſſarium eſt, vt Λ ſatis exiguum reddatur hincque maior claritatis gradus obtineatur; quem in finem ſuficere videtur, dummodo ſtatuatur $A=6$; hinc enim fit $\mathfrak{A}=\frac{6}{7}$ ideoque $\frac{\lambda}{\mathfrak{A}^3}=\frac{343}{216}\lambda$, qui valor ſumto $\lambda=1$ non multum ſuperat $\frac{3}{2}$, qui per $\mu<1$ multiplicatus certe infra $\frac{3}{2}$ reducitur; vnde iam ſatis notabilis valor pro x reſultat. Si igitur ſtatuatur $A=6$, videamus quantum ſumi oporteat C, ne s fiat nimis paruum etiam pro inſigni multiplicaplicatione $m = 960$. Quia itaque tum fit $s = \frac{48 C}{960}$ dig. $= \frac{C}{20}$ dig. haec diſtantia non infra $\frac{1}{4}$ dig. deprimetur, dummodo $C = 5$ quare ſi ſtatuamus $C = 6$, vt ſit $\mathfrak{C} = \frac{6}{7}$, ex hac parte nihil erit metuendum; tum vero tertium interuallum euadit $\frac{36.13.a}{7(\zeta-1)}$, omiſſo altero membro, ſiue $\frac{2.36.a}{\zeta-1} = \frac{72\,a}{\zeta-1}$; vnde ſi diſtantia obiecti ſit dimidii digiti, hoc interuallum erit $\frac{36}{\zeta-1}$ dig. quod ergo ſumto $\zeta = 3$ vel $\zeta = 4$ iam ſit tam modicum, vt nulla poſſit eſſe ratio de eo conquerendi.

CASVS II.

quo $\mathfrak{r} = 0$.

Hoc ergo caſu erit $\frac{1}{k} = k\omega$ ideoque $P = \mathfrak{M}\omega$ tum vero $M = \frac{1-\omega}{\mathfrak{M}-1}$. Hinc aequationes ex campo deductae erunt

I. $\mathfrak{B}\omega = \frac{(1-\mathfrak{M}\omega)(1-\omega)}{\mathfrak{M}-1}$.

II. $\mathfrak{C}\mathfrak{r} = -\frac{(\mathfrak{M}k\omega+1)(1-\omega)}{\mathfrak{M}-1} + \omega = 0$.

Cum igitur $P = \mathfrak{M}\omega$, erit $\omega = \frac{P}{\mathfrak{M}}$ et $1 - \omega = \frac{\mathfrak{M}-P}{\mathfrak{M}}$; vnde patet, P minus eſſe debere, quam $\mathfrak{M}$. Hic autem valor in aequatione poſteriore ſubſtitutus dabit $k = \frac{\mathfrak{M}(P-1)}{P(\mathfrak{M}-P)}$, vnde cum $k > 0$ patet eſſe debere $P > 1$. hinc autem porro ſequitur, fore $A > 0$, hincque $\mathfrak{A} < 1$; deinde vero reperitur

$$B = -\frac{(P-1)(\mathfrak{M}-P)}{\mathfrak{M}(2P-1)-P^2};\quad \mathfrak{B} = -\frac{(P-1)(\mathfrak{M}-P)}{P(\mathfrak{M}-1)};$$

 vnde

vnde B etiam negatiuum valorem obtinet, vti rei natura postulat. Denique erit

$$k' = \frac{\mathfrak{M}}{Pk} = \frac{\mathfrak{M} - P}{P - 1} \text{ et } M = \frac{\mathfrak{M} - P}{\mathfrak{M}(\mathfrak{M} - 1)},$$

vnde campus cognoscitur; hinc igitur patet, quo minus capiatur P, eo maiorem proditurum esse campum et cum P vnitatem superare debeat, semper erit $M < \frac{1}{\mathfrak{M}}$ His igitur valoribus inuentis habebimus

1°. distantias focales

$$p = \mathfrak{A} a; \; q = \frac{A(P-1)(\mathfrak{M}-P)a}{(\mathfrak{M}-1)};$$

$$r = \frac{A\mathfrak{C}.(\mathfrak{M}-P)^2.a}{\mathfrak{M}(\mathfrak{M}(2P-1)-P^2)} \text{ et } s = \frac{AC.(P-1)(\mathfrak{M}-P)a}{\mathfrak{M}(\mathfrak{M}(2P-1)-P^2)}$$

et interualla lentium

$$1^{mum} = Aa\left(1 - \frac{1}{P}\right)$$

$$2^{dum} = \frac{A(\mathfrak{M}-P).a}{\mathfrak{M}P};$$

$$3^{tium} = \frac{AC.(\mathfrak{M}-P)(\mathfrak{M}-1).a}{\mathfrak{M}(\mathfrak{M}(2P-1)-P^2)}.$$

Tum vero oculi distantia erit

$$O = \frac{s}{M\mathfrak{M}} = \frac{\mathfrak{M}-1}{\mathfrak{M}-P}.s$$

ac denique spatii in obiecto conspicui erit semidiameter $z = \frac{\mathfrak{M}-P}{\mathfrak{M}(\mathfrak{M}-1)}.a\xi$. Aperturam vero lentis obiectiuae ex aequatione nota definire oportet, pro aperturis vero sequentium lentium notetur esse $\omega = \frac{P}{\mathfrak{M}}$ et $\mathfrak{r} = 0$. Vnde colligitur semidiameter aperturae

lentis $II^{dae} = \frac{1}{P}x + \frac{Pq}{4\mathfrak{M}}$;

lentis $III^{tiae} = \frac{x}{Pk} + 0 = \frac{\mathfrak{M}-P}{\mathfrak{M}(P-1)}.x$;

lentis $IV^{tae} = \frac{x}{\mathfrak{M}} + \frac{1}{4}s$.

Coroll.

Coroll. 1.

236. Quoniam campus postulat, vt P satis paruum accipiatur, pro maioribus multiplicationibus licebit P prae $\mathfrak{M}$ negligere, vnde si P vnitatem non multum superet, distantiae focales ita exprimentur

$$p = \mathfrak{A}\, a;\ q = \frac{A.(P-1).a}{P};$$

$$r = \frac{A\mathfrak{C}.}{2P-1}.\, a \text{ et } s = \frac{AC}{m(2P-1)}.\, a.$$

deinde interualla lentium

$$\mathrm{I}^{mum} = A\, a\, (1 - \tfrac{1}{P});$$

$$\mathrm{II}^{dum} = \frac{A a.}{P};$$

$$\mathrm{III}^{tium} = \frac{AC.a}{2P-1.};$$

et distantia oculi $O = s.$

Coroll. 2.

