CAPVT V.

DE VLTERIORE TELESCOPIORVM PRIMI GENERIS PERFECTIONE VNA PLVRIBVSVE LENTIBVS ADJICIENDIS.

Problema I.

152.

Si huiusmodi telescopium primi generis ex tribus lentibus a se inuicem separatis sit conficiendum, inuestigare momenta, quibus ei maximus perfectionis gradus conciliari queat.

Solutio.

Manentibus perpetuo omnibus elementis vti in principio sunt constituta, consideremus primo aequationem $m = \frac{\alpha}{b} \cdot \frac{\beta}{c}$ in qua ambae fractiones $\frac{\alpha}{b}$ et $\frac{\beta}{c}$ debent esse negatiuae ac praeterea interualla $\alpha + b$ ac $\beta + c$ positiua et quoniam nunc non debet esse $\frac{\alpha}{b} = -1$, ne binae priores lentes coalescant, statuamus $\frac{\alpha}{b} = -k$, vt fiat $m = -k \cdot \frac{\beta}{c}$ hincque $\beta = \frac{-mc}{k}$ siue $c = \frac{-\beta k}{m}$ et $\alpha = -bk$ vnde ob $\beta = B b$ omnes hae distantiae per α sequenti modo determinantur, $b = -\frac{\alpha}{k}$; $\beta = -\frac{B\alpha}{k}$ et et $c = + \frac{B\alpha}{m}$ existente $\gamma = \infty$. Hinc igitur esse oportebit $\alpha (1 - \frac{1}{k}) > 0$; $\alpha B (\frac{1}{m} - \frac{1}{k}) > 0$. seu, quia m et k sunt positiua $\alpha (k - 1) > 0$ et $\alpha B (k - m) > 0$ adeoque etiam $\frac{B(k-m)}{k-1}$ debet esse > 0, quocirca duo casus erunt perpendendi, *prior Casus*, quo α est quantitas positiua, tum debet esse $k > 1$; tum vero vel $k > m$ si B sit positiuum vel $k < m$, si $B < 0$. *Altero casu*, quo α est negatiuum, debet esse $k < 1$; tum vero vel $k > m$, si B sit negatiuum vel $k < m$, si B sit positiuum, vbi ob $m > 1$ illa conditio $k > m$ sponte cadit. His igitur praemissis primo ad nihilum redigamus formulam pro semidiametro confusionis supra datam:

$$0 = \mu\lambda + \frac{\mu' q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right) + \frac{\mu''\lambda''}{B^3 m}$$

siue ob $q = -\frac{\alpha\mathfrak{B}}{k}$ et $p = \alpha$

$$0 = \mu\lambda - \frac{\mu'}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right) + \frac{\mu''\lambda''}{B^3 m}$$

quae redit ad hanc formam:

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3 k} + \frac{\mu''\lambda''}{B^3 m} - \frac{\mu'\nu'}{\mathfrak{B}Bk}$$

Deinde vt margo coloratus tollatur ob $O = 0$ haec habetur aequatio

$$0 = \frac{dn}{n-1} B\pi' - \frac{dn'}{n'-1}\cdot\frac{1}{k}((B+1)\pi'' - \pi)$$

atque vt haec confusio penitus euertatur habetur ex §. 54.

$0 =$

$$o = \frac{dn}{n-1} \cdot \frac{1}{p} + \frac{dn'}{n'-1} \cdot \frac{1}{k^2} \cdot \frac{a}{q} + \frac{dn''}{n''-1} \cdot \frac{1}{m^2} \cdot \frac{a}{r}$$

Ad has aequationes resoluendas primo ratio inter π et π' debet definiri, id quod facillime praestabitur per formulas fundamentales in ipso initio propositas: $\frac{\pi}{\Phi} = \frac{\alpha + b}{q} = \frac{1-k}{\mathfrak{B}}$ et $\frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = \frac{B\alpha}{c} = m$ ex quibus colligitur $\pi = \frac{(1-k)\Phi}{\mathfrak{B}}$ et $\pi' = (m-1)\Phi + \pi = \frac{1-k+(m-1)\mathfrak{B}}{\mathfrak{B}}\Phi$ ita, vt sit $\pi : \pi' = 1-k : 1-k+(m-1)\mathfrak{B} = 1 : 1 + \frac{m-1}{1-k} \cdot \mathfrak{B}$. deinde breuitatis gratia statuamus

$$\frac{dn}{n-1} = N; \quad \frac{dn'}{n'-1} = N'; \quad \frac{dn''}{n''-1} = N''.$$

atque hinc IIda et IIItia aequatio transformabuntur in sequentes:

$$\text{II.} \quad o = N.B\left(1 - k + (m-1)\mathfrak{B}\right) - N'.\tfrac{1}{k}\left(B(1-k) + (B+1)(m-1)\mathfrak{B}\right)$$

siue

$$o = (m-1)N\mathfrak{B} + (1-k)N - \frac{m-k}{k}N'.$$

$$\text{III.} \quad o = N - \frac{N'}{k\mathfrak{B}} + \frac{N''}{mB}$$

Ex vtraque harum aequationum definiri potest valor ipsius $\mathfrak{B}$.

Ex IIda

$$\mathfrak{B} = \frac{m-k}{(m-1)k} \cdot \frac{N'}{N} - \frac{1-k}{m-1}$$

Ex IIItia vero sequitur

$$\mathfrak{B} = \frac{mN' - N''}{k(mN - N'')}$$

videa-

Videamus, an poſterior valor ipſius $\mathfrak{B} = \frac{mN' - kN''}{k(mN - N'')}$ cum conditione ante inuenta $\frac{B(k-m)}{k-1} > 0$ ſubſiſtere poſſit. Hunc in finem ob $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$ quaeramus $1 - \mathfrak{B}$ fietque $1 - \mathfrak{B} = \frac{m(kN - N')}{k(mN - N'')}$ eritque $B = \frac{mN' - kN''}{m(kN - N')}$ vnde conditio noſtra poſtulat, vt ſit $\frac{(mN' - kN'')(k-m)}{m(kN - N')(k-1)} > 0$ quae ſi eſſet $N = N' = N''$ abiret in hanc $\frac{-(k-m)^2}{m(k-1)^2} > 0$ quod eſt impoſſibile, eatenus igitur tantum haec conditio locum habere poterit, quatenus litterae N, N', N'' ſunt inaequales, id quod eueniet, ſi numerator prodeat poſitiuus, quod ſit, ſi vterque eius factor vel fiat poſitiuus vel vterque negatiuus; priori caſu $mN' - kN'' > 0$ adeoque $k < m \cdot \frac{N'}{N''}$ et $k > m$, quod fieri poteſt, ſi modo ſit $\frac{N'}{N''} > 1$ ſiue $N' > N''$.

Pro altero vero caſu, quo vterque factor eſt negatiuus, erit $k < m$ et $k > \frac{N'}{N''} \cdot m$, quod fieri poteſt, ſi modo ſit $\frac{N'}{N''} < 1$ ſeu $N' < N''$; vnde patet pro vtroque caſu litteras N' et N'' inaequales eſſe debere ſeu lentem IIdam et IIItiam ex diuerſis vitri ſpeciebus confici debere. In genere autem patet, k non multum ab m differre poſſe. Sequuntur haec: ſi numerator ſtatuatur poſitiuus; ſi vero numerator ſit negatiuus, etiam denominatorem oportet eſſe negatiuum, pro quo etiam duos caſus habemus. Pro priori caſu ſi ſit $k > 1$ debet eſſe $kN < N'$ adeoque $k < \frac{N'}{N}$ pro poſteriori ſi $k < 1$ debet ſimul eſſe $k > \frac{N'}{N}$, pro quorum vtroque prima et ſecunda lens debent eſſe ex diuerſo verſo vitro formatae. Verum ex his quatuor caſibus eum eligi conuenit, qui ambos valores pro $\mathfrak{B}$ inuentos proxime aequales reddat; denique autem poſtquam $\mathfrak{B}$ et k conuenienter definiuerimus, ex prima aequatione ſiue λ ſiue λ' quaeri debet, quia λ'' iam inde datur, quod lens ocularis debeat eſſe vtrinque aequaliter concaua.

Coroll. 1.

153. Quatuor illi caſus pro determinatione litterae k facile ad duas ſequentes conditiones reducuntur, nam

vel 1° k ſumi debet intra limites 1 et $\frac{N'}{N}$

vel 2° k ſumi debet intra limites m et $\frac{N''}{N'}$. m

ita vt numerus iſte k proxime vel vnitati vel multiplicationi m aequalis accipi debeat, quoniam fractiones $\frac{N'}{N}$ et $\frac{N''}{N'}$ parumper tantum ab vnitate differunt.

Coroll. 2.

154. Operae igitur pretium erit inueſtigare, caſus, quibus k ipſi alterutri limiti aequalis ſtatuitur

1°. Si $k = 1$. foret interuallum inter primam et ſecundam lentem $= 0$ et $\mathfrak{B} = \frac{mN' - N''}{mN - N''}$ et $B = \frac{mN' - N''}{m(N - N')}$ et inter 2 et 3tiam lentem $= B\alpha\left(\frac{1-m}{m}\right)$ vnde colligitur vtrum α poſitiuum an negatiuum ſumi debeat.

 2°. Si

2°. Si $k = \frac{N'}{N}$ fit interuallum inter primam et secundam lentem $= \frac{N' - N}{N}\alpha$ quod cum positivum esse debeat, patet, vtrum α positiue an negatiue sumi oporteat tum vero erit $\mathfrak{B} = 1$ et $B = \infty$ vnde $\beta + c$ seu distantia inter 2 et 3 lentem fieret $= \infty$.

3°. Si $k = m$ interuallum secundae et tertiae lentis euanescet fietque $\mathfrak{B} = \frac{N' - N''}{mN - N''}$; et $B = \frac{N' - N''}{mN - N'}$ interuallum vero inter 1 et 2 lentem $= \alpha(k - 1) = \alpha(m - 1)$, vbi manifesto α debet esse quantitas positiua.

4°. Si $k = \frac{N'}{N''} \cdot m$, fiet $\mathfrak{B} = 0$ et $B = 0$; vnde fieret distantia inter 2 et 3 lentem $= 0$.

Cum igitur neque lentium distantias nullas neque infinitas admitti conueniat numerum k nulli limitum prorsus aequalis sumi poterit.

Coroll. 3.

155. Quod porro ad campum apparentem attinet, qui pendet a formula $\pi' - \pi$ quia inuenimus $\pi : \pi' = 1 : 1 + \frac{m-1}{1-k}\mathfrak{B}$ erit pro memoratis quatuor casibus

1°. Si $k = 1$ erit $\pi : \pi' = 1 : \infty$ hinc $\pi = 0$; ita, vt pro campo apparente haberetur $\Phi = \frac{\pi'}{m-1}$

2°. Si

2°. Si $k = \frac{N'}{N}$; fiet $\pi : \pi' = 1 : 1 + \frac{mN - N}{N - N'} = 1 : \frac{mN - N'}{N - N'}$; hincque $\pi = \frac{N - N'}{mN - N'} . \pi'$ et $\pi' - \pi = \frac{N(m - 1)}{mN - N'} \pi'$ sicque pro campo apparente fiet $\Phi = \frac{N}{mN - N'} \pi'$. sicque Φ maius euadet, si $\frac{N}{mN - N'} > \frac{1}{m - 1}$ hoc est, si $N < N'$, quod ergo eueniet, si prima lens ex vitro coronario, secunda ex chrystallino paretur.

3°. Si $k = m$, erit $\pi : \pi' = 1 : \frac{mN - N'}{mN - N'}$, seu $\pi = \frac{mN - N'}{mN - N'} . \pi'$. Vnde colligitur campum apparentem maiorem fieri, quam in capite praecedente, si fuerit $\pi < 0$; quod cum hic fieri nequeat, in hoc casu campus maior non est exspectandus.

4°. Si $k = \frac{N'}{N''} m$ erit $\pi : \pi' = 1 : 1$, vnde $\pi = \pi'$ et $\pi' - \pi = 0$, quo ergo casu campus apparens plane euanesceret.

Coroll. 4.

156. Hinc ergo concludimus, vt maiorem campum obtineamus, quam ante, necessario requiri, vt sit $N' > N$ atque k capi debere intra limites 1 et $\frac{N'}{N}$, qui posterior limes cum sit vnitate maior, etiam k erit maius vnitate; ex quo sequitur distantiam α capi debere positiuam, quia $\alpha(k - 1) > 0$.

Corol-

Coroll. 5.

157. Cum igitur ob eam caussam potissimum plures lentes adhibeamus, vt maiorem campum obtineamus, ex pluribus illis casibus, prout litterae N, N′, N″ inter se variare possunt, hic vnicus nobis relinquitur, quo $N' > N$ atque k inter limites 1 et $\frac{N'}{N}$ sumitur.

Scholion. 1.

158. In his corollariis vsi sumus eo valore ipsius 𝔅, quem ex tertia aequatione deduximus. Supra autem iam obseruauimus, hanc aequationem ita esse comparatam, vt de ea nunquam omnino certi esse queamus; cum enim valores litterarum N, N′, N″ etc. ex nulla theoria adhuc definiri possint, sed tantum per experimenta, qualia a Dollondo sunt instituta, concludantur; quantacunque cura et sollertia in iis adhibeatur, nunquam tamen tantum praecisionis gradum sperare licet, vt non error satis notabilis sit pertimescendus; quam ob caussam etiam valor ipsius 𝔅 inde deductus pro vero haberi non poterit, sed contentos nos esse oportet, si modo hunc valorem propemodum cognouerimus; id quod ipsa etiam rei natura confirmatur, quia enim aequatio nostra tertia spatium diffusionis, per quod imagines diuersicolores sunt diffusae, prorsus ad nihilum redigit; facile intelligitur, ad praxin sufficere, dummodo hoc spatium reddatur satis exi-

exiguum, praecipue postquam id praestiterimus, vt margo coloratus dispareat inprimis igitur valor litterae $\mathfrak{B}$ ex secunda aequatione determinari debet, qui si ita fuerit comparatus, vt tantum praeterpropter tertiae aequationi satisfaciat, confusio inde oriunda eo magis negligi poterit, quod etiam in telescopiis ex vna vitri specie paratis non adeo nocere deprehenditur. Verum ex secunda aequatione valorem ipsius $\mathfrak{B}$ pro eo etiam casu definire licet, quo omnes lentes ex eadem vitri specie essent confectae, ita, vt foret $N = N' = N''$ tum enim concluderetur

$$\mathfrak{B} = \frac{m-k}{(m-1)k} - \frac{1-k}{m-1} = \frac{m-2k+k^2}{(m-1)k}$$

quo valore si velimus vti, vt conditio supra praescripta $B.\frac{k-m}{k-1} > 0$ adimpleatur, cum inde sit

$$1 - \mathfrak{B} = \frac{mk-m+k-k^2}{(m-1)k} = \frac{(k-1)(m-k)}{(m-1)k}$$

erit $B = \frac{m-2k+k^2}{(k-1)(m-k)}$ hincque conditio $\frac{-m+2k-k^2}{(k-1)^2} > 0$; in qua cum denominator certe sit positiuus, etiam numerator talis esse debet, adeoque $1-m-(k-1)^2 > 0$, quod fieri nequit. Ex quo perspicuum est, hoc casu marginem coloratum plane tolli non posse. Videamus igitur, si diuerso vitro vtamur, num hoc vitium effugere queamus. Hunc in finem ponamus breuitatis gratia $\frac{N'}{N} = \xi$ vt ξ sit numerus vnitatem vel tantillum superans vel ab ea deficiens, et cum sit

$$\mathfrak{B} = \frac{(m-k)\xi + k(k-1)}{(m-1)k} \text{ erit } B = \frac{(m-k)\xi + k(k-1)}{(m-k)(k-\xi)}$$

 vnde

vnde conditio noſtra poſtulat, vt ſit $\frac{(k-m)\xi - k(k-1)}{(k-1)(k-\xi)} > 0$. Hic duo caſus ſunt conſiderandi.

