CAPUT III.

DE RESOLUTIONE AEQUATIONUM DIFFERENTIALIUM PRIMI GRADUS.

Problema 79.

469.

Si pro functione v trium variabilium x, y, z, posito
$$\partial v = p\,\partial x + q\,\partial y + r\,\partial z, \text{ fuerit}$$
$$\alpha p + \beta q + \gamma r = 0,$$
indolem functionis v definire.

Solutio.

Cum sit
$$\gamma\,\partial v = \gamma p\,\partial x + \gamma q\,\partial y - (\alpha p + \beta q)\,\partial z, \text{ erit}$$
$$\gamma\,\partial v = p(\gamma\,\partial x - \alpha\,\partial z) + q(\gamma\,\partial y - \beta\,\partial z),$$
ideoque ponendo
$$\gamma x - \alpha z = t \text{ et } \gamma y - \beta z = u,$$
habebitur
$$\gamma\,\partial v = p\,\partial t + q\,\partial u;$$
unde patet quantitatem v aequari functioni cuicunque binarum variabilium t et u, ita ut sit
$$v = \Gamma:(t \text{ et } u),$$
et restitutis valoribus assumtis
$$v = \Gamma:(\overline{\gamma x - \alpha z} \text{ et } \overline{\gamma y - \beta z}),$$

quae ergo est solutio problematis, si inter formulas differentiales proponatur haec conditio, ut sit

$$\alpha\left(\frac{\partial v}{\partial x}\right)+\beta\left(\frac{\partial v}{\partial y}\right)+\gamma\left(\frac{\partial v}{\partial z}\right)=0,$$

cujus itaque aequationis integrale clarius ita exhibetur

$$v=\Gamma:\left(\frac{x}{\alpha}-\frac{z}{\gamma}\text{ et }\frac{y}{\beta}-\frac{z}{\gamma}\right).$$

Corollarium 1.

470. Evidens est, hoc integrale etiam ita exprimi posse

$$v=\Gamma:\left(\frac{x}{\alpha}-\frac{y}{\beta}\text{ et }\frac{y}{\beta}-\frac{z}{\gamma}\right);$$

quandoquidem in genere ut supra observavimus, est

$$\Gamma:(x\text{ et }y)=\Delta:(t\text{ et }u),$$

siquidem t et u utcunque per x et y determinentur.

Corollarium 2.

471. Quin etiam affirmare licet, constitutis his tribus formulis

$$\frac{x}{\alpha}-\frac{y}{\beta},\ \frac{y}{\beta}-\frac{z}{\gamma},\ \frac{z}{\gamma}-\frac{x}{\alpha},$$

quantitatem v esse functionem quamcunque trium harum formularum; siquidem unaquaeque jam per binas reliquas datur, ac propterea v nihilominus functioni duarum tantum quantitatum variabilium aequatur.

Problema 80.

472. Si posito

$$\partial v=p\,\partial x+q\,\partial y+r\,\partial z,$$

haec conditio requiratur, ut sit

$$p x + q y + r z = n v, \text{ seu}$$
$$n v = x \left(\frac{\partial v}{\partial x}\right) + y \left(\frac{\partial v}{\partial y}\right) + z \left(\frac{\partial v}{\partial z}\right),$$

indolem hujus functionis v investigare.

Solutio.

Ex conditione praescripta capiatur valor $r = \frac{nv - px - qy}{z}$, quo substituto fit

$$\partial v - \frac{nv\partial z}{z} = p\left(\partial x - \frac{x\partial z}{z}\right) + q\left(\partial y - \frac{y\partial z}{z}\right), \text{ seu}$$
$$\partial v - \frac{nv\partial z}{z} = p z \partial . \frac{x}{z} + q z \partial . \frac{y}{z}.$$

Quo primum membrum integrabile reddatur, multiplicetur per $\frac{1}{z^n}$ ita ut jam habeamus

$$\partial . \frac{v}{z^n} = \frac{pz}{z^n} \partial . \frac{x}{z} + \frac{qz}{z^n} \partial . \frac{y}{z}.$$

