CAPVT III.

DE VLTERIORI TELESCOPIORVM SECVNDI GENERIS PERFECTIONE, DIVERSAS VITRI SPECIES ADHIBENDO.

Problema I.

273.

Si telescopium ex tribus lentibus sit componendum, inuenire momenta, ad eius perfectionem facientia.

Solutio.

Incipiendum igitur est a duabus fractionibus, quae methodo ante exposita ponantur $\frac{\alpha}{b} = -P$; et $\frac{\beta}{c} = -Q$; ita vt litterarum P et Q altera sit positiua, altera vero negatiua, ita, vt sit $PQ = -m$. Tum igitur erunt distantiae determinatrices

$$b = -\frac{\alpha}{P};\ \beta = -\frac{B\alpha}{P};\ c = \frac{B\alpha}{PQ}$$

distantiae focales

$$p = \alpha;\ q = \frac{-\mathfrak{B}\alpha}{P};\ r = \frac{B\alpha}{PQ} = \frac{-B\alpha}{m}$$

et bina interualla

$$\alpha + b = \alpha\left(1 - \frac{1}{P}\right);\ \beta + c = -\frac{B\alpha}{P}\left(1 - \frac{1}{Q}\right)$$

Deinde cum ſit pro campo $\Phi = \frac{\pi - \pi'}{m+1}$ et media lens parum confert, ſtatuamus $\pi = \omega \xi$ et $\pi' = -\xi$, vt fiat $\Phi = \frac{\omega+1}{m+1} . \xi$ vnde pro diſtantia oculi habetur

$$O = -\frac{\pi'}{\Phi} . \frac{r}{m} = -\frac{(m+1)}{\omega+1} . \frac{B\alpha}{m m}$$

$$= -\frac{B\alpha}{m} . \frac{m+1}{m(1+\omega)}$$

ita, vt nunc $-B\alpha$ debeat eſſe poſitiuum; ſeu his tribus conditionibus erit ſatisfaciendum:

$$1^{\circ}.\ \alpha\left(1-\frac{1}{P}\right) > 0$$

$$2^{\circ}.\ -\frac{B\alpha}{P}\left(1-\frac{1}{Q}\right) > 0$$

$$3^{\circ}.\ -B\alpha > 0$$

hincque $\frac{1}{P}\left(1-\frac{1}{Q}\right) > 0$.

Porro autem fiet

$$\mathfrak{B}\omega = (1-P)M;\ B = \frac{(1-P)M}{\omega-(1-P)M}$$

exiſtente $M = \frac{\omega+1}{m+1}$.

Iam pro margine colorato tollendo aequatio eſt, ſiquidem pro fractionibus $\frac{dn}{n-1}$; $\frac{dn'}{n'-1}$ et $\frac{dn''}{n''-1}$ litteras N, N′, N″ ſtatuamus.

$$0 = N'. \omega . \frac{1}{P} + N''. \frac{1}{PQ} \text{ ſeu}$$

$$0 = N'\omega + N''. \frac{1}{Q};\ \text{vnde fit } Q = -\frac{N''}{N'\omega}.$$

Vt autem haec confuſio penitus tollatur, requiritur, vt ſit

$$0 = N.\alpha - \frac{N'.\alpha}{\mathfrak{B}P} + \frac{N''\alpha}{B.PQ}$$

quae

quae loco Q valore ſubſtituto ſit

$$0 = N - \frac{N'}{P}\left(\frac{1}{\mathfrak{B}} + \frac{\omega}{B}\right)$$

eſt vero $\omega = \frac{1-P}{(m+1)\mathfrak{B}+P-1}$

in qua ſi ponatur valor ipſius ω, erit

$$0 = N - \frac{N'}{P}\left(\frac{m+P}{(m+1)\mathfrak{B}+P-1}\right)$$

deinde aequatio pro Q inuenta ob $PQ = -m$ dabit quoque

$$m = \frac{N''.P((m+1)\mathfrak{B}+P-1)}{N'(1-P)}$$

ex qua ſi in praecedente aequatione pro $(m+1)\mathfrak{B}+P-1$ ſcribatur valor ipſius $\frac{N'.m(1-P)}{N''.P}$ orietur haec aequatio,

$$0 = NP((m+1)\mathfrak{B}+P-1) - N'(m+P)$$

$$0 = \frac{N.N'm(1-P)}{N''} - N'(m+P)$$

indeque porro

$P = \frac{m.(N-N'')}{Nm+N''}$ ideoque cum ſit

$\frac{mN'(1-P)}{N''P} = (m+1)\mathfrak{B} + P - 1$, colligitur

$$\mathfrak{B} = \frac{N'}{N-N''} + \frac{N''}{Nm+N''} \text{ ſiue } \mathfrak{B} = \frac{NN'm+NN''+N'N''-N''N''}{(N-N'')(Nm+N'')}$$

hinc $1-\mathfrak{B} = \frac{NNm-NN'm-NN''m-N'N''}{(N-N'')(Nm+N'')}$

adeoque $B = \frac{NN'm+NN''+N'N''-N''N''}{NNm-NN'm-NN''m-N'N''}$

Iam videamus, quomodo hae determinationes cum ſuperioribus conditionibus ſubſiſtere queant et cum

 eſſe

esse debeat $\frac{1}{P}(1-\frac{1}{Q}) > 0$; siue ob $Q = -\frac{m}{P}$, $\frac{1}{P}+\frac{1}{m} > 0$ erit $\frac{N}{N-N''} > 0$; et $N > N''$.

Primum autem interuallum $\alpha(1-\frac{1}{P}) > 0$ abit in hoc $\frac{-N''(m+1)}{m(N-N'')}.\alpha$; ideoque $\frac{-N''.\alpha}{N-N''} > 0$. et quia denominator iam inuentus est positiuus, restat, vt sit $-N''.\alpha > 0$. Conditio vero $-B\alpha > 0$ dabit nunc $B > 0$ vnde etiam fiet $\mathfrak{B} > 0$, quare quum sit $N-N'' > 0$ erit quoque $\frac{NN'm+NN''+N'N''-N''N''}{Nm+N''} > 0$, adeoque necesse est vt in hac fractione, tam numerator quam denominator simul sit aut positiuus aut negatiuus, poni autem nequit $Nm+N'' < 0$, quia tum foret $m < -\frac{N''}{N}$, neque $NN'm+NN''+N'N''-N''N'' > 0$ nam inde sequeretur esse $m < \frac{N''(N''-N-N')}{NN'}$, quod cum sit impossibile, etiam impossibile est, vt ope trium lentium haec duo commoda, quibus altera confusio penitus tollitur, obtineantur.

Scholion.

274. Hoc ergo problema resolui nequit siquidem posteriorem confusionem penitus tollere velimus. Omissa autem vltima aequatione, solutio facilis fuisset, sed tum plus non essemus consecuti, quam in praecedente capite, vbi vnica vitri specie sumus vsi. Quoniam igitur non conuenit, duas vitri species adhibere, ad telescopia conficienda, quae ex vnica specie aeque felici successu obtineri possunt: huic inuestigationi non immorabimur, sed tantum eiusmodi in medium

dium producere conabimur, quae praeter superiores qualitates etiam omni confusione, quae ibi erat relicta, destituantur. Caussa autem, cur ista inuestigatio hic non successit, in eo manifesto consistit, quod numerus litterarum indefinitarum erat nimis paruus, siquidem ad tres aequationes adimplendas tantum tres litterae praesto erant. Quare si plures lentes constituamus, plures etiam habebimus eiusmodi litteras, quibus non solum his tribus aequationibus, sed reliquis etiam conditionibus satisfieri poterit.

Problema 2.

275. Si Telescopium ex quatuor lentibus sit componendum determinare momenta, ad eius perfectionem facientia.

Solutio.

Tres fractiones hic considerandae ponantur

$$\frac{\alpha}{b} = -P; \ \frac{\beta}{c} = -Q; \ \frac{\gamma}{d} = -R$$

ita, vt harum litterarum P, Q, R vna sit negatiua et $m = -PQR$. vnde erunt distantiae determinatrices:

$$b = -\frac{\alpha}{P}; \ \beta = -\frac{B\alpha}{P}; \ c = \frac{B\alpha}{PQ}$$

$$\gamma = \frac{BC\alpha}{PQ}; \ d = -\frac{BC\alpha}{PQR}$$

distantiae autem focales

$$p = \alpha; \ q = \frac{-\mathfrak{B}\alpha}{P}; \ r = \frac{B\mathfrak{C}\alpha}{PQ}; \ s = \frac{-BC\alpha}{PQR}$$

et interualla lentium

$$\alpha + b = \alpha\left(1 - \frac{1}{P}\right); \quad \beta + c = -\frac{B\alpha}{P}\left(1 - \frac{1}{Q}\right)$$

$$\gamma + d = \frac{BC\alpha}{PQ}\left(1 - \frac{1}{R}\right)$$

Pro campo autem apparente $\Phi = \frac{\pi - \pi' + \pi''}{m+1}$, ſtatuatur $\pi = \omega\xi$; $\pi' = -i\xi$ et $\pi'' = \xi$, exiſtente ξ valore maximo, quem hae litterae accipere poſſunt, ſcilicet $\frac{1}{4}$; ita vt ſit $\Phi = \frac{\omega + i + 1}{m+1} \cdot \xi$, ſiue $\Phi = M\xi$, poſito $M = \frac{\omega + i + 1}{m+1}$. Ex his igitur obtinemus

$$\mathfrak{B}\omega = (1 - P)M; \quad \mathfrak{C}i = (1 - PQ)M - \omega.$$

Pro loco autem oculi erit

$$O = \frac{\pi''}{\Phi} \cdot \frac{s}{m} = \frac{m+1}{m^2} \cdot \frac{BC.\alpha}{\omega + i + 1} = \frac{BC.\alpha}{m^2.M};$$

quod vt fiat poſitiuum debet eſſe $BC\alpha > 0$.

His poſitis tribus ſequentibus aequationibus ſatisfieri debet

$$\text{I. } 0 = \frac{N'\omega}{P} + \frac{N''i}{PQ} + \frac{N'''}{PQR}$$

$$\text{II. } 0 = N - \frac{N'}{\mathfrak{B}P} + \frac{N''}{B\mathfrak{C}PQ} - \frac{N'''}{BCPQR}$$

$$\text{III. } 0 = \mu\lambda - \frac{\mu'}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right) + \frac{\mu''}{B^3\mathfrak{C}.PQ}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu''}{C}\right) - \frac{\mu'''.\lambda'''}{B^3C^3.PQR}$$

quae reſolutio quo facilius inſtitui poſſit, conſideremus primo caſum, quo duae priores lentes ſibi immediate iunguntur, vt ſupra de lentibus duplicatis aſſumſimus.