I°. Si denominator ſit poſitiuus, quod ſit vel ſi $k > \xi$ et $k > 1$ vel ſi $k < \xi$ et $k < 1$. Tum enim eſſe debet $(k - m)\xi - k(k - 1) > 0$

$$\text{ſiue } \frac{(1+\xi)^2}{4} - m\xi > (k - \tfrac{1}{2}(1 + \xi))^2$$

quod cum m notabiliter ſuperet vnitatem, ξ vero ab vnitate parum differat, manifeſto fieri nequit.

II°. Si denominator ſit negatiuus quod ſit, ſi k continetur intra limites ξ et 1. Tum vero numerator debet etiam eſſe negatiuus ſeu $(k-m)\xi - k(k-1) < 0$

$$\text{ſiue } \frac{(1+\xi)^2}{4} - m\xi < (k - \tfrac{1}{2}(1 + \xi))^2$$

quod ſponte euenit, cum pars prior manifeſto ſit negatiua. Hic igitur caſus, vt iam notauimus, ſolus eſt, qui attentionem meretur, cum hoc modo etiam tertiae aequationi ſaltim proxime ſatisfiat.

Scholion 2.

159. Quodſi ergo nobis propoſitum ſit, marginem coloratum tollere, quae proprietas potiſſimum deſiderari ſolet, primo tenendum eſt, hoc nullo modo lentibus ex vna vitri ſpecie factis praeſtari poſſe, ſed ſaltem primam et ſecundam lentem ex diuerſo vitro conſtare debere, ita, vt poſito $\frac{N'}{N} = \xi$ ſiue $N = 1$, $N' = \xi$, littera ξ ab vnitate differat, dum pro N'' ſiue

ſiue vnitas ſiue ξ pro lubitu accipi poterit, deinde vidimus, numerum k intra limites 1 et ξ ſumi debere, quo facto erit $B = \frac{(m-k)\xi + k(k-1)}{(m-k)(k-\xi)}$ et $\mathfrak{B} = \frac{(m-k)\xi + k(k-1)}{(m-1)k}$; vnde diſtantiae determinatrices erunt

$$b = -\frac{\alpha}{k}$$

$$\beta = \frac{-(m-k)\xi - k(k-1)}{k(m-k)(k-\xi)}\,\alpha$$

$$c = \frac{(m-k)\xi + k(k-1)}{m(m-k)(k-\xi)} \cdot \alpha$$

hincque lentium interualla

$$\alpha + b = \alpha\left(\frac{k-1}{k}\right)$$

$$\beta + c = \frac{-(m-k)\xi - k(k-1)}{(m-k)(k-\xi)} \cdot \frac{m-k}{mk} \cdot \alpha$$

$$= \frac{+(m-k)\xi + k(k-1)}{mk(\xi-k)} \cdot \alpha$$

hincque tota teleſcopii longitudo erit

$$= \frac{m-1}{m}\left(\frac{\xi-k+1}{\xi-k}\right)\alpha$$

Porro maxime intereſt in campum apparentem inquirere, quod fit determinando valorem $\pi = \frac{\pi'}{1 + \frac{m-1}{1-k}\mathfrak{B}}$, qui abit in ſequentem $\pi = \frac{k(1-k)}{(m-k)\xi} \cdot \pi'$. vnde adipiſcimur $\Phi = \frac{\pi'-\pi}{m-1} = \frac{\pi'}{m-1}\left(1 + \frac{k(k-1)}{(m-k)\xi}\right)$. Cum igitur maxime interſit, campum, quantum fieri poteſt, augeri, hinc obtinemus iſtam concluſionem, numerum k vnitate maiorem eſſe debere, vnde cum k contineatur intra limites 1 et ξ, haec porro regula obſeruetur, *litteram*

ξ *vnitate maiorem esse debere*; vnde sequitur, lentem secundam ex vitro chrystallino, primam vero ex communi esse parandam; quo pacto alter casus, quo fieret $\xi < 1$ penitus e praxi excluditur. Quare cum sit $k > 1$ distantia α, quae adhuc incerta est relicta, debet esse positiua.

Nunc demum consideremus aequationem tertiam, qua confusio colorum penitus tollitur, et videamus, quanta ea nunc sit proditura. Illa autem tertia aequatio nunc fit

$$0 = 1 - \frac{\xi}{k\mathfrak{B}} + \frac{N''}{mB}$$

quae nunc induet hanc formam:

$$0 = \frac{-m(k-1)(\xi-k) - (m-k)(\xi-k)N''}{m([m-1]\xi + k[k-1])}$$

quae quantitas vtique non erit aequalis nihilo, sed cum k, ξ et N'' parum ab vnitate differant, semper erit valde parua, id quod clarius inde perspicitur, quod numerator habeat factorem minimum $\xi - k$, denominator autem semper sit satis magnus eoque maior, quo maior fuerit multiplicatio. Ex quo manifestum est hanc confusionem nunquam fore perceptibilem. Praeterea autem cum haec confusio plane euanesceret, si caperetur $\xi = k$, consultum quidem videtur, numerum k limiti ξ propiorem capere, quam vnitati, quandoquidem ipsi limiti ξ aequari nequit, quia interuallum inter IIdam et IIItiam lentem fieret infinitum, vti et longitudo telescopii; quare ne ea nimis magna prodeat, contrarium potius suadendum est, vt littera k a limite ξ, quantum fieri potest remoueatur et vnitati propius capiatur. Consequenter vnicus casus, qui euolui meretur, in hoc consistet, vt numero k valor vnitati proximus assignetur et excessus tam sit exiguus, quam crassities lentium admittere solet. Si enim k ipsi vnitati aequaretur, haberemus casum praecedentis capitis, quo interuallum lentium plane nullum est positum, quod incommodum hic euitare constituimus.

Problema 2.

160. Si prima lens ex vitro coronario, secunda vero ex chrystallino paretur, et inter eas interuallum tam exiguum statuatur, quam crassities lentium admittit, regulas determinare, quas in constructione huius telescopii obseruare oportet.

Solutio.

Hic ergo ex Dollondi experimentis statui debebit $\xi = \frac{10}{7}$ et quia k limiti 1 propius accipi conuenit, quam alteri limiti $\frac{10}{7}$, sumamus $k = \frac{8}{7}$ et quae in praecedentibus scholiis sunt tradita, sequentes nobis suppeditant determinationes.

I. Pro distantiis determinatricibus.

$$b = -\tfrac{7}{8}\,\alpha; \quad \beta = \frac{35(7m-8)+28}{8(7m-8)}\,\alpha$$

$$\text{vel } \beta = \frac{7(35m-36)}{8(7m-8)}\cdot\alpha$$

$$c = \frac{-35m+36}{m(7m-8)}\cdot\alpha$$

II. Pro interuallis lentium.

$$\alpha + b = \tfrac{1}{8}\alpha$$

$$\mathfrak{B} + c = \frac{35m - 36}{8m}\cdot\alpha$$

et longitudo telescopii $= \frac{9(m-1)}{2m}\cdot\alpha$

Hic obseruandum est, cum α sit distantia focalis primae lentis eiusque semidiameter aperturae esse debeat $x = my = \frac{m}{50}$. dig. istam distantiam α minorem esse non posse, quam $5x$ seu $\frac{m}{10}$. dig. ita vt sit $\alpha > \frac{m}{10}$ dig. quare si capiatur verbi gratia $m = 50$, longitudo telescopii prodiret maior, quam $\frac{9\cdot 49}{20}$; maior quam 22 dig. et si fieri debeat $m = 100$, ea maior esse deberet, quam $\frac{9\cdot 99}{20}$. dig. maior quam 44 dig. quae distantia cum facile tolerari queat, manifestum est, haec telescopia etiam ad maiores multiplicationes adhiberi posse; pro minoribus autem multiplicationibus eximium certe vsum praestant, cum si statuatur $m = 5$, longitudo prodeat $> \frac{36}{20}$ dig. maior, quam $\frac{9}{5}$ dig., sumtoque $m = \frac{5}{2}$, ea prodeat $> \frac{27}{45}$ dig.

Pro campo autem apparente habebimus eius semidiametrum

$$\Phi = \frac{\pi'}{m-1}\left(1 + \frac{4}{5(7m-8)}\right)$$

ideoque aliquantum maior, quam casu praecedente, praesertim si multiplicatio fuerit exigua. Notentur etiam distantiae focales harum lentium p, q, r et cum sit

$\mathfrak{B} =$

$$\mathfrak{B} = \frac{35m - 36}{28(m-1)} \text{ et } B = \frac{35m - 36}{-7m + 8}$$

erit $p = \alpha$; $q = -\frac{(35m - 36)\alpha}{32(m-1)}$; $r = \frac{-(35m - 36)\alpha}{m(7m-8)}$

et ſemidiameter aperturae ſecundae lentis ex §. 23

$$= \frac{m(35m - 36).\pi'}{400(m-1)(7m-8)} + \frac{14m(m-1)}{25}$$

Denique pro conſtructione harum lentium numeri λ, λ' et λ'' ita accipi debent, vt ſatisfiat primae noſtrae aequationi, quae erat

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3 k} + \frac{\mu''\lambda''}{B^3.m} - \frac{\mu'\nu'}{\mathfrak{B}Bk}$$

vbi notandum eſt, vt lens ocularis vtrinque fiat aeque concaua, ſtatui debere $\lambda'' = 1.60006$. ſi haec lens ſit ex vitro coronario ideoque $\mu'' = \mu$ ſin autem ſit ex vitro chryſtallino ideoque $\mu'' = \mu'$, fore $\lambda'' = 1.67445$. haec autem aequatio non niſi caſibus particularibus pro data multiplicatione euolui poterit; vbi meminiſſe iuuabit, fore,

$\mu = 0.9875$; $\mu' = 0.8724$.

$\nu' = 0.2529$; $\mu'\nu' = 0,2206$.

Exempl. I.

161. Si multiplicatio ſit $m = \frac{5}{2}$, teleſcopium huius generis ex tribus lentibus conſtans deſcribere.

Cum ſit $m = \frac{5}{2}$, erunt diſtantiae determinatrices

$b = -\frac{7}{8}.\alpha$; $\mathfrak{C} = \frac{721}{152}.\alpha$

$c = -\frac{206}{95}.\alpha$. $B = -\frac{103}{19}$

et

et interualla lentium

$$\alpha + b = \tfrac{1}{8}\cdot\alpha;\ \mathfrak{b} + c = \tfrac{103}{40}\cdot\alpha$$

et longitudo telescopii $= \frac{27}{20}\cdot\alpha$ atque pro campo apparente fiet $\Phi = \frac{2\pi'}{3}\left(1 + \frac{8}{93}\right)$ sumto $\pi' = \frac{1}{4}$ et multiplicando per 3437 minut. erit angulus $\Phi = 10^\circ\, 21'\, 2''$

Cum nunc sit $B = \frac{-103}{19}$ erit $\mathfrak{B} = \frac{103}{84}$

et habebimus

$$\text{Log.}\,(-B) = 0.7340836\ (-)$$

$$\text{et Log. } \mathfrak{B} = 0.0885580$$

aequatio autem pro confusione prima tollenda, si lentem ocularem ex vitro coronario faciamus, vt sit

$$\mu'' = \mu \text{ et } \lambda'' = 1.60006, \text{ erit}$$

$$0 = 0.9875\cdot\lambda - 0.4140\ \lambda' - 0.00396$$
$$+ 0.02903$$

$$0 = 0.9875\cdot\lambda - 0.4140\ \lambda' + 0.02507$$

vnde quaeratur λ', et habebitur

$$\lambda' = 2,3852\cdot\lambda + 0.06055$$

Si ergo hic capiatur $\lambda = 1$, fiet

$$\lambda' = 2,4457$$

vnde fit $\lambda' - 1 = 1,4457$

et log. $\sqrt{(\lambda' - 1)} = 0.0800391$

vnde constructio singularum lentium sequenti modo se ſe habebit, ſiquidem radii facierum primae lentis ſint F et G; ſecundae F' et G' et tertiae F'' et G''.

I. Pro prima lente ex vitro coronario.

$$F = \frac{\alpha}{\sigma} = 0.6023.\ \alpha$$

$$G = \frac{\alpha}{\varrho} = 4.4131.\ \alpha$$

II. Pro ſecunda lente ex vitro chryſtallino.

$$\frac{1}{F'} = \frac{\varrho\beta + \sigma b \mp \tau.(b+\beta)\sqrt{\lambda' - 1}}{b\beta}$$

$$\frac{1}{G'} = \frac{\sigma\beta + \varrho b \pm \tau(b+\beta)\sqrt{\lambda' - 1}}{b\beta}$$

cum nunc ſit $\log. \frac{b}{\alpha} = \log. (-\frac{7}{8}) = 9.9420080(-)$

et $\log. \frac{\beta}{\alpha} = 0.6760917.$

et $\log. \frac{(b+\beta)}{\alpha} = \log. \frac{147}{38} = 0.5875336.$

$\log. \sigma = 0.1993986.$

$\log. \varrho = 9.1501422.$

$\log. \tau = 9.9432471.$

Vnde inuenitur

$$\frac{1}{F'} = \frac{-0.7147 \mp 4.0815}{b\beta}.\ \alpha$$

$$\frac{1}{G'} = \frac{+7.3838 \pm 4.0815}{b\beta}.\ \alpha$$

Vt maiores numeri euitentur, ſumantur ſigna inferiora, fietque

$$F' = \frac{b\beta}{3.3668} = -1,2327.\ \alpha$$

$$G' = \frac{b\beta}{3.3023} = -1,2568.\ \alpha$$

Pro lente oculari ex vitro coronario paranda, cum ea vtrinque ſit aeque concaua, eiusque diſtantia focalis ſit $c = -\frac{206}{95} \cdot \alpha$, radius concauitatis pro vtraque facie erit $= 2(n-1)c = -\frac{106 \cdot 206}{95} \alpha = -2,2985 \cdot \alpha$.

Prima lens admittit aperturam, cuius ſemidiameter $x = 0,1506 \cdot \alpha$. Nunc vero claritas poſtulat, vt ſit $x = \frac{m}{50} \cdot$ dig. $= \frac{1}{20}$. dig. vnde α maius, quam $\frac{1}{3}$ dig. Sumatur ergo $\alpha = \frac{1}{2}$ dig. et conſtructio teleſcopii ita ſe habebit:

I. Pro lente prima

rad. faciei $\begin{cases} \text{anter.} = +0,3012. \text{ dig.} \\ \text{poſter.} = +2,2065. \text{ dig.} \end{cases}$ Crown Glaſſ.

