CALCULI INTEGRALIS
LIBER POSTERIOR.

PARS PRIMA,

SEU

INVESTIGATIO FUNCTIONUM DUARUM VARIABILIUM EX DATA DIFFERENTIALIUM CUJUSVIS GRADUS RELATIONE.

SECTIO TERTIA,

INVESTIGATIO DUARUM VARIABILIUM FUNCTIONUM EX DATA DIFFERENTIALIUM TERTII ALTIORUMQUE GRADUUM RELATIONE.

CAPUT I.

DE RESOLUTIONE AEQUATIONUM SIMPLICISSIMARUM UNICAM FORMULAM DIFFERENTIALEM INVOLVENTIUM.

Problema 61.

379.

Indolem functionis binarum variabilium x et y indagare, si ejus quaepiam formula differentialis tertii gradus evanescat.

Solutio.

Sit z functio illa quaesita, et cum ejus sint quatuor formulae differentiales tertii gradus

$$\left(\frac{\partial^3 z}{\partial x^3}\right),\ \left(\frac{\partial^3 z}{\partial x^2 \partial y}\right),\ \left(\frac{\partial^3 z}{\partial x\, \partial y^2}\right) \text{ et } \left(\frac{\partial^3 z}{\partial y^3}\right),$$

prout quaelibet harum nihilo aequalis statuitur, totidem habemus casus evolvendos.

I. Sit igitur primo $\left(\frac{\partial^3 z}{\partial x^3}\right) = 0$, et sumta y constante prima integratio praebet

$$\left(\frac{\partial \partial z}{\partial x^2}\right) = \Gamma : y,$$

tum simili modo secunda integratio dat

$$\left(\frac{\partial z}{\partial x}\right) = x\,\Gamma : y + \Delta : y,$$

unde tandem fit

$$z = \tfrac{1}{2} x x\,\Gamma : y + x\,\Delta : y + \Sigma : y,$$

ubi $\Gamma : y$, $\Delta : y$ et $\Sigma : y$ denotant functiones quascunque ipsius y, ita ut ob triplicem integrationem tres functiones arbitrariae in calculum sint ingressae, ut rei natura postulat.

II. Sit $\left(\frac{\partial^3 z}{\partial x^2 \partial y}\right) = 0$, ac primo bis integrando per solius x variabilitatem reperitur ut ante

$$\left(\frac{\partial z}{\partial y}\right) = x\,\Gamma' : y + \Delta' : y,$$

nunc autem sola y pro variabili habita, adipiscimur

$$z = x\,\Gamma : y + \Delta : y + \Sigma : x,$$

quandoquidem apices signis functionum inscripti hic semper hunc habent significatum, ut sit

$$\int \partial y\, \Gamma' : y = \Gamma : y \text{ et } \int \partial y\, \Delta' : y = \Delta : y.$$

III. Sit $\left(\frac{\partial^3 z}{\partial x \partial y^2}\right) = 0$, et quia hic casus a praecedente non differt, nisi quod binae variabiles x et y inter se sint permutatae, integrale quaesitum est

$$z = y\,\Gamma : x + \Delta : x + \Sigma : y.$$

IV. Sit $\left(\frac{\partial^3 z}{\partial y^3}\right) = 0$, et ob similem permutationem ex casu primo intelligitur fore

$$z = \tfrac{1}{2} y^2\, \Gamma : x + y\, \Delta : x + \Sigma : x.$$

Corollarium 1.

380. Tres functiones arbitrariae, hic per triplicem integrationem ingressae, sunt vel ipsius x, vel ipsius y tantum; omnes tres sunt ipsius y tantum casu primo $\left(\frac{\partial^3 z}{\partial x^3}\right) = 0$, ipsius x vero tantum casu quarto $\left(\frac{\partial^3 z}{\partial y^3}\right) = 0$; duae vero sunt ipsius y et una ipsius x casu secundo $\left(\frac{\partial^3 z}{\partial x^2 \partial y}\right) = 0$; contra autem duae ipsius x et una ipsius y casu tertio $\left(\frac{\partial^3 z}{\partial x \partial y^2}\right) = 0$,

Corollarium 2.

381. Porro observasse juvabit, si ejusdem variabilis puta x duae pluresve occurrant functiones arbitrariae, unam quidem absolute poni, alteram per y multiplicari, tertiam vero si adsit per $\frac{1}{2}yy$, seu quod eodem redit, per yy multiplicatam accedere.

Corollarium 3.