Primus-

Primus casus, quo $\alpha + b = 0$ ideoque $P = 1$, et $\omega = 0$ tum littera $\mathfrak{B}$ manet indeterminata hincque etiam B; quo facto resolutio facile institui poterit.

Prima enim aequatio dat: $0 = N''i + \frac{N'''}{R}$ vnde sequitur $R = -\frac{N'''}{N''i}$, ita vt R sit quantitas negatiua, siquidem i sit positiuum, id quod ratio campi postulat. Hinc ergo cum sit

$P = 1$, erit $PQR = \frac{-QN'''}{N''.i} = -m$

ideoque $Q = \frac{N''m.i}{N'''}$.

Secunda autem aequatio, in qua iam duo postremi termini euadunt valde parui, siquidem multiplicatio m sit magna, statim dat $0 = N - \frac{N'}{\mathfrak{B}}$, adeoque $\mathfrak{B} = \frac{N'}{N}$; et hinc fit $B = \frac{N''}{N - N'}$; vnde, si libuerit, valor ipsius $\mathfrak{B}$ adcuratius definiri poterit, habebitur nempe

$$\frac{1}{\mathfrak{B}} = \frac{N}{N'} + \frac{N'''(N - N')}{N'N'\mathfrak{C}.m.i} + \frac{N'''(N - N')}{N'N'.C.m.}$$

plerumque autem sufficit, his duabus aequationibus proxime satisfecisse.

Tertia autem adcurate resolui debet, cuius secundus terminus cum sit negatiuus, reliquis existentibus positiuis, vt mox videbimus, ille reliquis aequalis esse debet: erit enim $\mathfrak{C}i = \frac{N''' - N''mi}{N'''} . \frac{i+1}{m+1}$ ideoque $\mathfrak{C}$ est negatiuum, simulque etiam C vnde conditiones supra memoratae sunt perpendendae.

Primum autem interuallum est per hypothesin $= 0$. Secun-

Secundum fit $\beta + c = \frac{-N'(N'' m i - N''')}{(N - N')N'' m i} . \alpha$ pro quo si α fit positiuum, debet esse $-\frac{N'}{N - N'}$ positiuum, seu $N < N'$; contra autem si α fit negatiuum, debet esse $N > N'$.

Tertium porro interuallum est $\frac{N' C \alpha}{(N - N') Q}(1 + \frac{N'' i}{N'''})$; quia hic Q est positiuum; C negatiuum requiritur, vt fit $-\frac{N' \alpha}{N - N'}$ positiuum vti pro secundo interuallo, ita, vt si secundum interuallum fuerit positiuum, tertium sponte euadat positiuum.

Denique formula pro loco oculi $O = \frac{BC\alpha}{m i M}$ etiam fit positiua sub conditionibus iisdem. Ex quibus sequitur, si lens prima fit ex vitro coronario, secunda vero ex chrystallino, siue $N' > N$, tunc debere esse α positiuum; seu $p > 0$; $q < 0$; $r > 0$; $s > 0$.

Sin autem primam lentem ex vitro chrystallino, secundam vero ex coronario faciamus, ita, vt fit $N' < N$ debebit esse α negatiuum ideoque $p < 0$; $q > 0$; $r > 0$; $s > 0$. quare pro vtroque casu facile erit tertiam aequationem resoluere. His conditionibus perpensis, quae etiam nunc locum habebunt, dummodo ω fit fractio quam minima, statuamus primum interuallum $\alpha(1 - \frac{1}{P}) = \eta \alpha$ existente η fractione minima, siue positiua, si $\alpha > 0$, siue negatiua, si $\alpha < 0$ eritque $P = \frac{1}{1 - \eta}$; deinde maneat $\mathfrak{B}$ adhuc indefinita et quaeratur ω; eritque

$$\mathfrak{B} \omega = \frac{-\eta}{(1 - \eta)} \frac{(\omega + i + 1)}{(m + 1)};$$

in

in quo postremo factore, ω tuto omittitur; ita, vt hinc fit $\omega = \frac{-\eta}{(1-\eta)} \cdot \frac{i+1}{(m+1)\mathfrak{B}}$ qui valor ob duplicem caussam diminuitur, 1°. enim η est valde paruum, deinde ea diuiditur per $m+1$ numerum satis magnum; porro vero tam $\mathfrak{B}$. quam $i+1$ ab vnitate parum discrepant; quam ob caussam, valor ω recte pro euanescente haberi poterit; vnde prima aequatio nobis dabit, vt ante,

$$o = N'' i + \frac{N'''}{R}; \text{ et } R = \frac{-N'''}{N'' i};$$

quem valorem siquis adhuc adcuratius desideret, erit,

$$-\frac{1}{R} = \frac{N' \omega Q}{N'''} + \frac{N'' i}{N'''};$$

ita, vt nunc P et R sint quantitates cognitae in primo termino vtpote minimo sufficit Q proxime nosse, quem adeo ex casu praecedente desumere licet quia ω iam est definitum et $\mathfrak{B}$ mox definietur. Hinc igitur $Q = \frac{-m}{PR}$. Secunda aequatio iterum, vt in casu praecedente, proxime dabit, vt ante, $\mathfrak{B} = \frac{N'}{N}$; si quis eum vero exactius desideret, erit ei hac aequatione vtendum:

$$\frac{N'}{\mathfrak{B}} = N P + \frac{N''}{B\mathfrak{C}Q} - \frac{N'''}{BCQR}$$

vbi pro B sufficit valorem prope verum nosse, nempe $\mathfrak{B} = \frac{N'}{N-N'}$. Tum vero habebitur

$$\mathfrak{C} i = (1 + \frac{m}{R})(\frac{\omega + i + 1}{m+1}) - \omega$$

quem valorem manifestum est propter valorem ipsius R esse negatiuum, ideoque etiam C. Quocirca conditioditiones praescriptae iisdem casibus implentur, vt in praecedente, vbi $\omega = 0$; ita, vt nunc tantum superfit, aequationem tertiam resoluere; si modo meminerimus, ob $\pi'' = \xi$ quartam lentem fieri debere aeque conuexam; quae forma etiam tertiae lenti tribui deberet, si esset $i = 1$. Verum si sumeretur $i = 1$ vnde haec lens fieret vtrinque aeque conuexa, ob $\mathfrak{C} = -1$ propemodum, pro hac lente statui deberet

$$\lambda'' = 1 + N^2 (1 - 2 \mathfrak{C})^2 = 1 + N^2 . 9$$

vbi, vt ante sumsimus, est $N = \frac{\sigma - \rho}{2\tau}$ sicque numerus λ'' satis magnum obtineret valorem; quod incommodum euitabimus, sumendo $i < 1$. et plerumque sufficiet statuere $i = \frac{1}{2}$.

Corollarium.

276. Hic ergo differentia refractionis vitri tantum in duabus prioribus lentibus in considerationem venit ideoque sufficiet, vnicam tantum lentem ex vitro chrystallino conficere, et reliquas omnes ex vitro coronario sicque duos tantum casus habebimus euoluendos; alterum, quo prima lens ex vitro chrystallino conficitur; alterum, quo secunda.

CASUS I.

277. Sit igitur prima lens chrystallina, reliquae omnes ex vitro coronario factae erit $n = 1,58$; et $n' = n'' = 1,53$ tum vero secundum Dollondi experimenta

rimenta $N = 10$, $N' = N'' = N''' = 7$. His positis et sumto $i = \frac{1}{2}$ erit α negatiuum ideoque etiam η. Sumatur autem $\eta = -0,03$ hinc ergo habebimus

$P = \frac{1}{1,03} = \frac{100}{103}$; et quia erit proxime

$\mathfrak{B} = \frac{7}{10}$ et $B = \frac{7}{3}$, inuenimus $\omega = \frac{45}{721(m+1)}$.

Deinde cum sit proxime $R = -2$; ideoque $Q = \frac{103.m}{200}$ adcuratius habebimus

$$-\frac{1}{R} = \frac{1}{2} + \frac{27.m}{36050(m+1)} \text{ seu}$$

$$R = \frac{-36050(m+1)}{18952m+18025}$$

qui est valor correctus ipsius R, ex quo etiam adcuratius Q definiri poterit scilicet $Q = \frac{-m}{PR}$.

Vt nunc etiam $\mathfrak{B}$ adcuratius definiatur, erit

$$\frac{1}{\mathfrak{B}} = \frac{10.P}{7} + \frac{3.200}{7.103.m.\mathfrak{C}} + \frac{3.200}{7.2.103.m.C}.$$

Est vero $\mathfrak{C} = (1 + \frac{m}{R})(\frac{3+2\omega}{m+1}) - 2\omega$ et $C = \frac{\mathfrak{C}}{1-\mathfrak{C}}$; vnde etiam B definiri potest.

His igitur valoribus definitis, tertia aequatio principalis resolui debet, statuendo $\lambda''' = 1 + (\frac{\sigma-\varrho}{2\tau})^2$, vt quarta lens aeque conuexa vtrinque reddatur. Pro tertia autem lente videtur statui posse $\lambda'' = 1$.

Denique inuentis singulis litteris λ etiam singularum lentium constructio habebitur. Vtemur autem methodo iam aliquoties vsitata, scilicet pro casu quodam determinato, puta $m = 25$, deinde pro casu $m = \infty$ euolutionem instituamus, indeque constructionem pro quacunque multiplicatione deriuemus.

Exempl. I.

278. Si prima lens ex vitro chryſtallino, tres ſequentes autem ex coronario ſint parandae, pro multiplicatione $m = 25$ conſtructionem teleſcopii inueſtigare.