II. Pro lente ſecunda

rad. faciei $\begin{cases} \text{anter.} = -0,6163. \text{ dig.} \\ \text{poſter.} = -0,6284. \text{ dig.} \end{cases}$ Flint Glaſſ.

III. Pro lente tertia

radius vtriusque faciei $= -1,1492$. dig.
quae paratur ex Crown Glaſſ.

Tum interuallum ſtatuatur

Inter

I. et II. $= \frac{1}{16}$. dig. $= 0,0625$. dig.

II. et III $= \frac{103}{80}$ dig. $= 1,2875$. dig.

ita, vt tota teleſcopii longitudo ſit futura
$= 1,3500$. dig. $= 1\frac{1}{3}$ dig.

ſpatii vero viſi ſemidiameter erit $= 10^\circ. 21' 2''$.

Exem-

Exemplum II.

162. Si multiplicatio $m = 5$. telescopium huius generis ex tribus lentibus constans describere.

Cum sit $m = 5$, erit $7m - 8 = 27$ et $35m - 36 = 139$, vnde distantiae determinatrices fient

$$b = -\tfrac{7}{8}.\alpha = -0,8750.\alpha$$

$$\beta = \tfrac{973}{216}\alpha = +4,5046.\alpha$$

$$c = \tfrac{-139}{5.27}\alpha = -1,0296.\alpha$$

Ex quibus fiunt interualla

$$\alpha + b = \tfrac{1}{8}\alpha;\ \beta + c = 3,4750.\alpha$$

vnde telescopii longitudo $= 3,6.\alpha$.

Pro campo autem apparente fiet $\Phi = \frac{\pi'}{4}\left(1 + \frac{4}{5.27}\right)$ sumtoque $\pi' = \frac{1}{4}$ et multiplicando per 3437 min. erit $\Phi = 3^\circ\,41'$.

Cum iam sit $\mathfrak{B} = \frac{139}{112}$ et $B = \frac{-139}{27}$, sumtisque logarithmis

$$\text{Log. } \mathfrak{B} = 0.0937968$$

$$\text{Log. } B = 0.7116510\ (-)$$

et aequatio pro confusione prima tollenda, si lentem ocularem ex vitro coronario paremus, vt sit $\mu'' = \mu$ et $\lambda'' = 1.60006$, erit

$$0 = 0.9875.\lambda - 0.39933.\lambda' - 0.002316$$
$$+ 0.030211$$
$$\overline{+ 0.027895}$$

ex qua iterum quaeratur

$$\lambda' = 2,4729\,\lambda + 0.06985.$$

Hic non, vt ante sumamus $\lambda = 1$ sed, vt prima lens maximae aperturae fiat capax, ideoque distantia α minor accipi possit, capiatur $\lambda = 1,60006$, vt haec lens fiat vtrinque aequaliter conuexa, habebiturque

$$\lambda' = 4.0266 \text{ et } \lambda' - 1 = 3.0266.$$

$$\text{et Log. } \sqrt{(\lambda' - 1)} = 0.2404775.$$

atque hinc obtinebimus:

I. Pro prima lente ex vitro coronario.

radius vtriusque faciei $= 2(n-1).\alpha = 1,06.\alpha$; quae aperturam admittit, cuius semidiameter $x = 0,26.\alpha$.

II. Pro secunda lente ex vitro chrystallino.

$$\text{ob Log. } \frac{(b+\beta)}{\alpha} = 0.5598588.$$

calculus ita se habebit:

$$\frac{1}{F'} = \frac{-0,7479 \pm 0,5409}{b\beta}.\alpha$$

$$\frac{1}{G'} = \frac{+7.0057 \mp 5,5400}{b\beta}.\alpha$$

valeant hic signa superiora eritque

$$F' = \frac{b\beta}{4,7937\alpha} = -0.8223.\alpha$$

$$G' = \frac{b\beta}{1,4648\alpha} = -2.6908.\alpha$$

III. Pro

III. Pro tertia lente ex vitro coronario.

erit radius vtriusque faciei $= 2(n-1)c = 1,06\, c$ $= -1.0920.\ \alpha.$

Cum nunc ob claritatem esse debeat $x = \frac{m}{50}$ dig. $= \frac{1}{10}$ dig. fiet α maius, quam $\frac{2}{5}$ dig.

Sumi igitur poterit $\alpha = \frac{1}{2}$ dig. et constructio telescopii ita se habebit.

I. Pro lente prima Crown Glass.

rad. faciei vtriusque $= 0,5300$. dig.

II. Pro lente secunda Flint Glass.

radius faciei $\begin{cases} \text{anter.} = -0.4111. \text{ dig.} \\ \text{poster.} = -1.3454. \text{ dig.} \end{cases}$

III. Pro lente tertia Crown Glass.

rad. vtriusque faciei $= -0,5460$. dig.

Tum vero statuatur interuallum

I. et II. $= \frac{1}{10}$ dig.

II. et III. $= 1,7375$. dig.

ita, vt tota longitudo sit $= 1,8$. dig. ideoque nondum duos adaequet digitos.

Spatii tandem visi semidiameter erit $= 3^\circ\, 41'$.

Corollarium.

163. Telefcopia igitur in his duobus exemplis conftructa aptiffima videntur ad vfum vulgarem quoniam ea facile quis fecum gerere poteft iisque in fpectaculis praefertim vti. Sequentia autem exempla ad maiores multiplicationes accommodemus.

Exempl. III.

164. Sit multiplicatio $m = 25$, telefcopium huius generis tribus lentibus conftans defcribere.

Cum fit $m = 25$, erit $7m - 8 = 167$ et $35m - 36 = 839$, eruntque diftantiae determinatrices

$$b = -\tfrac{7}{8}.\alpha = -0,875.\alpha;$$

$$\beta = \tfrac{7.839}{8.167}.\alpha = 4,3960.\alpha;$$

$$\text{Log.}\ \tfrac{-b}{\alpha} = 9.9420081\ (-)$$

$$\text{Log.}\ \tfrac{\beta}{\alpha} = 0.6430535.$$

$$c = -\tfrac{839.\alpha}{35.167} = -0.2009.\alpha$$

$$b + \beta = 3.5210.\alpha$$

$$\text{Log.}\ (b + \beta) = 0.5466660.$$

Hinc prodeunt lentium interualla

$$\alpha + b = \tfrac{1}{8}\alpha;\ \beta + c = 4.1951.\alpha$$

et tota longitudo $= 4,3201.\alpha$

Pro

Pro campo autem apparente fiet $\Phi = \frac{\pi'}{24}(1 + \frac{4}{5,1057})$ hincque angulus $\Phi = 36$ min. prim. circiter.

Cum iam porro fit $\mathfrak{B} = \frac{839}{26,24}$

et Log. $\mathfrak{B} = 0.0963927$

Log. $-B = 0.7010454(-)$

peruenietur ad fequentem aequationem

$$0 = 0,9875\lambda - 0.3922\lambda' - 0,0004984 + 0.03077$$

feu $$0 = 0.9875\lambda - 0.3922\lambda' + 0,03028.$$

Quoniam vidimus, valorem $\lambda = 1,60006$ longitudinem telefcopii haud mediocriter diminuiffe, ftatim ponamus $\lambda = 1,60006$ eritque

$$0 = 1,6103 - 0,3922\lambda'.$$

vnde prodit

$$\lambda' = 4,1057; \text{ et } \lambda' - 1 = 3.1057$$

$$\text{et } \log. \sqrt{(\lambda' - 1)} = 0,2460797.$$

vnde conftructio fingularum lentium ita fe habebit:

I. Pro prima lente ex vitro coronario radius vtriusque faciei $= 1,06\,\alpha$ quae ergo aperturam admittit, cuius femidiameter $x = 0,265.\,\alpha$.

II. Pro fecunda lente

calculus ita fe habebit

$$\frac{1}{F'} = \frac{-7,7633 \pm 5,1449}{b\beta}\alpha.$$

$$\frac{1}{G'} = \frac{6,8738 \mp 5,1449}{b\beta}\alpha.$$

Valeant

Valeant ſigna ſuperiora, eritque

$F' = \frac{b\beta}{1,7816\alpha} = -0,8216\,\alpha$

$G' = \frac{b\beta}{1,3889.\alpha} = -2,7694.\alpha$

III. Pro tertia lente ex vitro coronario.

radius vtriusque faciei $= 1{,}06 . c = -0{,}21295 . \alpha$

Claritas autem poſtulat $= x = \frac{m}{50}$ dig. $= \frac{1}{2}$ dig. vnde concluditur $\alpha > 1,88$ ſumatur ergo $\alpha = 2$ et conſtructio haec erit

I. Pro lente prima

rad. vtriusque faciei $= 2, 12$ dig. Crown Glaſſ.

II. Pro lente ſecunda

rad. faciei $\begin{cases} \text{anter.} = -1,6432 \text{ dig.} \\ \text{poſter.} = -5,5388 \text{ dig.} \end{cases}$ Flint Glaſſ.

III. Pro lente tertia

radius vtriusque faciei $= -0, 42590$ dig. Crown Gl.

Tum ſtatuatur interuallum lentium

I. et II. $= \frac{1}{4}$ dig.

II. et III. $= 8. 3902.$ dig.

et tota longitudo $= 8. 64.$ dig. campique apparentis ſemidiameter $x = 36'$. circiter.

Exem-

Exempl. IV.

165. Si m debeat esse $= 50$, erit $7m - 8 = 342$; $35m - 36 = 1714$ adeoque distantiae

$$b = -\tfrac{7}{8}\alpha = -0,875 \cdot \alpha$$

$$\beta = 4,3852 \cdot \alpha;\ c = -0,10023 \cdot \alpha$$

$$\log. \tfrac{\beta}{\alpha} = 0.6419928$$

$$\log. \tfrac{-b}{\alpha} = 9.9420081\ (-)$$

$$\log. \tfrac{(b+\beta)}{\alpha} = 0.5453319.$$

$$\log. \tfrac{b\beta}{\alpha^2} = 0.5840009\ (-)$$

Tum vero interualla lentium erunt

$$\alpha + b = \tfrac{1}{8}\alpha = 0,125\ \alpha$$

$\beta + c = 4,2850 \cdot \alpha$ hincque

tota longitudo $= 4,4100\ \alpha$.

Porro reperitur

Log. $-$ B $= 0,6999847$.

Log. $\mathfrak{B} = 0,0966567$.

Pro campo apparente reperitur $\Phi = \frac{\pi'}{45}\left(1 + \frac{4}{5.342}\right)$ seu angulus $\Phi = 17\tfrac{1}{2}$ minut.

Pro confusione tollenda statuatur statim in aequatione inuenta $\lambda = 1,60006$ eritque

$$0 = 1,5801 - 0.3915 \cdot \lambda' - 0,00025 + 0,03083$$

fiue $0,3915\,\lambda' = 1,6107$

vnde $\lambda' = 4,1141$; hinc $\lambda' - 1 = 3.1141$ et

log. $\sqrt{(\lambda' - 1)} = 0.2466663$

vnde constructio singularum lentium ita se habebit.

I. Pro lente prima

radius vtriusque faciei $= 1,06\,\alpha$ quae ergo aperturam admittit, cuius semidiameter $= 0,265.\,\alpha$.

II. Pro lente secunda

$$\frac{1}{F'} = \frac{-0.7653 \pm 5.4355}{b\beta}\alpha$$

$$\frac{1}{G'} = \frac{+6.8169 \mp 5.4355}{b\beta}\alpha$$

Valeant ergo signa superiora eritque

$$F' = \frac{b\beta}{4.6702\,\alpha} = -0,8216.\,\alpha$$

$$G' = \frac{b\beta}{1.3814.\,\alpha} = -2,7776.\,\alpha$$

III. Pro lente tertia

erit radius vtriusque faciei $=$

$$2(n-1)c = 1,06.\,c = -0,10624.\,\alpha$$

Claritas autem postulat, $x = \frac{m}{55} = 1$ dig. vnde sequitur $\alpha > 3,8$, sumto ergo $\alpha = 4$, habebitur sequens telescopii constructio.

I. Pro lente prima: Crown Gl.

radius vtriusque faciei $= 4,24$. dig.

II. Pro

II. Pro lente ſecunda

radius faciei $\left\{ \begin{array}{l} \text{anter.} = -3,2864 \\ \text{poſter.} = -11,1104 \end{array} \right\}$ Flint Glaſſ.

III. Pro lente tertia

radius vtriusque faciei $= -0,42496$ Crown Glaſſ.

Tum vero ſtatui debet interuallum lentium

I. et II. $= 0,5$ dig.

II. et III. $= 17,1400$ dig.

adeoque teleſcopii longitudo $= 17,6400$ dig.

Campi denique viſi ſemidiameter inuentus eſt $17\frac{1}{2}$ min.

Scholion.

166. Cum in his ſolutionibus littera λ indeterminata ſit relicta, in tribus poſterioribus exemplis eius loco non vnitatem poſuimus, vt ante fecimus, ſed potius ei tribuimus illum valorem, quo ambae eius facies inter ſe aequales redderentur hocque modo inſigne commodum ſumus nacti, vt lens prima fere duplo maiorem aperturam admitteret hincque diſtantia α fere ad dimidium reduci poſſet. Vt autem in genere quaepiam lens cuius diſtantiae determinatrices ſunt a et α ambas ſuas facies obtineat aequales, ſupra vidimus, capi debere $\sqrt{(\lambda - 1)} = \frac{(\sigma - \varrho)(a - \alpha)}{2\tau(a + \alpha)} = \frac{2(nn - 1)}{n \cdot \sqrt{(4n - 1)}} \cdot \left(\frac{1 - A}{1 + A}\right)$ ob

ob $\alpha = A a$, vnde fit $\lambda = 1 + \frac{4(nn-1)^2}{n^2(4n-1)} \cdot \frac{(1-A)^2}{(1+A)^2}$ quare si vel a vel α fuerit infinitum, vti fit tam in lente obiectiua, quam in lente oculari, habebitur $\lambda = 1 + \frac{4(nn-1)^2}{n^2(4n-1)}$. Sin autem velimus, vt alia quaepiam lens obtineat ambas suas facies inter se aequales; tum ob $\frac{(1-A)^2}{(1+A)^2} = 1 - \frac{4A}{(1+A)^2}$ capere debemus $\lambda = 1 + \frac{4(nn-1)^2}{n^2(4n-1)} - \frac{16(nn-1)^2 \cdot A}{n^2(4n-1)(1+A)^2}$. Cum autem in nostra expressione pro semidiametro confusionis tum occurrat talis forma $\lambda(A+1)^2 + \nu A$, valor istius formulae fiet $= (A+1)^2 + \frac{4(nn-1)^2(A+1)^2}{n^2(4n-1)} - \frac{16(nn-1)^2 \cdot A}{nn(4n-1)} + \frac{4(n-1)^2 \cdot A}{4n-1}$.

Commodius autem erit, hoc casu valorem ipsius λ pro facilitate calculi ita exprimere $\lambda = 1 + \frac{(\sigma-\rho)^2(1-A)^2}{4\tau^2(1+A)^2}$.