237. Si ergo statuamus $P = 2$, fient distantiae focales

$$p = \mathfrak{A}\, a;\ q = \tfrac{1}{2} A\, a.;$$

$$r = \tfrac{1}{3}.\, A\, \mathfrak{C}.\, a \text{ et } s = \tfrac{1}{3}.\, \frac{AC}{\mathfrak{M}}.\, a.$$

et interualla

$$\mathrm{I}^{mum} = \tfrac{1}{2} A\, a;$$

$$\mathrm{II}^{dum} = \frac{Aa}{2};$$

$$\mathrm{III}^{tium} = \tfrac{1}{3}.\, A\, C.\, a;$$

et pro campo $z = \frac{a\xi}{\mathfrak{M}}.$

Coroll. 3.

238. Si, vt supra pro telescopiis fecimus statuamus $P = \sqrt{\mathfrak{M}}$ (quoniam quod ibi erat m, hic nobis est $\mathfrak{M}$) distantiae focales ita exprimentur:

$$p = \mathfrak{A}\, a; \quad q = \frac{A.(\sqrt{\mathfrak{M}} - 1)}{\mathfrak{M} + \sqrt{\mathfrak{M}}}. a;$$

$$r = \frac{A\mathfrak{C}.(\sqrt{\mathfrak{M}} - 1)}{2\mathfrak{M}}. a; \quad s = \frac{AC(\sqrt{\mathfrak{M}} - 1)}{2\mathfrak{M}\sqrt{\mathfrak{M}}}. a;$$

et interualla lentium

$$I^{mum} = A\, a\left(\frac{\sqrt{\mathfrak{M}} - 1}{\sqrt{\mathfrak{M}}}\right);$$

$$II^{dum} = \frac{A a.(\sqrt{\mathfrak{M}} - 1)}{\mathfrak{M}};$$

$$III^{tium} = \frac{A C. a.(\mathfrak{M} - 1)}{2\mathfrak{M}.\sqrt{\mathfrak{M}}}.$$

Pro campo autem apparente erit

$$z = \frac{1}{\mathfrak{M} + \sqrt{\mathfrak{M}}}. a\, \xi$$

et pro Oculi loco

$$O = \frac{\sqrt{\mathfrak{M}} + 1}{\sqrt{\mathfrak{M}}}. s = s\left(1 + \frac{1}{\sqrt{m}}\right).$$

Scholion.

239. Casus in Coroll. vltimo euolutus apprime conuenit cum eo, quem supra in telescopiis tractauimus, vbi praecedentes casus, in quibus litterae p minores valores sunt tributi, penitus exclusimus, idque ob eam rationem, quia interuallum tertium enormiter magnum prodiisset. Cum enim pro telescopiis sit $b = a = \infty$, necesse est, vt sit $\mathfrak{A} = 0 = A$, ita

ita tamen, vt fiat $\mathfrak{A}\, a = A\, a = p$ et $\mathfrak{M} = m$. Tum autem in genere erit tertium interuallum

$$= \frac{C.(\mathfrak{M} - P)(\mathfrak{M} - 1)p}{\mathfrak{M}(\mathfrak{M}(2P - 1) - P^2)}$$

quod, si P prae $\mathfrak{M}$ quasi euanescat, fiet $= \frac{C}{2P-1} \cdot p$. quare cum C debeat esse numerus praemagnus, hoc solum interuallum multis partibus excessurum esset distantiam focalem p, ideoque longitudo telescopii prodiret enormiter magna; quos igitur casus merito supra exclusimus. Nunc autem, vbi de microscopiis agitur, haec ratio penitus cessat, neque enim longitudo instrumenti ob tertium interuallum adeo enormiter magna euadit. Si enim vt ante notauimus pro magnis etiam multiplicationibus sumatur $A = 6$ et $C = 6$; tum tertium interuallum erit $= \frac{36.a}{2P-1}$ ac si a, vt fieri solet, capiatur $\frac{1}{2}$ dig. hoc interuallum fiet $\frac{18}{2P-1}$ dig. vnde si modo sit $P = 2$, id reducitur ad 6 dig. quod in praxi vtique admitti potest. Quocirca in hac de microscopiis tractatione casum in tertio Coroll. euolutum excludi conueniet seruato eo, vbi erat $P = 2$, siquidem hoc modo campus multo maior obtinetur; quin etiam, si lubuerit, sumi poterit $P = 3$. vt prodeant distantiae focales:

$$p = \mathfrak{A}\, a; \quad q = \tfrac{2}{3} A\, a;$$

$$r = \tfrac{4}{5} A\, \mathfrak{C}. a \text{ et } s = \tfrac{1}{5} \cdot \frac{AC}{\mathfrak{M}} \cdot a;$$

et

et interualla lentium

$$1^{mum} = \tfrac{2}{3} A a;$$

$$2^{dum} = \tfrac{1}{3} A a;$$

$$3^{tium} = \tfrac{1}{3} A C . a;$$

manenta $O = s$ proxime et $z = \frac{\mathfrak{M} - 1}{\mathfrak{M}(\mathfrak{M} - 1)} a \xi$.

Nunc autem ne s pro magnis multiplicationibus nimis fiat exiguum, litterae C vtique maior valor tribui debebit, ita, vt iam nulla ratio suadeat, cur litterae P potius valorem 3, quam 2 tribuere velimus, quandoquidem ponendo $P = 3$ tertium interuallum vix diminuitur.

Scholion 2.