Cum nunc quantitates p et q non sint determinatae, quoniam in genere ex tali aequatione

$$\partial V = P \partial X + Q \partial Y$$

sequitur

$$V = \Gamma : (X \text{ et } Y),$$

pro nostro casu colligimus

$$\frac{v}{z^n} = \Gamma : \left(\frac{x}{z} \text{ et } \frac{y}{z}\right), \text{ seu}$$
$$v = z^n \Gamma : \left(\frac{x}{z} \text{ et } \frac{y}{z}\right).$$

Si scilicet functio quaecunque binarum quantitatum $\frac{x}{z}$ et $\frac{y}{z}$ per z^n, seu etiam quod eodem redit per x^n vel y^n multiplicetur, oritur valor idoneus pro functione v conditioni praescriptae satisfaciens.

Corollarium 1.

473. Perspicuum autem est, formam $\Gamma : \left(\frac{x}{z} \text{ et } \frac{y}{z}\right)$ exprimere ejusmodi functionem, in qua tres variabiles x, y, z ubique constituant nullum dimensionum numerum, ac vicissim omnes hujusmodi functiones in forma illa contineri.

Corollarium 2.

474. Multiplicatione autem porro facta per z^n, oritur functio homogenea trium variabilium x, y, z, cujus dimensionum numerus est $= n$; unde solutio nostri problematis ita enunciari potest, ut quantitas quaesita v sit functio homogenea trium variabilium x, y et z, dimensionum numero existente $= n$.

Corollarium 3.

475. Quodsi ergo conditio praescripta sit

$$p x + q y + r z = 0, \text{ seu}$$

$$x \left(\frac{\partial v}{\partial x}\right) + y \left(\frac{\partial v}{\partial y}\right) + z \left(\frac{\partial v}{\partial z}\right) = 0,$$

quantitas v erit functio homogenea nullius dimensionis trium variabilium x, y et z.

Scholion.

476. Simili modo solutio succedit, si conditio praescripta postulet, ut sit

$$\alpha p x + \beta q y + \gamma r z = n v, \text{ seu}$$

$$\alpha x \left(\frac{\partial v}{\partial x}\right) + \beta y \left(\frac{\partial v}{\partial y}\right) + \gamma z \left(\frac{\partial v}{\partial z}\right) = n v,$$

tum enim ob

$$r = \frac{nv - \alpha p x - \beta q y}{\gamma z}, \text{ fit}$$

$$\partial v - \frac{nv\,\partial z}{\gamma z} = p\left(\partial x - \frac{\alpha x\,\partial z}{\gamma z}\right) + q\left(\partial y - \frac{\beta y\,\partial z}{\gamma z}\right),$$

quae aequatio sequenti forma exhibeatur

$$\frac{\gamma \partial v}{v} - \frac{n \partial z}{z} = \frac{px}{v}\left(\frac{\gamma \partial x}{x} - \frac{\alpha \partial z}{z}\right) + \frac{qy}{v}\left(\frac{\gamma \partial y}{y} - \frac{\beta \partial z}{z}\right),$$

ex qua concludimus, integrale primi membri $\gamma\, l v - n\, l z$ aequari functioni cuicunque binarum quantitatum

$$\gamma\, l x - \alpha\, l z \text{ et } \gamma\, l y - \beta\, l z,$$

et logarithmorum numeris sumtis fore

$$\frac{v^\gamma}{z^n} = \Gamma : \left(\frac{x^\gamma}{z^\alpha} \text{ et } \frac{y^\gamma}{z^\beta}\right).$$

Ponamus $\alpha = \frac{1}{\lambda}$, $\beta = \frac{1}{\mu}$ et $\gamma = \frac{1}{\nu}$, ut conditio praescripta sit

$$\frac{px}{\lambda} + \frac{qy}{\mu} + \frac{rz}{\nu} = n\, v,$$

et solutio reducetur ad hanc formam

$$v = z^{\nu n}\, \Delta : \left(\frac{x^\lambda}{z^\nu} \text{ et } \frac{y^\mu}{z^\nu}\right).$$