Exempl. V.

167. Si multiplicatio m debeat esse valde magna vel saltim maior, quam 25, huius generis telescopia ex tribus lentibus constantia describere. Hic statim obseruo, sumta prima lente vtrinque aequaliter conuexa, fore radium vtriusque curuaturae, vt ante, $= 1,06. \alpha$, quae admittet aperturam, cuius semidiam. $= \frac{1}{4}. \alpha = x$, cum autem ob claritatem sumi debeat $x = \frac{m}{50}$. dig. hinc intelligimus, semper statui posse $\alpha = \frac{2m}{25} = 0,08. m$. dig. et pro campo apparente $\Phi = \frac{\pi'}{m-1}$; sumtoque $\pi' = \frac{1}{4}$, erit $\Phi = \frac{859}{m-1}$. min.

Nunc

Nunc autem ante, quam reliquas partes constructionis definiamus, contemplemur casum, quo $m = \infty$ eritque

$$b = -\tfrac{7}{8}\alpha;\ \mathfrak{C} = \tfrac{35}{8}.\alpha;\ c = \tfrac{-5}{m}.\alpha; = -\tfrac{2}{5}.\text{dig.}$$

Distantiae porro lentium $\alpha + \mathfrak{C} = \frac{1}{8}\alpha.$

et $\mathfrak{C} + c = \left(\frac{35.\alpha}{8} - \frac{2}{5}\right)$ dig.

et $\mathfrak{B} = \frac{5}{4};\ \mathrm{B} = -5.$

Pro sequente calculo statim sumamus $\lambda = 1,60006.$ et aequatio prodibit

$$0 = 1,5801 - 0.3908.\ \lambda' + 0.03088$$

vnde inuenitur

$$\lambda' = 4.1220.\ \text{et}\ \lambda' - 1 = 3.1220.$$

$$\text{et log. } \sqrt{(\lambda' - 1)} = 0.2472164$$

Hinc ob $\text{Log.} - \frac{b}{\alpha} = 9.9420081\ (-)$

$$\text{Log.} \frac{\beta}{\alpha} = 0.6409781.$$

$$\text{Log.} \frac{b+\beta}{\alpha} = 0.5440680.$$

$$\text{Log.} \frac{b\beta}{\alpha^2} = 0.5829862 -$$

Ex quibus pro secunda lente habebimus

$$\frac{1}{F'} = \frac{-0.7662 \pm 5.4266}{b\beta}.\alpha$$

$$\frac{1}{G'} = \frac{+6.8006 \mp 5.4266}{b\beta}.\alpha$$

seu sumtis signis superioribus

$$F' = \frac{b\beta}{4{,}0004.\alpha} = -0.8214.\alpha$$

$$G' = \frac{b\beta}{1{,}3740.\alpha} = -2.7861.\alpha$$

qui valores pro multiplicatione infinita locum habent; at nunc pro multiplicatione quacunque m statuatur

$$F' = -(0.8214 + \tfrac{f}{m})\alpha$$

$$G' = -(2{,}7861 + \tfrac{g}{m})\alpha$$

vbi valores litterarum f et g ex casu praecedente $m = 50$ vel etiam, sed minus tuto, ex casu $m = 25$ erui debent, hocque modo reperitur $f = 0.01$ et $g = -0.4250$ ita, vt sit in genere

$$F' = -(0{,}8214 + \tfrac{0.01}{m})\alpha$$

$$G' = -(2{,}7861 - \tfrac{0.4250}{m})\alpha$$

Deinde cum supra iam inuenta sit distantia focalis lentis tertiae $= -\frac{2}{5}$ dig. pro $m = \infty$, statuamus pro quauis multiplicatione m esse $c = -\frac{2}{5} - \frac{b}{m}$ eritque

$$c = -(\tfrac{2}{5} + \tfrac{1{,}2480}{m})\ \text{dig.}$$

cuius ergo radius vtriusque faciei erit

$$-(0.4240 + \tfrac{1{,}3228}{m})\ \text{dig.}$$

Cum igitur sit $\alpha = 0.08\, m.$ dig.

Con-

Conſtructio teleſcopii ſequenti modo ſe habebit

I. Pro lente prima Crown Glaſſ.

rad. faciei vtriusque $= 0.0848 . m.$ dig.

II. Pro lente ſecunda Flint Glaſſ.

rad. faciei $\begin{cases} \text{anter.} = -(0.0657 . m + 0.0008) \text{ dig.} \\ \text{poſter.} = -(0.2228 . m - 0.0340) \text{ dig.} \end{cases}$

III. Pro lente tertia Crown Glaſſ.

radius vtriusque faciei $= -(0.4240 + \frac{1.7228}{m})$

Tum vero interualla erunt

$\alpha + b = 0.01 . m;$

$\beta + c = (0,35\, m - 0.36)$ dig.

hincque tota longitudo

$= (0,36\, m - 0.36)$ dig.

campique viſi ſemidiameter $= \frac{859}{m-1}$. minut. prim.

Corollarium.

168. Si ergo teleſcopium deſideretur, quod centies multiplicet, id ita ſe habebit

I. Pro lente prima. Crown Glaſſ.

radius vtriusque faciei $= 8,48$. dig.

II. Pro

II. Pro lente secunda. Flint Glass.

rad. faciei $\begin{cases} \text{anter.} = -6.57. \text{ dig.} \\ \text{poster.} = -22,24 \text{ dig.} \end{cases}$

III. Pro lente tertia.

radius vtriusque faciei $= -0.43$. dig.

Interuallum erit lentis

I. et II. $= 1$. dig.

II. et III. $= 34,64$. dig.

hincque longitudo telescopii

$= 35,64$. dig.

campique visi semidiameter

$= 8\frac{1}{2}$ min.

Problema 3.

169. Si huiusmodi telescopium primi generis ex quatuor lentibus a se inuicem separatis sit construendum, inuestigare momenta, quibus ei maximus perfectionis gradus conciliatur.

Solutio.

Hic igitur istarum trium fractionum $\frac{\alpha}{b}$; $\frac{\beta}{c}$; et $\frac{\gamma}{d}$ singulae debent esse negatiuae; ponamus ergo $\frac{\alpha}{b} = -k$ et $\frac{\beta}{c} = -k'$. et cum sit $m = -\frac{\alpha}{b} \cdot \frac{\beta}{c} \cdot \frac{\gamma}{d}$ habebimus

$b =$

$b = -\frac{\alpha}{k}$; $\beta = -\frac{B\alpha}{k}$; $c = -\frac{\beta}{k'} = +\frac{B\alpha}{k.k'}$ et $\gamma = +\frac{BC\alpha}{k.k'}$ et $m = \frac{-k.k'.\gamma}{d}$ hinc $d = -\frac{BC\alpha}{m}$; vnde interualla lentium $\alpha + b = \alpha\left(1 - \frac{1}{k}\right)$ $\beta + c = B\alpha\left(\frac{1}{kk'} - \frac{1}{k}\right)$ et $\gamma + d = BC\alpha\left(\frac{1}{kk'} - \frac{1}{m}\right)$ quae cum debeant esse positiua aeque ac numeri k, k' et m, bina posteriora per primum diuisa dabunt has duas conditiones

1°. $\frac{B(1-k')}{k'(k-1)} > 0.$

2°. $\frac{BC(m-kk')}{mk'(k-1)} > 0.$

Iam consideremus aequationem, qua margo coloratus tollitur, pro casu, quo distantia O est negatiua: ponendo, vt ante $\frac{dn}{n-1} = N$; $\frac{dn'}{n'-1} = N'$, $\frac{dn''}{n''-1} = N''$. $\frac{dn'''}{n'''-1} = N'''$ eritque

$$0 = N.\,BC\pi''.\,\alpha + N'.\,b\left((B+1)C\pi'' - \pi\right) + N''.\,c\left(\frac{(C+1)\pi'' - \pi'}{B}\right)$$

seu $$0 = NBC\pi'' - \frac{N'}{k}\left((B+1)C\pi'' - \pi\right) + \frac{N''}{k.k'}\left((C+1)\pi'' - \pi'\right)$$

quem in finem inuestigare oportet relationes inter litteras π, π', π'', est vero ex capite I.

Imo. $\frac{\mathfrak{B}\pi - \Phi}{\Phi} = -k$

vnde $\pi = \frac{1-k}{\mathfrak{B}}.\ \Phi.$

IIdo. $\frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = \frac{B\alpha}{c} = kk'$

vnde $\pi' = \left(\frac{1}{\mathfrak{B}} - \frac{k}{\mathfrak{B}} + kk'\right)\frac{\Phi}{\mathfrak{C}}.$

IIItio. $\frac{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}{\Phi} = \frac{BC\alpha}{d} = -m;$

vnde ob $\mathfrak{D} = 1$ fiet

$$\pi'' = \left(-m + \frac{1}{BC} - \frac{k}{\mathfrak{B}C} + \frac{k.k'}{\mathfrak{C}}\right)\Phi$$

vnde aequatio noſtra erit

$$0 = N\left(-BCm + 1 - \frac{Bk}{\mathfrak{B}} + \frac{BC.k.k'}{\mathfrak{C}}\right)$$

$$-\frac{N'}{k}\left(\frac{-BCm}{\mathfrak{B}} - \frac{Bk}{\mathfrak{B}} + \frac{BC.kk'}{\mathfrak{B}\mathfrak{C}}\right)$$

$$+\frac{CN''}{\mathfrak{C}kk'}(-m + kk');$$ vnde fit

$$C = N - N(B+1)k + NBkk' + N'(B+1) - N'(B+1)k'$$

$$-\frac{N''.m}{kk'} + N''$$

diuiſum per

$$NBm - NBkk' - \frac{N'(B+1)m}{k} + N'(B+1)k'$$

$$+\frac{N''m}{kk'} - N''$$

vel ſuccinctius

$$C = Nkk'(1 - k - Bk(1 - k')) + N'kk'(B+1)(1 - k')$$

$$-N''(m - kk')$$

diuiſum per

$$(m - kk')(Nkk'B - N'k'(B+1) + N'')$$

adeoque

$$1 + C = Nkk'(1 - k + B(m - k))$$

$$-N'(B+1)k'(m - k)$$

diui-

diuisum per

$$(m - k k')(N k k' B - N' k' (B + 1) + N'')$$

atque hinc

$$\mathfrak{C} = N k k' (1 - k - B k (1 - k')) + N' k k' (B + 1)(1 - k') - N'' (m - k k')$$

diuisum per

$$N k k' (1 - k + B (m - k)) - N' (B + 1) k' (m - k)$$

sed facile patet, hoc modo nobis vix vlterius progredi licere ob harum formularum complicationem, nisi pro k et k' et pro N, N', N'' valores substituantur interim tamen haec methodus etiam successura videtur, si eam ad plures adhuc lentes applicare vellemus, ceterum haud abs re erit hoc negotium etiam alio modo tentasse.

Ex praecedentibus scilicet aequationibus non litteras π, π', π'' quaeri, sed potius his quasi datis spectatis litteras $\mathfrak{B}$ et $\mathfrak{C}$ definiri conueniet; vnde statim obtinemus

$$\mathfrak{B} = \frac{1 - k}{\pi} \cdot \Phi; \quad \mathfrak{C} = \frac{(k k' - 1)\Phi + \pi}{\pi'}$$

ex tertia denique aequatione ob $\mathfrak{D} = 1$ colligitur

$$\Phi = \frac{-\pi + \pi' - \pi''}{m - 1};$$

ita, vt et Φ quasi datum spectari queat. Hinc cum sit

B =

$$B = \frac{\mathfrak{B}}{1-\mathfrak{B}} \text{ et } C = \frac{\mathfrak{C}}{1-\mathfrak{C}}$$

habebimus

$$B = \frac{(1-k)\Phi}{\pi-(1-k)\Phi}\,;\; C = \frac{(kk'-1)\Phi+\pi}{\pi'-\pi-(kk'-1)\Phi}.$$

Nunc cum prior conditio poſtulet, vt ſit $\frac{B(1-k')}{k'(k-1)} > 0$; altera vero per hanc diuiſa $\frac{C(m-kk')}{1-k'} > 0$ valoribus illis ſubſtitutis hae duae conditiones abibunt in ſequentes:

$$1^\circ.\; \frac{(k'-1)\Phi}{(\pi-(1-k)\Phi)k'} > 0$$

ſeu $\frac{(k'-1)\Phi}{\pi-(1-k)\Phi} > 0$

$$2^\circ.\; \frac{((kk'-1)\Phi+\pi)(m-kk')}{(1-k')(\pi'-\pi-(kk'-1)\Phi)} > 0$$

quae per illam multiplicata dat

$$\frac{-\Phi(m-kk')(\pi+(kk'-1)\Phi)}{(\pi-(1-k)\Phi)(\pi'-\pi-(kk'-1)\Phi)} > 0$$

ſi hic loco Φ eius valor ſubſtituatur, qui cum ſemper ſit poſitiuus ob $m-1$ etiam poſitiuum, dat primo

$$-\pi+\pi'-\pi'' > 0;$$

tum vero binae iſtae conditiones dabunt

$$1^\circ.\; \frac{k'-1}{(m-k)\pi+(k-1)\pi'-(k-1)\pi''} > 0$$

$$2^\circ.\; \frac{(m-kk')((m-kk')\pi+(kk'-1)\pi'-(kk'-1)\pi''}{(k'-1)((m-kk')\pi-(m-kk')\pi'-(kk'-1)\pi''} > 0$$

tum vero B et C ita definientur:

$$B = \frac{(k-1)(\pi-\pi'+\pi'')}{(m-k)\pi+(k-1)\pi'-(k-1)\pi''}$$

$$C = \frac{(m-kk')\pi+(kk'-1)\pi'-(kk'-1)\pi''}{-(m-kk')\pi+(m-kk')\pi'+(kk'-1)\pi''}$$

Verum

Verum ſi hos valores ſubſtituere vellemus ſiue in aequatione pro margine colorato vitando ſiue in primis pro ſemidiametro confuſionis ad nihilum redigendo in multo maiores ambages incideremus, quam priore methodo euenit; quocirca aliam adhuc methodum quaerere debemus; nulla autem alia nobis relinquitur, niſi vt ex ſuperioribus aequationibus litteras k et k' inueſtigemus; quo pacto noſtra inueſtigatio ſatis plana reddetur.