240. Euolutione horum duorum casuum attentius considerata, poterimus simili modo solutionem generalem instituere, posito enim breuitatis gratia $\frac{P - 1}{\mathfrak{B}} = -\zeta$ siue $\mathfrak{B} = -\frac{(P - 1)}{\zeta}$, habebimus statim $\omega = \zeta M$; deinde cum sit $P k = \mathfrak{M}(k \omega + \mathfrak{r})$, erit $P k = \zeta \mathfrak{M} M k + \mathfrak{M} \mathfrak{r}$, qui valor in altera aequatione, quae est

$$\mathfrak{C} \mathfrak{r} = \zeta M - M - P k M,$$

substitutus dat

$$\mathfrak{C} \mathfrak{r} = \zeta M - M - \zeta \mathfrak{M} M^2 k - \mathfrak{M} M \mathfrak{r}$$

ex quo reperitur

$$\mathfrak{r} = \frac{(\zeta - 1) M - \zeta \mathfrak{M} M^2 k}{\mathfrak{M} M + \mathfrak{C}}$$

hinc-

hincque

$$1 - \omega + \mathfrak{r} = - \frac{\zeta \mathfrak{M} (k + 1) M^2 + (\mathfrak{M} - 1 + \zeta (1 - \mathfrak{C})) M + \mathfrak{C}}{\mathfrak{M} M + \mathfrak{C}}$$

Cum igitur ſit $M = \frac{1 - \omega + \mathfrak{r}}{\mathfrak{M} - 1}$ erit

$$1 - \omega + \mathfrak{r} = M (\mathfrak{M} - 1)$$

vnde ſequens ſuppeditatur aequatio

$$\mathfrak{M} (\mathfrak{M} - 1 + \zeta + \zeta k) M^2 - (1 - \mathfrak{C})(\mathfrak{M} - 1 + \zeta) M - \mathfrak{C} = 0.$$

ex qua, niſi numeri ζ et k fuerint ſatis magni, ita, vt eos prae $\mathfrak{M}$ tuto negligere liceat, ſequitur fore ſaltim proxime

$$\mathfrak{M}^2 M^2 - \mathfrak{M} M (1 - \mathfrak{C}) - \mathfrak{C} = 0$$

quae in hos factores reſoluitur:

$$(\mathfrak{M} M - 1)(\mathfrak{M} M + \mathfrak{C}) = 0.$$

vnde manifeſto colligitur $M = \frac{1}{\mathfrak{M}}$; quo valore, etſi tantum prope vero, vti poterimus, quoniam parum refert, vtrum campus aliquanto ſit maior minorue, quam calculus indicat. Probe autem haec conditio obſeruetur, quod tam ζ quam k ſint numeri ſatis exigui, ſaltim multo minores, quam $\mathfrak{M}$. Si enim ζk tantus ſit numerus, vt eum prae $\mathfrak{M}$ reiicere non liceat, tum littera M multo minorem nanciſcetur valorem, quam $\frac{1}{\mathfrak{M}}$, ſicque campus inſignem pateretur diminutionem, quae ſola cauſſa ſufficit, vt maiores valores pro litteris ζ et k penitus excludantur haecque regula ſtabiliatur, vt nunquam litteris ζ et k valores tribuantur, qui binarium ſuperent vel vt ſaltim ζk quaternarium non ſuperet. Cum igitur ſit $\mathfrak{P} k k' = \mathfrak{M}$ et k numerus ab vnitate non multum diſcrepans, euidens eſt, vel $\mathfrak{P}$ vel k' eſſe debere numerum ſatis magnum vel adeo vtrumque. His ergo obſeruatis, ita, vt ſit $M = \frac{1}{\mathfrak{M}}$ habebimus

$$\omega = \frac{\zeta}{\mathfrak{M}} \text{ et } \mathfrak{r} = \frac{\zeta - \zeta k - 1}{\mathfrak{M}(1 + \mathfrak{C})};$$

quibus valoribus ſubſtitutis fit

$$\mathfrak{P} k = k + \frac{\zeta - \zeta k - 1}{1 + \mathfrak{C}} \text{ hincque}$$

$$\mathfrak{P} = 1 + \frac{\zeta - \zeta k - 1}{(1 + \mathfrak{C}) k} = \frac{(\zeta - 1)(1 - k) + \mathfrak{C} k}{(1 + \mathfrak{C}) k}$$

hoc igitur valore ipſius $\mathfrak{P}$ notato, erunt diſtantiae focales

$$p = \mathfrak{A} a; \; q = \frac{A(\mathfrak{P} - 1) a}{\mathfrak{P} \zeta};$$

$$r = \frac{A(\mathfrak{P} - 1) \mathfrak{C} a}{(\zeta + \mathfrak{P} - 1) \mathfrak{P} k}; \; s = \frac{A(\mathfrak{P} - 1) C a}{(\zeta + \mathfrak{P} - 1) \mathfrak{M}};$$

et lentium interualla

$$1^{mum} = A a \left(1 - \frac{1}{\mathfrak{P}}\right);$$

$$2^{dum} = \frac{A(\mathfrak{P} - 1) a}{(\zeta + \mathfrak{P} - 1) \mathfrak{P} k} (k + 1) \text{ et}$$

$$3^{tium} = \frac{A(\mathfrak{P} - 1) C a}{(\zeta + \mathfrak{P} - 1)} \left(\frac{1}{\mathfrak{P} k} + \frac{1}{\mathfrak{M}}\right)$$

et diſtantia oculi $O = s$.

ac denique $\zeta = \frac{1}{4} \cdot \frac{a}{\mathfrak{M}}$.

Datis ergo diſtantia obiecti $= a$ et multiplicatione

$= m$

$= m$ siue $\mathfrak{M} = \frac{ma}{b}$; arbitrio nostro relinquuntur sequentes quantitates

1°. $\mathfrak{A}$ quam vnitate non multo minorem assumi conuenit; hanc enim conditionem claritas postulat.

2°. Numerus ζ, qui esse debet positiuus ac tantus, vt $\zeta + P - 1$ fiat numerus positiuus.

3°. Littera C, quam autem ita definiri conuenit, vt distantia focalis ne fiat nimis exigua; sin autem haec littera sit valde magna, euidens est, litteram $\mathfrak{C}$ ad vnitatem proxime esse accessuram.

4°. Littera denique k, quam, vt vidimus, admodum paruam accipi conuenit.

Ratione autem valoris P obseruari oportet, semper esse debere $\zeta + P - 1 > 0$; vnde haec conditio adhuc implenda erit $\zeta \mathfrak{C} k + \zeta - 1 > 0$, quae est fere eadem quantitas, quam supra prae $\mathfrak{M}$ neglexitimus; ex quo cauendum est, ne ea aliquot vnitates superet.

Problema 3.

241. Si noua lens inter imaginem primam et secundam disponatur, omnia momenta ita definire, vt margo coloratus euanescat simulque maximus campus obtineatur.

 Solutio.

Solutio.