382. Perpetuo autem tenendum est has functiones ita arbitrio nostro relinqui, et etiam functiones discontinuae seu nulla continuitatis lege contentae non excludantur. Scilicet si libero manus tractu linea quaecunque describatur, applicata respondens abscissae x hujusmodi functionem $\Gamma : x$ referet.

Scholion 1.

383. Minus hic immorandum arbitror transformationi formularum differentialium altioris gradus, dum loco binarum variabilium x et y aliae quaecunque in calculum introducuntur, quoniam in genere expressiones nimis fierent complicatae vixque ullum usum habiturae, tum vero imprimis quod methodus has transformationes inveniendi jam supra (§. 229) satis luculenter est tradita. Casum tantum simpliciorem, quo binae novae variabiles t et u loco x et y introducendae ita accipiuntur, ut sit

$$t = \alpha x + \beta y \text{ et } u = \gamma x + \delta y,$$

hic quoque ad formulas differentiales altiores accommodabo. Cum igitur viderimus esse,

pro formulis primi gradus

$$\left(\frac{\partial z}{\partial x}\right) = \alpha \left(\frac{\partial z}{\partial t}\right) + \gamma \left(\frac{\partial z}{\partial u}\right),$$
$$\left(\frac{\partial z}{\partial y}\right) = \beta \left(\frac{\partial z}{\partial t}\right) + \delta \left(\frac{\partial z}{\partial u}\right),$$

et pro formulis secundi gradus

$$\left(\frac{\partial\partial z}{\partial x^2}\right) = \alpha^2 \left(\frac{\partial\partial z}{\partial t^2}\right) + 2\,\alpha\,\gamma \left(\frac{\partial\partial z}{\partial t\partial u}\right) + \gamma^2 \left(\frac{\partial\partial z}{\partial u^2}\right),$$

$$\left(\frac{\partial\partial z}{\partial x\partial y}\right) = \alpha\,\beta \left(\frac{\partial\partial z}{\partial t^2}\right) + (\alpha\,\delta + \beta\,\gamma) \left(\frac{\partial\partial z}{\partial t\partial u}\right) + \gamma\,\delta \left(\frac{\partial\partial z}{\partial u^2}\right),$$

$$\left(\frac{\partial\partial z}{\partial y^2}\right) = \beta^2 \left(\frac{\partial\partial z}{\partial t^2}\right) + 2\,\beta\,\delta \left(\frac{\partial\partial z}{\partial t\partial u}\right) + \delta^2 \left(\frac{\partial\partial z}{\partial u^2}\right),$$

erit pro formulis tertii gradus

$$\left(\frac{\partial^3 z}{\partial x^3}\right) = \alpha^3 \left(\frac{\partial^3 z}{\partial t^3}\right) + 3\alpha^2\gamma \left(\frac{\partial^3 z}{\partial t^2\partial u}\right) + 3\alpha\gamma^2 \left(\frac{\partial^3 z}{\partial t\partial u^2}\right) + \gamma^3 \left(\frac{\partial^3 z}{\partial u^3}\right),$$

$$\left(\frac{\partial^3 z}{\partial x^2\partial y}\right) = \alpha^2\beta \left(\frac{\partial^3 z}{\partial t^3}\right) + (\alpha^2\delta + 2\alpha\beta\gamma) \left(\frac{\partial^3 z}{\partial t^2\partial u}\right) + (\beta\gamma^2 + 2\alpha\gamma\delta) \left(\frac{\partial^3 z}{\partial t\partial u^2}\right) + \gamma^2\delta \left(\frac{\partial^3 z}{\partial u^3}\right),$$

$$\left(\frac{\partial^3 z}{\partial x\partial y^2}\right) = \alpha\beta^2 \left(\frac{\partial^3 z}{\partial t^3}\right) + (\beta\beta\gamma + 2\alpha\beta\delta) \left(\frac{\partial^3 z}{\partial t^2\partial u}\right) + (\alpha\delta^2 + 2\beta\gamma\delta) \left(\frac{\partial^3 z}{\partial t\partial u^2}\right) + \gamma\delta^2 \left(\frac{\partial^3 z}{\partial u^3}\right),$$

$$\left(\frac{\partial^3 z}{\partial y^3}\right) = \beta^3 \left(\frac{\partial^3 z}{\partial t^3}\right) + 3\beta^2\delta \left(\frac{\partial^3 z}{\partial t^2\partial u}\right) + 3\beta\delta^2 \left(\frac{\partial^3 z}{\partial t\partial u^2}\right) + \delta^3 \left(\frac{\partial^3 z}{\partial u^3}\right),$$