Quodsi porro scribamus

$$x^\lambda = X,\; y^\mu = Y \text{ et } z^\nu = Z, \text{ fiet}$$

$$v = Z^n\, \Delta : \left(\frac{X}{Z} \text{ et } \frac{Y}{Z}\right),$$

ideoque quantitas quaesita v est functio homogenea, in qua tres variabiles X, Y et Z ubique eundem dimensionum numerum $= n$ adimplent.

Problema 81.

477. Si posito

$$\partial v = p\, \partial x + q\, \partial y + r\, \partial z,$$

haec conditio praescribatur ut sit

$$p\, x + q\, y + r\, z = n v + S,$$

existente S functione quacunque data variabilium x, y, z, investigare naturam functionis quaesitae v.

Solutio.

Cum conditio praescripta praebet

$$r = \frac{nv + S - px - qy}{z}, \text{ erit}$$

$$\partial v - \frac{nv\partial z}{z} = \frac{S\partial z}{z} + p\left(\partial x - \frac{x\partial z}{z}\right) + q\left(\partial y - \frac{y\partial z}{z}\right),$$

seu

$$\partial \cdot \frac{v}{z^n} = \frac{S\partial z}{z^{n+1}} + \frac{p}{z^{n-1}}\,\partial \cdot \frac{x}{z} + \frac{q}{z^{n-1}}\,\partial \cdot \frac{y}{z}.$$

Sit $x = tz$ et $y = uz$, ut jam S fiat functio trium variabilium t, u et z, et formula differentialis $\frac{S\partial z}{z^{n+1}}$ ita integretur, ut quantitates t et u constantes habeantur, quo integrali posito $= V$, erit

$$v = Vz^n + z^n \Gamma : \left(\frac{x}{z} \text{ et } \frac{y}{z}\right),$$

ubi pars posterior significat functionem homogeneam trium variabilium x, y, z, numero dimensionum existente $= n$.

Corollarium 1.

478. Si S sit quantitas constans $= C$, erit

$$V = \int \frac{C\partial z}{z^{n+1}} = -\frac{C}{nz^n},$$

hincque primum integralis membrum

$$Vz^n = -\frac{C}{n},$$

ex quo perspicuum est eundem valorem proditurum fuisse, quantitatibus x, y, z inter se permutatis.

Corollarium 2.

479. Si S sit functio homogenea ipsarum x, y, z, dimensionum numero existente $= m$, quia tum posito $x = tz$ et $y = uz$, sit $S = Mz^m$, ita ut M tantum quantitates t et u involvat, ideo-

que pro constante sit habenda, prodit

$$V = \int Mz^{m-n-1}\partial z = \frac{Mz^{m-n}}{m-n} = \frac{S}{(m-n)z^n},$$

sicque primum integralis membrum erit $= \frac{S}{m-n}$.

Corollarium 3.

480. At si hoc casu sit $m = n$, fit

$$V = Mlz + C = Mlaz,$$

et primum integralis membrum

$$Mz^n laz = Slaz.$$

Pari jure id autem erit

$$Slby \text{ vel } Slax;$$

id quod satis est manifestum, cum horum valorum differentia fiat functio homogenea n dimensionum, ideoque in altero integralis membro contineatur.

Scholion.