Quoniam hic iterum quatuor habentur lentes earumque duae intra imaginem primam et fecundam cadant, litterarum P, Q, R prima et tertia hic erunt negatiuae, ftatuatur igitur $P = -k$ et $R = -k'$. vnde diftantiae focales erunt.

$$p = \mathfrak{A} a; \; q = \tfrac{A\mathfrak{B}}{k} . a;$$

$$r = -\tfrac{AB\mathfrak{C}}{kQ} . a \text{ et } s = -\tfrac{ABC}{kQk'} . a = -\tfrac{ABC.a}{\mathfrak{M}}$$

ob $kQk' = \mathfrak{M} = \frac{ma}{b}$.

Interualla vero lentium erunt

$$I^{mum} = Aa\left(1 + \tfrac{1}{k}\right)$$

$$II^{dum} = \tfrac{ABa}{k}\left(1 - \tfrac{1}{Q}\right)$$

$$III^{tium} = -\tfrac{ABC.a}{kQ}\left(1 + \tfrac{1}{k'}\right)$$

hincque fequitur

$$A > 0. \; B\left(1 - \tfrac{1}{Q}\right) > 0 \text{ et } BC < 0.$$

Porro erit

$$M = \tfrac{\mathfrak{q} + \mathfrak{r} + \mathfrak{s}}{\mathfrak{M} - 1}. \text{ vt fiat } z = Ma\xi.$$

Hincque diftantia Oculi $O = \frac{\mathfrak{s} s}{M\mathfrak{M}}$.

vbi, vt campus reddatur maximus, fumi conueniet $\mathfrak{s} = 1$. fi fcilicet lens ocularis vtrinque aequalis paretur. Tum autem erit

$$\mathfrak{B}\mathfrak{q} = -(1 + k) M \text{ et}$$

$$\mathfrak{C}\mathfrak{r} = -(1 + Qk) M - \mathfrak{q}.$$

Si

Si hic vt ante breuitatis gratia scribatur

$$\frac{1+k}{\mathfrak{B}} = \zeta \text{ vt sit } \mathfrak{q} = -\zeta M;$$

tum igitur erit

$$\mathfrak{C}\mathfrak{r} = -(1+Qk)M + \zeta M \text{ et}$$

$$\mathfrak{r} = \frac{(\zeta - Qk - 1)M}{\mathfrak{C}} \text{ hincque}$$

$$\mathfrak{q} + \mathfrak{r} = \frac{(\zeta(1-\mathfrak{C}) - Qk - 1)M}{\mathfrak{C}} \text{ et}$$

$$\mathfrak{q} + \mathfrak{r} + 1 = \frac{\mathfrak{C} + (\zeta(1-\mathfrak{C}) - Qk - 1)M}{\mathfrak{C}}.$$

Inde vero est

$$\mathfrak{q} + \mathfrak{r} + 1 = M(\mathfrak{M} - 1)$$

vnde sequitur

$$M = \frac{\mathfrak{C}}{(\mathfrak{M}-1)\mathfrak{C} - \zeta(1-\mathfrak{C}) + Qk + 1}$$

vbi ergo haec quantitas

$$\mathfrak{M} - 1 - \zeta\frac{(1-\mathfrak{C})}{\mathfrak{C}} + \frac{Qk+1}{\mathfrak{C}}$$

debet esse positiua et tam parua, quam circumstantiae permittunt.

Deinde vero vt margo coloratus euanescat, habetur haec aequatio:

$$0 = \frac{\mathfrak{q}}{P} + \frac{\mathfrak{r}}{PQ} + \frac{1}{PQR}$$

ex qua reperitur

$$\frac{1}{k'} = Q\mathfrak{q} + \mathfrak{r}$$

quae quantitas debet esse positiua. Cum igitur sit

$$kQk' = \mathfrak{M} \text{ erit } \mathfrak{M} = \frac{kQ}{Q\mathfrak{q}+\mathfrak{r}}$$

 hincque

hincque

$$k\mathrm{Q} = \mathfrak{M}(\mathrm{Q}\mathfrak{q} + \mathfrak{r})$$

Ante quam autem hanc formulam prosequamur, plurimum intererit inuestigare, num forte $\mathfrak{r}$ possit poni $= 1$. Statuamus igitur $\mathfrak{r} = 1$. vt sit

$$\mathrm{M} = \frac{\mathfrak{q}+2}{\mathfrak{M}-1} \text{ vnde ob } \mathfrak{q} = -\zeta\mathrm{M} \text{ fiet}$$

$$\mathfrak{q} = -\frac{2\zeta - \zeta\mathfrak{q}}{\mathfrak{M}-1} \text{ adeoque } \mathfrak{q} = -\frac{2\zeta}{\mathfrak{M}+\zeta-1},$$

hincque $\mathrm{M} = +\frac{2}{\mathfrak{M}+\zeta-1}$;

altera vero aequatio iam dabit

$$\mathfrak{C} = -\frac{2(1+\mathrm{Q}k)+2\zeta}{\mathfrak{M}+\zeta-1} = \frac{2(\zeta-\mathrm{Q}k-1)}{\mathfrak{M}+\zeta-1}.$$

Deinde cum sit $k\mathrm{Q} = \mathfrak{M}\mathrm{Q}\mathfrak{q} + \mathfrak{M}$ fiet nunc

$$k\mathrm{Q} = \mathfrak{M} - \frac{2\mathfrak{M}\zeta\mathrm{Q}}{\mathfrak{M}+\zeta-1},$$

vnde inuenitur k, dummodo sit

$$1 > \frac{2\zeta\mathrm{Q}}{\mathfrak{M}+\zeta-1} \text{ hoc est } \mathrm{Q} < \frac{\mathfrak{M}+\zeta-1}{2\zeta}.$$

Porro autem fiet

$$\mathfrak{B} = \frac{1+k}{\zeta} \text{ et } \mathfrak{C} = \frac{4\mathfrak{M}\zeta\mathrm{Q}}{(\mathfrak{M}+\zeta-1)^2} - 2$$

si igitur fuerit $\mathfrak{B} > 1$, vt sit $\mathrm{B} < 0$ sumi debet $\mathrm{Q} < 1$; tum vero C debet esse positiuum ideoque etiam $\mathfrak{C} > 0$ et $\mathfrak{C} < 1$; quocirca debet esse

$$1°.\ 2\mathfrak{M}\zeta\mathrm{Q} > (\mathfrak{M}+\zeta-1)^2$$

$$2°.\ 2\mathfrak{M}\zeta\mathrm{Q} < \tfrac{3}{2}(\mathfrak{M}+\zeta-1)^2$$

quae

quae conditio illi, qua debet esse $Q < 1$, manifesto repugnat. Debet ergo esse $B > 0$, hincque $Q > 1$, tum autem esse debet $C < 0$; quod eueniet, si fuerit $\mathfrak{C}$ etiam negatiuum, id est, si fuerit