Sumto igitur $\eta = -0,03$, vt interuallum duarum priorum lentium fiat $-\frac{3\alpha}{100}$ ob α negatiuum, habemus ſtatim $P = \frac{100}{103} = 0.97087$

Log. $P = 9.9871628.$

hinc $\omega = 0,00240$

deinde $R = \frac{-360506.26}{18952.25 + 18025}$

ſeu $R = \frac{-36050.26}{491825} = -1,90576$

Log. $R = 0,2800680. (-)$

hincque $Q = 13,51160$

Log. $Q = 1,1307092$

nunc pro $\mathfrak{C}$ inueniendo noretur eſſe $PQ = 13,11810$

Log. $PQ = 1,1178720.$

hincque erit

$$\mathfrak{C} = -12,1181 \frac{(1,00180)}{26} - 0,00480$$

ſeu $\mathfrak{C} = -1,40528$

Log. $\mathfrak{C} = 0,1477628 (-)$

hincque $C = -0,58425.$

Log. $C = 9,7665972 (-).$

Nunc de ique pro B inueniendo nulla approximatione vtamur, quia ob $\frac{1}{\mathfrak{B}} = \frac{1}{B} + 1$ adcurate habemus

1 —

$1 - \frac{1}{CQ} + \frac{1}{CQR} = (\frac{10}{7} P - 1) B$ sive

$1 + 0,05266 + 0,06647 = (1,38696 - 1) B$

adeoque $1,11913 = 0,38696. B.$

ideoque $B = 2,89210.$

Log. $B = 0,4612145.$

consequenter

$\mathfrak{B} = \frac{2,89210}{3,89210} = 0,74307$

Log. $\mathfrak{B} = 9.8710305$

His valoribus definitis primo quaeramus nostra elementa, ac reperiemus

$b = - 1,03000. \alpha.$

Log. $\frac{b}{\alpha} = 0.0128371 \; (-)$

Log. $B = 0,4612145$

Log. $\frac{\beta}{\alpha} = 0,4740517 (-)$; $\beta = -2,97887 \alpha$

Log. $Q = 1,1307092$

Log. $\frac{c}{\alpha} = 9,3433425$; $c = + 0,22046 \alpha$

Log. $C = 9,7665972 \; (+)$

Log. $\frac{\gamma}{\alpha} = 9,1099397$; $\gamma = - 0,12880 \alpha$

Log. $R = 0,2800680$

Log. $\frac{d}{\alpha} = 8.8298717$; $d = - 0,06759 \alpha$

 Hinc

Hinc porro etiam diſtantias focales

$$p = \alpha;\ q = -0,76536.\alpha$$

$$\text{Log. } q = 9,8838677\ (-)$$

$$r = -0,30982\,\alpha$$

$$\text{Log. } r = 9,4911053\ (-)$$

$$s = -0,06759.\alpha$$

$$\text{Log. } s = 8,8298717.$$

Porro interualla lentium erunt.

$$\alpha + b = -0,03000.\alpha.$$

$$\beta + c = -2,75841.\alpha.$$

$$\gamma + d = -0,19639.\alpha.$$

Diſtantia denique oculi ab vltima lente

$$O = -1,12480.\frac{m+1}{mm}.\alpha$$

ideoque interuallum inter primam et vltimam lentem $= -2,98480.\alpha$.

Tertiam iam conſideremus aequationem, quae reſoluta et per μ diuiſa pro hoc caſu dabit

$$0 = \lambda - \frac{\mu'}{\mu}.\frac{\lambda'}{\mathfrak{B}^3 P} + \frac{\mu'}{\mu}.\frac{\lambda''}{B^3 \mathfrak{C}^3 PQ}$$
$$- \frac{\mu'}{\mu}.\frac{\nu'}{B\mathfrak{B}P} + \frac{\mu'}{\mu}.\frac{\nu'}{B^3 C\mathfrak{C}PQ} - \frac{\mu'}{\mu}.\frac{\lambda'''}{B^3.C^3.P.Q.R}$$

vbi $\mu = 0,8724$, et $\mu' = 0,9875$ et $\nu' = 0,2196$, vnde habebitur haec aequatio ad numeros reducta

0 = λ

$$0 = \lambda - 2,84162\,\lambda' - 0,00128\,\lambda''$$

(Log. 7,1090175)

$$- 0,00938\,\lambda''' - 0,11913$$

(Log. 7,9724463) $+ 0,00095$.

Vt nunc hanc aequationem resoluamus, primo notandum est, quartam lentem esse vtrinque aeque conuexam ideoque $\lambda''' = 1,60006$ vnde cum eius distantia focalis sit $s = - 0,06759 . \alpha$; erit radius vtriusque faciei $= 1,06 . s = - 0,07164 . \alpha$. Pro tertia autem lente, cuius distantia focalis est $r = \mathfrak{C} c$, quia eius semidiameter aperturae esse debet $= \frac{1}{8} \mathfrak{C} c$ cui ergo quarta pars minoris radii aequalis esse debet; inde minor radius esse debet $\frac{1}{2} \mathfrak{C} c$, ex quo λ'' definiri oportet. Hunc in finem hanc lentem nunc definiamus. Eius autem radius anterioris faciei est

$$= \frac{c\gamma}{\varrho\gamma + \sigma c \pm \tau(c+\gamma)\sqrt{(\lambda''-1)}}$$

$$= \frac{Cc}{\varrho C + \sigma \mp \tau(1+C)\sqrt{\lambda''-1}}$$

et radius faciei posterioris

$$= \frac{Cc}{\sigma C + \varrho \pm \tau(1+C)\sqrt{\lambda''-1}}$$

ita, vt habeamus

$$\text{rad. fac.} \begin{cases} \text{anter.} = \dfrac{Cc}{1,5277 \mp 0,41575\,\tau\sqrt{\lambda''-1}} \\ \text{poster.} = \dfrac{Cc}{-0,7432 \pm 0,41575\,\tau\sqrt{\lambda''-1}} \end{cases}$$

vbi

vbi ſigna ſuperiora valere debent; tum vero prior radius eſt minor; ideoque ponatur $= \frac{1}{2}\mathfrak{C}c$; vnde colligitur

$$\frac{c}{1.5277 - 0.41575\,\tau\sqrt{\lambda''-1}} = \frac{1}{2}\mathfrak{C}$$

ſicque erit $0,41575\,\tau\sqrt{\lambda''-1} = 0,6961$ adeoque

$$\text{rad. fac.}\begin{cases}\text{anter.} = \frac{\gamma}{0,8316} = -0,15489.\alpha\\ \text{poſter.} = \frac{\gamma}{-0,0471} = +2,73475.\alpha\end{cases}$$

cuius ergo lentis ſemidiameter aperturae erit $= 0,03872.\alpha$ at ex valore ipſius x colligemus $\sqrt{(\lambda''-1)} = 1,80969$; hincque $\lambda'' = 4,27497$.

Nunc revertamur ad noſtram aequationem, quae erit

$$\lambda - 2,84162\,\lambda' - 0,00549 - 0,11818 - 0,01501$$

quia nunc λ' vnitate minus eſſe nequit, ſtatuamus $\lambda' = 1$, eritque

$$\lambda = 2,98030;\ \lambda - 1 = 1,98030$$

$$\text{et } \tau\sqrt{\lambda-1} = 1,23484;$$

ex quo prima lens ita erit conſtruenda

$$F = \frac{\alpha}{\sigma \pm 1.23484} = \frac{\alpha}{0.3479} = 2,87439.\alpha$$

$$G = \frac{\alpha}{\varrho \mp 1.23484} = \frac{\alpha}{1.3762} = 0,72664.\alpha$$

Tandem

Tandem pro ſecunda lente habemus

$$F = \frac{b\beta}{\varrho\beta + \sigma b} = \frac{Bb}{2,3157} = -1,28638.\alpha$$

$$G = \frac{b\beta}{\sigma\beta + \varrho b} = \frac{Bb}{5,0279} = -0,59247.\alpha$$

cuius minoris radii pars quarta ſeu $0,14812.\alpha$ dat ſemidiametrum aperturae tam pro lente prima, quam pro lente ſecunda. Quare hinc deducitur ſequens

Conſtructio Teleſcopii
pro multiplicatione $m = 25$.

I. Pro lente prima. Flint Glaſſ.

radius faciei $\begin{cases} \text{anter.} = 2,87439.\alpha \\ \text{poſter.} = 0,72664.\alpha \end{cases}$

Semidiameter aperturae $= 0,14812.\alpha$

Interuallum ad ſecundam $= -0,03.\alpha$

II. Pro ſecunda lente. Crown Glaſſ.

radius faciei $\begin{cases} \text{anter.} = -1,28638.\alpha \\ \text{poſter.} = -0,59247.\alpha \end{cases}$

Semidiameter aperturae vt ante.

Interuallum $= -2,75841.\alpha$

III. Pro tertia lente. Crown Glaſſ.

radius faciei $\begin{cases} \text{anter.} = -0,15489.\alpha \\ \text{poſter.} = +2,73475.\alpha \end{cases}$

Semidiameter aperturae $= 0,03872\,\alpha$.

Interuallum $= -0,19639.\alpha$.

IV. Pro quarta lente. Crown Glaſſ.

radius vtriusque faciei $= -0,07164 . \alpha$.

Semidiameter aperturae $= 0,01791 . \alpha$.

Diſtantia ad oculum $= -1,1248 . \frac{m+1}{m^2} \alpha$

ſeu $O = -1,1248 (1 + \frac{1}{m}) . \frac{\alpha}{m}$

$O = -0,04679 . \alpha$

vbi notandum eſt, eſſe α negatiuum ac ſi ſemidiametrum aperturae ſumamus $= \frac{m}{50}$ dig. $= \frac{1}{2}$ dig. fiet $\alpha = -\frac{7}{2}$ dig. circiter vel maius, et longitudo teleſcopii $= 10\frac{1}{2}$ dig. Denique ſemidiameter campi erit $\Phi = 49\frac{1}{2}$ min. circiter.

Exempl. II.

279. Si prima lens ex vitro chryſtallino, reliquae ex coronario ſint parandae, pro multiplicatione maxima conſtructionem teleſcopii inueſtigare.

Sit iterum $\eta = -0,03$; habebimus

vt ante, $P = 0,97087$

Log. $P = 9.9871628$

tum vero $\omega = 0$. et $R = -\frac{36050}{18952}$

ſeu $R = -1,90217$

Log. $R = 0,2792503 (-)$

$Q = 0,54148 . m$

Log. $\frac{Q}{m} = 9.7335868$

Porro

Porro $\mathfrak{C} = \frac{1}{R} = -1,57714$

Log. $\mathfrak{C} = 0.1978710\,(-)$

et $C = -0,61197$.

Log. $C = 9,7867330\,(-)$

Pro B autem inueniendo habetur haec aequatio

$$1 - \frac{1}{\mathfrak{C}Q} + \frac{1}{CQR} = (\tfrac{10}{7}P - 1)B$$

quae quia termini per Q diuisi euanefcunt, abit in hanc $1 = 0,3869.B$

hinc $B = 2,58464$.