481. Principium hujus solutionis in hoc lemmate latissime patente continetur, quod si fuerit

$$\partial V = S\partial Z = P\partial X + Q\partial Y,$$

ubi S denotat functionem datam, P et Q vero functiones indefinitas, futurum sit

$$V = \int S\partial Z + \Gamma : (X \text{ et } Y),$$

at hic non sufficit indicasse in integratione formulae $S\partial Z$, solam quantitatem Z pro variabili haberi, sed insuper notari convenit, binas X et Y tanquam constantes tractari debere. Quare si forte S sit proposita functio aliarum trium variabilium x, y, z, ex quibus hae X, Y, Z, quarum ratio hic est habenda, certo modo nascantur, primum loco x, y, z istae X, Y et Z introduci debent, ut fiat S funtio harum X, Y et Z; tum vero demum binis X et Y pro constantibus solaque Z pro variabili sumta, integrale $\int S\partial z$ est capiendum. Ita in casu problematis pro integrali $\int \frac{S\partial z}{z^{n+1}}$, quantitates $\frac{x}{z}$ et $\frac{y}{z}$ ut constantes sunt spectandae, sola z pro variabili sumta; ex quo in functione S statui oportet $x = tz$ et $y = uz$, ut S fiat functio ipsarum z, $t = \frac{x}{z}$ et $u = \frac{y}{z}$, quarum binae posteriores pro constantibus sunt habendae. Hoc ergo casu insignis error committeretur, si quis, sumta z variabili, reliquas x et y ut constantes tractare voluerit, quoniam ambae x et y etiam variabilem z involvere sunt censendae. Quod autem variabilibus permutatis primum integralis membrum idem resultare debeat, ut sit

$$z^n \int \frac{S\partial z}{z^{n+1}} = x^n \int \frac{S\partial x}{x^{n+1}}.$$

inde patet, quod posito $x = tz$ et $\partial x = t\partial z$, ob t constantem sumendam fiat

$$x^n \int \frac{S\partial x}{x^{n+1}} = t^n z^n \int \frac{St\partial z}{t^{n+1} z^{n+1}} = z^n \int \frac{S\partial z}{z^{n+1}};$$

in utraque enim integratione rationes variabilium $\frac{x}{z}$, $\frac{y}{z}$, $\frac{x}{y}$, pro constantibus sunt habendae, hincque in reductione facta quantitas $t = \frac{x}{z}$ recte ut constans spectatur.

Problema 82.

432. Si posito

$$\partial v = p\partial x + q\partial y + r\partial z,$$

haec conditio praescribatur, ut esse debeat

$$pL + qM + rN = 0,$$

existentibus L, M, N functionibus datis respective variabilium x,

y et z, nempe L ipsius x, M ipsius y et N ipsius z tantum, naturam functionis quaesitae v definire.

Solutio.

Ob $r = \frac{-pL - qM}{N}$ aequatio principalis fit

$$\partial v = p\left(\partial x - \frac{L\partial z}{N}\right) + q\left(\partial y - \frac{M\partial z}{N}\right), \text{ vel}$$

$$\partial v = pL\left(\frac{\partial x}{L} - \frac{\partial z}{N}\right) + qM\left(\frac{\partial y}{M} - \frac{\partial z}{N}\right).$$

Statuatur

$$t = \int\frac{\partial x}{L} - \int\frac{\partial z}{N} \text{ et } u = \int\frac{\partial y}{M} - \int\frac{\partial z}{N},$$

ut fiat

$$\partial v = pL\,\partial t + qM\,\partial u;$$

et manifestum est, quantitatem v aequari debere functioni cuicunque binarum variabilium t et u, quas ita quoque describere licet, ut positis formulis tribus integralibus $\int\frac{\partial x}{L}$, $\int\frac{\partial y}{M}$, et $\int\frac{\partial z}{N}$, pro t et u sumi oporteat differentias inter binas earum.

Scholion 1.