$$Q < \frac{(\mathfrak{M} + \zeta - 1)^2}{2\mathfrak{M}\zeta}$$

quae conditio cum praecedente $Q > 1$ facile consistere potest. Tum autem esse debet $\mathfrak{B} < 1$ ideoque $\zeta > 1 + k$. Tantum igitur campum obtinebimus, si capiamus $\zeta > 1 + k$, litteram Q vero intra limites 1 et $\frac{(\mathfrak{M} + \zeta - 1)^2}{2\mathfrak{M}\zeta}$, dummodo fuerit

$$kQ = \mathfrak{M} - \frac{2\mathfrak{M}\zeta Q}{\mathfrak{M} + \zeta - 1} \text{ adeoque } Q < \frac{\mathfrak{M} + \zeta - 1}{2\zeta},$$

quia est kQ positiuum; quod sponte fit per conditionem praecedentem; qua est $Q < \frac{(\mathfrak{M} + \zeta - 1)^2}{2\mathfrak{M}\zeta}$.

Praeterea autem debet esse

$$k = \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M} + \zeta - 1}$$

quia autem esse debet $\zeta > 1 + k$ habebimus nunc

$$\zeta > 1 + \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M} + \zeta - 1}$$

vnde sequitur haec conditio

$$Q > \frac{\mathfrak{M}(\mathfrak{M} + \zeta - 1)}{\zeta^2 + \zeta(3\mathfrak{M} - 2)}$$

quae conditio cum illa,

$$Q < \frac{(\mathfrak{M} + \zeta - 1)^2}{2\mathfrak{M}\zeta} \text{ egregie consistit.}$$

Cum

Cum autem praeterea esse debeat $\zeta > 1 + k$, loco k substituendo eius valorem fiet

$$\zeta > 1 + \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M}+\zeta-1}$$

vnde concluditur esse debere

$$Q > \frac{\mathfrak{M}(\mathfrak{M}+\zeta-1)}{\zeta^2+\zeta(3\mathfrak{M}-2)-\mathfrak{M}+1}$$

quia autem modo vidimus, esse

$$Q < \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta}$$

hic limes illo debet esse maior; vnde colligitur

$$\zeta^3+\zeta^2(4\mathfrak{M}-3)+\zeta(\mathfrak{M}^2-6\mathfrak{M}+3) > (\mathfrak{M}-1)^2$$

vnde patet, si $\mathfrak{M}$ sit numerus praemagnus, esse debere $\zeta > 1$. Statuamus ergo $\zeta = 1 + \frac{\alpha}{\mathfrak{M}}$; vnde fit

$$\mathfrak{M}^2+\mathfrak{M}(\alpha-2)+2\alpha+1 > \mathfrak{M}^2-2\mathfrak{M}+1$$

hincque porro $\alpha(\mathfrak{M}+2) > 0$, quod cum semper eueniat, patet, dummodo $\zeta > 1$, solutionem semper locum habere; ac si in illis formulis $\mathfrak{M}$ vt numerus praemagnus spectetur, limites pro Q erunt

$$Q > \frac{\mathfrak{M}}{3\zeta-1} \quad \text{et} \quad Q < \frac{\mathfrak{M}}{2\zeta}$$

sumtaque Q his limitibus conuenienter erit porro

$$k = \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M}+\zeta-1}$$

hincque reliqua elementa omnia facillime definientur. Ceterum in euolutione sequentium casuum haec clariora reddentur; quod denique ad lentium aperturas attinet, eas pro quouis casu ex cognitis fomulis facile definire licet.

Coroll.

Coroll. 1.

242. Videamus vero, quomodo omnes hae conditiones clarius euolui queant, ac primo quidem statim ac statuimus

$$\mathfrak{v} = 1. \text{ fit } M = \frac{2+q}{\mathfrak{M}-1}$$

ideoque

$$2 + q = M.(\mathfrak{M} - 1).$$

Posito autem

$$\frac{1+k}{\mathfrak{B}} = \zeta, \text{ vt fit } \mathfrak{B} = \frac{1+k}{\zeta}$$

fiet $q = -\zeta M$; qui valor ibi substitutus dat

$$2 - \zeta M = M(\mathfrak{M} - 1)$$

hincque

$$M = \frac{2}{\mathfrak{M}+\zeta-1} \text{ et } q = -\frac{2\zeta}{\mathfrak{M}+\zeta-1}$$

deinde vero inuenimus

$$kQ = \mathfrak{M}Qq + \mathfrak{M}.$$

vbi valor ipsius q substitutus dat

$$kQ = \mathfrak{M} - \frac{2\mathfrak{M}Q\zeta}{\mathfrak{M}+\zeta-1} \text{ siue}$$

$$kQ(\mathfrak{M}+\zeta-1) + 2\mathfrak{M}Q\zeta = \mathfrak{M}(\mathfrak{M}+\zeta-1)$$

vnde commode deducitur

$$Q = \frac{\mathfrak{M}.(\mathfrak{M}+\zeta-1)}{k(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta}$$

vnde fit

$$1 + kQ = \frac{k(\mathfrak{M}+1)(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta}{k(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta}$$

 Cum

Cum igitur ſit

$$\mathfrak{C} = M(\zeta - Qk - 1) \text{ erit}$$

$$\mathfrak{C} = \frac{2\mathfrak{M}\zeta(\zeta-1) - k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)}{k(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta}\cdot M$$

ſeu

$$\mathfrak{C} = \frac{4\mathfrak{M}\zeta(\zeta-1) - 2k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)}{(k(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta)(\mathfrak{M}+\zeta-1)}.$$

Coroll. 2.

243. Iam ratione litterae $\mathfrak{B}$ duo caſus ſunt conſiderandi; alter, quo $\mathfrak{B} > 1$ ideoque $B < 0$. alter vero, quo $\mathfrak{B} < 1$. ideoque $B > 0$. Priori caſu erit $\zeta < 1 + k$ et quia B eſt minus nihilo, ob ſecundum interuallum debet eſſe $Q < 1$; vnde fit

$$\mathfrak{M}.(\mathfrak{M}+\zeta-1) < k(\mathfrak{M}+\zeta-1) + 2\mathfrak{M}\zeta$$

id quod fieri nequit, cum ſit $\mathfrak{M}$ numerus valde magnus.

Coroll. 3.