Log. $B = 0,4124012$

et $\mathfrak{B} = 0,72103$

Log. $\mathfrak{B} = 9.8579557$.

Hinc habebimus diftantias determinatrices

$b = -1,03000.\alpha$; Log. $b = 0,0128371\,(-)$

$\beta = -2,66218\,\alpha$; Log $\beta = 0,4252383\,(-)$

$c = +4,91645\,\frac{\alpha}{m}$; Log. $c = 0.6916515$

$\gamma = -3,00874\,\frac{\alpha}{m}$. Log. $\gamma = 0,4783845\,(-)$

$d = -1,58173.\frac{\alpha}{m}$; Log. $d = 0,1991342\,(-)$

atque interualla lentium

$\alpha + b = -0,03000.\alpha$

$\beta + c = -2,66218.\alpha - 4,91645.\frac{\alpha}{m}$

$\gamma + d = -4,59047.\frac{\alpha}{m}$

 et

et diſtantia oculi poſt vltimam lentem

$$O = -0,63269 \cdot \frac{\alpha}{m}$$

tum vero diſtantias focales

$$p = \alpha;\ q = -0,74267 \cdot \alpha;$$
$$r = -7,75394 \cdot \frac{\alpha}{m};\ s = -1,58173 \cdot \frac{\alpha}{m}.$$

Nunc igitur conſideremus aequationem tertiam

$$0 = \lambda - \frac{\mu'}{\mu} \cdot \frac{\lambda'}{\mathfrak{B}^3 P} - \frac{\mu' \nu'}{\mu \cdot \mathfrak{B} B P}$$
$$0 = \lambda - 3,11022\ \lambda' - 0,13738$$

quae ſumto, vt ante, $\lambda' = 1$, dat $\lambda = 3,24760$; hinc $\lambda - 1 = 2,24760$ et $\tau \sqrt{\lambda - 1} = 1,31555$; vnde lentium conſtructio ſequenti modo expedietur.

I. Pro prima lente ex vitro chryſtallino.

$$F = \frac{\alpha}{\sigma - 1,31555} = \frac{\alpha}{0,2672} = 3,74251 \cdot \alpha$$
$$G = \frac{\alpha}{\varrho + 1,31555} = \frac{\alpha}{1,4569} = 0,68639 \cdot \alpha$$

II. Pro ſecunda lente ex vitro Coronario

$$F = \frac{b\beta}{\varrho\beta + \sigma b} = \frac{\beta}{2,2461} = -1,18525 \cdot \alpha$$
$$G = \frac{b\beta}{\sigma\beta + \varrho b} = \frac{\beta}{4,5175} = -0,58931 \cdot \alpha$$

III. Pro tertia lente ex Crown Glaſſ.

$$F = \frac{Cc}{C\varrho + \sigma \mp x};\ G = \frac{Cc}{C\sigma + \varrho \pm x}$$
$$F = \frac{\gamma}{1,5214 \mp x};\ G = \frac{\gamma}{-0,7892 \pm x}$$

Cum

Cum nunc, vt ante, radius faciei anterioris fiat minor, is femiffi diftantiae focalis $= \frac{1}{2} \mathfrak{C} c$ aequalis ponatur, eritque

$$1,5214 - x = \frac{2C}{\mathfrak{C}} = 2(C+1) \text{ feu } x = 0,7454$$

Vnde ftatim habetur

$$F = -3,87697 \cdot \frac{\alpha}{m}$$

$$G = \frac{\gamma}{-0,0438} = +68,69266 \cdot \frac{\alpha}{m}.$$

IV. Denique pro quarta lente,

cuius diftantia focalis $s = -1,58173 \frac{\alpha}{m}$

radius vtriusque faciei erit

$$= 1,06 \cdot s = -1,67663 \cdot \frac{\alpha}{m}$$

ficque omnia momenta pro hoc cafu funt definita.

Exempl. III.

280. Si prima lens ex vitro chryftallino, reliquae ex coronario fint parandae, pro multiplicatione quacunque conftructionem telefcopii exponere.

Solutio.

Ex comparatione duorum exemplorum praecedentium fingula momenta methodo fupra indicata facile definiemus. Primo pro diftantiis determinatricibus erit

$b = -1,0300 \cdot \alpha$; pro reliquis autem ftatuatur

$\beta = - 2,6622\, \alpha - \beta'. \frac{\alpha}{m}$; erit $\beta' = 7,9150$

$c = + 4,9164. \frac{\alpha}{m} + \frac{c'}{m}. \frac{\alpha}{m}$; $c' = 14,8812$

$\gamma = - 3,0087. \frac{\alpha}{m} - \frac{\gamma'}{m}. \frac{\alpha}{m}$; $\gamma' = 5,281$

$d = - 1,5817 \frac{\alpha}{m} - \frac{d'}{m}. \frac{\alpha}{m}$; $d' = - 3,531$

simili modo pro distantiis focalibus est $p = \alpha$; pro reliquis statuatur

$q = - 0,7427. \alpha - \frac{q'}{m}. \alpha$;

$r = - 7,7539. \frac{\alpha}{m} - \frac{r'}{m}. \frac{\alpha}{m}$;

$s = - 1,5817. \frac{\alpha}{m} + \frac{3,531}{m}. \frac{\alpha}{m}$;

eritque $q' = 0,5665$; $r' = - 0,2062$

vnde lentium interualla sunt

$a + b = - 0,0300. \alpha$

$\beta + c = - 2,6622. \alpha - \frac{2,098}{m}. \alpha + \frac{14,88}{m}. \frac{\alpha}{m}$

$\gamma + d = - 4,5904. \frac{\alpha}{m} - \frac{1,750}{m}. \frac{\alpha}{m}$

Pro loco oculi statuatur

$O = - 0,63269. \frac{\alpha}{m} - \frac{O'}{m}. \frac{\alpha}{m}$

erit $O' = 13,431$.

I. Pro lente prima statuatur radius faciei

anter. $= 3,7425. \alpha + F'. \frac{\alpha}{m}$;

poster. $= 0,6864. \alpha + G'. \frac{\alpha}{m}$

erit $F' = - 21,70$; $G' = 1,005$.

II. Pro

II. Pro lente secunda statuatur

radius faciei

anter. $= -1,1853 \cdot \alpha - F' \cdot \frac{\alpha}{m};$

poster. $= -0,5893 \cdot \alpha - G' \frac{\alpha}{m}$

erit $F' = 2,52$; $G' = 0,08$.

III. Pro lente tertia

radius faciei

anter. $= -3,8769 \cdot \frac{\alpha}{m} - \frac{F'}{m} \cdot \frac{\alpha}{m}$

poster. $= +68,6926 \cdot \frac{\alpha}{m} + \frac{G'}{m} \cdot \frac{\alpha}{m}$

erit $F' = -0,116$; $G' = -8,096$.

IV. Denique pro quarta lente

radius vtriusque faciei $= -1,6766 \cdot \frac{\alpha}{m} - \frac{H}{m} \cdot \frac{\alpha}{m}$

erit $H = +2,862$. ex quibus conficitur sequens.

Constructio Telescopii pro multiplicatione quacunque m.

I. Pro prima lente Chrystall. Glass.

radius faciei

anter. $= 3,7425 \cdot \alpha - 21,70 \cdot \frac{\alpha}{m}$

poster. $= 0,6864 \cdot \alpha + 1,005 \cdot \frac{\alpha}{m}$

Eius distantia focalis $= \alpha$

Semidiameter aperturae $= \frac{m}{50}$ dig.

Interuallum ad lentem secundam $= -0,03 \cdot \alpha$.

II. Pro

II. Pro ſecunda lente. Crown Glaſſ.

radius faciei

anter $= - 1,1853\,\alpha - 2,52\frac{\alpha}{m}$

poſter. $= - 0,5893\,\alpha - 0,08.\frac{\alpha}{m}$

Diſtantia focalis $= - 0,7427.\,\alpha - 0,\frac{5665\,\alpha}{m}$

Semidiameter aperturae quoque $= \frac{m}{50}$ dig.

Interuallum ad tertiam

$= - 2,6622\,\alpha - 2,998.\frac{\alpha}{m} + \frac{14,88}{m}.\frac{\alpha}{m}.$

III. Pro tertia lente Crown Glaſſ.

radius faciei

anter. $= - 3,8769\frac{\alpha}{m} + 0,116.\frac{\alpha}{mm}$

poſter. $= + 68,6926.\frac{\alpha}{m} - 8,096.\frac{\alpha}{mm}$

Diſtantia focalis $= - 7,7539.\frac{\alpha}{m} + 0,206.\frac{\alpha}{mm}$

cuius pars octaua dat ſemidiametrum aperturae

Interuallum ad quartam

$- 4,5904.\frac{\alpha}{m} - 1,750.\frac{\alpha}{mm}$

At diſtantia imaginis realis poſt hanc lentem

$= \gamma = - 3,0087.\frac{\alpha}{m} - 5,281.\frac{\alpha}{mm}.$

IV. Pro quarta lente Crown Glaſſ.

radius vtriusque faciei $= - 1,6766\frac{\alpha}{m} - \frac{2,862\,\alpha}{mm},$

Eius diſtantia focalis $= - 1,5817.\frac{\alpha}{m} - \frac{3,57.\alpha}{mm},$

cuius pars quarta dat ſemidiametrum aperturae.

Et interuallum ad oculum

$O = - 0,6326.\frac{\alpha}{m} - 13,43.\frac{\alpha}{mm}.$

Hinc

Hinc totius Telescopii longitudo erit

$$=-0,03\alpha-2,998\frac{\alpha}{m}+14,88\frac{\alpha}{mm}$$
$$-2,6622.\alpha-4,590\frac{\alpha}{m}-1,75\frac{\alpha}{mm}$$
$$-0,632\frac{\alpha}{m}-13,43\frac{\alpha}{mm}$$

$$=-2,6922.\alpha-8,220\frac{\alpha}{m}-0,30.\frac{\alpha}{mm}$$

Semidiameter vero campi apparentis erit $=\frac{1289}{m+1}$ min.

Hic vero notandum est, α esse negatiuum, cuius valor definietur ex radio posteriore lentis secundae, cuius pars quarta $0,1473.\alpha$ seu circiter $=\frac{1}{7}.\alpha$ ipsi $\frac{m}{50}$. aequalis posita dabit $\alpha=-\frac{7.m}{50}$ statui igitur poterit $\alpha=-\frac{m}{7}$.