483. Solutio etiam successisset, dummodo $\frac{L}{N}$ fuisset functio ipsarum x et z, et $\frac{M}{N}$ ipsarum y et z tantum; tum enim multiplicatores P et Q ad integrationem apti quaeri debuissent, ut fieret

$$P\left(\partial x - \frac{L\partial z}{N}\right) = \partial t \text{ et } Q\left(\partial y - \frac{M\partial z}{N}\right) = \partial u,$$

et ob

$$\partial v = \frac{p\partial t}{P} + \frac{q\partial u}{Q}, \text{ foret}$$

$$v = \Gamma:(t \text{ et } u).$$

Permutandis vero variabilibus x, y et z, etiam alii casus resolubiles prodeunt. Quando autem quantitates L, M, N aliter sunt comparatae, via non patet certa ad solutionem perveniendi, quae certe haud parum abstrusa videtur, cum pro hoc casu satis simplici

$$(y + z) p + (x + z) q + (x + y) z = 0$$

per plures ambages tandem ad hanc pervenerim solutionem, ut posito

$$t = (x + y + z)(x - z)^2 \text{ et } u = (x + y + z)(y - z)^2,$$

fiat $v = \Gamma : (t \text{ et } u)$; quoniam igitur binae quantitates t et u, quarum functio quaecunque loco v posita conditioni satisfacit, hoc casu tantopere sunt complicatae, generaliter multo minus solutionem expectare licebit.

Scholion 2.

484. Ad plures autem alios casus solutio extendi potest. Si functiones datae L, M, N ita fuerint comparatae, ut alias E, F, G, H reperire liceat, quibus fiat

$$\mathrm{E}\left(\partial x - \frac{\mathrm{L}\partial z}{\mathrm{N}}\right) + \mathrm{F}\left(\partial y - \frac{\mathrm{M}\partial z}{\mathrm{N}}\right) = \partial t \text{ et}$$

$$\mathrm{G}\left(\partial x - \frac{\mathrm{L}\partial z}{\mathrm{N}}\right) + \mathrm{H}\left(\partial y - \frac{\mathrm{M}\partial z}{\mathrm{N}}\right) = \partial u,$$

tum enim posito

$$p = \mathrm{PE} + \mathrm{QG} \text{ et } q = \mathrm{PF} + \mathrm{QH}; \text{ fiet}$$

$$\partial v = \mathrm{P}\partial t + \mathrm{Q}\partial u,$$

ubi P et Q sunt functiones indefinitae loco p et q introductae, quantitas v aequabitur functioni cuicunque binarum variabilium t et u, seu erit

$$v = \Gamma : (t \text{ et } u).$$

Totum ergo negotium huc redit, ut pro datis functionibus L, M, N, functiones E, F et G, H inveniantur, quod quidem semper praestari posse videtur, sed haec ipsa quaestio plerumque difficilior evadit quam ipsa proposita. Sufficit autem binas ejusmodi functiones

E et F indeque quantitatem t investigasse; quia deinceps permutandis variabilibus x, y, z, una cum respondentibus L, M, N, sponte idoneus valor pro u elicitur. Ita in exemplo ante allato

$$L = y + z, \quad M = x + z, \quad N = x + y,$$

postquam invenerimus

$$t = (x + y + z)(x - z)^2,$$

sola permutatio statim praebet

$$u = (x + y + z)(y - z)^2,$$

vel etiam

$$u = (x + y + z)(x - y)^2,$$

Problema 83.

485. Si posito

$$\partial v = p\partial x + q\partial y + r\partial z$$

haec conditio praescribatur, ut sit $pqr = 1$, naturam functionis v investigare.

Solutio.

Ob $r = \frac{1}{pq}$, erit

$$\partial v = p\partial x + q\partial y + \frac{\partial z}{pq},$$

unde colligimus

$$v = px + qy + \frac{z}{pq} - \int \left(x\partial p + y\partial q - \frac{z\partial p}{ppq} - \frac{z\partial q}{pqq}\right),$$

qua transformatione id sumus assecuti, ut formula integralis bina tantum differentialia ∂p et ∂q involvat. His igitur in locum principalium introductis concludimus, illam formulam integralem aequari debere functioni cuicunque binarum variabilium p et q. Sit S talis functio, ut fiat

$$v = px + qy + \frac{z}{pq} - S,$$

et jam superest, ut cum litterae p et q in calculo retineantur aliae duae elidentur, id quod inde est petendum, quod sit

$$\partial S = (x - \tfrac{z}{ppq})\,\partial p + (y - \tfrac{z}{pqq})\,\partial q,$$

ideoque

$$x - \tfrac{z}{ppq} = (\tfrac{\partial S}{\partial p}) \text{ et } y - \tfrac{z}{pqq} = (\tfrac{\partial S}{\partial q}).$$