Hanc viam ſequentes ſtatim habemus $k = \frac{\Phi - \mathfrak{B}\pi}{\Phi}$ et $k\,k' = \frac{\Phi - \pi + \mathfrak{C}\pi'}{\Phi}$; exiſtente $\Phi = \frac{-\pi + \pi' - \pi''}{m - 1}$ vnde cum k et k' ſint numeri poſitiui, pariter atque angulus Φ habemus ſtatim iſtas conditiones:

$$\Phi - \mathfrak{B}\,\pi > 0.$$

$$\Phi - \pi + \mathfrak{C}\,\pi' > 0.$$

$$-\pi + \pi' - \pi'' > 0.$$

Porro cum hinc interualla lentium fiant

$$1^{\circ}.\ \alpha + b = \frac{-\mathfrak{B}\pi}{\Phi - \mathfrak{B}\pi} . \alpha > 0$$

$$2^{\circ}.\ \beta + c = \frac{(\mathfrak{B}\pi - B\,\mathfrak{C}\pi')\alpha\Phi}{(\Phi - \pi + \mathfrak{C}\pi')(\Phi - \mathfrak{B}\pi)} > 0$$

$$3^{\circ}.\ \gamma + d = \frac{(m-1)\Phi + \pi - \mathfrak{C}\pi'}{m(\Phi - \pi + \mathfrak{C}\pi')} . B\,C\,\alpha > 0.$$

inde colligimus has nouas conditiones:

$$-\mathfrak{B}\,\pi . \alpha > 0.$$

$$(\mathfrak{B}\,\pi - B\,\mathfrak{C}\,\pi')\,\alpha > 0.$$

$$((m-1)\,\Phi + \pi - \mathfrak{C}\,\pi')\,B\,C\,\alpha > 0.$$

ideoque etiam harum quoti positiui esse debent

$$\frac{\mathfrak{B}\pi - B\mathfrak{C}\pi'}{-\mathfrak{B}\pi} > 0$$

$$\frac{((m-1)\Phi + \pi - \mathfrak{C}\pi')BC}{\mathfrak{B}\pi - B\mathfrak{C}\pi'} > 0$$

quae posterior ob $(m-1)\Phi = -\pi + \pi' - \pi''$ abit in hanc

$$\frac{(\mathfrak{C}\pi' - C\pi'')B}{\mathfrak{B}\pi - B\mathfrak{C}\pi'} > 0.$$

sicque quinque habentur conditiones ab α liberae, quibus satisfieri oportet. Nunc autem aequatio pro destruendo margine colorato ita se habebit:

$$0 = NBC\pi'' - \frac{N'\Phi}{\Phi - \mathfrak{B}\pi}((B+1)C\pi'' - \pi)$$
$$+ \frac{N''\Phi}{\Phi - \pi + \mathfrak{C}\pi'}((C+1)\pi'' - \pi')$$

His quomodocunque obseruatis perpendatur aequatio vltima pro confusione penitus destruenda

$$0 = N - \frac{N'}{k\mathfrak{B}} + \frac{N''}{kk'B\mathfrak{C}} - \frac{N'''}{m.BC.}$$

ob $p = \alpha$, $q = \mathfrak{B}b = -\frac{\mathfrak{B}\alpha}{k}$; $r = c = \frac{B\mathfrak{C}\alpha}{kk'}$

et $s = d = -\frac{BC\alpha}{m}$

num ei vel absolute vel saltim proxime satisfieri queat.

Denique vt etiam confusio prior tollatur satisfiat huic aequationi:

$$\mu\lambda - \frac{\mu'\lambda'}{k\mathfrak{B}^3} + \frac{\mu''\lambda''}{kk'B^3\mathfrak{C}^3} - \frac{\mu'''\lambda'''}{B^3C^3.m} - \frac{\mu'\nu'}{k\mathfrak{B}B} + \frac{\mu''\nu''}{kk'B^3C\mathfrak{C}} = 0.$$

Scho-

Scholion.

170. Cum hic in genere vix vlterius progredi liceat, ad casus particulares erit descendendum, et quia in capite praecedente lentes perfectae triplicatae id optatum visum non praestiterant, quod confusio a lente oculari oriunda ab iis non destruebatur, hic lentem obiectiuam iterum triplicatam statuamus, vt bina priora interualla euanescant, eius vero tres lentes ita definiamus, vt iis etiam confusio a lente oculari oriunda destruatur; quo facto deinceps forte via patebit inter tres lentes priores exigua interualla statuendi. Semper enim in huiusmodi disquisitionibus arduis expedit a casibus facilioribus exordiri, quoniam inde ratio perspicitur difficultates superandi, quae primo intuitu inuincibiles erant visae.

Problema 4.

171. Si tres lentes priores inter se immediate iungantur, vt lentem obiectiuam triplicatam constituant, quarta vero lens sit ocularis, regulas pro constructione huiusmodi telescopii exponere.

Solutio.

Cum hic sint interualla tam $\alpha + b = 0$, quam $\beta + c = 0$ fiet statim $k = 1$ et $k' = 1$, vnde sequuntur distantiae

$b =$

$$b = -\alpha;\ \beta = -B\alpha;\ c = B\alpha,$$
$$\gamma = BC\alpha \text{ et } d = -\frac{BC\alpha}{m}$$

hincque interuallum

$$\gamma + d = BC\alpha\left(1 - \frac{1}{m}\right) = \frac{m-1}{m}.\, BC\alpha$$

quod debet esse positiuum. Pro litteris autem π, π', π'' habebimus

$$1^\circ.\ \pi = 0;\quad 2^\circ.\ \pi' = 0.$$
$$3^\circ.\ \pi'' = -(m-1)\Phi.$$

Atque hinc aequatio pro tollendo margine colorato erit

$$0 = NBC - N'(B+1)C + N''(C+1)$$

vnde elicimus

$$C = -\frac{N''}{NB - N'(B+1) + N''}$$

vnde interuallum $\gamma + d$ fit

$$= -\frac{(m-1)}{m}.\ \frac{N''.B\alpha}{NB - N'(B+1) + N''}$$

quod cum esse debeat positiuum, duo casus sunt perpendendi.

Alter, quo $\alpha > 0$. tum esse debet

$$\frac{N''.B}{NB - N'(B+1) + N''} < 0.$$

ideoque

$$N - \frac{N'}{B}(B+1) + \frac{N''}{B} < 0.$$

siue

siue $N - N' + \frac{1}{B}(N'' - N') \lessgtr 0.$ contrarium euenire debet,

Altero casu, si $\alpha \lessgtr 0$; scilicet

$$N - N' + \frac{1}{B}(N'' - N') \gtrless 0.$$

Consideretur nunc aequatio, qua ista confusio penitus tollitur, scilicet

$$0 = N - \frac{N'}{\mathfrak{B}} + \frac{N''}{B\mathfrak{C}} - \frac{N'''}{mBC}$$

ex qua per $B\,C$ multiplicata, vt fit

$$0 = N\,B\,C - N'(B+1)\,C + N''(C+1) - \frac{N'''}{m}$$

quoniam a praecedente aequatione non differt, nisi vltimo termino $\frac{N'''}{m}$ qui prae reliquis est valde paruus, concludimus, si illi fuerit satisfactum, simul quoque huic proxime satisfieri idque eo magis, quo maior fuerit multiplicatio m, quae conclusio nititur fundamento, quod numeri N, N', N'', N''' parum ab vnitate differunt.

Pro priore autem confusione tollenda insuper satisfieri debet huic aequationi

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3} + \frac{\mu''\lambda''}{B^3\mathfrak{C}^3} - \frac{\mu'''\lambda'''}{mB^3C^3} - \frac{\mu'\nu'}{\mathfrak{B}B} + \frac{\mu''\nu''}{B^3C\mathfrak{C}}$$

in qua loco C eius valor supra inuentus

$$C = \frac{-N''}{B(N-N') - N' + N''}$$

substitui debet, id quod in genere ad formulam valde molestam deduceret, quare solutio non nisi casibus particularibus absolui poterit.

Coroll. 1.

172. Etsi conditiones pro littera B sunt datae, haec tamen littera prorsus indeterminata relinquitur, dummodo notetur

1°. si fuerit

$$N - N' + \tfrac{1}{B}(N'' - N') < 0$$

tum capi debere α positiuum.

2°. Sin autem fuerit

$$N - N' + \tfrac{1}{B}(N'' - N') > 0$$

tum capi debere α negatiuum.

Corollarium 1.

Cum inuenerimus $C = \frac{-N''}{B(N-N') - N' + N''}$

erit $1 + C = \frac{B(N-N') - N'}{B(N-N') - N' + N''}$

hincque $\mathfrak{C} = \frac{-N''}{B(N-N') - N'}$.

Coroll. 2.

173. Si velimus loco B introducere $\mathfrak{B}$ ponendo $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$; tunc consequemur

$$C = \frac{-N''(1-\mathfrak{B})}{(N-N'')\mathfrak{B} - N' + N''} \quad \text{et} \quad \mathfrak{C} = \frac{-N''(1-\mathfrak{B})}{N\mathfrak{B} - N'}$$

quibus obseruatis substitutio postrema facilius expedietur: fiet enim postrema aequatio

$0 =$

$$
0 = \begin{cases} \mu \lambda \mathfrak{B}^3 - \mu' \lambda' - \mu' \nu' \mathfrak{B}(1-\mathfrak{B}) \\ - \frac{\mu'' \lambda'' (N\mathfrak{B} - N')^3}{(N'')^3} \\ + \frac{\mu'' \nu'' (1-\mathfrak{B})(N\mathfrak{B} - N')((N-N'')\mathfrak{B} - N' + N'')}{(N'')^2} \\ + \frac{\mu''' \lambda''' ((N-N'')\mathfrak{B} - N' + N'')^3}{m(N'')^3}. \end{cases}
$$

Coroll. 3.

174. Respectu campi apparentis cum sit $\pi'' = -(m-1)\Phi$, si statuamus, vt hactenus $\pi'' = -\frac{1}{4}$; prodibit semidiameter $\Phi = \frac{1}{4(m-1)}$ et in min. primis $\Phi = \frac{859}{m-1}$ min. prim. siquidem lens ocularis fiat vtrinque aequaliter concaua, quod vti ostendimus fiet si $\lambda''' = 1,60006$. hac scilicet lente ex vitro coronario parata. Sin autem eam ex vitro chrystallino parare velimus, poni debet $\lambda''' = 1.67445$.

Exemplum I.

175. Si prima et tertia lens fuerit ex vitro coronario, media ex chrystallino, ex hisque lens obiectiua constituatur, lens vero ocularis ex vitro coronario paretur, pro quauis data multiplicatione telescopium construere.

Hoc exemplum ideo affero, quod hic casus in capite praecedente est praetermissus, quem autem hic alio modo tractabo, vt longitudo minor prodeat.

Cum igitur hic ſit $n = 1.53$; $n' = 1.58$; $n'' = 1.53$; $n''' = 1.53$ erit, vti vidimus, $N = 7$, $N' = 10$; $N'' = 7$; $N''' = 7$ ita, vt ſit $\mu'' = \mu$; $\nu'' = \nu$; $\mu''' = \mu$. Ex his colligitur $C = +\frac{7}{3}(1 - \mathfrak{B})$ $\mathfrak{C} = \frac{-7(1 - \mathfrak{B})}{7\mathfrak{B} - 10}$. Tantum ergo reſtat haec aequatio reſoluenda

$$0 = \begin{cases} \mu\lambda\mathfrak{B}^3 - \mu'\lambda' - \mu'\nu'\mathfrak{B}(1 - \mathfrak{B}) \\ - \frac{\mu\lambda''(7\mathfrak{B} - 10)^3}{7^3} \\ - \frac{3\mu\nu(1 - \mathfrak{B})(7\mathfrak{B} - 10)}{7^2} - \frac{27 \cdot \mu\lambda'''}{7^3 \cdot m} \end{cases}$$

quodſi ergo ſtatuamus $\lambda'' = \lambda$ et $\lambda''' = 1.60006$. haec aequatio induet hanc formam

$$\begin{aligned} &\mu\lambda\left(\tfrac{30}{7}\mathfrak{B}^2 - \tfrac{300}{49}\mathfrak{B} + \tfrac{1000}{343}\right) \\ &- \mu'\lambda' + \mu'\nu'(\mathfrak{B}^2 - \mathfrak{B}) \\ &+ \mu\nu\left(\tfrac{3}{7}\mathfrak{B}^2 - \tfrac{51}{49}\mathfrak{B} + \tfrac{30}{49}\right) \\ &- \tfrac{27 \cdot \mu\lambda'''}{343 \cdot m} = 0. \end{aligned}$$

quae tantum eſt aequatio quadratica, ex qua valor ipſius $\mathfrak{B}$ erui debet; terminis igitur ſecundum poteſtates ipſius $\mathfrak{B}$ diſpoſitis habebitur:

$$\begin{aligned} &\mathfrak{B}^2\left(\tfrac{30}{7}\mu\lambda + \mu'\nu' + \tfrac{3}{7}\mu\nu\right) \\ &+ \mathfrak{B}\left(-\tfrac{300}{49}\mu\lambda - \mu'\nu' - \tfrac{51}{49}\mu\nu\right) \\ &+ \tfrac{1000}{343}\mu\lambda - \mu'\lambda' + \tfrac{30}{49}\mu\nu \\ &- \tfrac{27 \cdot \mu\lambda'''}{343 \cdot m} = 0. \end{aligned}$$

Reſo-

Resolutionem autem huius aequationis ita instituamus, vt lens obiectiua maiorem aperturam admittat, quem in finem, non vt ante, $\lambda = 1$, sed $\lambda = 1.60006$ statuamus, vt prima lens vtrinque sibi similis euadat; quare cum sit

$$\log. \mu = 9.9945371$$

$$\log. \mu\nu = 9.3360593. \quad l. \mu' = 9.9407157$$

$$\log. \mu'\nu' = 9.3436055. \quad \mu'\nu' = 0.2206$$

$$\text{et } \operatorname{Log}. \lambda = \operatorname{Log}. \lambda''' = 0.2041363$$

at pro secunda lente ponatur non, vt ante, $\lambda = 1$, sed hanc litteram indeterminatam relinquamus; vnde nostra aequatio in numeris ita erit comparata:

$$0 = 7.0852\,\mathfrak{B}^2 - 10.1201\,\mathfrak{B} + 4.7393 - \mu'\lambda' - 0.\frac{12437}{m}.$$

quae reducitur ad hanc

$$\mathfrak{B}^2 = \frac{10.1201}{7.0852}\mathfrak{B} - \frac{4.7393}{7.0852} + \frac{\mu'\lambda'}{7.0852} + \frac{0.12437}{m.\,7.0852.}$$

$$\mathfrak{B}^2 = 1.4283\,\mathfrak{B} - 0.6689 + 0.1231\,\lambda' + 0.\frac{01755}{m}.$$

Vnde inuenitur

$$\mathfrak{B} = 0.7142 \pm \sqrt{\left(-0.1589 + 0.1231\,\lambda' + \frac{0.01755}{m}\right)}$$

Vnde patet, λ' capi debere vnitate maius. Statuatur ergo $\lambda' = 1\frac{1}{2}$ eritque

$$\mathfrak{B} = 0.7142 \pm \sqrt{\left(0.0257 + \frac{0.01755}{m}\right)}$$

hinc autem vlterius progredi non licet, nisi litterae m valores determinatos tribuendo; quem in finem sequentes casus adiungimus.

Casus I.

$m = 10.$

176. Erit hoc casu $\mathfrak{B} = 0.7142 \pm 0.1657$ sumtoque signo inferiore

$$\mathfrak{B} = 0.5485.$$

vel sumto superiore signo

$$\mathfrak{B} = 0.8799.$$

Sin autem velimus, vt pro $\mathfrak{B}$ vnicus valor $= 0.7142$ prodeat, capi deberet

$$\lambda' = \frac{0.1572}{0.1231} = 1\frac{341}{1231}$$

hocque casu hic vtamur.