Corollarium.

281. Si igitur statuamus $\alpha=-\frac{m}{7}$, habebitur sequens

Constructio determinata telescopii in ratione $m:1$ multiplicantis:

I. Pro prima lente. Flint Glass.

radius faciei in digitis

anter. $=-0,5346.m+3,10$.
poster. $=-0,0981.m-0,14$.

Distantia focalis $=-\frac{m}{7}$
Semidiameter aperturae $=\frac{m}{50}$
Interuallum $=0,0043.m$.

II. Pro secunda lente. Crown Glass.

radius faciei

anter. $= + 0,1693 \cdot m + 0,36.$

poster. $= + 0,0842 \cdot m + 0,01.$

Distantia focalis $= + 0,1061 \cdot m + 0,080.$

Semidiameter aperturae $= \frac{m}{50}$ dig.

Interuallum $= + 0,3803 \cdot m + 0,43 - \frac{2,12}{m}.$

III. Pro tertia lente. Crown Glass.

radius faciei

anter. $= + 0,55 - \frac{0,01}{m}.$

poster. $= - 9,81 + \frac{1,1}{m}.$

Distantia focalis $= + 1,11 - \frac{0,03}{m}.$

Interuallum $= 0,65 + \frac{0,3}{m}.$

Distantia imaginis realis post hanc lentem

$= 0,43 + \frac{0,7}{m}.$

IV. Pro quarta lente. Crown Glass.

radius vtriusque faciei $= + 0,24 - \frac{0,4}{m}.$

Distantia focalis $= 0,22 + \frac{0,6}{m}.$

Semidiameter aperturae $= 0,05 + \frac{0,1}{m}.$

Interuallum vsque ad oculum $= 0,09 - \frac{1,9}{m}.$

et

et tota telescopii longitudo

$$=0,3846.m+1,18+\frac{0,04}{m}.$$

et semidiameter campi $=\frac{1289}{m+1}$ min.

Scholion.

282. Haec itaque telescopia multo fiunt longiora, quam ea, quae praecedente capite sunt inuenta, pro eadem vtrinque multiplicatione. Hic enim pro multiplicatione $m=100$ prodit longitudo $=40$ dig., dum in praecedente capite sufficiebat longitudo $=13\frac{1}{2}$ dig. hoc est triplo minor. Hic igitur merito quaeritur, vtrum qualitas, qua etiam spatium diffusionis a diuersa radiorum refrangibilitate oriundae ad nihilum redigitur, tanti sit habenda, vt longitudo telescopii triplicetur, quae quaestio non nisi per praxin dijudicari poterit quae autem eo erit difficilior, quo minus accuratissimam exsecutionem horum praeceptorum exspectare licet. Quocirca nisi haec conditio praescripta felicissime succedat, semper praestabit, telescopia praecedentis capitis praeferre; ita, vt duplici vitri specie carere possimus. Ceterum in euolutione casus secundi huiusmodi euolutionibus numericis non immorabimur, cum quia methodus tales calculos tractandi iam satis est explicata, tum vero quia propositum est, huic operi singularem librum subiungere, in quo sola praecepta pro praxi dirigenda in gratiam artificum colligentur.

 CASVS II.

CASVS II.

283. Sit iam ſecunda lens ex vitro chryſtallino, reliquae vero ex vitro coronario, ita, vt ſit $n' = 1,58$ et $n = n'' = n''' = 1,53$, indeque etiam $N = N'' = N'''$ et ponatur $\frac{N'}{N} = \zeta$, vt ſit ſecundum Dollondum $\zeta = \frac{10}{7}$. His poſitis et ſumto $i = \frac{1}{2}$ erit α poſitiuum ideoque etiam η. Sumatur igitur $\eta = 0,03$, vnde habebimus $P = \frac{1}{1-\eta}$. Quia nunc, vt vidimus, eſt $\mathfrak{B} = \zeta$ proxime, erit

$$\omega = \frac{-\eta}{1-\eta} \cdot \frac{i+1}{(m+1)\zeta} = \frac{-9}{194 \cdot \zeta (m+1)}.$$

Deinde cum ſit proxime $R = -2$, ideoque $Q = \frac{m}{2P}$, vbi $P = +\frac{100}{97}$; adcuratius habebimus ex prima aequatione, $0 = \zeta Q \omega + i + \frac{1}{R}$, quae abit in hanc

$$-\frac{1}{R} = \frac{1}{2} - \frac{9m}{388 \cdot P(m+1)} \text{ ſeu } -\frac{1}{R} = \frac{1}{2} - \frac{9m}{400(m+1)}$$

ex quo etiam Q adcuratius definiri poteſt, cum ſit $Q = \frac{-m}{PR}$.

Vt nunc etiam $\mathfrak{B}$ adcuratius definiatur, colligimus ex aequatione ſecunda,

$$0 = 1 - \frac{\zeta}{\mathfrak{B}P} + \frac{1}{B\mathfrak{C}PQ} - \frac{1}{BCPQR}$$

quae ob $\frac{1}{\mathfrak{B}} = \frac{1}{B} + 1$ dabit

$$-B(\zeta - P) = \zeta - \frac{1}{\mathfrak{C}Q} + \frac{1}{CQR}$$

Erat autem proxime $B = \frac{-\zeta}{\zeta - 1}$, ideoque negatiuum. Ex valore porro adcurato ipſius B erit $\mathfrak{B} = \frac{B}{B+1}$.

His

His igitur definitis tertia aequatio erit

$$0 = \lambda - \frac{\mu'.\lambda'}{\mu \mathfrak{B}^3 P} + \frac{\lambda''}{B^3 \mathfrak{C}^3 PQ} - \frac{\lambda'''}{B^3 C^3 PQR}$$
$$- \frac{\mu' \nu'}{\mu B \mathfrak{B} P} + \frac{\nu}{B^3 C \mathfrak{C} PQ}.$$

Hic duo primi termini vtpote valde magni dabunt proxime $\lambda' = \frac{\mu}{\mu'} . \mathfrak{B}^3 P. \lambda$ vnde intelligere licet pro lente concaua eius valorem λ maiorem prodire, quam casu primo ita, vt casus primus ad praxin sit aptior iudicandus.

Vt nunc etiam de longitudine telescopii iudicare possimus, quia praecedente casu ea nimis magna prodierat, considerari debet eius pars praecipua, quae est littera β, cuius valor ante fuerat $= -2,6622 . \alpha$ circiter, qui autem nunc ob $\beta = \frac{-B\alpha}{P}$ et $B = \frac{-\zeta}{\zeta - P}$ proxime seu $B = -\frac{10}{3}$ et $P = 1$ reperitur $\beta = \frac{10}{3} . \alpha = 3,33 . \alpha$, ita, vt telescopia hinc nata adhuc fiant longiora, quae propterea hic fusius euoluere operae non erit pretium.

Scholion.

284. Quia longitudo telescopii, quae hinc tanta resultat, perfectioni non parum obstat, disquirendum est, vtrum huic incommodo non aliquod remedium adferri possit, quod autem ex hypothesi hic facta, qua lens obiectiua quasi ex duabus lentibus constare est assumta, neutiquam sperare licet. Quamobrem lentem obiectiuam quasi ex tribus lentibus constantem assumi conueniet, quarum vel vna vel duae

sint concauae et ex vitro chrystallino formatae. Ne autem nouam inuestigationem circa maiorem campum suscipere cogamur, hic statim quoque tres lentes quasi oculares introducamus, vt hoc modo si negotium successerit, non solum breuiora telescopia obtineantur, sed etiam simul campus apparens notabile incrementum accipiat.

Problema 2.

285. Si tres lentes priores ad obiectiuam constituendam referantur, tum vero tres quasi lentes oculares adiungantur, definire momenta, ad telescopii constructionem necessaria.

Solutio.

Cum igitur hic occurrant sex lentes, statuamus
$\frac{\alpha}{b} = -P$; $\frac{\beta}{c} = -Q$; $\frac{\gamma}{d} = -R$; $\frac{\delta}{e} = -S$; $\frac{\epsilon}{f} = -T$.
vt sint nostra elementa

$$b = -\frac{\alpha}{P};\ c = \frac{B\alpha}{PQ};\ d = \frac{-BC\alpha}{PQR};\ e = \frac{BCD\alpha}{PQRS}$$

$$\beta = \frac{-B\alpha}{P};\ \gamma = \frac{BC\alpha}{PQ};\ \delta = \frac{-BCD\alpha}{PQR};\ \epsilon = \frac{BCDE\alpha}{PQRS}$$

$$\text{et } f = \frac{-BCDE\alpha}{PQRST}$$

adeoque distantiae focales

$$p = \alpha;\ q = \frac{-\mathfrak{B}\alpha}{P};\ r = \frac{B\mathfrak{C}\alpha}{PQ};\ s = \frac{-BC\mathfrak{D}\alpha}{PQR};$$

$$t = \frac{BCD\mathfrak{E}\alpha}{PQRS};\ u = \frac{-BCDE\alpha}{PQRST};$$

existente $m = -PQRST$.

Horum

Horum iam numerorum P, Q, R, S, T vnicus debet eſſe negatiuus; interualla autem lentium ita exprimentur

$$\alpha + b = \alpha\left(1 - \tfrac{1}{P}\right);\quad \beta + c = \tfrac{-B\alpha}{P}\left(1 - \tfrac{1}{Q}\right)$$

$$\gamma + d = \tfrac{BC\alpha}{PQ}\left(1 - \tfrac{1}{R}\right);\quad \delta + e = \tfrac{-BCD\alpha}{PQR}\left(1 - \tfrac{1}{S}\right)$$

$$\text{et } \varepsilon + f = \tfrac{BCDE\alpha}{PQRS}\left(1 - \tfrac{1}{T}\right)$$

quod vltimum interuallum ob $\varepsilon = -Tf$, etiam ita exhibetur $\varepsilon + f = f(1 - T)$; vnde quia f debet eſſe quantitas poſitiua, ſtatim patet, eſſe debere $T < 1$.

