CAPVT IV.

DE TELESCOPIIS CATADIOPTRICIS MINORE SPECVLO CONVEXO INSTRVCTIS.

Problema I.

§. 50.

Constructionem huiusmodi telescopiorum describere, quibus obiecta situ inuerso repraesententur, seu vbi vnica imago realis occurrat.

Solutio.

Cum in hoc genere distantia amborum speculorum sit $AB = (1 - \varepsilon)p$, ideoque $b = -\varepsilon p$ ob $\alpha = p$ erit $P = -\frac{\alpha}{b} = +\frac{1}{\varepsilon}$, vbi ε designat fractionem aliquanto minorem, quam ratio foraminis ad speculum maius $\frac{y}{x}$ designat, ita, vt posito $y = \delta x$ sit $\varepsilon < \delta$, ob rationem ante allegatam §. 49. qua scilicet obtinetur, vt etiam radii obliqui a minore speculo excipiantur. Interim tamen semidiameter aperturae minoris speculi maneat $= \delta x = y$, ita, vt hoc speculum foramini aequetur, vti initio assumsimus. Nunc statim consideremus aequationem, qua margo colo-

coloratus deſtruitur, quae ſi praeter ſpecula duae lentes adhibeantur, reducitur ad hanc formam: $0 = \mathfrak{r} + \frac{\mathfrak{s}}{R}$, vnde vt ambae litterae $\mathfrak{r}$ et $\mathfrak{s}$ valores poſitiuos habere queant, vti ratio campi poſtulat, conueniet litterae R valorem tribui negatiuum et quidem vnitate non minorem, vt ſumto $\mathfrak{s} = 1$ prior lens C, cuius apertura iam per foramen determinatur, campum non reſtringat. Ponamus igitur $R = -i$ et cum ex data multiplicatione m ob repraeſentationem inuerſam ſit $PQR = -m$ fiet hinc $PQ = \frac{m}{i}$ et $Q = \frac{\varepsilon m}{i}$. Eſt vero $Q = -\frac{\beta}{c}$ et quia eſt

$$\beta + c = BC = AB = (1 - \varepsilon)p;$$

hinc colligimus

$$c = -\frac{i(1-\varepsilon)p}{\varepsilon m - i} \text{ et } \beta = \frac{m.\varepsilon(1-\varepsilon)p}{\varepsilon m - i}$$

quare cum in genere ſit $\frac{1}{q} = \frac{1}{b} + \frac{1}{\beta}$ erit

$$\frac{1}{q} = -\frac{(1-2\varepsilon)m - i}{m\varepsilon(1-\varepsilon)p} \text{ hincque } q = -\frac{m\varepsilon(1-\varepsilon)p}{(1-2\varepsilon)m + i}.$$

Porro ex valoribus b et β colligimus

$$B = \frac{\beta}{b} = \frac{-m(1-\varepsilon)}{\varepsilon m - i} \text{ et } \mathfrak{B} = \frac{+m(1-\varepsilon)}{(1-2\varepsilon)m + i}.$$

Deinde cum ſit $C = \frac{\gamma}{c}$ et $\mathfrak{C}c = r$ hinc inuenimus

$$\mathfrak{C} = \frac{r}{c} = -\frac{(\varepsilon m - i)r}{i(1-\varepsilon)p} \text{ ideoque}$$

$$C = \frac{-(\varepsilon m - i)r}{i(1-\varepsilon)p + (\varepsilon m - i)r} = \frac{\gamma}{c}$$

ex quo porro colligitur

$$\gamma = \frac{i(1-\varepsilon)pr}{i(1-\varepsilon)p + (\varepsilon m - i)r}.$$

Deni-

Denique cum ſit $R = -\frac{\gamma}{d} = -\frac{\gamma}{s}$ ob $s = d$, erit

$$s = -\frac{\gamma}{R} = \frac{\gamma}{i} = \frac{(1-\varepsilon)pr}{i(1-\varepsilon)p + (\varepsilon m - i)r}$$

hincque tertium interuallum

$$CD = \gamma + s = \frac{(1+i)(1-\varepsilon)pr}{i(1-\varepsilon)p + (\varepsilon m - i)r}.$$

Nunc autem aperturae praebent has aequationes

1°. $\mathfrak{B}\mathfrak{q} = (P-1)M$, vnde fit

$$\mathfrak{q} = \frac{((1-2\varepsilon)m + i)M}{\varepsilon m};$$

2°. $\mathfrak{C}\mathfrak{r} = (PQ-1)M - \mathfrak{q} = \frac{\varepsilon m^2 - (1-\varepsilon)im - i^2}{\varepsilon i m} M$

ſeu $\mathfrak{C}\mathfrak{r} = \frac{(m+i)(\varepsilon m - i)}{\varepsilon i m}.M$

vnde elicitur $\mathfrak{r} = -\frac{(m+i)(1-\varepsilon)p}{\varepsilon m r}.M$

vnde cum ſit $\mathfrak{s} = i\mathfrak{r}$ ideoque $\mathfrak{r} + \mathfrak{s} = (1+i)\mathfrak{r}$, erit

$$\mathfrak{q} + \mathfrak{r} + \mathfrak{s} = \frac{(1-2\varepsilon)mr + ir - (1+i)(m+i)(1-\varepsilon)p}{\varepsilon m r}.M$$

$= M(m-1)$, ſicque facta diuiſione per M inueniemus

$$r = \frac{-(m+i)(1+i)(1-\varepsilon)p}{\varepsilon m^2 - (1-\varepsilon)m - i}$$

qui valor cum ſit negatiuus, ex eo etiam prodibit interuallum CD negatiuum, vnde patet hunc valorem in praxi locum habere non poſſe.

Verum cum ſaepenumero problemata duas pluresue ſolutiones admittant, idem etiam hic vſu venit, hocque problema praeter ſolutionem hic inuentam inſuper aliam complectitur, quam per diuiſionem ex calculo expulimus. Qnod quod facilius appareat, calculum ita inſtituamus; cum primo ſit

$$\mathfrak{q} = \frac{(1-2\varepsilon)m+i}{\varepsilon m}.\ \mathrm{M}\ \text{deinde}\ \mathfrak{s} = i\,\mathfrak{r},\ \text{erit}$$

$$\mathfrak{q}+\mathfrak{r}+\mathfrak{s} = \frac{(1-2\varepsilon)m+i}{\varepsilon m}\mathrm{M}+(i+1)\mathfrak{r} = \mathrm{M}(m-1)$$

vnde colligitur

$$\mathrm{M} = \frac{\varepsilon m(1+i)\mathfrak{r}}{\varepsilon m^2-(1-\varepsilon)m-i}\ \text{ideoque}$$

$$\mathfrak{q} = \frac{(1+i)((1-2\varepsilon)m+i)\mathfrak{r}}{\varepsilon m^2-(1-\varepsilon)m-i},$$

altera vero aequatio dabit

$$\mathfrak{C}\,\mathfrak{r} = \frac{(m-i)\varepsilon m(1+i)\mathfrak{r}-i(1+i)((1-2\varepsilon)m+i)\mathfrak{r}}{\varepsilon i m^2-(1-\varepsilon)i m-i^2}$$

vnde fit

$$\mathfrak{C} = \frac{(1+i)(m+i)(\varepsilon m-i)}{i(\varepsilon m^2-(1-\varepsilon)m-i)}$$

ſupra vero iam inuenimus

$$\mathfrak{C} = \frac{-(\varepsilon m-i)r}{i(1-\varepsilon)p}$$

vnde patet aequalitatem horum duorum valorum duplici modo obtineri poſſe 1°. ſcilicet, ſi fuerit $i=\varepsilon m$, quo quippe vterque valor euaneſcit; 2°. autem, quo facta diuiſione per

$$\varepsilon m - i\ \text{fit}\ \frac{(1+i)(m+i)}{\varepsilon m^2-(1-\varepsilon)m-i} = \frac{-r}{(1-\varepsilon)p}$$

haecque eſt ſolutio incongrua ante inuenta. Statuamus igitur nunc $i=\varepsilon m$ fietque $\mathfrak{C}=0$, littera vero r hinc

r hinc plane non determinatur, et noſtra ſolutio ſequenti modo ſe habebit:

$$\alpha = p;\ b = -\varepsilon p;\ c = -\infty;\ d = \frac{r}{\varepsilon m};$$
$$\beta = \infty;\quad \gamma = r;$$

vbi notetur, fore $\beta + c = (1 - \varepsilon) p$.

Hinc porro erit

$$\mathrm{B} = \infty;\ \mathfrak{B} = 1.\ \mathrm{C} = 0;\ \mathfrak{C} = 0.$$

tum vero

$$\mathrm{P} = \frac{1}{\varepsilon};\ \mathrm{Q} = 1;\ \mathrm{R} = -\varepsilon m;$$

ita, vt ſit $\mathrm{PQR} = -m$.

Quia vero $\mathrm{B} = \infty$ et $\mathrm{C} = \mathfrak{C} = 0$, productum in ſe manet indefinitum; verum cum ſit

$$r = \frac{\mathrm{B}\mathfrak{C}}{\mathrm{PQ}} \cdot p = \varepsilon \mathrm{B} \mathfrak{C} p,$$ hinc viciſſim erit $\mathrm{B}\mathfrak{C} = \frac{r}{\varepsilon p}$.

Praeterea vero erunt diſtantiae focales

$$q = -\varepsilon p;\ \text{et}\ s = d = \frac{r}{\varepsilon m}$$

atque interualla

$$\mathrm{AB} = \mathrm{BC} = (1 - \varepsilon) p\ \text{et}\ \mathrm{CD} = r\left(1 + \frac{1}{\varepsilon m}\right)$$

Denique cum ſit

$$\mathfrak{q} = \frac{(1 + \varepsilon m)(1 - \varepsilon)\mathfrak{r}}{\varepsilon m - 1}\ \text{et}\ \mathfrak{s} = \varepsilon m \mathfrak{r}\ \text{erit}$$
$$\mathrm{M} = \frac{\varepsilon(\varepsilon m + 1)\mathfrak{r}}{\varepsilon m - 1} = \frac{(\varepsilon m + 1)\mathfrak{s}}{m(\varepsilon m - 1)}$$

ideoque ſemidiameter campi apparentis

$$\Phi = \frac{1}{4} \cdot \frac{(\varepsilon m + 1)\mathfrak{s}}{m(\varepsilon m - 1)} = 859 \cdot \frac{(\varepsilon m + 1)\mathfrak{s}}{m(\varepsilon m - 1)}\ \text{minut.}$$

vbi

vbi sumere licebit $\theta = 1$, si modo lens ocularis vtrinque fiat aeque conuexa. Oculi vero post hanc lentem distantia reperitur

$$O = \frac{\theta s}{M m} = \frac{\varepsilon m - 1}{\varepsilon m + 1} \cdot s.$$

Quia autem lentis C semidiameter aperturae maior esse nequit, quam $y = \delta x$, ponamus $\frac{1}{4} \mathfrak{r}\, r = \delta x$ siue $\frac{\theta r}{4 \varepsilon m} = \delta x$; vnde, sumto $\theta = 1$, definitur $r = 4 \delta \varepsilon m x$ hincque $s = 4 \delta x$. Verum etiam ad aperturam minoris speculi est attendendum, cuius semidiameter reuera est $= \delta x$ et qui ob campum esse deberet $= \frac{1}{4} \mathfrak{q} q$; quam ob caussam necesse est sit

$$\frac{(1 - \varepsilon)(\varepsilon m + 1) \theta . p}{4 m (\varepsilon m - 1)} < \delta x \text{ ideoque } \theta < \frac{4 m . (\varepsilon m - 1) \delta x}{(1 - \varepsilon) \cdot (\varepsilon m + 1) p}.$$

Tuto igitur sumere licebit $\theta = 1$, si modo fuerit

$$4 m (\varepsilon m - 1) \delta x > (1 - \varepsilon)(\varepsilon m + 1) p.$$

Contra vero θ vnitate minus accipi deberet. Tantum igitur superest, vt ex formula semidiametri confusionis definiamus distantiam focalem speculi principalis p, quae ita reperitur expressa

$$p = k x \sqrt[3]{m \left(\frac{1 - \varepsilon}{8} + \mu . \frac{\varepsilon^4 p^3}{r^3} . \lambda'' + \mu . \frac{\varepsilon^3 p^3}{m r^3} \lambda''' \right)}$$

siquidem ambobus speculis figura sphaerica inducatur, at si ambo habeant figuram parabolicam, debebit esse

$$r = k \varepsilon x \sqrt[3]{\mu m \left(\varepsilon \lambda'' + \frac{\lambda'''}{m} \right)}$$

ita, vt iam aliter non definiatur, nisi ex quantitate speculi, cum sine dubio semper esse debeat p multo maius

maius, quam x. Quia vero iam ante definiuimus, $r = 4\,\delta\,\varepsilon\, m\, x$, habebitur nunc

$$4\,\delta\, m = k \sqrt[3]{\mu\, m}\,(\varepsilon\,\lambda'' + \frac{\lambda'''}{m})$$

Cum nunc ſit proxime $\mu = 1$. ſumique poſſit $\lambda'' = 1$. et λ''' binario ſit minus, k vero infra 50 capi non debeat, valorem ipſius ε aeſtimare poterimus; tantus enim eſſe debet, vt numerus $\frac{4\,\delta\, m}{\sqrt[3]{(\varepsilon\, m + 2)}}$ non minor prodeat, quam 50; vnde patet pro ε ſumi debere fractionem valde paruam, ſi enim eſſet $\delta = \frac{1}{2}$ et $m = 100$, colligitur circiter $\varepsilon = \frac{1}{50}$.