Cum igitur sit $\lambda' = 1\frac{341}{1231}$ erit $\lambda' - 1 = \frac{341}{1231}$.

$$\text{Log.}\ \sqrt{\lambda' - 1} = 9.7208976$$

$$\text{Log.}\ (\lambda' - 1) = 9.4417952.$$

Cum nunc pro omni multiplicatione sit $\mathfrak{B} = 0.7142$, si quidem capiamus

$\lambda' =$

$$\lambda' = 1.2908 - \frac{0.1425}{m}$$

$$\lambda' - 1 = 0.2908 - \frac{0.1425}{m}$$

hincque erit $1 - \mathfrak{B} = 0.2857$

$$\mathrm{B} = 2.4998. \quad \mathrm{C} = 0.6666 = \tfrac{2}{3}.$$

Vnde obtinemus diſtantias

$$b = -\alpha; \ \beta = -2.4998. \alpha = -2\tfrac{1}{2}\alpha$$

$$c = +2.4998. \alpha.$$

$$\gamma = 1.6665. \alpha, \ d = -0.16665\, \alpha$$

Cum nunc ſit $\lambda = 1.60006$.

et $\lambda' = 1.2766$

$$\lambda'' = 1.60006 = \lambda'''.$$

erit

I. Pro Ima lente vtrinque aequaliter conuexa radius vtriusque faciei $= 1.06. \alpha$.

II. Pro IIda lente ex vitro chryſtallino

$$\frac{1}{F'} = \frac{\varrho\beta + \sigma b \mp \tau(b + \beta)\sqrt{\lambda' - 1}}{b\beta}$$

$$\frac{1}{G'} = \frac{\sigma\beta + \varrho b \pm \tau(b + \beta)\sqrt{\lambda' - 1}}{b\beta}$$

$$\frac{1}{F'} = \frac{-1.9360 \mp 1.6152}{b\beta}\, \alpha$$

$$\frac{1}{G'} = \frac{-4.0080 \pm 1.6152}{b\beta}.\, \alpha$$

ſum-

ſumtisque ſignis ſuperioribus erit

$$F' = \frac{-b\beta}{0.5512\,\alpha} = -0.7039\,\alpha$$

$$G' = \frac{-b\beta}{2.4828\,\alpha} = -1.0070\,\alpha.$$

III. Pro tertia lente ex vitro coronario.

Cum hic ſit $\tau . \sqrt{\lambda'' - 1} = \frac{\sigma - \rho}{2}$

tum vero $c = 2\frac{1}{2}\alpha$. et $\gamma = 1.\frac{2}{3}\alpha$. erit $c + \gamma = 4\frac{1}{6}\alpha$.

$$\frac{1}{F'} = \frac{4.5280 \pm 0.9870}{c\gamma}\,\alpha$$

$$\frac{1}{G'} = \frac{7.3335 \mp 0.9870}{c\gamma}\,\alpha$$

et ex ſignis inferioribus

$$F' = \frac{c\gamma}{1.5410\,\alpha} = 2.7039\,\alpha$$

$$G' = \frac{c\gamma}{6.3205\,\alpha} = 0.6592.\,\alpha.$$

Hae ergo tres lentes ſibi iunctae aperturam admittent, cuius ſemidiameter aeſtimari poteſt

$x = 0.1648.\,\alpha. = \frac{1}{7}\alpha$ circiter.

Cum autem ob claritatem eſſe debeat $x = \frac{m}{50}$ dig. $= \frac{1}{5}$ dig. capi debebit circiter $\alpha = \frac{7}{5}$ dig. vnde teleſcopii longitudo $= 1.4999\,\alpha = 1\frac{1}{2}\alpha = 2,1.$ dig.

IV. Pro quarta lente aequaliter vtrinque concaua. erit rad. vtriusque faciei $=$

$$1,06.\,d. = -1,06.\,(0.1666)\,\alpha$$
$$= -0.1766.\,\alpha$$
$$= -0.2472.\,\text{dig.}$$

Co-

Corollarium 1.

177. Si ergo hoc modo valor ipſius λ' definiatur, praecedentes determinationes pro omnibus multiplicationibus valebunt, excepta ſola lente ſecunda; tum autem pro quarta lente ſemidiameter vtriusque eius faciei capi debet $= -(1.06.).\frac{5}{3}.\frac{\alpha}{m}$ ſiue $= -1.7666.\frac{\alpha}{m}$.

Coroll. 2.

178. Conſtructio autem ſecundae lentis a multiplicatione pendebit, quia valor litterae λ' multiplicationem inuoluit, cum ſit $\lambda' = 1.2908 - \frac{0.1425}{m}$.

Scholion.

179. Haud difficile autem erit, pro quauis multiplicatione ſecundam lentem definire, poſtquam ea iam pro caſu $m = 10$ eſt inuenta; ſtatuatur enim $m = \infty$ erit $\lambda' = 1.2908$. hinc $\lambda' - 1 = 0.2908$. et Log. $V(\lambda' - 1) = 9.7317972$ vnde membrum ambiguum erit $= 1.6562\,\alpha$. vnde pro lente ſecunda erit

$$\frac{1}{F'} = \frac{-1.9360 \mp 1.6562}{b\beta}.\ \alpha$$

$$\frac{1}{G'} = \frac{-1.0080 \mp 1.6562}{b\beta}.\ \alpha$$

ſumtis ergo ſignis ſuperioribus

$$F' = \frac{-b\beta}{3.5922.\alpha} = -0.6959.\alpha$$

$$G' = \frac{-b\beta}{2.4418.\alpha} = -1.0239.\alpha$$

Nunc igitur ponamus pro multiplicatione quacunque m esse

$$F' = -(0.6959 + \tfrac{f}{m})\alpha$$

$$G' = -(1.0239 + \tfrac{g}{m})\alpha$$

et quia posito $m = 10$.

$$0.6959 + \tfrac{f}{10} = 0.7039$$

et $$1.0239 + \tfrac{g}{10} = 1.0070$$

reperitur $f = 0.0800$

$$g = -0.1690$$

quibus inuentis adipiscimur sequentem telescopii constructionem.

I. Pro prima lente Crown Glass.

radius vtriusque faciei $= + 1.06\,\alpha$.

II. Pro secunda lente Flint Glass.

rad. faciei $\begin{cases} \text{anter.} = -(0.6959 + \frac{0.0800}{m})\alpha \\ \text{poster.} = -(1.0239 - \frac{0.1690}{m})\alpha \end{cases}$

III. Pro tertia lente Crown Glass.

rad. faciei $\begin{cases} \text{anter.} = + 2.7039\,\alpha \\ \text{poster.} = + 0.6592.\,\alpha \end{cases}$

IV. Pro quarta lente Crown Glass.

rad. vtriusque faciei $= - 1.7666.\,\frac{\alpha}{m}$

quibus

quibus lentibus paratis ternae priores sibi invicem iungantur, post quas tertia collocetur intervallo $= \frac{m-1}{m} \cdot \frac{5}{3} \cdot \alpha$.

Cum porro sit $x = \frac{m}{50}$ dig et invenerimus $x = 0.1648\,\alpha$, hinc colligitur fore $\alpha = \frac{m}{8.2400}$, ita, vt statui possit $\alpha = \frac{4}{33} \cdot m$ seu $\alpha = \frac{12}{100} \cdot m$. Quare habetur

Constructio telescopii primi generis:

I. Pro prima lente Crown Glass.

rad. fac. vtriusque $= 0.1272 \cdot m$.

II. Pro secunda lente Flint Glass.

rad. fac. $\begin{cases} \text{anter.} = -0.0835 \cdot m - 0.0096 \\ \text{poster.} = -0.1229 \cdot m + 0.0202. \end{cases}$

III. Pro tertia lente Crown Glass.

rad. fac. $\begin{cases} \text{anter.} = 0.3244 \cdot m \\ \text{poster.} = 0.0791 \cdot m \end{cases}$

quibus tribus lentibus immediate iunctis postea intervallo $= \frac{1}{5}(m-1)$ dig. statuatur lens ocularis.

IV. Pro quarta lente. Crown Glass.

radius vtriusque faciei $= -0.2119$.

CASUS 2.

180. Cum pro omnibus multiplicationibus constructio telescopii sit tradita, solutionem exempli supra allati alio modo expediamus. Scilicet cum hic

 pro

pro $\mathfrak{B}$ valorem vnitate minorem ſimus conſecuti, qui ſupra vnitate maior prodierat, notatu dignus videtur caſus $\mathfrak{B} = 1$, quem hic euoluamus. Tum autem erit $B = \infty$ et quia diſtantiae determinatrices ſunt a; $b = -a$; $\beta = -B\alpha$; $c = B\alpha$; $\gamma = BC\alpha$, $d = \frac{-BC\alpha}{m}$ neceſſe eſt, vt ſit BC quantitas finita ideoque $C = 0$ et $\mathfrak{C} = C = 0$. Quare ſtatuamus $BC = \vartheta$, vt fiat $\gamma = \vartheta\alpha$ et $d = \frac{-\vartheta\alpha}{m}$ hincque teleſcopii longitudo $= \frac{m-1}{m}.\vartheta\alpha$. His poſitis aequatio pro margine colorato tollendo dabit ob $N = 7$, $N' = 10$, $N'' = N''' = 7$ et $\pi = 0$, $\pi' = 0$, $\pi'' = -(m-1)\Phi$.

$$0 = N\vartheta - N'\vartheta + N''$$

$$0 = 7\vartheta - 10\vartheta + 7; \text{ hincque } \vartheta = \tfrac{7}{3}.$$

Nunc autem aequatio pro confuſione primae ſpeciei tollenda fiet

$$0 = \mu\lambda - \mu'\lambda' + \frac{27\mu\lambda''}{343} - \frac{27\mu\lambda''''}{343\, m}$$

quae per μ diviſa ob $\frac{\mu'}{\mu} = 0.8834$ abit in hanc

$$0 = \lambda - 0.8834\,\lambda' + 0.0787\,\lambda'' - \frac{0.0787.\lambda''''}{m}$$

facta autem lente oculari vtrinque aequali erit

$$\lambda'''' = 1.60006 \text{ et}$$

$$0 = \lambda - 0.8834\,\lambda' + 0.0787.\lambda'' - \frac{0.1259}{m}$$

ex qua litteras λ ita definiri conuenit, vt vnitatem minimum ſuperent ſtatuamus ergo $\lambda = 1$ et $\lambda'' = 1$ erit

erit: $0.8834\,\lambda' = 1.0787 - \frac{0.1259}{m}$

vnde colligitur

$$\lambda' = 1.2210 - \frac{0.1428}{m}$$

sola ergo lens secunda a multiplicatione m pendet, quam deinceps seorsim euoluamus

Calculum ergo pro prima, tertia et quarta instituamus

Pro prima autem est:

$$F = \frac{\alpha}{\sigma} = 0.6023.\,\alpha$$

$$G = \frac{\alpha}{\rho} = 4.4111.\,\alpha$$

Pro tertia lente ob $\lambda'' = 1.$ et $c = \infty$

$$F = \frac{\gamma}{\sigma} = 1.4055.\,\alpha$$

$$G = \frac{\gamma}{\rho} = 10.2925.\,\alpha$$

Pro quarta lente

radius vtriusque faciei $= 1.06.\,d$
$$= -\frac{2.4733.\,\alpha}{m}$$

et tota telescopii longitudo $= \frac{m-1}{m}.\,\frac{7}{3}.\,\alpha.$

Pro secunda autem lente cum in genere sit:

$$F = \frac{b\beta}{\rho\beta + \sigma b \pm \tau(b+\beta)\sqrt{\lambda - 1}}.$$

$$G = \frac{b\beta}{\sigma\beta + \rho b \mp \tau(b+\beta)\sqrt{\lambda - 1}}.$$

 ob

ob $\beta = \infty$ et $b = -\alpha$ erit

$$F = \frac{-\alpha}{\varrho \pm \tau \sqrt{(\lambda - 1)}}$$

$$G = \frac{-\alpha}{\sigma \mp \tau \sqrt{(\lambda - 1)}}$$

pro quo duo casus sunt euoluendi alter, quo $m = 10$ et alter, quo $m = \infty$.

Priore erit ob $m = 10$

$\lambda' = 1.2068$; $\lambda' - 1 = 0.2068$.

$$\begin{array}{rl} \text{et Log.} \sqrt{(\lambda' - 1)} = & 9.6577753 \\ \text{Log.}\ \tau = & \underline{9.9432471} \\ & 9.6010224 \end{array}$$

cui logarithmo respondet 0.3990 ideoque

$$F = \frac{-\alpha}{0.1414 \pm 0.3990}$$

$$F = \frac{-\alpha}{0.5404} = -1.8505.\alpha$$

$$G = \frac{-\alpha}{1.5827 \mp 0.3990}$$

$$G = \frac{-\alpha}{1.1837} = -0.8467.\alpha.$$

Altero casu ob $m = \infty$ erit

$\lambda' = 1.2210$ et $\lambda' - 1 = 0.2210$

$$\begin{array}{rl} \text{et Log.} \sqrt{(\lambda' - 1)} = & 9.6721961 \\ \text{Log.}\ \tau = & \underline{9.9432471} \\ & 9.6154432 \end{array}$$

hinc $\tau.\sqrt{\lambda' - 1} = 0.4125$.

Vnde

Vnde fit

$$F = \frac{-\alpha}{0.1414 + 0.4125} = \frac{-\alpha}{0.5539}$$

$$G = \frac{-\alpha}{1.5827 + 0.4125} = \frac{-\alpha}{1.1708}$$

hincque

$$F = -1.8054 \cdot \alpha$$

$$G = -0.8545 \cdot \alpha$$

quare ftatuamus pro multiplicatione quacunque m

$$F = -\left(1.8054 + \frac{f}{m}\right)\alpha$$

$$G = -\left(0.8545 + \frac{g}{m}\right)\alpha$$

et ex cafu $m = 10$ elicimus

$$f = 0.4510;\ g = -0.0780$$

ita, vt fit

$$F = -\left(1.8054 + \frac{0.4510}{m}\right)\alpha$$

$$G = -\left(0.8545 - \frac{0.0780}{m}\right)\alpha$$

pro α autem definiendo confideretur radius minimus in lente hac obiectiua triplicata occurrens 0.6023α, cuius pars quarta $0.1506.\alpha = \frac{m}{50}$; ficque prodibit $\alpha = \frac{m}{7.5300}$ dig. Sumatur ergo $\alpha = \frac{2m}{15}$ dig. et habetur fequens

Con-

Constructio Telescopii primi generis.

I. Pro prima lente Crown Gl.

radius faciei $\begin{cases} \text{anter.} = +0.0803.m.\text{dig.} \\ \text{poster.} = +0.5882.m.\text{dig.} \end{cases}$

II. Pro secunda lente Flint Glaß.

rad. fac. $\begin{cases} \text{anter.} = (-0.2407.m - 0.0601)\text{ dig.} \\ \text{poster.} = (-0.1139.m + 0.0104)\text{ dig.} \end{cases}$

III. Pro tertia lente Crown Glaß.

radius faciei $\begin{cases} \text{anter.} = +0.1874.m.\text{dig.} \\ \text{poster.} = +1.3723.m.\text{dig.} \end{cases}$

IV. Pro quarta lente Crown Glaß.

radius vtriusque faciei $= 0.3298.$ dig.