Quia nunc tres priores lentes exiguis interuallis a ſe inuicem diſtare debent, ſtatuamus vtrumque interuallum $= \eta\alpha$ hincque habebimus

$$P = \tfrac{1}{1 - \eta} \text{ et } Q = \tfrac{B}{B + \eta P}.$$

Nunc vero cum ſit campi ſemidiameter

$$\Phi = \frac{\pi - \pi' + \pi'' - \pi''' + \pi''''}{m + 1}$$

ſtatuamus $\pi = \omega\xi$; $\pi' = -\omega'\xi$; $\pi'' = i\xi$; $\pi''' = -\xi$; et $\pi'''' = \xi$, ita, vt ſit

$$\Phi = \frac{\omega + \omega' + i + 2}{m + 1}\,\xi = M\,\xi; \text{ exiſtente}$$

$$M = \frac{\omega + \omega' + i + 2}{m + 1} \text{ ſeu proxime } M = \frac{i + 2}{m + 1}$$

ob ω et ω' fractiones minimas, vt mox videbimus. Hincque ob $\frac{\pi''''}{\Phi} = \frac{1}{M}$, ſtatim oritur diſtantia oculi

$$O = \frac{1}{M}\cdot\frac{t}{m} = \frac{m + 1}{m(i + 2)}\cdot t$$

Porro

Porro confiderari oportet fequentes aequationes

$$\mathfrak{B}\,\omega = (1-P)\,M;\ \mathfrak{C}\,\omega' = (1-PQ)\,M - \omega$$

$$\mathfrak{D}\,i = (1-PQR)\,M - \omega' - \omega$$

$$\mathfrak{E} = (1-PQRS)\,M - \omega' - \omega - i.$$

Ex harum aequationum duabus primis quaerantur particulae ω et ω', fcilicet

$$\omega = \frac{(1-P)M}{\mathfrak{B}} = \frac{-\eta . M}{(1-\eta)\mathfrak{B}}$$

$$\omega' = \frac{(1-PQ)M - \omega}{\mathfrak{C}} \text{ feu}$$

$$\omega' = \frac{\eta(1-B).M}{(B+(1-B)\eta)\mathfrak{C}} + \frac{\eta M}{(1-\eta)\mathfrak{B}\mathfrak{C}} \text{ feu}$$

$$\omega' = \frac{\eta M}{\mathfrak{C}}\left(\frac{2B+\eta(1-B)}{(B+\eta(1-B))(1-\eta)B}\right.$$

quare cum η fit fractio valde parua erit proxime

$$\omega = \frac{-\eta M}{\mathfrak{B}};\ \omega' = \frac{2\eta M}{B\mathfrak{C}}$$

atque litterae $\mathfrak{B}$ et $\mathfrak{C}$ etiamnum manent indeterminatae, dum fequentes $\mathfrak{D}$ et $\mathfrak{E}$ per formulas hic allatas determinantur. Nunc igitur confiderentur tres aequationes, quas adimpleri oportet

$$\text{I. } 0 = +\frac{N'\omega.}{P} + \frac{N''\omega'}{PQ} + \frac{N'''i}{PQR} + \frac{N''''}{PQRS} + \frac{N'''''}{PQRST}$$

$$\text{II. } 0 = N - \frac{N'}{\mathfrak{B}P} + \frac{N''}{B\mathfrak{C}PQ} - \frac{N'''}{BC\mathfrak{D}PQR}$$

$$+ \frac{N''''}{BCD\mathfrak{E}.PQRS} - \frac{N'''''}{BCDE\mathfrak{F}PQRST}$$

III. o =

$$\text{III.}\quad 0 = \mu\lambda - \frac{\mu'}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right)$$
$$+ \frac{\mu''}{B^3\mathfrak{C}PQ}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu''}{C}\right)$$
$$- \frac{\mu'''}{B^3C^3\mathfrak{D}PQR}\left(\frac{\lambda'''}{\mathfrak{D}^2} + \frac{\nu'''}{D}\right)$$
$$+ \frac{\mu''''}{B^3C^3D^3.\mathfrak{E}.PQRS}\left(\frac{\lambda''''}{\mathfrak{E}^2} + \frac{\nu''''}{E}\right)$$
$$- \frac{\mu'''''.\lambda'''''}{B^3C^3D^3E^3.PQRST}.$$

In prima autem aequatione duo membra priora prae ſequentibus manifeſto ſunt valde parua, dum etiam per m multiplicata adhuc multo minora manent ſequentibus. Iis omiſſis habebitur

$$0 = N'''i + \frac{N''''}{S} + \frac{N'''''}{ST}$$

vbi quia tres poſteriores lentes ex eadem vitri ſpecie fieri conuenit erit $0 = iST + T + 1$; vnde patet, vel S vel T eſſe debere quantitatem negatiuam. Hanc ob rem ſtatuatur, $S = -k$ vt vnica harum lentium ante imaginem realem cadat, vnde fiet $k = \frac{T+1}{iT}$; ante autem iam vidimus, eſſe $T < 1$ ideoque $K > 2$ ob $i < 1$ et ſi $i = \frac{1}{2}$ fiet adeo $K > 4$ ex quo erit

$$KT = \frac{1+T}{i} = -ST.$$

Iam ob P et Q vnitati proxime aequales erit quoque proxime $RST = -m$ ideoque $R = \frac{im}{1+T}$ adeoque numerus magnus. Ex his autem valoribus proximis facile erit valores adcuratos ex eadem aequatione prima deducere. Iam in aequatione ſecunda ſatis euidens eſt, tres terminos priores multo maiores eſſe ſequentibus.

His ergo omissis habebitur

$$o = N - \frac{N'}{\mathfrak{B}P} + \frac{N''}{B\mathfrak{C}PQ}$$

vnde ob P et Q proxime $= 1$, colligitur fore etiam proxime

$$o = N - \frac{N'}{\mathfrak{B}} + \frac{N''}{B\mathfrak{C}}$$

vnde colligitur

$$\mathfrak{C} = \frac{N''.\mathfrak{B}}{B(N' - N\mathfrak{B})} = \frac{N''(1-\mathfrak{B})}{N' - N\mathfrak{B}} \text{ hineque}$$

$$C = \frac{N''(1-\mathfrak{B})}{N' - N\mathfrak{B} - N'' + N''\mathfrak{B}}.$$

$$C = \frac{N''(1-\mathfrak{B})}{N' - N'' + \mathfrak{B}(N'' - N)}$$

vbi littera $\mathfrak{B}$ adhuc arbitrio nostro permittitur.

Pro $\mathfrak{C}$ autem sequentes casus sunt notandi;

1°. Si $\mathfrak{B} > 1$ et $\mathfrak{B} < \frac{N'}{N}$; tunc erit $\mathfrak{C}$ negatiuum, adeoque etiam C negatiuum.

2°. Si $\mathfrak{B} > 1$ et $\mathfrak{B}$ simul $> \frac{N'}{N}$; tunc erit $\mathfrak{C}$ positiuum.

3°. Si fuerit $\mathfrak{B} < 1$ et $\mathfrak{B} < \frac{N'}{N}$; tunc erit $\mathfrak{C}$ positiuum.

4°. Si fuerit $\mathfrak{B} < 1$ et $\mathfrak{B} > \frac{N'}{N}$; tunc erit $\mathfrak{C}$ negatiuum; adeoque et C negatiuum.

Pro secundo autem casu erit C negatiuum si $N' > N$. Sin autem fuerit $N' < N$, erit C positiuum. Pro casu tertio erit C positiuum, si sit $N' > N$; sin autem sit $N' < N$, erit C negatiuum.

Circa

Circa duas autem reliquas litteras $\mathfrak{D}$ et $\mathfrak{E}$ notandum eſt, fore $\mathfrak{D}$ negatiuum, ideoque et D, tum vero $\mathfrak{E}$ eſſe poſitiuum; fit enim proxime

$$\mathfrak{E} = \frac{i+2}{1} - i. \text{ ideoque}$$

$$E = \frac{\frac{i+2}{T} - i}{1 + i - \frac{i+2}{T}}; \text{ ideoque E negatiuum.}$$

Examinemus iam interualla lentium ac primo quidem cum ſit $d < \gamma$ neceſſe eſt, vt ſit γ poſitiuum ideoque debet eſſe $BC\alpha > 0$.

Eſt vero $BC = \frac{N''\mathfrak{B}}{N' - N'' + \mathfrak{B}(N'' - N)}$.

Cum igitur diſtantia γ maximam partem totius longitudinis contineat, $\mathfrak{B}$ ita accipi debet, vt quantitas BC vnitatem vix ſuperet, quia ante hic coefficiens modo binarium tum vero et ternarium ſuperauerat.

Statim autem ac γ reddita eſt quantitas poſitiua, erit d negatiuum et manifeſto $\gamma + d > 0$.

Porro vero fit δ poſitiuum, nempe $\delta = Dd$ atque etiam e poſitiuum; ita, vt ſit $\delta + e$ poſitiuum.

Denique $\varepsilon = Ee$ adeoque negatiuum et f poſitiuum; eritque etiam $\varepsilon + f > 0$ ob $T < 1$, vt iam ante notauimus; ex quo etiam diſtantia oculi fit poſitiua, ſicque omnes conditiones ſunt adimpletae.

Denique in tertia aequatione etiam manifeſtum eſt, tres tantum terminos priores potiſſimum in compuputum venire, vtpote prae reliquis multo maiores; vnde fit

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3} + \frac{\mu''\lambda''}{B^3\mathfrak{C}^3}$$

vbi efficiendum quoque est, vt nulla litterarum λ, λ', λ'' vnitatem notabiliter superet. Quemadmodum igitur haec omnia commodissime praestentur, diuersos casus euolui oportet, prouti inter tres lentes priores vel vna vel duae chrystallinae occurrant et quo loco.

CASVS I.

286. Quo prima lens chrystallina, secunda et tertia vero ex vitro coronario est parata. Erit ergo $N = 10$, $N' = 7$, et $N'' = 7$. adeoque primo

$$\mathfrak{C} = \frac{7(1-\mathfrak{B})}{7-10\mathfrak{B}} \text{ et } C = \frac{-7(1-\mathfrak{B})}{3\mathfrak{B}} \text{ ideoque}$$

$$B\mathfrak{C} = \frac{7\mathfrak{B}}{7-10\mathfrak{B}} \text{ et } BC = -\frac{7}{3}$$

hinc ergo fit $\gamma = -\frac{7}{3}\alpha$; vnde euidens est, sumi debere α negatiuum, qui valor cum non minor sit, quam in problemate praecedente; hunc casum vlterius non prosequimur.

CASVS II.