Exemplum.

§. 51. Ponamus $m = 100$, $x = 2$ dig. $y = \frac{1}{2}$ dig. ideoque $\delta = \frac{1}{4}$ et vt $\frac{4\,\delta\, m}{\sqrt[3]{(\varepsilon\, m + 2)}}$ ſatis magnum obtineat valorem, ſumamus $\varepsilon = \frac{1}{20}$ ſic enim prodit $k = \frac{100}{\sqrt[3]{7}}$ ſeu $k > 50$ hinc ergo erit $r = 10$. dig. et $s = 2$ dig. Deinde cum pro ſpeculo minore debeat eſſe

$$8000 > 57 \cdot p. \text{ erit } p < \frac{8000}{57}.$$

Vnde tuto ſumi poterit $p = 25$. dig. ſicque erit $q = -\frac{5}{4}$ dig. et interuallum $AB = BC = 23\frac{3}{4}$ dig. et $CD = 12$. dig. Oculi vero diſtantia $O = \frac{4}{3}$ dig. at campi apparentis ſemidiameter $\Phi = 12'\, 53''$, vbi probe notandum, hic ambo ſpecula aſſumi perfecte parabolica.

 Scho-

Scholion.

§. 52. Quamuis autem haec constructio perfecte succedat, tamen tale telescopium tam insigni vitio erit praeditum, vt omni vsu destituatur; cum enim radii a minore speculo reflexi iterum fiant inter se paralleli, radii peregrini circa hoc speculum transeuntes et in lentem C incidentes cum illis refractionem communem patientur, simulque cum iis in oculum deferentur, ita, vt verum obiectum cum vicinis prorsus permixtum visioni repraesentetur neque vllo modo separari poterunt. Cum igitur huius vitii caussa in eo sit sita, quod radii a minore speculo reflexi fiant paralleli seu interuallum $\beta = \infty$, ne hoc fiat, diligenter erit cauendum, quod fiet, si distantia β minor fuerit interuallo BC, ita, vt in hoc interuallum imago realis incidat litteraque Q negatiuum obtineat valorem. Praeterea vero quia etiam R negatiuum valorem habere debet ob marginem coloratum, duae iam habebuntur imagines reales et obiecta situ erecto cernentur. Neque vero duabus tantum lentibus adhibendis scopo nostro satisfacere poterimus, sed tertiam insuper lentem in subsidium vocari oportebit, quae commodissime ita instrui poterit, vt aperturam quam minimam requirat, siquidem hoc modo segregatio radiorum peregrinorum felicissime succedet, quemadmodum in sequente problemate ostendemus.

Pro-

Problema 2.

§. 52. Huiusmodi telescopium cum speculo minore conuexo et tribus lentibus vitreis construere, quod obiecta situ erecto distincte repraesentet.

Solutio.

Maneat, vt ante, $y = \delta x$ et interuallum speculorum $AB = (1 - \varepsilon) p = BC$. vt sit $b = -\varepsilon p$. Iam cum debeat esse $\beta < (1 - \varepsilon) p$ et tamen superare debeat eius semissem $\frac{1}{2}(1 - \varepsilon) p$, statuamus $\beta = \zeta (1 - \varepsilon) p$, ita, vt ζ inter limites 1 et $\frac{1}{2}$ contineatur, hinc ergo fiet Tab. III. Fig. 9.

$$q = \frac{\zeta \varepsilon (1 - \varepsilon)}{\varepsilon - \zeta (1 - \varepsilon)} \cdot p = \frac{-\zeta \varepsilon (1 - \varepsilon)}{\zeta - \varepsilon (\zeta + 1)} \cdot p$$

Tum vero erit

$$\mathrm{B} = \frac{\beta}{b} = \frac{-\zeta (1 - \varepsilon)}{\varepsilon} \text{ et } \mathfrak{B} = \frac{\zeta (1 - \varepsilon)}{\zeta - \varepsilon (\zeta + 1)}.$$

Porro vero erit $c = (1 - \varepsilon)(1 - \zeta) p$ sicque habebimus

$$\mathrm{P} = \frac{1}{\varepsilon};\ \mathrm{Q} = \frac{-\beta}{c} = \frac{-\zeta}{1 - \zeta}.$$

Statuatur igitur praeterea $\mathrm{R} = -k$ fiatque

$$\mathrm{PQRS} = m = \frac{\zeta k}{\varepsilon (1 - \zeta)} \cdot \mathrm{S},$$

vnde reliquae distantiae focales erunt

$$r = (1 - \varepsilon)(1 - \zeta)\,\mathfrak{C}.\,p;$$

$$s = \frac{(1 - \varepsilon)(1 - \zeta)\,\mathfrak{C}\mathfrak{D}}{k} \cdot p \text{ et}$$

$$t = \frac{-\zeta (1 - \varepsilon) \cdot \mathfrak{C}\mathfrak{D}}{\varepsilon m} \cdot p.$$

reliquaque interualla

$$CD = (1-\varepsilon)(1-\zeta)(1+\tfrac{1}{k})\,C.\,p$$
$$DE = (1-\varepsilon)(1-\zeta)(1-\tfrac{1}{S})\,CD.\,p.$$

vnde intelligimus esse debere $C > 0$ ideoque $\mathfrak{C} < 1$; et $(1-\frac{1}{S})D > 0$. Vt vero fiat $t > 0$, debet esse $D < 0$ ideoque $S < 1$. Consideretur nunc aequatio pro margine colorato tollendo, quae est

$$0 = \mathfrak{r} + \frac{\mathfrak{s}}{R} + \frac{\mathfrak{t}}{RS} \text{ siue } \mathfrak{r} = \frac{\mathfrak{s}}{k} + \frac{\mathfrak{t}}{kS};$$

vt iam secunda lens nulla apertura indigeat, statuatur

$$\mathfrak{s} = 0. \text{ eritque } \mathfrak{r} = \frac{\mathfrak{t}}{kS}$$

aequationes autem, pro litteris $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ posito

$$M = \frac{q+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}}{m-1} = \frac{q+(1+kS)\mathfrak{r}}{m-1}, \text{ sunt}$$

$$1^{\circ}.\ \mathfrak{B}q = \frac{1-\varepsilon}{\varepsilon}.\,M$$
$$2^{\circ}.\ \mathfrak{C}\mathfrak{r} = \frac{-((1-\varepsilon)\zeta+\varepsilon)}{\varepsilon(1-\zeta)}\,M - q$$
$$3^{\circ}.\ 0 = \frac{\zeta(\varepsilon+k)-\varepsilon}{\varepsilon(1-\zeta)}.\,M - q - \mathfrak{r}.$$

Ex prima autem habetur

$$q = \frac{\zeta-\varepsilon(\zeta+1)}{\varepsilon\zeta}.\,M$$

Ex tertia autem fit

$$q = \frac{\zeta(\varepsilon+k)-\varepsilon}{\varepsilon(1-\zeta)}\,M - \mathfrak{r}$$

qui duo valores inter se aequati dant

$$M = \frac{\varepsilon\zeta(1-\zeta)\mathfrak{r}}{\zeta^2(1+k)-\zeta(1+\varepsilon)+\varepsilon}, \text{ hincque}$$
$$q = \frac{(1-\zeta)(\zeta-\varepsilon(1+\zeta))\mathfrak{r}}{\zeta^2(1+k)-\zeta(1+\varepsilon)+\varepsilon}.$$

Tum

Tum vero ob $M = \frac{q + r(1 + kS)}{m - 1}$ reperietur etiam

$$M = \frac{kSr}{m - 1 - \frac{-\zeta(\varepsilon + k) + \varepsilon}{\varepsilon(1 - \zeta)}}$$

ex quorum valorum aequalitate ob $m = \frac{\zeta k}{\varepsilon(1 - \zeta)}$. S reperitur tandem

$$\zeta(\zeta k S - \varepsilon(1 - \zeta) - \zeta(\varepsilon + k) + \varepsilon) =$$
$$k S(\zeta^2(1 + k) - \zeta(1 + \varepsilon) + \varepsilon) \text{ seu}$$
$$\zeta^2 = S(\zeta(1 + \varepsilon) - k\zeta^2 - \varepsilon)$$

vnde concludimus esse debere

$$\zeta(1 + \varepsilon) > k\zeta^2 + \varepsilon \text{ siue } k < \frac{\zeta(1 + \varepsilon) - \varepsilon}{\zeta^2}.$$

Praeterea vero vt ex secunda aequatione pro $\mathfrak{C}$ prodeat valor positiuus, necesse est, vt sit $q < 0$. ideoque etiam $\mathfrak{B} < 0$. vnde speculum minus foret concauum; verum vt fiat $\mathfrak{B} < 0$, debet esse $\zeta < \varepsilon(\zeta + 1)$ seu $\varepsilon > \frac{\zeta}{\zeta + 1}$. Hoc vero non sufficit, sed insuper necesse est, vt sit

$$-q > \frac{(1 - \varepsilon)\zeta + \varepsilon}{-\varepsilon(1 - \zeta)}.\ M \text{ seu } \frac{-\zeta + \varepsilon(\zeta + 1)}{\zeta} > \frac{(1 - \varepsilon)\zeta + \varepsilon}{(1 - \zeta)}$$

vnde sequitur $\varepsilon > \frac{\zeta}{1 - \zeta}$, quod cum nullo modo fieri queat, quia ζ intra limites 1 et $\frac{1}{2}$ continetur et ε vnitate minus esse debet, nunc demum intelligimus, hunc casum locum habere non posse.

Alia Solutio.