Tribus prioribus lentibus inuicem iunctis quartae ab iis interuallum erit

$$= \tfrac{14}{45}(m - 1)\text{ dig.}$$

et campi apparentis semidiameter erit, vt hactenus, $\Phi = \frac{859}{m-1}$. min. prim.

Scholion.

181. Quia in hac solutione posuimus $\lambda = 1$ et $\lambda'' = 1$, consuluimus potissimum artifici, quia hoc casu errores in exsecutione commissi non admodum negotium turbant, sed longitudo horum telescopiorum prodiit aliquanto maior, propterea quod radius satis exi-

exiguus in determinatione lentis obiectiuae occurrebat, si pro huic autem incommodo medelam afferemus, si pro prima et tertia lente statuamus $\lambda = 1.60006$, quo facto obtinebitur $\lambda' = 1.9536 - \frac{0.1425}{m}$ qui numerus tantum in secundam lentem influit, cuius constructionem deinceps inuestigemus.

Iam vero erit

Pro prima lente Crown Glass.

radius vtriusque faciei $= 1.06.\alpha$.

Pro tertia lente Crown Glass.

radius vtriusque faciei $= 1.06.\gamma$
$= 2.473.\alpha$.

Pro quarta lente

radius vtriusque faciei $= -\frac{2.4733.\alpha}{m}$.

Restat igitur, vt secundam lentem euoluamus, vt ante.

Scilicet duos casus contemplabimur, alterum, quo $m = 10$, alterum, quo $m = \infty$.

Sit igitur primo $m = 10$ eritque $\lambda' = 1.93937$ et $\lambda' - 1 = 0.93937$ et $\tau.\sqrt{(\lambda' - 1)} = 0.85048$.

Quare

$$F = \frac{-\alpha}{0.1434 \pm 0.85048}.$$

$$G = \frac{-\alpha}{1.5827 + 0.85048}.$$

ſeu

$$F = \frac{-\alpha}{0.9919} = -1.0082.\alpha$$

$$G = \frac{-\alpha}{0.7322} = -1.3657.\alpha$$

Sit nunc, $m = \infty$, erit

$$\lambda' = 1.9536$$

$$\tau \surd (\lambda' - 1) = 0.8569$$

hincque

$$F = \frac{-\alpha}{0.1414 \pm 0.8569}$$

$$G = \frac{-\alpha}{1.5827 \mp 0.8569}$$

$$F = \frac{-\alpha}{0.9983} = -1.0017.\alpha$$

$$G = \frac{-\alpha}{0.7258} = -1.3778.\alpha$$

Nunc pro multiplicatione quacunque m ſtatuatur

$$F = -(1.0017 + \frac{f}{m})\alpha$$

$$G = -(1.3778 + \frac{g}{m})\alpha$$

et ex priore caſu $m = 10$ colligitur

$$f = 0.065;\ g = -0.121$$

ita, vt ſit pro lente ſecunda

$$\text{rad. fac.} \begin{cases} \text{anter.} = -(1.0017 + \frac{0.065}{m})\alpha \\ \text{poſter.} = -(1.3778 - \frac{0.121}{m})\alpha. \end{cases}$$

Hic

Hic iam tuto sumi potest $x = \frac{1}{4}\alpha = \frac{m}{50}$; hinc obtinetur $\alpha = \frac{8}{100} \cdot m$

Hinc ergo orietur sequens

Telescopii primi generis Constructio:

I. Pro lente prima: Crown Glass.

radius vtriusque faciei $= 0.0848 \cdot m$. dig.

II. Pro lente secunda

$$\text{rad. fac.} \begin{cases} \text{anter.} = (-0.08014\,m. - 0.0052)\ \text{dig.} \\ \text{poster.} = (-0.110224\,m. + 0.0097)\ \text{dig.} \end{cases}$$

III. Pro lente tertia Crown Glass.

radius vtriusque faciei $= 0.19784 \cdot m$. dig.

IV. Pro lente quarta.

radius vtriusque faciei $= -0.19786$. dig.

Tribus lentibus prioribus sibi immediate iunctis ad interuallum $= (0,187)(m-1)$ dig. collocetur lens quarta, cui oculus immediate adplicatus cernet campum, cuius semidiameter erit $\frac{859}{m-1}$. min. prim.

Scholion.

182. Hic casus inprimis est omni attentione dignus, quoniam pro quauis multiplicatione huius generis telescopia breuissima suppeditat: si enim multiplicationem adeo centuplam desideremus, longitudo vix superabit $18\frac{1}{2}$ digitos. Haec igitur methodus,

qua posuimus $\mathfrak{B} = 1$ vtique mereretur, vt etiam ad alias vitri species seu vbi pro lentibus alia combinatio vitri coronarii et chrystallini statueretur, seorsim adplicaretur. Sed quia ea etiam ad problema nostrum generale soluendum aeque felici successu in vsum vocari potest eiusque beneficio insignes difficultates supra commemoratae euanescunt, expediet sequens problema generalius tractasse.

Problema 5.

183. Si telescopium ex quatuor lentibus sit construendum, duae priores vero lentes ita debeant esse comparatae, vt radii per eas transmissi iterum inter se fiant paralleli, regulas pro constructione describere.

Solutio.

Cum igitur radii per secundam lentem refracti iterum fiant axi paralleli, erit $\beta = \infty$ ideoque $\frac{\beta}{b} = B = \infty$ et $\mathfrak{B} = 1$, erunt distantiae determinatrices

$$b = -\frac{\alpha}{k};\ \beta = -\frac{B\alpha}{k} = \infty$$

$$c = \frac{B\alpha}{kk'};\ \gamma = \frac{BC\alpha}{k.k'};\ d = -\frac{BC\alpha}{m}.$$

hicque iam notari oportet, vt distantia inter primam et secundam lentem $\beta + c$ fiat finita, debere ob $\beta = \infty$ esse $c = -\infty$, vnde fit $k' = 1$.

Quo

Quo autem rem clarius explicemus, statuatur haec distantia $= \eta\alpha$, vt fit $B\alpha(\frac{1}{kk'} - \frac{1}{k}) = \eta\alpha$ vnde fit $k' = \frac{B}{B + \eta k}$, quae ob $B = \infty$ fit $k' = 1$; interim tamen conueniet, illam expressionem $k' = \frac{B}{B + \eta k}$ in vsum sequentem notasse.

Deinde quia $c = \infty$, γ vero finita quantitas, erit $\frac{\gamma}{c} = C = 0$, hincque etiam $\mathfrak{C} = \frac{C}{1 + C} = C = 0$; interim tamen productum $B\,C$ debet esse finitum. Sit igitur $B\,C = \vartheta$, vt fiat $\gamma = \frac{\vartheta\alpha}{k}$ et $d = \frac{-\vartheta\alpha}{m}$; cum illa autem aequatione coniungi debet ista, qua summo rigore est $C = \frac{\gamma}{c} = \frac{\theta}{B}$, hincque $\mathfrak{C} = \frac{\theta}{B + \theta}$. His notatis erunt interualla lentium

$$\alpha + b = \alpha\left(\frac{k-1}{k}\right)$$

$$\beta + c = \eta\alpha$$

$$\gamma + d = \left(\frac{1}{k} - \frac{1}{m}\right)\vartheta . \alpha$$

$$= \frac{m-k}{km} . \vartheta\alpha .$$

Vnde hae fractiones $\frac{\eta k}{k-1}$ et $\frac{m-k}{m(k-1)}\vartheta$ debent esse positiuae, seu $\frac{\eta}{k-1} > 0$; $\frac{m-k}{k-1} . \vartheta > 0$ seu $\frac{m-k}{\eta} . \vartheta > 0$. Iam inquiramus in valores litterarum π, π' et π'', ex tribus sequentibus aequationibus definiendos

I. $\mathfrak{B}\pi - \Phi = -k\Phi$

II. $\mathfrak{C}\pi' - \pi + \Phi = kk'\Phi$

III. $(m-1)\Phi = -\pi + \pi' - \pi''$

 qua-

quarum prima ſtatim dat ob $\mathfrak{B} = 1$

$$\pi = (1 - k)\Phi = -(k - 1)\Phi$$

vnde vt hic valor campo augendo inſeruiat, π numerus negatiuus eſſe debet ideoque $k > 1$.

Secunda autem aequatio ob $\mathfrak{C} = 0$ et $k' = 1$. daret $-\pi + \Phi = k\Phi$, vnde pro π' nihil concludere liceret, quare pro $\mathfrak{C}$ valores illos exactiores ſcribi oportebit fietque

$$\frac{\theta}{B+\theta}.\pi' - \pi + \Phi = \frac{Bk}{B+\eta k}\Phi$$

quae ob $\pi = -(k - 1)\Phi$ abit in hanc

$$\frac{\theta}{B+\theta}.\pi' + k\Phi = \frac{Bk}{B+\eta k}\Phi$$

$$\text{ſeu } \frac{\theta}{B+\theta}\pi' = \frac{-\eta k^2}{B+\eta k}.\Phi$$

quae ergo ob $B = \infty$ dat

$$\pi' = \frac{-\eta k^2}{\theta}.\Phi$$

quia autem conuenit ſumere $k > 1$ debet eſſe $\alpha > 0$ ideoque et $\eta > 0$, hic valor π' erit negatiuus, ſi fuerit $\vartheta > 0$; ſin autem $\vartheta < 0$, is erit poſitiuus, vbi autem meminiſſe oportet eſſe debere $(m - k)\vartheta > 0$.

Tertia denique aequatio abit in hanc formam:

$$(m - 1)\Phi = +(k - 1)\Phi - \frac{\eta k^2}{\theta}\Phi - \pi''$$

$$\text{hincque } \pi'' = (k - m - \frac{\eta k^2}{\theta})\Phi$$

$$\text{ſiue } \pi'' = -(m - k + \frac{\eta k^2}{\theta})\Phi$$

quae

quae formula cum etiam inferuiat campo definiendo, si capiatur $\pi'' = -\frac{1}{4}$; reperitur

$$\Phi = \frac{859.}{m - k + \frac{\eta k^2}{\theta}} \text{ minut.}$$

$$\Phi = \frac{85\;\cdot\theta}{(m-k)\theta + \eta k^2}$$

quare curandum est, vt $\frac{\eta k^2}{\theta}$ quam minimum reddatur, quod facile praestatur faciendo interuallum secundae et tertiae lentis quam minimum adeoque euanescens, quo casu erit $\Phi = \frac{85\cdot}{m-k}$ qui eo maior fit, quo maior sumitur k. Nunc igitur aequationem pro margine colorato tollendo consideremus, quae erit

$$0 = N\vartheta\pi'' - \frac{N'}{k}(\vartheta\pi'' - \pi)$$
$$+ \frac{N''}{k}(\pi'' - \pi')$$

quae substitutis valoribus dat

$$0 = -N((m-k)\vartheta + \eta k^2)$$
$$+ \frac{N'}{k}((m-k)\vartheta + \eta k^2 - k + 1)$$
$$- \frac{N''}{k}(m-k)$$

ex qua aequatione ϑ commode definiri potest reperieturque

$$\vartheta = \frac{-N\eta k^3 + N'\eta k^2 - N'(k-1) - N''(m-k)}{(m-k)(Nk - N')}$$

quia autem conuenit η quam minimum assumere ac praeterea non necesse est, vt isti aequationi summo rigore satisfiat, his terminis omissis habebimus $\vartheta =$

$$\vartheta = \frac{-N'(k-1) - N''(m-k)}{(m-k)(Nk-N')}$$

vnde fit

$$(m-k)\,\vartheta = \frac{-N'(k-1) - N''(m-k)}{Nk-N'}$$

quae quantitas cum debeat esse positiua, numerator autem manifesto sit negatiuus, etiam denominatorem negatiuum esse oportet ideoque $N' > Nk$. Quodsi ergo N' maximum habeat valorem ex vitro scilicet chrystallino, N vero minimum ex vitro coronario, vt sit $N = 7$ et $N' = 10$; numerus k non amplius nostro arbitrio relinquitur, sed ita capi debet, vt fiat $7k < 10$; et $k < \frac{10}{7}$ seu contineri debet intra limites 1 et $\frac{10}{7}$. Notetur hic, si caperetur $k = 1$, casum praecedentem esse oriturum, neque campum hinc auctum iri; Sin autem capiatur $k = \frac{10}{7}$ foret $\vartheta = \infty$ et longitudo telescopii fieret infinita; vnde conueniet k propius vnitati; quam alteri limiti assumere. His probe perpensis statuamus $k = \frac{8}{7}$, $N = 7$; $N' = 10$. $N'' = 7$, quo ϑ obtineat valorem minorem. Vnde fiet $\vartheta = \frac{49m - 46}{2(7m-8)}$ hincque $\frac{\eta k^2}{\theta}$ habebit hunc valorem $\frac{2 \cdot 8^2 (7m-8)}{7^2(49m-46)} \cdot \eta$ qui sumto $m = \infty$ fit $= \frac{128}{343} \cdot \eta$ ex quo colligitur, si modo η non excedat $\frac{1}{10}$ campi diminutionem non fore sensibilem.

Denique pro semidiametro confusionis ad nihilum redigendo satisfiat huic aequationi:

$$0 = \mu\lambda - \frac{\mu'\lambda'}{k} + \frac{\mu''\lambda''}{k\theta^3} - \frac{\mu'''\lambda'''}{m\theta^3}$$

ex

ex qua commodissime definiemus λ', qui erit ob $\mu' = \mu'' = \mu'''$

$$\lambda' = \frac{\mu}{\mu'}\left(k\lambda + \frac{\lambda''}{\theta^2} - \frac{k\lambda'''}{m\theta^3}\right)$$

sicque hoc problema feliciter est solutum.

Coroll. I.

184. Distantiae ergo determinatrices singularum lentium erunt

Pro prima: ∞ et α cum λ

Pro secunda: $b = \frac{-\alpha}{k}$; et $\beta = \infty$ cum λ'

Pro tertia: $c = \infty$ et $\gamma = \frac{\theta\alpha}{k}$ cum λ''

Pro quarta: $d = \frac{-\theta\alpha}{m}$; $\delta = \infty$ cum λ'''.

vbi notandum, primam, tertiam et quartam ex vitro coronario, secundam ex chrystallino esse parandam; tum vero fore interualla lentium

$$\alpha + b = \alpha\left(\frac{k-1}{k}\right) = \tfrac{1}{2}\alpha$$

$$\beta + c = \eta\alpha$$

de qua distantia notetur, eam statui debere quam minimam; ac denique

$$\gamma + d = \frac{m-k}{km}.\vartheta\alpha$$

vnde tota longitudo prodit

$$= \alpha\left(\frac{k-1}{k} + \eta + \frac{m-k}{km}.\vartheta\right).$$

Co-

Coroll. 2.

185. Pro litteris autem k et ϑ hos valores ſtatuimus, $k = \frac{8}{7}$, $\vartheta = \frac{49m - 46}{2(7m-8)}$, quae expreſſio cum adhuc m inuoluat, calculum non, vt ante, pro quauis multiplicatione in genere abſoluere licebit; interim tamen ſimili modo, qno ante vſi ſumus, poſtquam pro duabus tribusue multiplicationibus calculum abſoluerimus, interpolando formulas generaliores pro omni multiplicatione concludere poterimus.