Nunc igitur solutio ita se habebit. Introductis his ternis variabilibus p, q et z, sumtaque binarum p et q functione quacunque S, capiatur

$$x = \tfrac{z}{ppq} + (\tfrac{\partial S}{\partial p}) \text{ et } y = \tfrac{z}{pqq} + (\tfrac{\partial S}{\partial q}),$$

ac tum functio quaesita v ita definietur, ut sit

$$v = \tfrac{3z}{pq} + p\,(\tfrac{\partial S}{\partial p}) + q\,(\tfrac{\partial S}{\partial q}) - S.$$

Vel si malimus v per ipsas tres variabiles x, y, z exprimere, ex binis aequationibus

$$x = \tfrac{z}{ppq} + (\tfrac{\partial S}{\partial p}) \text{ et } y = \tfrac{z}{pqq} + (\tfrac{\partial S}{\partial q})$$

quaerantur valores ipsarum p et q, quibus in functione S substitutis erit

$$v = px + qy + \tfrac{z}{pq} - S,$$

sicque quaesito erit satisfactum.

Corollarium 1.

486. Si functio S sumatur quantitas constans C, ob

$ppq = \frac{z}{x}$ et $pqq = \frac{z}{y}$, erit

$pq = \sqrt[3]{\frac{zz}{xy}}$, hincque

$p = \sqrt[3]{\frac{yz}{xx}}$ et $q = \sqrt[3]{\frac{xz}{yy}}$; unde fit

$v = 3\sqrt[3]{xyz} - C$,

qui est valor particularis problemati satisfaciens.

Corollarium 2.

487. Quoniam in conditione praescripta

$$pqr = 1, \quad \text{seu} \quad \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right)\left(\frac{\partial v}{\partial z}\right) = 1$$

tantum differentialia trium variabilium x, y et z occurrunt, eas quantitatibus constantibus quibusvis augere licet, unde nascitur solutio aliquanto latius patens

$$v = 3\sqrt[3]{(x+a)(y+b)(z+c)} + C.$$

Scholion 1.

488. Alius datur praeterea casus facilem evolutionem admittens, ponendo $S = 2c\sqrt{pq}$, unde colligitur

$$p = \frac{\sqrt{y}}{\sqrt{x}}\sqrt[3]{\frac{z}{\sqrt{xy}-c}} \quad \text{et} \quad q = \frac{\sqrt{x}}{\sqrt{y}}\sqrt[3]{\frac{z}{\sqrt{xy}-c}},$$

ideoque $S = 2c\sqrt[3]{\frac{z}{\sqrt{xy}-c}}$.

Assequimur ergo

$$v = 3\sqrt[3]{z(\sqrt{xy}-c)^2},$$

et permutandis variabilibus simili modo habebimus

$$v = 3\sqrt[3]{y(\sqrt{xz}-b)^2} \quad \text{et} \quad v = 3\sqrt[3]{x(\sqrt{yz}-a)^2},$$

ubi porro pro x, y, z scribere licet $x+f$, $y+g$, $z+h$. Caeterum patet solutionem generalem perinde succedere, si quantitas r functioni cuicunque ipsarum p et q aequari debeat, seu si inter p, q, r aequatio quaecunque proponatur.

Scholion 2.