§. 53. Quoniam igitur hoc incommodum inde nascitur, quod sumsimus R negatiuum, consideremus

 alte-

alterum casum, quo S fit negatiuum, manente R positiuo, et quoniam Q positum est negatiuum, ponamus $Q = -i$ et $S = -k$, vt fit

$$PQRS = \frac{iRk}{\varepsilon} = m;$$

calculus autem commodior euadet, si littera i retineatur, et cum sit $i = \frac{\beta}{c}$ et $\beta + c = (1 - \varepsilon)p$, euidens est, capi debere $i > 1$, eritque

$$\beta = \frac{i(1-\varepsilon)}{1+i}p \text{ et } c = \frac{1-\varepsilon}{1+i}p. \text{ vnde fit}$$

$$B = +\frac{\beta}{b} = -\frac{i(1-\varepsilon)}{\varepsilon(1+i)} \text{ et}$$

$$\mathfrak{B} = +\frac{i(1-\varepsilon)}{i(1-2\varepsilon)-\varepsilon}; \text{ hincque } q = -\frac{i\varepsilon(1-\varepsilon)}{i(1-2\varepsilon)-\varepsilon}.p.$$

Reliquae vero distantiae focales erunt

$$r = +\frac{(1-\varepsilon).\mathfrak{C}}{(1+i)}.p.; \; s = -\frac{(1-\varepsilon).C\mathfrak{D}}{(1+i)R}.p \text{ et}$$

$$t = -\frac{(1-\varepsilon)CD}{(1+i)Rk}.p,$$

et duo reliqua interualla erunt

$$CD = +\frac{(1-\varepsilon)C}{1+i}(1-\tfrac{1}{R})p;$$

$$DE = -\frac{(1-\varepsilon)C.D}{(1+i)R}(1+\tfrac{1}{k})p.$$

Vt igitur fiat $t > 0$, debet esse CD negatiuum, quo ipso etiam vltimum interuallum fit positiuum. Vt vero et penultimum fiat positiuum, debet esse $C(1-\frac{1}{R})$ positiuum. Conditio porro marginis colorati sumto $\mathfrak{s} = 0$. praebet $\mathfrak{r} = \frac{t}{Rk}$ siue $t = Rk.\mathfrak{r}$. et cum sit

$$M = \frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{t}}{m-1} = \frac{\mathfrak{q}+(1+Rk)\mathfrak{r}}{m-1}$$

satis-

ſatisfieri oportet his tribus aequationibus

$$1^\circ.\ \mathfrak{B}\,\mathfrak{q} = \frac{1-\varepsilon}{\varepsilon}.\,M$$

$$2^\circ.\ \mathfrak{C}\,\mathfrak{r} = -\frac{(i+\varepsilon)}{\varepsilon}\,M - \mathfrak{q}$$

$$3^\circ.\ 0 = +\frac{(iR+\varepsilon)}{\varepsilon}\,M + \mathfrak{q} + \mathfrak{r}.$$

Ex tertia ergo fit

$$\mathfrak{q} + \mathfrak{r} = -\frac{(iR+\varepsilon)}{\varepsilon}\,M;\ \text{hincque}$$

$$\mathfrak{q} + \mathfrak{r}(1 + Rk) = -\frac{(iR+\varepsilon)}{\varepsilon}\,M + Rk\mathfrak{r} = M(m-1)$$

vnde colligitur

$$M = \frac{Rk.\,\mathfrak{r}}{m + \frac{iR}{\varepsilon}},\ \text{ſimulque}$$

$$\mathfrak{q} = -\frac{Rk(iR+\varepsilon)}{m\varepsilon + iR}\,\mathfrak{r} - \mathfrak{r} = -\frac{(kiR^2 + R(i+k\varepsilon) + m\varepsilon)}{m\varepsilon + iR}.\,\mathfrak{r}.$$

ex quo valor ipſius $\mathfrak{q}$ prodit negatiuus, qui cum ex prima forma prodeat poſitiuus, ſiquidem eſt $\mathfrak{B} > 0$, patet, etiam hanc ſolutionem locum habere non poſſe, ſiquidem ſecundum ſpeculum eſt conuexum, vti aſſumſimus.

Tertia Solutio.

§. 54. Pro repraeſentatione igitur erecta vnicus tantum caſus ſupereſt, quo ſumto Q poſitiuo ambae litterae R et S negatiuos obtinent valores. Statuamus igitur $Q = +i$; $R = -k$ et $S = -k'$, vt ſit

$$PQRS = m = \frac{ikk'}{\varepsilon}\ \text{hincque}\ k' = \frac{\varepsilon m}{ik}.$$

Porro

Porro erit

$$\mathfrak{C} = \frac{i(1-\varepsilon)}{i-1} \cdot p; \quad c = -\frac{(1-\varepsilon)}{i-1} \cdot p; \text{ vnde fit}$$

$$\mathfrak{B} = \frac{-i(1-\varepsilon)}{\varepsilon(i-1)} \text{ et } \mathfrak{B} = \frac{i(1-\varepsilon)}{i(1-2\varepsilon)-\varepsilon}$$

quare diſtantiae focales ſequenti modo ſe habebunt:

$$q = \frac{-\varepsilon i(1-\varepsilon)}{i(1-2\varepsilon)+\varepsilon} \cdot p; \quad r = \frac{-(1-\varepsilon)\mathfrak{C}}{i-1} \cdot p;$$

$$s = \frac{-(1-\varepsilon)C\mathfrak{D}}{(i-1)k} \cdot p. \text{ et } t = \frac{-(1-\varepsilon)CD}{(i-1)kk'} \cdot p.$$

$$= \frac{-i(1-\varepsilon).CD}{\varepsilon(i-1)m} \cdot p.$$

Interualla vero lentium erunt

$$CD = \frac{-(1-\varepsilon)C}{i-1}\left(1+\frac{1}{k}\right)p.$$

$$DE = \frac{-(1-\varepsilon)CD}{(i-1)k}\left(1+\frac{1}{k'}\right)p.$$

vnde intelligimus, eſſe debere $C \lessgtr 0$ et $D > 0$. ideoque $\mathfrak{D} \lessgtr 1$. et $\mathfrak{D} > 0$. Nunc autem conditio marginis colorati dabit $0 = \mathfrak{r} + \frac{t}{kk'}$, vnde patet, eſſe debere $\mathfrak{r} \lessgtr 0$. ſeu ob lentem C campum diminui. Ponamus ergo hic $\mathfrak{r} = -\omega$, vt fiat $t = \omega . kk' = \frac{\varepsilon m}{i}\omega$, quandoquidem etiam hic aſſumſimus $\mathfrak{s} = 0$; pro campo ergo apparente erit

$$M = \frac{iq + \omega(\varepsilon m - i)}{i(m-1)}$$

cui ſequentes tres aequationes ſunt adiungendae:

1°. $\mathfrak{B} q = \frac{1-\varepsilon}{\varepsilon} . M.$

2°. $-\mathfrak{C}\omega = \frac{i-\varepsilon}{\varepsilon} M - q.$

3°. $0 = -\frac{(ik+\varepsilon)}{\varepsilon} M - q + \omega.$

Ex

Ex hac vltima ergo concludimus

$$q = \omega - \frac{(ik+\varepsilon)}{\varepsilon} . M.$$

addatur vtrinque $\omega\left(\frac{\varepsilon m}{i} - 1\right)$, eritque

$$q + \omega\left(\frac{\varepsilon m}{i} - 1\right) = \frac{\varepsilon m}{i}\,\omega - \frac{(ik+\varepsilon)}{\varepsilon}\, M = M(m-1)$$

ex quo colligitur

$$M = \frac{\varepsilon^2 m\omega}{i(m\varepsilon+ik)}; \text{ vnde viciſſim}$$

$$q = \frac{\varepsilon m(i-ik-\varepsilon)+i^2 k}{i(\varepsilon m+ik)} . \omega.$$

Ex prima vero aequatione fit

$$q = \frac{(\varepsilon i(1-2\varepsilon)+\varepsilon^2)m\omega}{i^2(m\varepsilon+ik)}$$

quorum valorum aequalitas ſuppeditat hanc aequationem

$$\varepsilon i m(i - ik - \varepsilon) + i^3 k = \varepsilon m(i(1-2\varepsilon)+\varepsilon)$$

ſeu

$$\varepsilon m(i^2(1-k) - i(1-\varepsilon) - \varepsilon) + i^3 k = 0.$$

ex qua aequatione inuenimus

$$k = \frac{\varepsilon m(i^2 - i(1-\varepsilon) - \varepsilon)}{i^2(\varepsilon m - i)} \text{ ſeu } k = \frac{\varepsilon m(i+\varepsilon)(i-1)}{i^2(\varepsilon m - i)}$$

qui valor debet eſſe poſitiuus; quem in finem ſumi debet $i > 1$ et $i < \varepsilon m$. Iam ſubſtituto valore ipſius k reperitur

$$M = \frac{\varepsilon(\varepsilon m - i)\omega}{\varepsilon i m - i(1-\varepsilon) - \varepsilon}.$$

Ex ſecunda denique aequatione colligimus

$$\mathfrak{E} = -\frac{k(\varepsilon m - i)}{\varepsilon m + ik}$$

Secunda vero aequatio dat

$$\mathfrak{C} = -\frac{\varepsilon m(i+\varepsilon)(i-1)}{i^2(\varepsilon m+ik)}$$

qui valor ergo eſt negatiuus, ideoque et C < o, vti ſupra iam requirebatur. Litterae autem 𝔇 et D arbitrio noſtro manent permiſſae, dummodo D poſitiue capiatur; quod tandem ad ipſam quantitatem p attinet, eam ex confuſione definiri conuenit ope formulae notae, vbi inprimis diſpiciendum erit, vtrum ſpeculis figura ſphaerica inducta ſit an parabolica.

Coroll. 1.

§. 55. Si ergo littera t in calculum introducatur, quam licebit vnitati aequalem ſumere, pro campo apparente habebimus

$$M = \frac{i(\varepsilon m - i)}{m(\varepsilon i m - i(1-\varepsilon) - \varepsilon)} \cdot t$$

quippe qui valor per 859 min. multiplicatus dat ſemidiametrum campi Φ. Vidimus autem, litteram i intra limites 1 et εm capi debere.

Coroll. 2.

§. 56. Si caperetur $i = 1$, foret $\mathfrak{C} = \infty$ et radii a ſpeculo minore reflexi fierent inter ſe paralleli, vnde vitium ſupra memoratum oriretur, quod ſcilicet radii peregrini ita cum propriis permiſcerentur, vt nullo modo ſeparari poſſent, qui caſus cum ſit ſollicite euitandus, litteram i vnitate multo maiorem accipi conueniet, neque tamen alteri limiti εm aequa-

lis

lis assumi potest, quia alioquin campus prorsus euanesceret.

Coroll. 3.

§. 57. Calculum instituenti facile patebit, maximum in hac expressione M locum non habere et eius valorem eo magis diminutum iri, quo maior littera i accipiatur. Quare cum esse debeat $i > 1$, si sumamus $i = 2$, erit

$$M = \frac{2(\varepsilon m - 2).t}{m(2\varepsilon m + \varepsilon - 2)}$$

sicque pro magnis multiplicationibus $M = \frac{\varepsilon}{m}.t$ qui valor etiam prodit, si capiatur $i = 3$ vel 4 etc. dummodo i sit multo minus, quam εm, qui campus simplex censeri solet. Sin autem medium inter limites sumendo capiatur

$$i = \frac{\varepsilon m + 1}{2} \text{ fiet } M = \frac{(\varepsilon m + 1)t}{2m(\varepsilon m + \varepsilon + 1)}$$

et pro magnis multiplicationibus campus ad dimidium redigetur.

Coroll. 4.

§. 58. Idem etiam patet ex primitiuo valore ipsius M, qui est

$$M = \frac{q + r + t}{m - 1}, \text{ pro quo } r = -\omega = -\frac{it}{\varepsilon m}.$$

Etsi autem q addi debet, tamen ex superioribus patet, esse $q < \omega$; erat enim ex tertia aequatione

$$q = \omega - \frac{(ik + \varepsilon)}{\varepsilon} M.$$

Scholion.

§. 59. Circa campum autem inprimis eſt inquirendum, an loco t ſcribere liceat vnitatem, quod iudicium ex prima lente C eſt petendum, cuius ſemidiameter aperturae reuera eſt $= \delta x$ ob campum autem eſſe debet $= \frac{1}{4} . \mathfrak{r} r$. Cum igitur ſit $\mathfrak{r} = - \frac{it}{\epsilon m}$ et

$$r = \frac{-(1-\epsilon)\mathfrak{C}}{i-1} . p \text{ ſeu } r = \frac{\epsilon m (1-\epsilon)(i+\epsilon)}{i^2 (\epsilon m + ik)} . p.$$

Iam ſupra autem inuenimus eſſe

$$\epsilon m + ik = \frac{\epsilon m (i(\epsilon m + \epsilon - 1) - \epsilon)}{i(\epsilon m - i)} = \frac{\epsilon m (\epsilon i m - i(1-\epsilon) - \epsilon)}{i(\epsilon m - i)}$$

Quocirca erit

$$r = \frac{(\epsilon m - i)(1-\epsilon)(i+\epsilon)}{i(\epsilon i m - i(1-\epsilon) - \epsilon)} p$$

vnde, niſi fuerit

$$\frac{(\epsilon m - i)(1-\epsilon)(i+\epsilon)}{\epsilon m (\epsilon i m - i(1-\epsilon) - \epsilon)} . p > 4 \delta x$$

tum ſumere licebit $t = 1$. Contra vero t tanto minus vnitate capi debebit, vbi notaſſe iuuabit, eſſe $\delta > \epsilon$. Quoniam autem hae formulae nimis ſunt complicatae, quam vt in genere omnia momenta pro conſtructione teleſcopii commode exprimi queant: ſtatuamus $i = \frac{1}{2}(\epsilon m + 1)$ vt interuallum CD minus euadat, etſi campus ad ſemiſſem redigitur; deinde enim videbimus, quomodo campus amplificari poſſit. Poſito autem

$$i = \frac{\epsilon m + 1}{2} \text{ erit } k = \frac{2 \epsilon m (\epsilon m + 2\epsilon + 1)}{(\epsilon m + 1)^2},$$

qui valor abit in $k = 2$ pro magnis multiplicationibus;

Dein-

Deinde vero

$$\mathfrak{C} = \frac{-(\varepsilon m + 2\varepsilon + 1)(\varepsilon m - 1)^2}{2(\varepsilon m + 1)((\varepsilon m + 1)(\varepsilon m + \varepsilon - 1) - 2\varepsilon)}$$

vnde C reperitur.