Scholion.

186. Haec teleſcopia iis, quae modo ante deſcripſimus, ideo erunt praeferenda, quod in his nullae lentes ſibi immediate iunctae aſſumuntur, quippe quod in praxi locum habere nequit; tum vero etiam quod aliquod campi augmentum largiuntur. Ceterum haec teleſcopia aliquanto fiunt longiora, tam ob diſtantiam inter lentes primam et ſecundam, quam potiſſimum ob maiorem valorem ipſius ϑ, a quo interuallum tertiae et quartae lentis potiſſimum pendet. Interuallum autem medium $\eta\alpha$ hic merito negligimus. Quo tamen breuitati inſtrumenti quantum fieri licet, conſulamus, expediet ſine dubio, vt modo ante fecimus, tam primam et tertiam lentem, quam quartam vtrinque aequales formare, ita, vt ſit $\lambda = \lambda'' = \lambda'''$ $= 1.60006$; tum vero erit $\mu = 0.9875$; $\mu' = 0.8724$. vnde harum lentium conſtructio ſtatim ſequitur.

Erit ſcilicet radius vtriusque faciei.

I. Pro lente prima

$= 1.06\,\alpha.$

II. Pro lente tertia

$= 1.06.\frac{\theta\alpha}{k}.$

III. Pro lente quarta

$= -1.06.\frac{\theta\alpha}{m}.$

Nihil igitur aliud reſtat, niſi vt pro quibusdam multiplicationibus calculum expediamus; ac primo quidem conueniet, multiplicationem quandam exiguam $m = 5$ euoluere, vt pateat, quantum haec inueſtigatio in minimis teleſcopiis huius generis praeſtare poſſit; tum vero multiplicationem quandam maiorem veluti $m = 10$, indeque ſubito $m = \infty$ euoluamus, vt ex horum caſuum comparatione concluſionem pro quauis maiore multiplicatione formare queamus.

Exemplum I.

$m = 5.$

187. Teleſcopium pro multiplicatione $m = 5$ deſcribere.

Erit hoc caſu

$$\left.\begin{array}{r}\vartheta = \frac{199}{54} = 3.6852 \\ \text{Log.}\ \vartheta = 0.5664593\end{array}\right\}\ \text{et Log.}\ k = 0.0579920$$

hincque $b = -\frac{7}{8}\alpha$; $\beta = \infty$; $c = \infty$

 $\gamma =$

$$\gamma = 3,2246.\,\alpha$$
$$d = -0,7370.\,\alpha$$

hincque

$$\alpha + b = \tfrac{5}{8}\,\alpha;\ \beta + c = \eta\,\alpha = \text{minimo.}$$
$$\gamma + d = 2.4876.\,\alpha$$

ſicque longitudo tota teleſcopii erit $= 2.6126\,\alpha + \eta\alpha$.

Vnde tres lentes ex vitro coronario parandae ita ſe habebunt:

I. Pro prima lente

radius vtriusque faciei $= 1.06.\,\alpha$.

II. Pro tertia lente

radius vtriusque faciei $= 3.4181.\,\alpha$.

III. Pro quarta lente

radius vtriusque faciei $= -0.7812.\,\alpha$.

IV. Pro ſecunda lente. Flint Glaſſ.

ante omnia quaeri debet numerus λ'. ex formula

$$\lambda' = 1.\,\frac{60006.\mu'}{\mu''}\left(k + \frac{1}{\theta^3} - \frac{k}{m\,\theta^3}\right) \text{ vnde}$$

$$\lambda' = 2,0977;\ \text{ergo}\ \lambda' - 1 = 1,0977$$

hincque $\tau.\sqrt{(\lambda' - 1)} = 0.91936$.

Quare

Quare pro hac lente erit

$$F = \frac{b}{0.1414 \pm 0.51936} = \frac{b}{1.0608}$$

$$G = \frac{b}{1.5827 \mp 0.51936} = \frac{b}{0.6633}$$

$$F = -0.8248.\ \alpha;\ G = -1.3192.\ \alpha$$

Vnde fluit sequens

Constructio Telescopii

I. Pro lente prima Crown Glass.

radius vtriusque faciei $= +1.06\ \alpha$

Interuallum $= 0.125\ \alpha$.

II. Pro lente secunda Flint Glass.

radius faciei $\begin{cases} \text{anter.} = -0.8248\ \alpha \\ \text{poster.} = -1.3192.\ \alpha \end{cases}$

Interuallum minimum.

III. Pro lente tertia Crown Glass.

radius faciei vtriusque $= 3.4181.\ \alpha$

Interuallum $= 2.4876.\ \alpha$.

IV. Pro lente quarta Crown Glass.

radius vtriusque faciei $= -0.7812.\ \alpha$.

Lenti obiectiuae tribui potest apertura, cuius semidiameter $x = \frac{1}{4}\ \alpha$.

Cum autem ob claritatem statui debeat $x = \frac{m}{50}$. dig. $= \frac{1}{10}$ dig. vnde $\alpha = \frac{2}{5}$. dig.

 et

et telescopii longitudo $= 2.6126.\alpha + \eta\alpha$

et semidiameter campi $\Phi = 223$ min. $= 3^{\circ}\,43'$.

Exempl. II.

188. Si multiplicatio $m = 10$ desideretur, telescopium huius generis describere.

Ob $m = 10$ erit $\vartheta = \frac{444}{124} = 3.5806$

Log. $\vartheta = 0.5539613$.

Log. $\frac{1}{\vartheta} = 9.4460386$.

vnde $b = -\frac{7}{8}\alpha = -0,875\,\alpha$

$\beta = \infty = c$; $\gamma = 3.1331.\alpha$

$d = -0.35806.\alpha$.

Nunc euoluatur numerus λ', qui reperitur

$\lambda' = 2.1049$; $\lambda' - 1 = 1.1049$

hinc $\tau.\sqrt{(\lambda' - 1)} = 0.92132$

Vnde radii facierum

$$F = \frac{b}{0.1414 \pm 0.9213} = \frac{b}{1.0627}$$

$$G = \frac{b}{1.5827 \mp 0.9213} = \frac{b}{0.6614}$$

seu $F = -0.8234\,\alpha$

$G = -1.3230\,\alpha$

Vnde colligitur sequens

Con-

Constructio Telescopii

I. Pro prima lente Crown Glass.

radius vtriusque faciei $= 1.06.\,\alpha$

Interuallum $= 0.125.\,\alpha$

II. Pro secunda lente Flint Glass.

rad. fac. $\begin{cases} \text{anter.} = -0.8234.\,\alpha \\ \text{poster.} = -1.3230.\,\alpha \end{cases}$

Interuallum minimum.

III. Pro lente tertia: Crown Glass.

radius vtriusque faciei $= 3.3211.\,\alpha$

Interuallum $= 2.7751.\,\alpha$

IV. Pro lente quarta Crown Glass.

radius vtriusque faciei $= -0.3796.\,\alpha$

Vnde fit tota longitudo $= 2.9001\,\alpha$. Lenti primae autem apertura tribui debet, cuius semidiameter $x = \frac{m}{50} = \frac{1}{4}\,\alpha = \frac{1}{5}$ dig. Vnde sequitur $\alpha = \frac{4}{5}$ dig. siue maius. Campi autem visi semidiameter erit $\Phi = 94$ minut. $= 1^\circ\, 34'$.

Exempl. III.

189. Si multiplicatio m fuerit ∞, telescopium huius generis describere.

Ob $m = \infty$ erit $\vartheta = 3,5$.

et

et $\text{Log}.\ \vartheta = 0.5440680$

$\text{Log}.\ \frac{1}{\vartheta} = 9.4559319$

hincque $b = -0.875.\alpha$; $\beta = \infty = c$

$\gamma = 3.0625.\alpha$; $d = -3,5.\frac{\alpha}{m}$.

Pro lente autem ſecunda inuenimus

$\lambda' = 2,1120$; $\lambda' - 1 = 1,1120$

et $\tau.\sqrt{(\lambda' - 1)} = 0,9253$.

Ex quibus colligitur

$$F = \frac{b}{0,1414 \pm 0,9253} = \frac{b}{1,0667}$$

$$G = \frac{b}{1,5827 \mp 0,9253} = \frac{b}{0,6574}$$

$F = -0.8205.\alpha$

$G = -1,3309.\alpha$

Vnde colligitur ſequens

Conſtructio Teleſcopii

I. Pro prima lente Crown Glaſſ.

radius vtriusque faciei $= 1,06.\alpha$

Interuallum $= 0.125.\alpha$

II. Pro ſecunda lente Flint Glaſſ.

radius faciei $\begin{cases} \text{anter.} = -0.8205.\alpha \\ \text{poſter.} = -1,3309.\alpha \end{cases}$

Interuallum minimum.

III. Pro

III. Pro tertia lente Crown Glass.

radius vtriusque faciei $= 3.2462. \alpha$

Interuallum $= (3.0625 - \frac{3,5}{m})\alpha$

IV. Pro quarta lente Crown Glass.

radius vtriusque faciei $= -3.710. \frac{\alpha}{m}$

Hincque longitudo telescopii erit $= (3,1875 - \frac{3,5}{m}).\alpha$.

Lenti vero obiectiuae apertura tribuatur, cuius semidiameter $= \frac{1}{4}\alpha = \frac{m}{50}$ ita, vt capi possit

$\alpha = \frac{2}{25}. m.$ dig. $= \infty$.

Exempl. IV. generale.

190. Si multiplicatio fuerit quaecunque m, saltem denario maior, telescopium huius generis describere.

Cum pro casu $m = \infty$ inuenerimus $\vartheta = 3,5$ nunc in genere ponamus $\vartheta = 3,5 + \frac{e}{m}$ et quia pro $m = 10$ fuerat $\vartheta = 3,5806$, erit $e = 0.806$, ita vt sit $\vartheta = 3,5 + \frac{0,806}{m}$ vnde distantiae ita se habebunt:

$b = -0.875.\alpha$; $\beta = \infty = c$

$\gamma = (3,0625 + \frac{0,7053}{m})\alpha$

$d = -(3,5 + \frac{0,8060}{m})\frac{\alpha}{m}$

Pro lente autem secunda ponatur

$F = -(0.8205 + \frac{f}{m})\alpha$

$G = -(1.3309 + \frac{g}{m})\alpha$

Comparatione igitur instituta cum casu $m = 10$ erit $f = 0.0290$, $g = -0.0790$.

Constructio huius Telescopii

I. Pro prima lente. Crown Glass.

radius vtriusque faciei $= 1.06.\,\alpha$

Interuallum $= 0.125.\,\alpha$.

II. Pro secunda lente. Flint Glass.

$$\text{rad. faciei} \begin{cases} \text{anter.} = -(0.8205 + \frac{0.0290}{m})\alpha \\ \text{poster.} = -(1.3309 - \frac{0.0790}{m})\alpha \end{cases}$$

Interuallum minimum.

III. Pro tertia lente. Crown Glass.

radius vtriusque faciei $= +(3,2462 + \frac{0.7476}{m})\alpha$

Interuallum $= (3,0625 - \frac{2.7947}{m} - \frac{0.8060}{m^2})\alpha$.

IV. Pro quarta lente.

radius faciei vtriusque $= -(3,710 + \frac{0.8544}{m})\frac{\alpha}{m}$

sicque tota longitudo erit

$= (3,1875 - \frac{2.7947}{m} - \frac{0.8060}{m^2})\alpha$

deinde lentis obiectiuae semidiameter aperturae debet esse $x = \frac{m}{50}$ dig. vnde α capi debebit $\alpha = \frac{2}{25}m$. dig. siue maius, campique visi semidiameter

$$= \Phi = \frac{859}{m - \frac{8}{7}} \text{ min. prim.}$$

Scho-

Scholion.

191. En ergo insignem multitudinem variorum primi generis telescopiorum, quae adhuc in infinitum multiplicari possent, si litteris B et C alios valores tribuere vel etiam pluribus lentibus vti vellemus. Verum huiusmodi inuestigatio prorsus superflua videtur, cum maior perfectionis gradus exspectari nequeat ac plures lentes claritati semper obsint, neque etiam maior campus sperari possit. Inprimis autem obseruandum est in his telescopiis marginem coloratum aliter destrui non potuisse, nisi diuersis vitri speciebus adhibendis, ita, vt iam affirmare possimus, ex eadem vitri specie huiusmodi telescopia confici non posse, quae non vitio marginis colorati laborent, cum tamen in sequentibus generibus, lentibus ex vna vitri specie factis talis margo feliciter tolli possit, etiamsi tunc ipsum diffusionis spatium ad nihilum redigere non liceat. Haec restrictio etiam in caussa erat, quod campum apparentem vix notabiliter augere licuerat; sin autem marginem coloratum negligere vellemus, campus haud mediocriter augeri posset. Tum enim in casu vltimi problematis litterae k et ϑ manerent arbitrio nostro relictae et cum semidiameter campi esset $\Phi = -\frac{\pi''}{m-k}$, posito $\eta = 0$, videtur is ad lubitum augeri posse, dum tantum k parum ab m deficiens assumatur atque adeo sumto $k = m$ in infinitum abiret; quod tamen nullo modo praestari posse experientia abunde testatur. Quare hoc dubium soluisse operae erit pre-

pretium; ad quod tantum recordari oportet, litteris π, π' et π'' certum praescriptum esse terminum veluti $\frac{1}{k}$ quem transgredi nunquam debent; quare etsi hoc casu valor $-\pi'' = \frac{1}{k}$ enormem magnitudinem pro Φ praebet, tamen hic etiam ad valorem ipsius π spectari conuenit, qui cum ante iam inuentus esset $\pi = -\Phi(k-1)$, ideoque $\pi = \frac{\pi''(k-1)}{m-k}$, maxime cauendum est, ne hinc prodeat $\pi > \pi''$. quamobrem litteram k iam non pro lubitu augere licebit, sed eo usque tantum, quoad fiat $k - 1 = m - k$, siue $k = \frac{m+1}{2}$, quae positio campum duplo maiorem, quam ante, produceret, scilicet $\Phi = \frac{-\pi''}{m-k} = \frac{-2\pi''}{m-1}$, quem ergo obtinere possemus, si modo marginem coloratum despiciamus. Tum autem pro eodem casu vltimi problematis forent distantiae determinatrices $b = \frac{-2\alpha}{m+1}$; $\beta = \infty = c$ et $\gamma = \frac{+2\theta\alpha}{m+1}$ et $d = \frac{-\theta\alpha}{m}$; vnde fit postremum interuallum $\gamma + d = + \vartheta\alpha\left(\frac{2}{m+1} - \frac{1}{m}\right) = \frac{\theta\alpha(m-1)}{m(m+1)}$ vbi adhuc ϑ nostro arbitrio permittitur, dummodo positiue capiatur; verum quia hoc modo margo coloratus praemagnus esset prodituros; huiusmodi telescopia nullo modo commendari poterunt atque hoc praeceptum etiam in posterum obseruabimus, nullaque alia telescopia exceptis tantum simplicissimis proferemus, nisi quae saltem a margine colorato sint immunia, siquidem tota haec confusio non vitari queat.

LIBRI