287. Quo lens secunda est chrystallina; prima vero et tertia ex vitro coronario; erit $N = 7$, $N' = 10$; $N'' = 7$; atque hinc

$$C = \frac{7(1-\mathfrak{B})}{3} \text{ adeoque } BC = \frac{7}{3}\mathfrak{B}$$

$$\mathfrak{C} = \frac{7(1-\mathfrak{B})}{10-7\mathfrak{B}} \text{ et } B\mathfrak{C} = \frac{7\mathfrak{B}}{10-7\mathfrak{B}}$$

vnde

vnde fit $\gamma = \frac{7}{3}\mathfrak{B}.\alpha$. Quo igitur telescopium contrahatur, sumi debet $\mathfrak{B} < 1$; tum vero erit B positiuum; at ex tertia aequatione habebimus

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3} + \frac{\mu''\lambda''}{B^3\mathfrak{C}^3} \text{ siue}$$

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3} + \frac{\mu\lambda''(10-7\mathfrak{B})^3}{7^3.\mathfrak{B}^3} \text{ vnde fit}$$

$$\frac{\mu'\lambda'}{\mu} = \mathfrak{B}^3\lambda + \frac{(10-7\mathfrak{B})^3.\lambda''}{7^3}$$

hic ergo videamus, an litterae λ et λ' et λ'' possint ad vnitatem redigi; ad quod requiritur, vt sit

$$\frac{\mu'}{\mu} = \mathfrak{B}^3 + (\tfrac{10}{7} - \mathfrak{B})^3$$

cuius euolutio ob $\frac{\mu''}{\mu}$ propemodum $= 1$ dat

$$(\tfrac{10}{7})^3 - 3(\tfrac{10}{7})^2\mathfrak{B} + 3(\tfrac{10}{7})\mathfrak{B}^2 = 1$$

cuius radices sunt prior $\mathfrak{B} = 0,97$; qua autem hic nihil in longitudine lucramur; altera vero est $\mathfrak{B} = 0,46$ siue $\mathfrak{B} = \frac{5}{11}$ hocque modo fiet $\gamma = \frac{7}{9}\alpha$; quod insigne lucrum est qui ergo casus inprimis meretur, vt adcuratius euoluatur.

CASVS III.

288. Sit nunc tertia lens chrystallina, prima et secunda ex vitro coronario, vt sit $N = 7$, $N' = 7$, at $N'' = 10$. Hinc igitur, sequitur

$$\mathfrak{C} = \frac{10(1-\mathfrak{B})}{7-7\mathfrak{B}} = \frac{10}{7};\ C = -\frac{10}{3} \text{ indeque}$$

$$B\mathfrak{C} = \frac{10}{7}B;\ \text{et } BC = -\frac{10}{3}B$$

vnde fit $\gamma = -\frac{10}{3}B\alpha$. Deberet ergo esse $-B < 1$

 vel

vel $B > -1$. hinc $\mathfrak{B} < 1$. Hinc ergo tertia aequatio vnitate loco cuiuslibet λ scripta erit

$$0 = 1 - \frac{1}{\mathfrak{B}^3} + \frac{343(1-\mathfrak{B})^3}{1000\,\mathfrak{B}^3} \text{ seu}$$

$$0 = \mathfrak{B}^3 - 1 + \tfrac{343}{1000}(1-\mathfrak{B})^3 \text{ seu}$$

$$0 = 657 - 1029\,\mathfrak{B} + 1029\,\mathfrak{B}^2 + 657\,\mathfrak{B}^3$$

qui ergo casus etiam euolutione dignus videtur.

CASVS IV.

289. Si prima et secunda lens sint chrystallinae et tertia coronaria; erit

$$N' = N = 10; \text{ et } N'' = 7; \text{ hincque}$$

$$\mathfrak{C} = \frac{7(1-\mathfrak{B})}{10(1-\mathfrak{B})} = \tfrac{7}{10} \text{ et } C = \tfrac{7}{3} \text{ hincque } BC = \tfrac{7B}{3}.$$

Nunc quia est α negatiuum, γ autem positiuum, debet esse $\gamma = + \frac{7}{3} B\alpha$. Ponamus nunc $\frac{7}{3}B = -1$ erit $B = -\frac{3}{7}$ et $\mathfrak{B} = -\frac{3}{4}$ et $B\mathfrak{C} = -\frac{3}{10}$ vnde tertia aequatio fiet

$$0 = \lambda + \frac{\lambda'.4^3}{3^3} - \frac{\mu''}{\mu}.\frac{\lambda''.10^3}{3^3}$$

ita, vt esse debeat

$$\lambda + \frac{64\lambda'}{27} = \frac{\mu''}{\mu}.\frac{1000\lambda''}{27} + \text{etc.}$$

quod cum fieri nequeat, nisi pro λ et λ' numeri maximi accipiantur, hic casus nullo modo in praxi admitti potest.

CASVS V.

290. Sint prima et tertia lens chrystallinae, secunda ex vitro coronario, erit $N = N'' = 10$ et

et $N' = 7$, adeoque

$$\mathfrak{C} = \frac{10(1-\mathfrak{B})}{7-10\mathfrak{B}} \text{ et } C = -\frac{10(1-\mathfrak{B})}{3} \text{ hinc}$$

$$BC = -\tfrac{10}{3}\mathfrak{B} \text{ et } \gamma = -\tfrac{10}{3}\mathfrak{B}\alpha.$$

Ponatur ergo $\frac{10}{3}\mathfrak{B} = 1$ eritque $\mathfrak{B} = \frac{3}{10}$ et $B = \frac{3}{7}$ et $B\mathfrak{C} = \frac{3}{4}$; vnde aequatio tertia dabit

$$o = \lambda - \frac{\mu'}{\mu} \cdot \frac{\lambda' \cdot 1000}{27} + \frac{64\lambda''}{27} \text{ seu}$$

$$\lambda + \frac{64\lambda''}{27} = \frac{\mu'}{\mu} \cdot \frac{1000\lambda'}{27} + \text{etc.}$$

quae aeque parum ad praxin est idonea.

CASVS VI.

291. Sint secunda et tertia lens chrystallinae, prima ex vitro coronario confecta; erit $N = 7$, et $N' = N'' = 10$, vnde colligitur

$$\mathfrak{C} = \frac{10(1-\mathfrak{B})}{10-7\mathfrak{B}} \text{ et } C = \frac{10(1-\mathfrak{B})}{3\mathfrak{B}}; \text{ hincque}$$

$$BC = \tfrac{10}{3} \text{ et } \gamma = \tfrac{10}{3}\alpha,$$

qui ergo casus iam sponte cadit.

Evolutio vlterior casus secundi.

292. Quod ad valorem litterae η attinet, pro quauis multiplicatione m, quae lentis obiectiuae aperturam postulat, cuius semidiameter sit circiter $\frac{m}{50}$ dig. sumamus accipi $\alpha = \frac{m}{6}$ dig. quia lens plerumque fere est plano-conuexa; eritque huius lentis crassities circiter $\frac{1}{64}\alpha$, quare si interuallum binarum lentium priorum statuamus $\frac{1}{50}\alpha$; metuendum non est, ne duae lentes se mutuo tangant, sed satis relinquetur spatii, vt etiam

etiam quodammodo moueri possint. Ponamus ergo $\eta = \pm \frac{1}{50} = \pm 0,02$.

Quia nunc prima lens est ex vitro coronario, ideoque conuexa erit α positiuum et $\eta = + \frac{1}{50} = + 0,02$. Hinc reperimus statim $P = \frac{50}{49}$ et $Q = \frac{45 \cdot B}{49 B + 1}$, vbi de B infra dispiciemus. Hic notasse sufficiat, esse proxime $Q = 1$ et $P = 1$.

His praemissis sumta fractione $i = \frac{1}{2}$ et $T = \frac{1}{2}$, quandoquidem esse debet $T < 1$. prima aequatio nostra dabit $k = 6 = - S$ et ob $PQ = 1$ proxime $R = \frac{1}{3} m$; neque opus est, vt hic valor accuratius eruatur.

Secunda autem aequatio, cui pariter proxime tantum satisfecisse sufficit, quia hoc casu est $N = 7$, $N' = 10$, $N'' = N''' = N'''' = N''''' = 7$. nobis praebet

$$0 = 7 - \frac{10}{\mathfrak{B} P} + \frac{7}{B \mathfrak{C} P Q}.$$

quae sumto $P = 1$ et $PQ = 1$ dat

$$\mathfrak{C} = \frac{-7(1 - \mathfrak{B})}{(7\mathfrak{B} - 10)} = \frac{7(1 - \mathfrak{B})}{10 - 7\mathfrak{B}} \text{ indeque}$$

$$C = \frac{7(1 - \mathfrak{B})}{3} \text{ et } BC = \frac{7 \cdot \mathfrak{B}}{3}.$$

Cum dein ex primis elementis sit $\gamma = \frac{BC\alpha}{PQ}$, quae distantia praecipuam partem totius longitudinis continet, faciamus $\gamma = \alpha$ siue proxime saltem $= \alpha$ adeoque $BC = 1$; vnde sequitur $\frac{7}{3} \mathfrak{B} = 1$ et $\mathfrak{B} = \frac{3}{7}$; vnde porro $B = \frac{3}{4}$; $\mathfrak{C} = \frac{4}{7}$ et $C = \frac{4}{3}$ ideoque $B\mathfrak{C} = \frac{3}{7}$. Nunc

Nunc ex valore B inuento habebimus praeter $P = \frac{50}{49}$ etiam $Q = \frac{147}{151}$ et $PQ = \frac{150}{151}$ ficque adcurate iam erit $R = \frac{151 \cdot m}{3 \cdot 150}$.