Scholion 2.

§. 60. Quia vero valor $i = \frac{2m+1}{2}$ merito nimis magnus videri poteſt, pro i potius medium geometricum ſumamus ſitque $i = \sqrt{\varepsilon m}$ ac primo pro campo apparente fiet

$$M = \frac{\varepsilon}{\varepsilon m + \sqrt{\varepsilon m} + \varepsilon} \cdot t:$$

Deinde vero habebimus $k = \frac{\varepsilon + \sqrt{\varepsilon m}}{\sqrt{\varepsilon m}}$ hincque

$$B = \frac{-(1-\varepsilon)\sqrt{\varepsilon m}}{\varepsilon(\sqrt{\varepsilon m} - 1)} \text{ et } \mathfrak{B} = \frac{(1-\varepsilon)\sqrt{\varepsilon m}}{(1-2\varepsilon)\sqrt{\varepsilon m} + \varepsilon}.$$

$$\mathfrak{C} = \frac{-(\varepsilon + \sqrt{\varepsilon m})(\sqrt{\varepsilon m} - 1)}{\varepsilon m + \sqrt{\varepsilon m} + \varepsilon} \text{ et } C = \frac{-(\varepsilon + \sqrt{\varepsilon m})(\sqrt{\varepsilon m} - 1)}{2\varepsilon m + \varepsilon\sqrt{\varepsilon m}}$$

Ex his ſi ponamus $D = \vartheta$, vt ſit $\mathfrak{D} = \frac{\theta}{1+\theta}$ reperientur diſtantiae focales:

$$p = p;\; q = \frac{-\varepsilon(1-\varepsilon)\sqrt{\varepsilon m}}{(1-2\varepsilon)\sqrt{\varepsilon m} + \varepsilon} \cdot p.$$

$$r = \frac{(1-\varepsilon)(\varepsilon + \sqrt{\varepsilon m})}{\varepsilon m + \sqrt{\varepsilon m} + \varepsilon} \cdot p.$$

$$s = \frac{\theta}{1+\theta} \cdot \frac{(1-\varepsilon)}{2\sqrt{\varepsilon m} + \varepsilon} \cdot p.$$

$$t = \frac{\theta \cdot (1-\varepsilon)(\varepsilon + \sqrt{\varepsilon m})}{\varepsilon m(\varepsilon + 2\sqrt{\varepsilon m})} \cdot p.$$

Interualla vero lentium erunt

$$AB = BC = (1 - \varepsilon)\, p.$$

$$CD = \frac{(1-\varepsilon)(\varepsilon + 2\sqrt{\varepsilon m})}{2\varepsilon m + \varepsilon\sqrt{\varepsilon m}} \cdot p$$

$$DE = \frac{\theta \cdot (1-\varepsilon)(\varepsilon m + \sqrt{\varepsilon m} + \varepsilon)}{\varepsilon m(\varepsilon + 2\sqrt{\varepsilon m})} \cdot p.$$

Pro loco autem oculi erit

$$O = \frac{t.t}{Mm} = \frac{\varepsilon m + \sqrt{\varepsilon m} + \varepsilon}{\varepsilon m}.t. = t\left(1 + \frac{1}{\sqrt{\varepsilon m}} + \frac{\varepsilon}{m}\right).$$

Pro aperturis autem inuenimus

$$q = \frac{(1 - 2\varepsilon)\sqrt{\varepsilon m} + \varepsilon}{(\varepsilon m + \sqrt{\varepsilon m} + \varepsilon)\sqrt{\varepsilon m}}.t$$

$$r = -\frac{t}{\sqrt{\varepsilon m}}; \text{ et } \mathfrak{s} = 0.$$

Licebit autem sumere $t = 1$, nisi prodeat

$$\frac{(1-\varepsilon)(\varepsilon + \sqrt{\varepsilon m})}{(\varepsilon m + \sqrt{\varepsilon m} + \varepsilon)\sqrt{\varepsilon m}}.p > 4\delta x.$$

Lenti autem in D, pro qua est $\mathfrak{s} = 0$, apertura tribui debet, cuius semidiameter sit $= \frac{x}{PQR} = \frac{\varepsilon x}{\varepsilon + \sqrt{\varepsilon m}}$ ita, vt huius lentis apertura sit tam exigua, vt ad radios peregrinos arcendos apprime sit accommodata. Interim tamen quia campus apparens hic nimis est exiguus, vtique operae erit pretium, huic generi telescopiorum maiorem campum procurare, quod in sequente problemate praestabimus.

Problema 3.

§. 61. Telescopiorum generi in problemate praecedente descripto nouum gradum perfectionis addere, dum eius campus apparens amplificatur.

Solutio.

Fit hoc additione nouae lentis, ita, vt nunc telescopium ex duobus speculis et quatuor lentibus componatur. Maneat autem, vt ante,

$$P = \frac{1}{\varepsilon}; \; Q = i; \; R = -k \text{ et } S = -k';$$

quibus

quibus accedente littera T fit, $\frac{ikk'T}{\varepsilon} = m$ deinde fit etiam, vt ante,

$$B = \frac{-i(1-\varepsilon)}{\varepsilon(i-1)} \text{ hincque } \mathfrak{B} = \frac{i(1-\varepsilon)}{i(1-2\varepsilon)+\varepsilon};$$

ex quibus distantiae focales ita formabuntur;

$$q = -\frac{\mathfrak{B}}{P}p = -\varepsilon\mathfrak{B}p; \quad r = \frac{B\mathfrak{C}}{PQ}p = \frac{\varepsilon B\mathfrak{C}}{i}p;$$

$$s = \frac{\varepsilon.BC\mathfrak{D}.}{ik}p; \quad t = \frac{\varepsilon.BCD.\mathfrak{E}}{ikk'}.p; \text{ et}$$

$$u = -\frac{\varepsilon BCDE}{ikk'T}.p = -\frac{BCDE}{m}.p;$$

et interualla

$$AB = BC = (1-\varepsilon)p; \quad CD = \frac{\varepsilon BC}{i}(1+\tfrac{1}{k})p;$$

$$DE = +\frac{\varepsilon BCD}{ik}.(1+\tfrac{1}{k'})p., \text{ et}$$

$$EF = \frac{\varepsilon BCDE}{ikk'}(1-\tfrac{1}{T})p.$$

vbi cum fit $B < 0$, debet esse $C < 0$; deinde $D > 0$. Porro vt fiat u positiuum, debet esse $E < 0$ hincque ob vltimum interuallum $T < 1$. Nunc statuatur etiam $\mathfrak{r} = -\omega$; $\mathfrak{s} = 0$; et vt campus maximus euadat, $u = t$, vt fit $M = \frac{\mathfrak{q}-\omega+2t}{m-1}$. Vt vero margo coloratus euanescat, debet esse

$$\omega = \frac{t}{kk'} + \frac{u}{kk'T} = \frac{t}{kk'}(1+\tfrac{1}{T})$$

et quia debet esse $T < 1$, sumatur statim $T = \frac{1}{2}$ vt fit $m = \frac{ikk'}{2\varepsilon}$; hincque $kk' = \frac{2\varepsilon m}{i}$ tum igitur erit $\omega = \frac{3i}{2\varepsilon m}.t$, ac vicissim $t = \frac{2\varepsilon m\omega}{3i}$; vnde fit

$$M = \frac{\mathfrak{q} + \omega(\frac{4\varepsilon m}{3i} - 1)}{m-1}.$$

Nunc

Nunc autem confiderari oportet fequentes quatuor aequationes:

$$\text{I}^{\circ}.\ \mathfrak{B}\,\mathfrak{q} = \frac{1-\varepsilon}{\varepsilon}.\ M$$

$$\text{II}^{\circ}.\ -\mathfrak{C}\,\omega = \frac{i-\varepsilon}{\varepsilon}\,M - \mathfrak{q}$$

$$\text{III}^{\circ}.\ 0 = -\left(\frac{ik+\varepsilon}{\varepsilon}\right) M - \mathfrak{q} + \omega$$

$$\text{IV}^{\circ}.\ \mathfrak{C}\,t = \frac{ikk'-\varepsilon}{\varepsilon}\,M - \mathfrak{q} + \omega.$$

Ex tertia igitur habemus

$$\mathfrak{q} - \omega = -\left(\frac{ik+\varepsilon}{\varepsilon}\right) M$$

addatur vtrinque $\frac{4\varepsilon m\omega}{3i}$, ac prodibit

$$M(m-1) = \frac{4\varepsilon m\omega}{3i} - \left(\frac{ik+\varepsilon}{\varepsilon}\right) M$$

vnde inuenitur

$$M = \frac{4\varepsilon m\omega}{m+\frac{ik}{\varepsilon}} = \frac{4\varepsilon^2 m\omega}{3i(m\varepsilon+ik)}$$

feu fubftituto valore ipfius ω

$$M = \frac{2\varepsilon}{m\varepsilon+ik}.\ t;$$

atque infuper ex eadem aequatione erit

$$\mathfrak{q} = \frac{3i(m\varepsilon+ik) - 4\varepsilon m(ik+\varepsilon)}{3i(m\varepsilon+ik)}.\ \omega$$

at vero prima aequatio dat

$$\mathfrak{q} = \frac{4(1-\varepsilon)\varepsilon.m\omega}{3i(m\varepsilon+ik)\mathfrak{B}},$$

quorum valorum aequalitas praebet

$$3i(m\varepsilon+ik) - 4\varepsilon m(ik+\varepsilon)$$
$$= \frac{4\varepsilon(1-\varepsilon)m}{\mathfrak{B}} = \frac{4\varepsilon m(i(1-2\varepsilon)+\varepsilon)}{i}$$

vnde

vnde fit

$$ik(4\varepsilon m - 3i) = \varepsilon m(3i - 4\varepsilon) - \frac{4\varepsilon m(i(1-2\varepsilon)+\varepsilon)}{i}$$

$$= \frac{\varepsilon m}{i}(3i^2 - 4i(1-\varepsilon) - 4\varepsilon) \text{ seu}$$

$$ik = \frac{\varepsilon m(3i^2 - 4i(1-\varepsilon) - 4\varepsilon)}{i(4\varepsilon m - 3i)}$$

qui valor, vt sit positiuus, debet esse $i < \frac{4}{3}\varepsilon m$ simulque

$$i > \frac{2}{3}(1 - \varepsilon + \sqrt{1 + \varepsilon + \varepsilon^2})$$

Hinc autem valore ipsius k definito secunda aequatio dabit

$$\mathfrak{C} = \frac{-4\varepsilon m(i^2 - i(1-\varepsilon) - \varepsilon)}{3i^2(m\varepsilon + ik)} = \frac{-4\varepsilon m(i-1)(i+\varepsilon)}{3i^2(m\varepsilon + ik)}$$

siue ex altero valore ipsius q

$$\mathfrak{C} = \frac{-\varepsilon m(1+4k) + 3ik}{3(m\varepsilon + ik)}$$

erit ergo ob $\mathfrak{C} < 0$ etiam $C < 0$ vti requiritur, ex quorum valorum aequalitate idem valor pro k, qui ante, prodit. Notetur autem hic esse

$$\varepsilon m + ik = \frac{4\varepsilon m(\varepsilon im - i(1-\varepsilon) - \varepsilon)}{i(4\varepsilon m - 3i)}$$

vnde fit

$$M = \frac{i(4\varepsilon m - 3i).f.}{2m(\varepsilon im - i(1-\varepsilon) - \varepsilon)}.$$

Deinde vero littera D arbitrio nostro permittitur, dummodo sumatur positiua.