243. Cum igitur eſſe nequeat $\mathfrak{B} > 1$ ſtatuamus $\mathfrak{B} < 1$. ſiue $\zeta > 1 + k$ et quia iam $B > 0$, debebit eſſe $Q > 1$; id quod ſponte euenit pro maioribus ſcilicet multiplicationibus ad quas hic ſolas attendimus. Tum autem eſſe debet $C < 0$ id quod euenit, vel ſi fuerit $\mathfrak{C} < 0$ vel $\mathfrak{C} > 1$. Priori caſu ſi $\mathfrak{C} < 0$, debebit eſſe

$$k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1) > 2\mathfrak{M}\zeta(\zeta-1)$$

ſiue

siue $k > \frac{2\mathfrak{M}\zeta(\zeta-1)}{(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)}$.

Ex illa vero conditione $\zeta > 1 + k$ debet esse $k < \zeta - 1$; vnde porro colligitur esse debere

$$\zeta - 1 > \frac{2\mathfrak{M}\zeta(\zeta-1)}{(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)} \text{ seu}$$

$$(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1) > 2\mathfrak{M}\zeta,$$

quod etiam semper euenit; ita, vt littera ζ arbitrio nostro relinquatur, dummodo vnitate maior accipiatur. Cum autem sit $M = \frac{2}{\mathfrak{M}+\zeta-1}$; campi magnitudo postulat, vt ζ quam minime vnitatem superet.

Coroll. 4.

244. Examinandus restat alter casus, quo debet esse $\mathfrak{C} > 1$. tum ergo esse deberet ante omnia numerator positiuus seu

$$2\mathfrak{M}\zeta(\zeta-1) > k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)$$

$$\text{ideoque } k < \frac{2\mathfrak{M}\zeta(\zeta-1)}{(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)}$$

deinde vt etiam vnitatem superet, debet esse

$$4\mathfrak{M}\zeta(\zeta-1) - 2k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)$$
$$> k(\mathfrak{M}+\zeta-1)^2 + 2\mathfrak{M}\zeta(\mathfrak{M}+\zeta-1)$$

$$\text{ideoque } k < \frac{2\mathfrak{M}\zeta(\zeta-\mathfrak{M}-1)}{(\mathfrak{M}+\zeta-1)(3\mathfrak{M}-\zeta+1)}$$

quod cum sit absurdum ob $k > 0$ indicio est, hunc casum locum habere non posse, ita, vt nobis solus casus in coroll. praeced. euolutus relinquatur.

Scholion.

Exempl. I.

245. Quoniam positio $\mathfrak{r} = 1$, ad campum maxime est accommodata simulque solutionem tam facilem suppeditat, ea vtique sola meretur, vt ad praxin adplicetur. Hic autem primum obseruari conuenit, nequaquam sumi posse $\zeta = 1$; quia tum foret $k = 0$ statimque primum interuallum $= \infty$, vt reliqua incommoda taceamus, dum scilicet tam secunda quam tertia lens haberent distantias focales infinitas. Ex quo necesse est, pro ζ sumi numerum vnitate maiorem; ita, vt excessus non sit nimis paruus, quia alioquin ad eadem incommoda appropinquaremus. Quamobrem quo clarius appareat, quomodo in hoc negotio sit procedendum, sumamus $\zeta = 2$, vt fiat $M = \frac{2}{\mathfrak{M}+1}$. Tum vero k contineri debet intra hos limites

$$1 \text{ et } \frac{4\mathfrak{M}}{(\mathfrak{M}+1)(\mathfrak{M}-1)} \text{ vel } 1 \text{ et } \frac{4}{\mathfrak{M}}.$$

Neque autem k ad vnitatem nimis prope accedere debet, quia alioquin B hincque secundum interuallum nimis euaderet magnum. Sumamus igitur $k = \frac{5}{9}$ hinc erit

$$Q = \frac{2\mathfrak{M}(\mathfrak{M}+1)}{9\mathfrak{M}+1} = \frac{2}{9}\mathfrak{M} \text{ proxime.}$$

Deinde vero erit $\mathfrak{B} = \frac{3}{4}$ et $B = 3$ ac denique

$$\mathfrak{C} = \frac{16\mathfrak{M} - 2(\mathfrak{M}+1)(\mathfrak{M}-1)}{(9\mathfrak{M}+1)(\mathfrak{M}+1)} = -\frac{2}{9} + \frac{16}{\mathfrak{M}} = -\frac{2}{9} \text{ proxime}$$

hincque $C = -\frac{2}{11}$; vnde pro microscopii constructione habebimus has distantias focales $p =$

$$p = \mathfrak{A}\, a;\quad q = \tfrac{3}{2}\, \mathrm{A}\, a;$$
$$r = \tfrac{6}{\mathfrak{M}} \cdot \mathrm{A}\, a;\quad s = \tfrac{6}{11} \cdot \tfrac{\mathrm{A}\, a}{\mathfrak{M}};$$

et interualla

$$\mathrm{I}^{mum} = 3\, \mathrm{A}\, a;$$
$$\mathrm{II}^{dum} = 6\, \mathrm{A}\, a \left(1 - \tfrac{9}{2\mathfrak{M}}\right);$$
$$\mathrm{III}^{tium} = \tfrac{60}{11} \cdot \tfrac{\mathrm{A}\, a}{\mathfrak{M}};$$

cum vero diſtantia oculi

$$O = \frac{s}{\mathfrak{M}\, M} = \frac{s(\mathfrak{M}+1)}{2\mathfrak{M}} = \tfrac{1}{2} s \left(1 + \tfrac{1}{\mathfrak{M}}\right) = \tfrac{1}{2} s \text{ proxime.}$$

Pro campo autem apparente

$$z = M\, a\, \xi = \frac{2 a \xi}{\mathfrak{M}+1} = \tfrac{1}{2} \cdot \frac{a}{\mathfrak{M}+1} \text{ ob } \xi = \tfrac{1}{4}.$$

Quod vero ad litteram A attinet, quae ad primam lentem refertur, curandum eſt, vt A tantus fiat numerus, vt lens ocularis non fiat nimis exigua; vnde ſequitur $\mathfrak{A}$ eſſe debere fractionem parum ab vnitate deficientem, cuius valore ſtabilito apertura primae lentis definiri debet ex aequatione nota

$$\frac{1}{k^3} = \frac{\mu \mathfrak{M} x^3}{a^3} \left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{4\mathfrak{A}} + \text{etc.} \right)$$

ſicque obtinemus microſcopium ſatis notatu dignum, quod inſtar primi exempli ſpectari poteſt.