Nunc igitur ad aequationem tertiam progrediamur:

$$0 = \lambda - \frac{\mu'}{\mu} \cdot \frac{\lambda'}{\mathfrak{B}^3 P} + \frac{\lambda''}{B^3 \mathfrak{C}^3 PQ}$$

$$- \frac{\mu' \nu'}{\mu} \cdot \frac{\nu}{B \mathfrak{B} P} + \frac{\nu}{B^3 C \mathfrak{C} PQ}$$

$$- \frac{\lambda'''}{B^3 C^3 \mathfrak{D}^3 PQR} + \frac{\lambda''''}{B^3 C^3 D^3 \mathfrak{E}^3 PQRS}$$

$$- \frac{\nu}{B^3 C^3 D \mathfrak{D} PQR} + \frac{\nu}{B^3 C^3 D^3 E \mathfrak{E} PQRS}$$

$$+ \frac{\lambda'''''}{B^3 C^3 D^3 E^3 \cdot m}$$

quae vt in numeros euolui poffit, ante neceffe eft, valores litterarum D et E inueftigare, qui ex formulis fupra datis inueniuntur:

$$\tfrac{1}{2} \mathfrak{D} = (1 - PQR) M; \text{ feu}$$

$$\mathfrak{D} = (1 - \tfrac{1}{3} m) \frac{5}{m+1} = -\frac{5}{3}$$

ob m vt valde magnum affumtum praecipue cum $\mathfrak{D}$ tantum occurrat in numeris per fe minimis; adeoque $D = -\frac{5}{8}$. Simili modo erit

$$\mathfrak{E} = \frac{5(1 + 2m)}{2(m+1)} - \frac{1}{2} = \frac{9}{2} \text{ et } E = \frac{9}{7}.$$

His igitur valoribus substitutis habebimus

$$0 = \lambda - 10,9985\,\lambda' + 12,7884.\,\lambda''$$

$$(1,0413348)\quad(1,1068156)$$

$$- 0,68119 + 0,68775$$

$$(9,8332690)\quad(9,8374334)$$

$$+ \frac{0,64800.\lambda'''}{m} + \frac{0,02247.\lambda''''}{m}$$

$$(9,8115752)\quad(8,3516924)$$

$$- \frac{0,363245}{m} + \frac{0,07773}{m}$$

$$(9,8010248)\quad(8,8906053)$$

$$+ \frac{1,92720.\lambda'''''}{m}$$

$$(0,2849264)$$

ex qua aequatione si sumatur $\lambda = 1$ et $\lambda'' = 1$, colligitur

$$\lambda' = 0,09092 + 0,0589\,\lambda''' : m - \frac{0,05044}{m}$$

$$+ 1,16275 + 0,0020\,\lambda'''' : m$$

$$+ 0,00060 + 0,1752\,\lambda''''' : m$$

$$1,25427$$

Circa litteras λ''', λ'''', λ''''' obseruandum est, quia binae postremae plenam aperturam admittere debent, esse debere

$$\lambda''''' = 1,60006 \text{ et } \lambda'''' = 1 + 0,60006.$$

$$\text{et } \lambda''' = 1 + 0,60006\,(1 - 2\,\mathfrak{E})^2$$

$$= 1 + 0,60006.\,64$$

vnde hae duae lentes statim computari possunt.

Pro

Pro quarta autem lente in ipso radiorum calculo valor numeri λ''' definiatur. Tum vero lens prima et tertia quoque per calculum determinantur. Quo facto quaeratur valor ipsius λ', qui cum etiam m inuoluat, primo pro valore determinato ipsius m. v. g. $m = 25$, deinde pro $m = \infty$ radii facierum huius lentis inuestigentur ex iisque pro multiplicatione quacunque eorum valores concludantur, vt iam supra aliquoties est factum.

Interualla autem lentium cum distantiis focalibus sequenti modo se habebunt:

$b = -0,98.\alpha$

$\beta = -0,73500.\alpha.$ log. $\beta = 9,8662874;$

$c = 0,75500.\alpha;$

$\gamma = 1,00667.\alpha.$ log. $\gamma = 0,0028856$

$d = -\frac{3\alpha}{m}$

$\delta = + \frac{1,875}{m}.\alpha$

$e = + \frac{0,3125.\alpha}{m}$

$\varepsilon = - \frac{0,40178.\alpha}{m}$

$f = + \frac{0,80357.\alpha}{m}$

et distantiae focales

$p = \alpha;\ q = -0,42000.\alpha;\ r = 0,43144$

$s = \frac{5\alpha}{m};\ t = + \frac{1,40625.\alpha}{m};\ u = \frac{0,80357}{m}.\alpha$

 Hinc-

Hincque interualla lentium

$$\alpha + b = 0,02.\alpha; \quad \beta + c = 0,0200.\alpha$$

$$\gamma + d = 1,00667.\alpha - \tfrac{3}{m}.\alpha$$

$$\delta + e = 2,1875.\tfrac{\alpha}{m}; \quad \varepsilon + f = \frac{0,40178.\alpha}{m}$$

et pro loco oculi $O = 0,5626.\frac{\alpha}{m}$.

Conſtructio lentium.

Inueſtigemus primo conſtructionem pro ſingulis lentibus ex vitro coronario parandis poſitisque pro quauis lente

radiis faciei $\begin{cases} \text{anter.} = F. \\ \text{poſter.} = G. \end{cases}$

haec determinatio ſequenti modo ſe habebit

I. Pro prima lente

ob $\lambda = 1$ reperietur

$$F = \frac{\alpha}{\sigma} = \frac{\alpha}{1,6601} = 0,60237.\alpha$$

$$G = \frac{\alpha}{\varrho} = \frac{\alpha}{0,2267} = 4,41111.\alpha$$

III. Pro tertia lente.

ob $\lambda'' = 1$ erit

$$F = \frac{c\gamma}{\gamma\varrho + c\sigma} = \frac{Cc}{C\varrho + \sigma} = \frac{\gamma}{C\varrho + \sigma}$$

$$G = \frac{c\gamma}{\gamma\sigma + c\varrho} = \frac{Cc}{C\sigma + \varrho} = \frac{\gamma}{C\sigma + \varrho}$$

$$F = \frac{\gamma}{1,9624} = 0,51298.\alpha$$

$$G = \frac{\gamma}{2,4401} = 0,41255.\alpha$$

IV.

IV. Pro quarta lente

ob λ''' etiamnunc incognitum ponatur breuitatis gratia

$\tau(1+D)\sqrt{(\lambda'''-1)} = x$ eritque

$F = \frac{\delta}{D\varrho+\sigma\pm x}$; $G = \frac{\delta}{\varrho+D\sigma\mp x}$

adeoque

$F = \frac{\delta}{1,5184\pm x}$; $G = \frac{\delta}{-0,8109\mp x}$.

Vt nunc haec lens aperturam $\frac{1}{2}\xi$ admittat, hoc eueniet, si posterior facies fuerit plana, seu denominator $= 0$; valeant igitur signa inferiora et ponatur

$x = 0,8109$ vnde fiet

$G = \infty$ et $F = \frac{\delta}{0,7075}$ seu $F = \frac{2,650.\alpha}{m}$

vti debet esse, quia $F = (n-1)5$. Cum igitur sit

$\tau(1+D)\sqrt{(\lambda'''-1)} = 0,8109$ erit

$\sqrt{\lambda'''-1} = \frac{0,8109}{0,3469}$ hincque $\lambda''' = 6,4642$.

V. Pro quinta lente

est $\lambda'''' = 39,40384$ et quia haec lens est vtrinque aeque conuexa erit

radius faciei vtriusque $= 1,06.\,t = 1,4906.\frac{\alpha}{m}$.

VI. Pro sexta lente

est, vti vidimus, $\lambda''''' = 1,60006$ ideoque

radius vtriusque faciei $= 1,06.\,u = 0,8518.\frac{\alpha}{m}$.

II. Pro fecunda lente

reperietur nunc primo

$$\lambda' = 1,25427 + \frac{0,6894}{m}.$$

Statuamus nunc effe $m = 25$ eritque

$$\lambda' = 1,28184.$$

Quare cum pro fecunda lente fit

$$F = \frac{\beta}{B\varrho' + \sigma' \pm \tau'(1 + B)\sqrt{(\lambda' - 1)}}$$

$$G = \frac{\beta}{B\sigma' + \varrho' \mp \tau'(1 + B)\sqrt{(\lambda' - 1)}}$$

erit $\tau'(1 + B)\sqrt{\lambda' - 1} = 0,81524$

vnde colligitur

$$F = \frac{\beta}{1,6868 \pm 0,81524} = \frac{\beta}{\overline{0,8736}}$$

$$G = \frac{\beta}{1,3284 \mp 0,81524} = \frac{\beta}{\overline{2,1436}}$$

hinc $F = -0,84134 \cdot \alpha$

$G = -0,34286 \cdot \alpha$

fit nunc $m = \infty$ erit

$\lambda' = 1,25427$ et $\tau(1 + B)\sqrt{\lambda' - 1}$
$= 0,77434$, vnde radii facierum

$$F = \frac{\beta}{1,6888 \pm 0,7743} = \frac{\beta}{\overline{0,9145}}$$

$$G = \frac{\beta}{1,3284 \mp 0,7743} = \frac{\beta}{\overline{2,1027}}$$

hinc

hinc $F = -0,80373 \cdot \alpha$

$G = -0,34955 \cdot \alpha$

Ex his igitur duobus casibus pro multiplicatione quacunque concludimus

$$F = -0,80373 \cdot \alpha - \frac{f}{m}$$

$$F = -0,80373 \cdot \alpha - 0,940 \frac{\alpha}{m}$$

et $G = -0,34955\,\alpha - \frac{g}{m}$

$$G = -0,34955\,\alpha + 0,167 \cdot \frac{\alpha}{m}$$

Denique semidiameter campi visi erit

$$\Phi = \frac{2147}{m+1} \text{ minut.}$$

Scholion.

293. Quia ternae lentes priores communem postulant aperturam, cuius semidiameter sit $\frac{m}{50}$ dig. hic ad radium minimum istarum lentium, qui est $0,343 \cdot \alpha$; respici debet, cuius pars quarta $0,086 \cdot \alpha$ est circiter $\frac{1}{12}\alpha$ ipsi $\frac{m}{50}$ dig. aequalis posita dabit $\alpha = \frac{6}{25} m$ siue $\alpha = \frac{1}{4} m$ cum ante licuisset statuere $\alpha = \frac{1}{7} m$ neque ergo voti nostri compotes sumus facti, dum longitudinem telescopii contrahere sumus conati, etsi enim hic longitudo telescopii minorem tenet rationem ad α, tamen ipsa quantitas α fere tanto maior hic prodiit. Ex quo intelligitur, si omnes plane perfectiones desideremus, necesse prorsus esse, maiorem longitudinem admittere. Interim tamen longitudo hinc

hinc refultans aliquanto minor eft, quam fupra inuenta, fed probe hic eft perpendendum, hoc cafu elaborationem lentium multo maioribus difficultatibus effe obnoxiam, quam ante. Ita vt artifex non nifi poft plurima tentamina fcopum attingere poffit. Quocirca his inueftigationibus non vlterius immoror, cum ex calculis allatis facile fit huiusmodi telefcopiorum conftructionem in vfum artificum depromere.

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