Quarta denique aequatio nobis praebet valorem litterae

$$\mathfrak{E} = \frac{4\varepsilon i m - 2i(1-\varepsilon) - 2\varepsilon}{i(m\varepsilon + ik)}$$

quare vt E prodeat negatiuum, oportet esse $\mathfrak{E} > 1$ siue

$$4\varepsilon i m - 2i(1-\varepsilon) - 2\varepsilon > i(\varepsilon m + ik)$$

et valore ipsius $\varepsilon m + ik$ substituto

$$(4\varepsilon m - 3i)(4\varepsilon i m - 2i(1-\varepsilon) - 2\varepsilon)$$
$$> 4\varepsilon m(\varepsilon i m - i(1-\varepsilon) - \varepsilon)$$

quod vt fiat necesse est sit

$$6.\varepsilon^2 i m^2 - 2\varepsilon m(3i^2 + i(1-\varepsilon) + \varepsilon)$$
$$+ 3i^2(1-\varepsilon) + 3\varepsilon i > 0.$$

quod sponte euenit, cum certo sit $i < \varepsilon m$.

Tandem pro loco oculi habebimus

$$O = \frac{tu}{M.m} = \frac{2(\varepsilon i m - i(1-\varepsilon) - \varepsilon)}{i(4\varepsilon m - 3i)}.u \text{ siue}$$
$$O = \tfrac{1}{2}u\left(1 + \frac{3i^2 - 4i(1-\varepsilon) - 4\varepsilon}{i(4\varepsilon m - 3i)}\right)$$

Superest porro, vt diiudicemus, an pro t vnitas accipi queat, quod licebit, si fuerit

$$r < \frac{4\delta x}{\omega} \text{ seu } r < \frac{8\delta\varepsilon m x}{3i}.$$

Contra vero accipi debet $t = \frac{8\delta\varepsilon m x}{3ir}$ quo casu campus in eadem ratione diminuetur, in qua t ab vnitate deficit. Quod autem ad quantitatem p attinet, ea ea ex aequatione nota definiri debet, speculorum ratione habita, vtrum sint sphaerica an parabolica.

Corollarium.

§. 62. Quia lens in D, quam minimo foraminulo pertundi sufficit, a lente C distat interuallo

$$CD = \frac{\varepsilon . BC}{i}\left(1 + \frac{1}{k}\right)p$$

radii autem peregrini in lentem C incidentes post eam colliguntur ad distantiam $r = \frac{\varepsilon . B\mathfrak{C}}{i}p$; vt hi radii excludantur, necesse est, vt hae duae distantiae a se inuicem discrepent; seu notabilis differentia esse debet inter has quantitates $C\left(1 + \frac{1}{k}\right)$ et $\mathfrak{C}$, hoc est inter $1 + \frac{1}{k}$ et $1 - \mathfrak{C}$ seu inter $\frac{1}{k}$ et $-\mathfrak{C}$. Est vero

$$\frac{1}{k} = \frac{i^2(4\varepsilon m - 3i)}{\varepsilon m(3i^2 - 4i(1-\varepsilon) - 4\varepsilon)} \text{ et}$$

$$-\mathfrak{C} = \frac{(4\varepsilon m - 3i)(i-1)(i+\varepsilon)}{3i(\varepsilon i m - i(1-\varepsilon) - \varepsilon)}$$

quare cum ratio inter has quantitates debeat esse admodum inaequalis, haec fractio =

$$\frac{3i^3(\varepsilon i m - i(1-\varepsilon) - \varepsilon)}{\varepsilon m(i-1)(i+\varepsilon)(3i^2 - 4i(1-\varepsilon) - 4\varepsilon)}$$

plurimum ab vnitate discrepare debet; at differentia inter numeratorem et denominatorem satis est magna, vt aequalitas non sit metuenda.

Coroll. 2.

§. 63. Quodsi autem sumamus $i = 2$, fractio illa ab vnitate diuersa euadet $= \frac{6(2\varepsilon m + \varepsilon - 2)}{\varepsilon m(1+\varepsilon)(2+\varepsilon)}$, quae vtique ſatis ab vnitate diſcrepat, vt tranſitus radiorum peregrinorum neutiquam ſit metuendus. Campi autem ratio maxime exigit, vt ipſi i tam paruum valorem tribuamus, quam circumſtantiae permittunt. Ceterum multo magis ille tranſitus euitabitur, ſi capiatur $i > 2$.

Exemplum I.

§. 64. Pro multiplicatione $m = 50$. Ponamus hic $\delta = \frac{1}{4}$; $\varepsilon = \frac{1}{5}$, et quia haec multiplicatio poſtulat $x = 1$. dig. erit $y = \frac{1}{4}$ dig. Deinde ſtatuamus

$i = 3$. erit

$(i + \varepsilon)(i - 1) = 6,4$

$3i^2 - 4i(1 - \varepsilon) - 4\varepsilon = 16,6.$

$\varepsilon m = 10;$

$4\varepsilon m - 3i = 31.$

$B = -6$; $\mathfrak{B} = \frac{6}{5}$; $k = \frac{166}{93} = 1,785.$

$\varepsilon m + ik = 15,355;$

$\mathfrak{C} = -0,6175$; $C = -0,3817.$

$\mathfrak{E} = 2,4921$; $E = -1,6702$

vnde elementa primitiua ſequenti modo definientur, ponendo ϑ loco D, vt ſit $\mathfrak{D} = \frac{\theta}{1+\theta}$.

$\alpha = p$; $\beta = 1,2.p$; $\gamma = 0,1526.p$;

$b = -\frac{1}{5}p = -0,2.p$; $c = -0,4\,p$; $d = 0,0855.p$;

$\delta = 0,$

$\delta = 0,0855.\,\vartheta.\,p.$; $e = 0,0229.\,\vartheta\,p.$

$\varepsilon = -0,0382.\,\vartheta\,p$; $f = 0,0764.\,\vartheta.\,p.$

ex quibus interualla colliguntur

$AB = BC = 0,8.\,p$; $CD = 0,2381.\,p$;

$DE = 0,1084.\,\vartheta\,p$; $EF = 0,0382.\,\vartheta\,p$;

ſicque tubus foramini ſpeculi annectendus erit circiter $= \frac{1}{3}\,p.$

Diſtantiae vero focales erunt

$q = \mathfrak{B}\,b = -0,24.\,p$;

$r = \mathfrak{C}\,c = 0,247.\,p$;

$s = \mathfrak{D}\,d = 0,0855.\,\frac{\theta}{1+\theta}.\,p$;

$t = \mathfrak{E}\,e = 0,0571.\,\vartheta.\,p$;

$u = f = 0,0764.\,\vartheta.\,p.$

Praeterea pro hoc caſu habebimus

$$M = \frac{93}{2745}\,t = 0,0339.\,t$$
$$(8,5307323.)$$

Tum vero $\mathfrak{q} = 0,113.\,t$

$\mathfrak{r} = -\omega = -0,45.\,t.$

Nunc igitur videamus, an pro t ſumi poſſit vnitas, nec ne? quem in finem conſideremus valorem

$\mathfrak{r}\,r = 4\,\delta\,x$; ſeu $0,111.\,p.\,t = 1$ dig.

vnde fit $t = \frac{1}{0,111\,p} = \frac{9}{p}$; vnde apparet, ſi p fuerit nouem digitorum vel minus, tum ſumi poſſe $t = 1$. ſin autem fuerit $p > 9$ dig. tum ſumi debet $t = \frac{9}{p}$ et campus tanto fiet minor. Circa locum oculi vero notandum eſt, eſſe $O = \frac{1}{2} u \left(1 + \frac{16,6}{93}\right) = 0,58. u$. Nunc vero reſtat praecipua inueſtigatio diſtantiae focalis p, quae ex menſura confuſionis colligitur

$$p = kx \sqrt[3]{50} \Big(0,125 - 0,0283 \\ + 0,00131. \mu (\lambda + \nu. \mathfrak{C}. 1 - \mathfrak{C}) \\ + 0,0031. \mu \left(\frac{(1+\theta)^3 \lambda'}{\theta^3} + \frac{\nu.(1+\theta)}{\theta^2}\right) \\ + \frac{0,00005. \mu}{\theta^3} (\lambda'' + \nu. \mathfrak{C}. 1 - \mathfrak{C}) \\ + \frac{0,00036. \mu}{\theta^3}. \lambda''' \Big)$$

Circa hanc expreſſionem vero ſequentia obſeruemus:

I°. Si ſpeculum principale ſit parabolicum; primum membrum poſt ſignum radicale 0,125 omitti debet; ac ſi etiam minus ſpeculum eſſet parabolicum; tum quoque ſecundum terminum omittere liceret. Conſultius autem videtur ſolum primum ſpeculum parabolicum efficere; alteri vero figuram ſphaericam perfectam inducere, tum enim ſequentia membra ita inſtrui, ſiue litterae λ, λ′, λ″ cum littera ϑ ita aſſumi poterunt, vt iſta membra a ſecundo, quod eſt negatiuum, perfecte tollantur; ſicque tota confuſio ad nihilum redigatur. Quod ſi ſucceſſerit, ſufficiet litteram

p ex

p ex sola apertura definire, sumendo scilicet $p = 4x$ vel $6x$ vel $7x$, prouti visum fuerit. Hoc ergo casu ob $x = 1$ dig. distantia focalis p tuto minor, quam 9 dig. accipi poterit.

II°. Cum igitur sumi possit $p < 9$. dig. ponere licebit $t = 1$. et campi apparentis semidiameter erit $= 859$. M. minut. $= 29$. minut. Tum autem binas postremas lentes vtrinque aeque conuexas confici oportet, vnde si lentes ex vitro communi pro quo est $n = 1,55$ parentur, erit $\lambda''' = 1 + (\frac{\sigma - \varrho}{2\tau})^2 = 1,6299$. At $\lambda'' = 1 + 0,6299.(1 - 2\mathfrak{C}) = 10,9991$.

III°. Quia adeo capere liceret $p = 4$ dig. ne distantia focalis vltimae lentis fiat nimis parua, sufficiet statuere $\vartheta = 1$. atque hinc erit vltimum membrum nostrae formulae $= 0,00055$. Pro penultimo membro erit

$$\nu.\mathfrak{C}.1 - \mathfrak{C} = -0,8649; \text{ ideoque}$$

$$\lambda'' + \nu.\mathfrak{C}\,1 - \mathfrak{C} = 10,1342,$$

ac propterea totum membrum $= 0,00047$. Quocirca ambo postrema membra iunctim sumta dabunt $0,00102$.

IV°. Pro prima autem lente erit

$$\nu.\mathfrak{C}.1 - \mathfrak{C} = -0,2323;$$

vnde

vnde totum membrum inde natum fiet

$$= 0,00123 . \lambda - 0,00028.$$

Pro secunda autem lente erit

$$\frac{(1+\theta)^3}{\theta^3} \lambda' + \frac{\nu.(1+\theta)}{\theta^2} = 8 . \lambda' + 2\nu$$

hincque totum membrum erit

$$= 0,0232 . \lambda' + 0,00135.$$

V°. His ergo inuentis litteras λ et λ′ ita definiri oportet, vt fiat

$$0,0283 = 0,00123 . \lambda + 0,0232 . \lambda' + 0,00209$$

siue

$$0,0262 = 0,00123 . \lambda + 0,0232 . \lambda'$$

vbi notandum litteras λ et λ′ vnitate minores esse non posse statuamus ergo $\lambda' = 1$; et esse debebit $0,0030 = 0,00123\ \lambda$; hincque $\lambda = \frac{0,00300}{0,00123} = \frac{300}{123} = 2,44.$

Hinc igitur consequimur sequentem constructionem:

Telescopium Catadioptricum pro multiplicatione $m = 50$.

§. 65. Ex iis, quae modo euoluimus, obtinemus sequentes determinationes:

I°. Pro speculo principali, quod exactissime secundum figuram parabolicam elaborar debet, distan-

distantia focalis accipi posset $p = 4$ dig. Interim tamen litteram p quasi indeterminatam in calculo retineamus.
Semidiameter aperturae huius speculi $x = 1$ dig. et semidiameter foraminis $y = \delta x = \frac{1}{4}$ dig. Ante hoc speculum ad interuallum $= 0,8.p$ constituatur speculum Secundum Q B Q.