Exemplum II.

246. Conſideremus etiam caſum $\zeta = 3$ vt ſit $M = \frac{2}{\mathfrak{M}+2}$ et pro k habebuntur hi limites 2 et $\frac{12}{\mathfrak{M}}$.

Statua-

Statuamus ergo $k = 1$ vt fiat $\mathfrak{B} = \frac{2}{3}$ et $B = 2$. Tum vero erit $Q = \frac{\mathfrak{M}}{7}$ et $k' = 7$. Porro vero erit $\mathfrak{C} = -\frac{2}{7}$ et $C = -\frac{2}{9}$ vnde pro his microſcopiis erunt diſtantiae focales

$$p = \mathfrak{A}\, a;\ q = \tfrac{2}{3}\, A\, a;$$
$$r = 4 \cdot \frac{A a}{m} \text{ et } s = \frac{4}{9} \cdot \frac{A a}{\mathfrak{M}};$$

et interualla

$$I^{mum} = 2\, A\, a;$$
$$II^{dum} = 2 \cdot A\, a\left(1 - \frac{7}{\mathfrak{M}}\right) \text{ et}$$
$$III^{tium} = \frac{32}{9} \cdot \frac{A a}{\mathfrak{M}};$$

et diſtantia oculi

$$O = s \cdot \frac{\mathfrak{M} + 2}{2\mathfrak{M}} = \tfrac{1}{2} s\left(1 + \frac{2}{\mathfrak{M}}\right) = \tfrac{1}{2} s \text{ proxime.}$$

Pro campo vero $z = \frac{1}{2} \cdot \frac{a}{\mathfrak{M} + 2}$.

Circa aperturam modo allegata valent. Ration autem littera $\mathfrak{q}$ erit caſu exempli praecedentis $\mathfrak{q} = -\frac{4}{\mathfrak{M} + 2}$ et caſu huius exempli $\mathfrak{q} = -\frac{6}{\mathfrak{M} + 2}$; vnde patet, ſecundae lentis ſemidiametrum aperturae eſſe debere $= \frac{x}{\frac{1}{2}} = \frac{x}{k}$ ideoque priori caſu $= 2\, x$; hoc vero $= x$. Binae poſtremae lentes autem fieri debent vtrinque aeque conuexae.

Scholion.

247. Si haec ad teleſcopia referamus, quod nunc eo magis eſt neceſſarium, quoniam ſupra huius generis

neris casum tantum maxime particularem euoluimus, qui ne campum quidem maximum, vt hic faciamus, praebebat; tantum fecimus $a = \infty$ et $\mathfrak{M} = m$ ob $b = a$. Tum autem capi debet tam $\mathfrak{A} = 0$ quam $A = 0$, ita vt fiat $\mathfrak{A}a = Aa = p$, quare cum reliqua omnia maneant vt ante, ex exemplo priore posita lentis obiectiuae distantia focali $= p$ erunt reliquarum lentium distantiae focales

$$q = \tfrac{3}{2}p;\ r = \tfrac{6}{m}\cdot p \text{ et } s = \tfrac{6}{11}\cdot\tfrac{p}{m}.$$

Interuallo vero

$$1^{mum} = 3p;$$
$$2^{dum} = 6.p\left(1 - \tfrac{1}{2m}\right) \text{ et}$$
$$3^{tium} = \tfrac{60}{11m}.p \text{ et } O = \tfrac{1}{2}s.$$

Tum vero campi semidiameter

$$\Phi = \tfrac{1}{2}.\tfrac{1}{m+1} = \tfrac{1718}{m+1} \text{ min.}$$

Deinde vero ob

$$\mathfrak{B} = \tfrac{3}{4},\ B = 3,\ \mathfrak{C} = -\tfrac{2}{9} \text{ et } C = -\tfrac{2}{11}$$

distantia focalis p ex requisita apertura $x = \frac{1}{50}m$ dig. definiri debet ope huius aequationis:

$$p = m\sqrt[3]{\mu\left(\lambda + 1\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}}\right) - \frac{9}{mB^3}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right) - \frac{\lambda'''}{B^3C^3.m}\right)}$$

In casu autem alterius exempli erunt distantiae focales

$$q = \tfrac{2}{3}p;\ r = \tfrac{4}{m}.p;\ s = \tfrac{4}{9m}.p;$$

et

et interualla

$$1^{mum} = 2p;$$
$$2^{dum} = 2p\left(1 - \frac{7}{m}\right);$$
$$3^{tium} = \frac{32}{9m} \cdot p.$$

tum vero p ita definietnr, vt fit

$$p = m\sqrt[3]{\mu\left(\lambda + \frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}} - \frac{7}{mB^3}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right) - \frac{\lambda'''}{mB^3C^3}\right)}$$

reliqua vero erunt, vt ante. Hinc igitur loco communium telescopiorum terrestrium nanciscimur ex casu posteriore sequentem constructionem, siqutdem omnes 4 lentes ex vitro communi, cuius refractio sit $n = 1,55$, conficere velimus.

Constructio Telescopiorum loco vulgarium terrestrium substituendorum.

248. Pro data multiplicatione m quaeratur primo lentis obiectiuae distantia focalis $= p$; ex hac nempe formula, ad quam praecedens proxime reducitur: $p = \frac{2}{5}m$ dig. deinde constructio ita se habebit:

I. Pro prima lente, cuius distantia focalis est $\frac{2}{5}m$ dig. capiatur

radius faciei $\begin{cases} \text{anter.} = 0,6148.\,p \\ \text{poster.} = 5,2438.\,p \end{cases}$

eius aperturae semidiameter $x = \frac{1}{50}m$ dig.

et distantia ad lentem sequentem $= 2p$.

II. Pro

II. Pro ſecunda lente, cuius diſtantia focalis eſt $q = \frac{2}{3}p$, et numeri $\mathfrak{B} = \frac{2}{3}$ et $\lambda' = 1$, erit

$$\text{radius faciei} \begin{cases} \text{anter.} = \frac{q}{0,66806} = 0,99792 \cdot p \\ \text{poſter.} = \frac{q}{1,1432} = 0,58315 \cdot p \end{cases}$$

eius aperturae ſemidiameter $= x = \frac{1}{50} m$ dig.
et interuallum ad tertiam lentem $= 2p\left(1 - \frac{7}{m}\right)$.