489. Quodsi enim posito

$$\partial v = p\partial x + q\partial y + r\partial z,$$

inter ternas formulas

$$p = \left(\frac{\partial v}{\partial x}\right), \quad q = \left(\frac{\partial v}{\partial y}\right), \quad r = \left(\frac{\partial v}{\partial z}\right)$$

aequatio proponatur quaecunque, quae differentiata praebeat

$$P \partial p + Q \partial q + R \partial r = 0,$$

tum facto

$$S = \int (x \partial p + y \partial q + z \partial r),$$

ut sit

$$v = p x + q y + r z - S,$$

sumatur functio quaecunque trium quantitatum p, q, r, quae sit V, haecque differentiata praebeat

$$\partial V = L \partial p + M \partial q + N \partial r,$$

tum vero est

$$0 = P u \partial p + Q u \partial q + R u \partial r,$$

ideoque

$$\partial V = (L + P u) \partial p + (M + Q u) \partial q + (N + R u) \partial r,$$

quae forma ob novam introductam variabilem u latissime patet. Statuatur jam $S = V$, fietque

$$x = L + P u, \quad y = M + Q u, \quad z = N + R u,$$

ita ut nunc praeter variabiles p, q, r, quarum una per binas reliquas datur, nova habeatur u, ex quibus jam tres x, y et z ita definivimus, ut per eas vicissim hae p, q, r et u determinentur, tum vero erit

$$v = p x + q y + r z - V.$$

Quare pro V sumta quacunque functione trium quantitatum p, q, r, inter quas ejusmodi conditio praescribitur, ut sit

$$P \partial p + Q \partial q + R \partial r = 0,$$

sumatur

$$x = \mathrm{P}u + \left(\frac{\partial \mathrm{V}}{\partial p}\right), \; y = \mathrm{Q}u + \left(\frac{\partial \mathrm{V}}{\partial q}\right), \; z = \mathrm{R}u + \left(\frac{\partial \mathrm{V}}{\partial r}\right),$$

eritque

$$v = (\mathrm{P}p + \mathrm{Q}q + \mathrm{R}r)\,u + p\left(\frac{\partial \mathrm{V}}{\partial p}\right) + q\left(\frac{\partial \mathrm{V}}{\partial q}\right) + r\left(\frac{\partial \mathrm{V}}{\partial r}\right) - \mathrm{V},$$

quae solutio praecedenti ideo est anteferenda, quod in hanc tres quantitates p, q, r aequaliter ingrediuntur.

Problema 84.

490. Si posito

$$\partial v = p\,\partial x + q\,\partial y + r\,\partial z,$$

haec conditio praescribatur, ut esse debeat $pqr = \frac{v^3}{xyz}$, naturam functionis v definire.

Solutio.

Ponamus $p = \frac{\mathrm{P}v}{x}$, $q = \frac{\mathrm{Q}v}{y}$, $r = \frac{\mathrm{R}v}{z}$, et ob conditionem praescriptam debet esse $\mathrm{PQR} = 1$; tum vero erit

$$\frac{\partial v}{v} = \frac{\mathrm{P}\partial x}{x} + \frac{\mathrm{Q}\partial y}{y} + \frac{\mathrm{R}\partial z}{z}.$$

Statuamus nunc

$$lv = \mathrm{V}, \; lx = \mathrm{X}, \; ly = \mathrm{Y}, \; lz = \mathrm{Z},$$

et habebimus hanc aequationem

$$\partial \mathrm{V} = \mathrm{P}\,\partial \mathrm{X} + \mathrm{Q}\,\partial \mathrm{Y} + \mathrm{R}\,\partial \mathrm{Z},$$

pro qua esse debet $\mathrm{PQR} = 1$, quae quaestio cum non discrepet a problemate praecedente, eadem solutio huc quoque facillime transferetur.

Scholion.

491. Plures casus, quos forte in hoc capite expedire liceat, hic non evolvo, cum quia usus nondum perspicitur, tum vero im-

primis, quoniam hujus partis calculi integralis prorsus adhuc incognitae prima tantum principia adumbrare constitui. Pro formulis autem differentialibus altiorum graduum, quae in conditionem praescriptam ingrediantur, vix quicquam proferre licet, praeter quasdam observationes ad aequationes homogeneas pertinentes, quibus ergo hanc partem calculi integralis sum finiturus, simulque toti operi finem impositurus.