II°. Pro quo debet esse distantia focalis $q = -0,24.p$, ita, vt hoc speculum debeat esse conuexum et ad figuram sphaericam exacte elaboratum. Eius aperturae semidiameter $= \frac{1}{4}$ dig. Post hoc speculum in ipso foramine speculi maioris ad distantiam $BC = \frac{4}{5}.p = 0,8.p$. constituitur.

III°. Lens prima, ex vitro communi $n = 1,55$ paranda, cuius distantia focalis sit $r = 0,247.p$ capiendo

rad. fac. $\begin{cases} \text{ant.} = \dfrac{r}{\sigma - \mathfrak{C}(\sigma - \varrho) + \tau\sqrt{(\lambda - 1)}} = \dfrac{r}{1,4285} = 0,1729.p \\ \text{poster.} = \dfrac{r}{\varrho + \mathfrak{C}(\sigma - \varrho) \pm \tau\sqrt{\lambda - 1}} = \dfrac{r}{0,3896} = 0,6339.p \end{cases}$

Semidiameter aperturae $= \frac{1}{4}$ dig; vt foraminis, et interuallum vsque ad lentem secundam

$= 0,2381.p = CD.$

IV°. Pro secunda lente S D S, cuius distantia focalis $s = 0,0427.p$ ob $\mathfrak{D} = \frac{1}{2}$ et $\lambda' = 1$ capiatur

$$\text{rad. fac.} \begin{cases} \text{anter.} = \dfrac{s}{\sigma - \frac{1}{2}(\sigma - \varrho)} = \frac{s}{0,9090} = 0,04697 p. \\ \text{poster.} = \dfrac{s}{\varrho + \frac{1}{2}(\sigma - \varrho)} = \frac{s}{0,9090} = 0,04697 p. \end{cases}$$

Eius aperturae semidiameter:

$= \frac{x}{PQR} = \frac{1}{26,775} = 0,037.$ dig.

et interuallum ad tertiam lentem

$DE = 0,1084.\ p.$

V°. Pro tertia lente, cuius distantia focalis $t = 0,0571.\ p,$ capiatur radius vtriusque faciei $= 0,0628.\ p.$ eius aperturae semidiam. $= \frac{1}{4}t = 0,0142.\ p.$ et interuallum ad quartam lentem $= 0,0382.\ p.$

VI°. Pro quarta lente, cuius distantia focalis $u = 0,0764.\ p,$ capiatur radius vtriusque faciei $= 0,0840.\ p.$ eius aperturae semidiam. $= \frac{1}{4}u = 0,0191.\ p.$ et interuallum ad oculum

$= 0,58.\ u = 0,0445.\ p.$

VII°. Tubi ergo anterioris ambo specula continentis longitudo aliquanto maior est, quam $0,8.\ p.$ Tubi vero posterioris lentes continentis longitudo erit $= 0,4292.\ p.$ sicque totius instrumenti

menti longitudo erit circiter $= 1,4292.p$. ita, vt sumto $p = 5$. dig.; haec longitudo futura sit 7. dig.

VIII°. Campi autem apparentis semidiameter iam supra indicatus est $= 29$. minut., qui pro multiplicatione $m = 50$ satis est notabilis.

IX°. Diaphragmatis siue septis in locis imaginum realium collocandis hic plane non erit opus, cum secunda lens tam exiguam habeat aperturam, quae radios peregrinos omnes excludat. Interim tamen si in loco primae imaginis realis, quae post primam lentem cadit ad interuallum $\gamma = 0,1526.p$ collocetur diaphragma, eius foraminis semidiameter sumi debet $= 0,127.p$ hoc vero diaphragmate vix erit opus, cum radiorum peregrinorum in lentem primam incidentium imago cadat post hanc lentem ad distantiam $r = 0,247.p$, dum ea radiorum propriorum cadit ad distantiam $\gamma = 0,1526.p$, quod discrimen satis est notabile.

X°. Si quis metuat, ne a tam exiguo speculo, cuius semidiameter est $= 1$ dig. quodque adeo foramine est pertusum, nimis exigua luminis copia ad oculum transmittatur, is tantum mensuram digitorum pro lubitu augeat nihil enim impedit, quominus menſura digiti adeo duplicetur. Hoc enim modo claritas ad lubitum augeri poterit neque tamen inſtrumenti longitudo, quae per ſe eſt parua, ob hanc cauſſam enormis euadet.

Exemplum II.

Pro multiplicatione $m = 100$.

§. 66. Statuamus hic $\delta = \frac{1}{4}$ et $\varepsilon = \frac{1}{5}$ vt ſit $\varepsilon m = 20$. Tum vero ſumamus $i = 4$, quo tubus breuior euadat, atque habebimus

$$P = \tfrac{1}{\varepsilon} = 5;\ Q = i = 4;$$

$$R = -k = -\tfrac{43}{68} = -0,63235 \text{ ob}$$

$$3i^2 - 4i(1-\varepsilon) - 4\varepsilon = 34\tfrac{2}{5} \text{ et } 4\varepsilon m - 3i = 68$$

porro

$$S = -k' = -\tfrac{680}{43} = -15,814 \text{ et } T = \tfrac{1}{2} = 0,5.$$

Vnde fit

$$PQ = 20;\ PQR = -12,647;$$

$$PQRS = 200 \text{ et } PQRST = 100.$$

Reliquae vero litterae reperientur

$$\mathfrak{B} = \tfrac{16}{13} = 1,231.$$

$$B = -\tfrac{16}{3} = -5,333.$$

$$\mathfrak{C} = -\tfrac{17\cdot 21}{383} = -0,93211$$
$$(9,9694694)$$

C =

$$\mathfrak{C} = -\tfrac{357}{740} = -0,4824$$
$$(9,6834398)$$

et

$$\mathfrak{D} = \tfrac{\theta}{1+\theta}; \quad \mathfrak{E} = \tfrac{17 \cdot 783}{2,5 \cdot 383} = 3,4755$$
$$(0,5410119)$$
$$D = \vartheta; \quad E = -\tfrac{3,4755}{2,4755} = -1,4039$$
$$(0,1473490)$$

Vnde colligimus

$$\log. B\mathfrak{C} = 0,6964410; \quad \log. BC\mathfrak{E} = 0,9514233;$$
$$\log. BC = 0,4104114; \quad \log. BCE = 0,5577604(-)$$

His praemiſſis elementa noſtra erunt

$$\alpha = p; \quad b = -\tfrac{\alpha}{P} = -\tfrac{1}{5}\alpha = -0,2.p.$$
$$\beta = Bb = 1,0666.p; \quad c = -0,2666.p.$$
$$\gamma = Cc = 0,1286.p; \quad d = 0,20344.p.$$
$$\delta = Dd = 0,20344.\vartheta.p; \quad e = 0,01286.\vartheta.p.$$
$$\varepsilon = Ee = -0,01806.\vartheta.p; \quad f = 0,03612.\vartheta.p.$$

vnde ſtatim obtinemus interualla

$$AB = 0,8.p; \quad BC = 0,8.p; \quad CD = 0,3320.p.$$
$$DE = 0,2163.\vartheta.p; \quad EF = 0,01806.\vartheta.p.$$

Diſtantiae vero focales ita ſe habebunt:

$$q = \mathfrak{B}b = -0,246.p; \quad r = \mathfrak{C}c = 0,2485.p.$$
$$s = 0,2034.\tfrac{\theta}{1+\theta}.p; \quad t = \mathfrak{E}e = 0,0447.\vartheta.p.$$
$$u = f = 0,0361.\vartheta.p.$$

Praeterea vero erit $\omega = 0,3 . t = - \mathfrak{r}$ vnde aequatio $\mathfrak{r} r = 4 \delta x$ abit in hanc: $0,06455 t . p = x$; quare si sumatur $x = 2$ dig.; hinc fiet $t = \frac{2}{0,06455 . p}$. Dummodo igitur fuerit $p < 30$ dig. capere licebit $t = 1$. binasque vltimas lentes vtrinque aeque conuexas fieri oportet. Verum si etiam hic liceat totam confusionem ad nihilum redigere, ob $x = 2$. dig. sumi adeo posset $p = 8$. dig. etiamsi praestet ipsi p maiorem valorem tribuere; vnde patet tuto assumi posse $\vartheta = 1$.

Praeterea vero pro campo apparente habebitur $M = \frac{34}{1915} . t$; quare si capi poterit $t = 1$. semidiameter campi apparentis erit $\Phi = \frac{855 . 34}{1915}$. min. $= 15\frac{1}{4}$ min. et pro loco oculi habebimus

$$O = 0,563 . u . = 0,02037 . p.$$

Denique vt tota confusio euanescat, primum speculum perfecte parabolicum confici necesse est, atque tum esse debebit

$$\frac{\varepsilon(1 + B)(1 - B)^2}{8 B^3} = \frac{\mu}{B^3 \mathfrak{C}^3 P Q} (\lambda + \nu . \mathfrak{C} . 1 - \mathfrak{C})$$
$$- \frac{\mu}{B^3 C^3 P Q R} (8 . \lambda' + 2 \nu)$$
$$+ \frac{\mu}{B^3 C^3 \mathfrak{E}^3 P Q R S} (\lambda'' + \nu . \mathfrak{E} . 1 - \mathfrak{E})$$
$$- \frac{\mu}{B^3 C^3 . E^3 . m} \lambda'''.$$

vbi vt ante si refractio vitri sit

$n = 1,55$. erit $\lambda''' = 1,6299$ et

$\lambda'' = 1 + 0,6299 . (1 - 2 \mathfrak{E})^2 = 23,308$.

vnde

vnde aequatio noſtra praebebit

$$\begin{aligned} 0,02864 = 0,000382.\lambda &- 0,00016 \\ + 0,034843.\lambda' &+ 0,00200 \\ &- 0,00001 \\ &+ 0,00015 \\ &+ 0,00032 \end{aligned}$$

ſiue $0,02634 = 0,000382.\lambda + 0,03484.\lambda'$

quae aequalitas quia λ et λ' vnitate minores eſſe nequeunt, ſubſiſtere non poteſt. Quamobrem coacti ſumus ipſi ϑ maiorem valorem tribuere; ſit ergo $\vartheta = 2$, et noſtra aequatio fiet

$$\begin{aligned} 0,02809 = 0,000382.\lambda &- 0,00016 \\ + 0,01143.\lambda' &+ 0,00075 \\ &- 0,00001 \\ &+ 0,00002 \\ &+ 0,00004 \end{aligned}$$

ſiue $0,02744 = 0,000382.\lambda + 0,01143.\lambda'$

Ne hinc valor ipſius λ prodeat nimis magnus, ſumamus $\lambda' = 2$ eritque $0,00458 = 0,000382.\lambda$, hincque $\lambda = \frac{4580}{382} = 12$. Sin autem ſumſiſſemus $\lambda' = 2\frac{1}{3}$ obtinuiſſemus $\lambda = \frac{770}{382} = 2$.

Vtamur ergo his poſtremis valoribus $\lambda = 2$; et $\lambda' = 2\frac{1}{3}$, exiſtente $\vartheta = 2$; hincque $\mathfrak{D} = \frac{2}{3}$; vnde colligitur ſequens

Con-

Constructio Telescopii Catadioptrici pro $m = 100$.

§. 67. Haec ergo constructio constabit sequentibus determinationibus.

I°. Primum speculum perfecte secundum figuram parabolicam elaboretur, cuius distantia focalis sit $= p$, quam ad minimum 8 dig. statui oportet; eius aperturae semid. $= x = 2$. dig. foraminis autem semidiam. $= \frac{1}{2}$ dig. et distantia a speculo minore $AB = 0,8.p$.

II°. Minus speculum figuram sphaericam habeto, cuius distantia focalis sit $q = -0,246.p$; et semidiamet. aperturae $= \frac{1}{2}$ dig. indeque distantia ad primam lentem $BC = 0,8.p$.

III°. Pro prima lente, cuius distantia focalis

$r = 0,2485.p$, numeri vero

$\mathfrak{C} = -0,9321$ et $\lambda = 2$,

capiatur radius faciei

anter. $= \dfrac{r}{\sigma - \mathfrak{C}(\sigma - \varrho) \pm \tau\sqrt{\lambda - 1}} = \dfrac{r}{2,5666 - 0,9051} = 0,1205.p.$

poster. $= \dfrac{r}{\varrho + \mathfrak{C}(\sigma - \varrho) \mp \tau\sqrt{\lambda - 1}} = \dfrac{r}{-1,1485 + 0,9051} = -1,0210.p.$

Semidiam. apert. foramini aequalis $= \frac{1}{2}$ dig.

et distantia ad lentem secund. $CD = 0,3320.p$.