III. Pro tertia lente, cuius diſtantia focalis eſt $r = \frac{4}{m} p$. quoniam ea debet eſſe vtrinque aequaliter conuexa, capiatur eius vterque

$$\text{radius} = 1,1 \cdot r = 4,4 \cdot \frac{p}{m}$$

eius aperturae ſemidiameter $= \frac{p}{m}$
et diſtantia ad quartam lentem $= \frac{22}{9 \cdot m} \cdot p$.

IV. Pro quarta lente, cuius diſtantia focalis eſt $s = \frac{4}{9} \cdot \frac{p}{m}$ capiatur itidem vterque radius $= \frac{22}{45} \cdot \frac{p}{m}$
eius aperturae ſemidiameter $= \frac{1}{9} \cdot \frac{p}{m}$
et diſtantia ad oculum $= \frac{1}{2} s$.

V. Tum vero erit ſemidiameter campi

$$\Phi = \frac{1718}{m+2} \text{ min.}$$

Scholion.

249. Teleſcopia haec vtique inſigni vitio laborant propterea quod eorum longitudo ſit plane

enormis, maior ſcilicet quam $4p$. Huic autem vitio medela afferri poterit, litterae k maiorem valorem tribuendo; tum vero etiam littera ζ maior accipi debebit; vnde quidem campus aliquantillum diminuitur, qui tamen defectus in maioribus multiplicationibus vix percipietur. Sumamus igitur $\zeta = 6$, vt fiat $M = \frac{2}{\mathfrak{M}+5}$ et cum limites pro k ſint 5 et $\frac{60}{m}$ ſumamus $k = 4$ vt fiat $\mathfrak{B} = \frac{5}{6}$ et $B = 5$, tum vero erit $Q = \frac{m}{6}$ et $k' = \frac{5}{2}$, tandemque $\mathfrak{C} = -\frac{1}{2}$ et $C = -\frac{5}{3}$. hinc autem erit interuallum

$$1^{mum} = \tfrac{5}{4}p.$$
$$2^{dum} = \tfrac{5}{4}p\left(1 - \tfrac{6}{m}\right)$$
$$3^{tium} = \tfrac{25}{6} \cdot \tfrac{p^2}{m}$$

vnde longitudo prodiret quaſi $2\frac{1}{2}p$. quae adhuc nimis magna videri poteſt. Hanc longitudinem autem non mediocriter diminuere poterimus, ſumendo $\zeta = 12$ et $k = 9$, hinc enim fit $\mathfrak{B} = \frac{5}{6}$ et $B = 5$, vt ante; vnde ſequitur interuallum

$$1^{mum} = \tfrac{10}{9}p. \text{ et } 2^{dum} = \tfrac{5}{9}p \ldots$$

ita, vt tota longitudo quaſi fiat $1\frac{2}{3}p$. quae non excedit teleſcopia huius generis vulgaria. Si ſumſiſſemus $\zeta = 12$ et $k = 8$ vt fiat $\mathfrak{B} = \frac{3}{4}$ et $B = 3$, foret interuallum

$$1^{mum} = \tfrac{9}{8}p \text{ et } 2^{dum} = \tfrac{3}{8}p$$

ita, vt tota longitudo quaſi ſit $1\frac{1}{2}p$. quae vtique admitti

mitti poterit. Hic ergo casus meretur, vt plenius euoluatur.

Fiet autem porro $Q = \frac{m}{32}$ hincque $k' = 4$. Porro $\mathfrak{C} = -\frac{1}{2}$ et $C = +\frac{1}{3}$; vnde

$$q = \frac{3}{32} \cdot p; \quad r = 6 \cdot \frac{p}{m} \text{ et } s = \frac{p}{m};$$

tum vero interualla

$$1^{mum} = \frac{9}{8} p.$$

$$2^{dum} = \frac{13}{8} p \left(1 - \frac{32}{m}\right);$$

$$3^{tium} = 5 \cdot \frac{p}{m};$$

et pro loco oculi $O = \frac{5}{2}\left(1 + \frac{22}{m}\right)$

semidiameter campi $= \frac{1718}{m + 11}$ min.

Sumto igitur pro apertura lentis obiectiuae $x = \frac{m}{50}$ dig. et $k = 50$ capi debebit circiter

$$p = m \sqrt[3]{\frac{4}{3}} = \frac{11}{10} m \text{ dig.}$$

vnde conficitur sequens

Constructio Telescopii communis ex vitro communi.

I. Pro data multiplicatione m sumatur

$p = \frac{11}{10} \cdot m$ circ. siue etiam $p = m$ dig.

II. Pro prima lente cuius distantia focalis $= p$, capiatur

radius faciei $\begin{cases} \text{anter.} = 0,6148 \cdot p \\ \text{poster.} = 5,2438 \cdot p \end{cases}$

 eius

eius aperturae ſemidiameter $= \frac{m}{50}$ dig.
et diſtantia ad lentem ſecundam $= 1\frac{1}{8}p$.

III. Pro lente ſecunda, cuius diſtantia focalis eſt $q = \frac{3}{32}.p$ capiatur radius faciei

anter. $= \frac{q}{0,5499} = 0,17039.p.$

poſter. $= \frac{q}{1,2682} = 0,07388.p.$

eius aperturae ſemidiameter $= \frac{1}{8}x = \frac{m}{400}$ dig.
et diſtantia ad lentem tertiam $= \frac{3}{8}p\left(1 - \frac{32}{m}\right)$.

IV. Pro lente tertia, cuius diſtantia focalis eſt $r = 6.\frac{p}{m}$, capiatur vterque radius $= 6,6.\frac{p}{m}$ eique apertura maxima tribuatur, diſtantia vero a lente quarto erit $= s.\frac{p}{m}$.

V. Pro lente quarta, cuius diſtantia focalis $s = \frac{p}{m}$, capiatur vterque radius $= 1,1.\frac{p}{m}$.
eiusque ab oculo diſtantia $= \frac{5}{2}\left(1 + \frac{11}{m}\right)$.

VI. Longitudo erit $1\frac{1}{2}p - 6\frac{1}{2}.\frac{p}{m}$. Campi vero apparentis ſemidiameter erit $= \frac{1718}{m+11}$ min.

Hoc ergo Teleſcopium vulgaribus terreſtribus merito anteferendum videtur; notetur vero, id in praxi locum habere non poſſe, niſi ſit m notabiliter maius, quam 32. Hic autem iſtud caput finimus ad ſequens progreſſuri, vbi microſcopia magis compoſita huius generis inueſtigabimus.

CAPVT