IV°. Pro

IV°. Pro secunda lente, cuius distantia focalis $s = 0,1356.p$ et numeri $\mathfrak{D} = \frac{2}{3}$ et $\lambda' = 2,3333.$ capiatur radius faciei

anter. $= \frac{s}{\sigma - \mathfrak{D}(\sigma - \varrho) \pm \tau \surd(\lambda' - 1)} = \frac{s}{1,7147} = 0,0791.p.$

poster. $= \frac{s}{\varrho + \mathfrak{D}(\sigma - \varrho) \mp \tau \surd(\lambda' - 1)} = \frac{s}{0,1034} = 1,3114.p.$

Eius aperturae semidiam. $= \frac{x}{PQR} = 0,16.$ dig. et distantia a lente tertia $DE = 0,4326.p.$

V°. Pro lente tertia, cuius dist. focal. $t = 0,0894.p.$ capiatur radius vtriusque faciei $= 0,0983.p.$ eius aperturae semid. $= \frac{1}{4}t = 0,0246.p.$ et distantia ad lentem quartam $EF = 0,03612.p.$

VI°. Pro lente quarta, cuius dist. foc. $u = 0,0722.p.$ capiatur radius vtriusque faciei $= 0,0794.p.$ eius aperturae semidiam. $= \frac{1}{4}u = 0,0198.p.$ et distantia oculi $O = 0,563.u = 0,0204.p.$

VII°. Longitudo ergo tubi prioris aliquanto maior erit, quam $0,8.p.$ tubi autem affixi longitudo $= 0,8211.p$; hincque totius instrumenti circiter $= 1,6211.p.$

VIII°. Campi apparentis semidiameter $= 15\frac{1}{4}$ min. et quae supra obseruauimus praeterea, etiam hic locum habent.

Exempl. III.

Pro multiplicatione $m = 150$.

§. 68. Maneant, vt ante, $\delta = \frac{1}{4}$ et $\varepsilon = \frac{1}{5}$ vt fit $\varepsilon m = 30$. fumatur autem $i = 5$ et vt claritate fufficiente fruamur, fit $x = 3$. dig. vt fit $y = \frac{3}{4}$ dig. et hinc colligimus

$$P = 5;\ Q = 5;\ R = -k = -0,6652.$$

$$S = -k' = -1,8040;\ \text{et } T = \tfrac{1}{2}.\ \text{hinc}$$

$$PQ = 25;\ PQR = -16,63;$$

$$PQRS = 300 \text{ et } PQRST = 150.$$

inde vero reliquae litterae reperientur:

$$\mathfrak{B} = \tfrac{5}{4} = 1,25;\ B = -5;$$

$$\mathfrak{C} = -\tfrac{33,28}{33,326} = -0,9986$$
$$(9,9994001)$$

$$C = -\tfrac{0,9986}{1,9986} = -0,49966.$$
$$(9,6986742)$$

$$\mathfrak{D} = \tfrac{\theta}{1+\theta};\ D = 9;$$

$$\mathfrak{E} = \tfrac{118,32}{33,326} = 3,5504.$$
$$(0,5502750)$$

$$E = -\tfrac{3,5504}{2,5504} = -1,3921.$$
$$(0,1436667)$$

vnde colligimus

$$\log. B\mathfrak{C} = 0,6983701;$$

log.

$\log. BC = 0,3976442;$

$\log. BC\mathfrak{E} = 0,9479192;$

$\log. BCE = 0,5413109\,(-)$

His praemissis elementa nostra erunt

$\alpha = p;\ b = -0,2.p;\ \beta = p;\ c = -0,2.p.$

$\gamma = 0,099932.p;\ d = 0,15023.p.$

$\delta = 0,15023.\vartheta.p;\ e = 0,00833.\vartheta.p.$

$\varepsilon = -0,01159.\vartheta.p;\ f = +0,02318.\vartheta.p.$

vnde colligimus interualla

$AB = 0,8.p = BC;\ CD = 0,25016.p.$

$DE = 0,15856.\vartheta.p;\ EF = 0,01159.\vartheta.p.$

Distantiae vero focales ita se habebunt:

$q = -0,25.p.;\ r = 0,19972.p;$

$s = 0,15023.\frac{\theta}{1+\theta}.p;\ t = 0,02956.\vartheta.p;$ et

$u = 0,02318.\vartheta.p.$

Porro est $\omega = \frac{1}{4}t = -\mathfrak{r};$ vnde aequatio $\mathfrak{r}\,r = 4\delta\,x$ dabit

$$t = \frac{12}{0,19972.p} = \frac{60}{p}$$

proxime dum ergo p sit < 60, tuto sumere licebit $t = 1.$ et quia tum erit $M = \frac{2}{100,530};$ hincque semidiameter campi $\Phi = 10\frac{1}{3}$ min. et pro loco oculi

$$O = 0,555.u = 0,01285.\vartheta.p.$$

 Deni-

Denique ſi primum ſpeculum conficiatur parabolicum, omnis confuſio tolletur huic aequationi ſatisfaciendo

$$0,0288 = 0,00030144 . \lambda - 0,00013994 + 0,0036177 . \frac{\lambda' . (1+\theta)^3}{\theta^3} + 0,00084146 . \frac{1+\theta}{\theta^2} + \frac{0,0001001}{\theta^3} + \frac{0,00024176}{\theta^3}$$

ſiue

$$0,0289399 = 0,00030144 . \lambda + 0,0036177 . \frac{(1+\theta)^3}{\theta^3} \lambda' + 0,00084146 . \frac{1+\theta}{\theta^2} + \frac{0,0003419}{\theta^3}$$

Hic patet ſtatim, ſumi non poſſe $\vartheta = 1$. tentetur ergo poſitio $\vartheta = \frac{3}{2}$ eritque

$$0,0289399 = 0,00030144 \lambda + 0,0167487 . \lambda' + 0,00093495 + 0,0001013.$$

ſiue

$$0,0279037 = 0,00030144 . \lambda + 0,0167487 . \lambda'$$

quare ſi hic ſtatuatur $\lambda' = 1$; fiet

$$\lambda = \frac{0,0111 2 5500}{0,00030144} = \frac{11155}{301} = 37.$$

ſin autem ſumamus $\lambda = 1$. fiet

$$\lambda' = \frac{0,0276023}{0,0167487} = \frac{276023}{167487} = 1,648.$$

Sin autem λ ſtatueretur 2 vel 3, valor ipſius λ' vix inde mutaretur vnde pro vſu practico praeſtare videtur, ſi ipſi λ' certus quidam valor tribuatur quia

quia enim tum ob leuissimos errores λ multum variare potest, plures lentes pro variis valoribus λ parari poterunt; ex quibus aptissimam experientia declarabit. Statuamus ergo $\lambda' = \frac{3}{2}$ ac reperietur

$$\lambda = \frac{0,0027807}{0,0003014} = \frac{27807}{3014} = 9.$$

vnde in praxi ternae lentes parari poterunt ex valoribus $\lambda = 8; = 9; = 10.$

Posito ergo $\vartheta = \frac{3}{2}$; vt sit $\mathfrak{D} = \frac{5}{3}$ sumatur $\lambda' = \frac{3}{2}$ et $\lambda = 9.$ vnde colligitur sequens:

Constructio Telescopii Catadioptrici pro $m = 150.$

§. 69. Haec constructio sequentibus determinationibus continetur:

I°. Speculum obiectiuum accuratissime secundum figuram parabolicam elaboretur; cuius distantia focalis minor non sit duodecim digitis; quam hic littera p designemus. Eius aperturae semidiameter vero sit $x = 3$. dig. foraminis vero semidiameter $= \frac{3}{4}$ dig. et distantia ad speculum minus $AB = 0, 8\, p.$

II°. Speculum minus exactissime ad figuram sphaericam elaboretur, cuius distantia focalis sit $q = -0, 25.\, p.$ quippe quod est conuexum. Eius aperturae semidiameter $= \frac{3}{4}$ dig. et distantia ad primam lentem $BC = 0, 8.\, p.$

III°. Pro prima lente, cuius diſtantia focalis eſt $r = 0,19972.\,p.$ numerique $\mathfrak{C} = -0,9986$ et $\lambda = 9.$ capiatur radius faciei

$$\text{anter.} = \frac{r}{\sigma - \mathfrak{C}(\sigma - \varrho) \pm \tau\sqrt{8.}} = \frac{r}{0,5021} = 0,39777.\,p.$$

$$\text{poſter.} = \frac{r}{\varrho + \mathfrak{C}(\sigma - \varrho) \mp \tau.\sqrt{8.}} = \frac{r}{1,3160} = 0,15176.\,p.$$

Sin autem ſumeretur $\lambda = 10$, prodiret radius faciei

$$\text{anter.} = \frac{r}{0,3468} = 0,57589.\,p.$$

$$\text{poſter.} = \frac{r}{1,4713} = 0,13543.\,p.$$

vnde concludimus in genere ſumi poſſe radium faciei

$$\text{anter.} = (0,39777 \mp 0,17812.\,\omega)\,p.$$

$$\text{poſter.} = (0,15176 \pm 0,01633.\,\omega)\,p.$$

vbi ω per experientiam definiri conueniet.

Huius autem lentis ſemidiameter aperturae $= \frac{3}{4}$ dig et diſtantia ad lentem ſecundam

$$CD = 0,25016.\,p.$$

IV°. Pro ſecunda lente, cuius diſtantia focalis $s = 0,090138.\,p.$ et numeri $\mathfrak{D} = \frac{3}{5}$ et $\lambda' = 1,5$ capiatur radius faciei

$$\text{anter.} = \frac{s}{\sigma - \mathfrak{D}(\sigma - \varrho) \pm \tau\sqrt{0,5}} = \frac{s}{0,1254} = 0,72046.\,p.$$

$$\text{poſter.} = \frac{s}{\varrho + \mathfrak{D}(\sigma - \varrho) \mp \tau\sqrt{0,5}} = -\frac{s}{0,0313} = -2,8864.\,p.$$

Eius

Eius aperturae semidiameter

$= \frac{x}{PQR} = \frac{2}{11}$ dig. $= 0, 18.$ dig.

et distantia ad lentem tertiam

$DE = 0, 23784. p.$

V°. Pro tertia lente, cuius distantia focalis

$t = 0, 04434. p.$

sumatur radius faciei vtriusque $= 0,048774. p.$

eius aperturae semidiameter $= 0, 01108. p.$

et distantia ad lentem quartam $EF = 0, 01738. p.$

VI°. Pro lente quarta, cuius distantia focalis

$u = 0, 03477. p.$

capiatur radius vtriusque faciei $= 0,03824. p.$

eius aperturae semidiameter $= \frac{1}{4}u = 0,00869. p.$

et distantia ad oculum

$O = 0, 555. u = 0, 01927. p.$

VII°. Longitudo ergo tubi prioris specula continentis aliquantum superabit $0, 8. p$; posterioris vero erit $= 0, 52465. p.$ ita, vt totius instrumenti longitudo sit circiter $= 1, 32465. p.$ Tum vero semidiameter campi apparentis erit $= 10\frac{1}{3}$ minut.

Scholion.

§. 70. Remedium in subsidium praxeos, quod hic pro prima lente attulimus, etiam facile ad exempla prae-

praecedentia accommodatur. Ponamus enim pro hac lente inuentos esse radios facierum f et g, et nunc quaestio eo redit, quomodo hos radios variari oporteat, vt distantia focalis maneat eadem. Ponatur prior $= f + x$; posterior $= g - y$, et necesse est, vt fiat $\frac{fg}{f+g} = \frac{(f+x)(g-y)}{f+g+x-y}$ vnde sumto x pro lubitu siue negatiue siue positiue capi debebit $y = \frac{g^2 x}{f^2 + (f+g)x}$; quare cum x et y sint satis parua erit $y = \frac{g^2 x}{f^2}$, siue $x : y = f^2 : g^2$, ita, vt posito $x = f^2 \omega$ futurum sit $y = g^2 . \omega$. Pro lente ergo prima, cuius radii supra inuenti sint f et g, alias successiue substitui conueniet, quarum radii sint $f \pm f^2 . \omega$ et $g \mp g^2 . \omega$. Deinde hic etiam notasse iuuabit, pro lente prima minorem aperturam sufficere posse, quam hic assignauimus foramini aequalem. Sufficiet enim apertura, cuius semidiameter $= \frac{1}{4} v r . = \frac{1}{16} . r = 0{,}01248 . p .$ vnde si $p = 12$ dig. iste semidiameter foret $= 0,1497$. dig. $= \frac{1}{7}$ dig. circiter; ac si adeo esset $p = 20$. dig.; foret iste semidiameter $= \frac{1}{4}$ dig. ex quo concludimus, sufficere, si huic lenti apertura tribuatur, cuius semidiameter sit $\frac{1}{4}$ dig. quo pacto ingentem copiam radiorum peregrinorum ab introitu arcebimus, sicque reliqui eo felicius a secunda lente excludentur; etsi eius apertura non tam est exigua, vt in praecedentibus exemplis, cuius rei ratio est, quod litteram i in multo minore ratione auximus, quam multiplicationem m; quam ob caussam in sequente exemplo litterae i multo maiorem valo-

valorem tribuemus, quia inde nihil aliud eſt metuendum, niſi exigua diminutio campi.

Exemplum 4.
pro multiplicatione $m = 200$.

§. 71. Manentibus litteris $\delta = \frac{1}{4}$ et $\varepsilon = \frac{1}{5}$, capiatur $i = 10$ et vt ſufficiens claritatis gradus obtineatur, ſumamus $x = 5$ dig. vt ſit ſemidiameter foraminis $= \delta x = \frac{5}{4}$ dig. et $\varepsilon m = 40$. Hinc ergo colliguntur valores

$P = 5$; $Q = 10$; $R = -k = -0,8221$;
$S = -k' = -9,7312$ et $T = \frac{1}{2}$; hincque
$PQ = 50$; $PQR = -41,105$;
$PQRS = 400$ et $PQRST = 200$.

reliquae vero litterae ita determinabuntur

$\mathfrak{B} = \frac{40}{31} = 1,2903$; $B = -\frac{40}{9} = -4,4444$.
$\mathfrak{C} = -1,0153$; $C = -0,50381$.
$(0,0066052)(-)$; $(9,7022655)(-)$
$\mathfrak{E} = 3,2841$; $E = -1,4377$.
$(0,5164093)$ $(0,1576942)$

vnde colliguntur ſequentes logarithmi

log. $B\mathfrak{C} = 0,6544183$; l. $BC = 0,3500786$.
log. $BC\mathfrak{E} = 0,8664879$; l. $BCE = 0,5077728 -$

 hinc

hinc elementa sequenti modo definientur:

$\alpha = p$; $b = -0,2.p$; $\beta = 0,8889.p$;

$c = -0,0889.p$; $-\gamma = 0,04478$;

$d = 0,054473.p$; $\delta = 0,054473.\vartheta.p$;

$e = 0,005598.\vartheta.p$; $\varepsilon = -0,007798.\vartheta.p$;

et $f = 0,015596.\vartheta.p$.

ex quibus colliguntur interualla

$AB = 0,8.p = BC$; $CD = 0,09925.p$;

$DE = 0,060071.\vartheta.p$; $EF = 0,007798.\vartheta.p$.

Distantiae vero focales

$q = -0,2581.p$; $r = 0,09025.p$;

$s = 0,05447\frac{\vartheta}{1+\vartheta}.p$; $t = 0,01838.\vartheta.p$;

et $u = 0,015596.\vartheta.p$.

Porro est $\omega = -\mathfrak{r} = \frac{3}{1}t$; vnde aequatio $\mathfrak{r}r = 4\delta x$ dabit $t = \frac{160}{p}$. dig.; vnde patet, dummodo p minor fit, quam 160. dig. tuto sumi posse $t = 1$; at si liceat confusionem ad nihilum redigere, adeo sumere licebit $p = 20$. dig. tum autem fiet $M = \frac{1}{120}$; vnde semidiameter campi erit $\frac{859}{120}$ min. $= 7\frac{1}{6}$ min. Praeterea vero pro loco oculi habebitur $O = 0,6.u$. Tantum igitur superest, vt confusionem ad nihilum redigamus, quod fiet hac aequatione:

0,029

$$0,029074 = 0,00020418.\lambda - 0,0000972.$$
$$+ 0,0020329.\frac{(\theta+1)^3}{\theta^3}\lambda'$$
$$+ 0,00047286.\frac{1+\theta}{\theta^2}.$$
$$+ \frac{0,000111}{\theta^3}.$$
$$+ \frac{0,00000116}{\theta^3},$$

siue

$$0,029171 = 0,00020418.\lambda$$
$$+ 0,0020329.\frac{(1+\theta)^3}{\theta^3}.\lambda'$$
$$+ 0,0004729.\frac{1+\theta}{\theta^2}.$$
$$+ \frac{0,0001122}{\theta^3}.$$

vbi iam nihil obstat, quominus statuatur $\vartheta = 1$. hincque habebimus

$$0,028113 = 0,0002042.\lambda + 0,016264.\lambda'.$$

Ne igitur hinc valor ipsius λ prodeat nimis magnus, commode statui poterit $\lambda' = 1\frac{1}{2}$, atque reperietur $\lambda = \frac{3717}{204} = 18$. proxime. Commodius vero erit sumere $\lambda' = 1\frac{2}{3}$; vnde fiet $\lambda = \frac{1006}{204} = 5$. Retineamus igitur valores $\vartheta = 1$; $\lambda' = 1\frac{2}{3}$, vt fiat $\lambda = 5$, cui adiungere poterimus valores finitimos $\lambda = 4$ et $\lambda = 6$. quo praxi melius consulatur; atque hinc colligetur sequens

Constructio Telescopii Catadioptrici pro multiplicatione $m = 200$.

§. 72. Statuamus hic, vt hactenus, distantiam focalem speculi principalis $= p$, quum, vt vidimus,

 mino-

minorem quam 20 dig. assumi non conuenit. Praestabit autem eam haud mediocriter maiorem assumere.

I°. Speculum igitur primum adcuratissime forma parabolica elaboretur, cuius distantia focalis sit $= p$;
Eius aperturae semidiameter $x = 5$. dig.
et semidiameter foraminis $y = 1\frac{1}{4}$ dig.
Distantia vero ad speculum minus $AB = 0,8.p$.

II°. Pro secundo speculo minore conuexo eius figura accuratissime sphaerice elaboretur, vt sit eius distantia focalis $q = -0,2581.p$.
Eius aperturae semidiameter $= 1\frac{1}{4}$ dig.
et distantia ad primam lentem in foramine $= BC = 0,8.p$.

III°. Pro lente prima, cuius distantia focalis $r = 0,09025.p$. et numeri $\mathfrak{C} = -1,0153$. et $\lambda = 5$, capiatur radius faciei

$$\text{anter.} = \frac{r}{\sigma - \mathfrak{C}(\sigma - \varrho) \mp \tau\sqrt{4}} = \frac{r}{3,0861 \mp 1,8102}$$

$$\text{poster.} = \frac{r}{\varrho + \mathfrak{C}(\sigma - \varrho) \pm \tau\sqrt{4}} = \frac{r}{-1,2680 \mp 1,8102}$$

hinc radius faciei
anter. $= 0,070734.p$.
poster. $= 0,16645.p$.

Sin

Sin autem ſumeremus $\lambda = 4$, prodiret radius faciei

anter. $= \frac{r}{3,0861 + 1,5677} = 0,05944 . p.$

poſter. $= \frac{r}{-1,2680 + 1,5677} = 0,30113 . p.$

At ſi ſumeretur $\lambda = 6$. foret radius faciei

anter. $= \frac{r}{3,0861 + 2,0239} = 0,08496 . p.$

poſter. $= \frac{r}{-1,2680 + 2,0239} = 0,11940 . p.$

ex quibus caſibus deducimus in ſubſidium praxeos ſequentes concluſiones:

Prior: Si $\lambda = 5 - \omega$, denotante ω fractionem arbitrariam, erit radius faciei

anter. $= (0,07073 - 0,01129 . \omega) p$

poſter. $= (0,16645 + 0,13468 . \omega) . p.$

Poſter: Sin autem $\lambda = 5 + \omega$, erit radius faciei

anter. $= (0,07073 + 0,01423 . \omega) p.$

poſter. $= (0,16645 - 0,04705 . \omega) . p.$

Eius aperturae ſemidiameter $= 1 \frac{1}{4}$, dig.

et diſtantia ad lentem ſecundam

$CD = 0,09925 . p.$

IV°. Pro ſecunda lente, cuius diſtantia focalis eſt $s = 0,02723 . p.$ et numeri $\mathfrak{D} = \frac{1}{2}$, et

 $\lambda' =$

$\lambda' = 1,6667$. capiatur radius faciei

anter. $= \frac{s}{\frac{1}{2}(\sigma + \varrho) + \tau \sqrt{0,6667}} = \frac{s}{0,9090 + 0,7350}$

poster. $= \frac{s}{\frac{1}{2}(\sigma + \varrho) \pm \tau \sqrt{0,6667}} = \frac{s}{0,9050 \pm 0,7350}$

seu anter. $= 0,01652. p$.

poster. $= 0,16018. p$.

eius aperturae semidiam. $= \frac{x}{PQR} = \frac{1}{7}$ dig.

et distantia a lente tertia $DE = 0,06007. p$.

V°. Pro lente tertia, cuius distantia focalis

$t = 0,01838. p$.

capiatur radius faciei vtriusque $= 0,02022. p$.

Eius apert. semidiam. $= \frac{1}{4} t = 0,00459. p$.

et distantia a lente quarta $EF = 0,007798. p$.

VI°. Pro lente quarta, cuius distantia focalis

$u = 0,015596. p$.

capiatur radius faciei vtriusque $= 0,01715. p$

Eius aperturae semid. $= \frac{1}{4} u = 0,0039. p$.

et distantia ad oculum $= 0,6. u = 0,00936. p$.

VII°. Hinc ergo longitudo tubi prioris erit quasi $= p$, quia maior esse debet, quam $\frac{4}{5}. p$.

posterioris vero lentes continentis $= 0,17648. p$.

ita,

ita, vt tota longitudo futura ſit circiter

$= 1,17648 . p.$

Campi vero apparentis ſemidiameter erit

$= 7\frac{1}{6}$ minut.

VIII°. Si pro lente prima tantum ad claritatem ſpectemus, eius aperturae ſemidiameter deberet eſſe $= \frac{x}{PQ} = \frac{1}{10}$ dig. ſin autem ad campum ſpectemus, hic ſemidiameter eſſe debet

$= \frac{1}{4} \mathfrak{r} r = \frac{3}{32} . r = 0,00846 . p.$

qui, ſi adeo eſſet $p = 40$ dig. fieret

$0,3384$ dig. $= \frac{1}{3}$ dig.

Quare cum ſemidiameter foraminis $= 1\frac{1}{4}$ dig. tuto oram huius lentis obtegere licebit, donec eius aperturae ſemidiameter fiat $= \frac{1}{3}$ dig. quo pacto radii peregrini iam maximam partem excludentur.

IX°. Cum igitur ne opus quidem ſit tantam magnitudinem primae lenti tribuere, ipſum foramen maioris ſpeculi multo minus ſtatuere licebit, quam $1\frac{1}{4}$ dig. hocque modo dum ipſum hoc ſpeculum maiorem ſuperficiem adipiſcetur, etiam claritatis gradus augebitur, neque vero ideo neceſſe erit, et minoris ſpeculi magnitudinem imminuere, cum ſufficiens radiodiorum copia in ſpeculum cadere poſſit. Radii peregrini colliguntur poſt lentem C in diſtantia $r = 0,09025.p.$ radii vero proprii in diſtantia $\gamma = 0,0448.p.$

X°. Cum deinde prima imago realis poſt lentem primam cadat ad diſtantiam $\gamma = 0,0448.p.$ radii autem peregrini in hanc lentem incidentes ſuam imaginem forment ad diſtantiam $r = 0,09025.p$; quae cum illa plus quam duplo ſit maior, neutiquam metuendum erit, ne radii peregrini ad oculum vsque propagentur.