et pro formulis quarti gradus

	$\left(\frac{\partial^4 z}{\partial t^4}\right)$	$\left(\frac{\partial^4 z}{\partial t^3\partial u}\right)$	$\left(\frac{\partial^4 z}{\partial t^2\partial u^2}\right)$	$\left(\frac{\partial^4 z}{\partial t\partial u^3}\right)$	$\left(\frac{\partial^4 z}{\partial u^4}\right)$
$\left(\frac{\partial^4 z}{\partial x^4}\right)$	$=\alpha^4$	$+4\alpha^3\gamma$	$+6\alpha^2\gamma^2$	$+4\alpha\gamma^3$	$+\gamma^4$
$\left(\frac{\partial^4 z}{\partial x^3\partial y}\right)$	$=\alpha^3\beta$	$\alpha^3\delta+3\alpha^2\beta\gamma$	$3\alpha^2\gamma\delta+3\alpha\beta\gamma^2$	$+3\alpha\gamma^2\delta+\beta\gamma^3$	$+\gamma^3\delta$
$\left(\frac{\partial^4 z}{\partial x^2\partial y^2}\right)$	$=\alpha^2\beta^2$	$2\alpha^2\beta\delta+2\alpha\beta^2\gamma$	$\alpha^2\delta^2+4\alpha\beta\gamma\delta+\beta^2\gamma^2$	$2\alpha\gamma\delta^2+2\beta\gamma^2\delta$	$+\gamma^2\delta^2$
$\left(\frac{\partial^4}{\partial x\partial y^3}\right)$	$=\alpha\beta^3$	$3\alpha\beta^2\delta+\beta^3\gamma$	$3\alpha\beta\delta^2+3\beta^2\gamma\delta$	$\alpha\delta^3+3\beta\gamma\delta^2$	$+\gamma\delta^3$
$\left(\frac{\partial^4 z}{\partial y^4}\right)$	$=\beta^4$	$+4\beta^3\delta$	$+6\beta^2\delta^2$	$+4\beta\delta^3$	$+\delta^4$

unde simul lex pro altioribus gradibus elucet: pro formula scilicet generali $\left(\frac{\partial^{m+n} z}{\partial z^m \partial y^n}\right)$ hi coefficientes iidem sunt qui oriuntur ex evolutione hujus formae

$$(\alpha + \gamma\,v)^m\,(\beta + \delta\,v)^n,$$

siquidem termini secundum potestates ipsius v disponantur.

Scholion 2.

384. Haud alienum fore arbitror evolutionem istius formulae ex principiis ante stabilitis accuratius docere. Sit igitur

$$s = (\alpha + \gamma v)^m (\beta + \delta v)^n,$$

ac ponatur

$$s = A + Bv + Cv^2 + Dv^3 + Ev^4 + Fv^5 + \text{etc.}$$

ubi quidem primo patet esse $A = \alpha^m \beta^n$; pro reliquis vero coefficientibus inveniendis, sumtis differentialibus logarithmorum, habebimus

$$\frac{\partial s}{s \partial v} = \frac{m\gamma}{\alpha + \gamma v} + \frac{n\delta}{\beta + \delta v}, \text{ ideoque}$$

$$\frac{\partial s}{\partial v}[\alpha\beta + (\alpha\delta + \beta\gamma)v + \gamma\delta vv]$$

$$- s[m\beta\gamma + n\alpha\delta + (m+n)\gamma\delta v] = 0,$$

ubi si loco s series assumta substituatur, orietur haec aequatio

$$\begin{array}{llll}
0 = \alpha\beta B + 2\alpha\beta C v & + 3\alpha\beta D v^2 & + 4\alpha\beta E v^3 & + 5 A\beta F v^4 + \text{etc.} \\
\quad + \alpha\delta B & + 2\alpha\delta C & + 3\alpha\delta D & + 4\alpha\delta E \\
\quad + \beta\gamma B & + 2\beta\gamma C & + 3\beta\gamma D & + 4\beta\gamma E \\
 & + \gamma\delta B & + 2\gamma\delta C & + 3\gamma\delta D \\
- m\beta\gamma A - m\beta\gamma B & - m\beta\gamma C & - m\beta\gamma D & - m\beta\gamma E \\
- n\alpha\delta A \;\; - n\alpha\delta B & - n\alpha\delta C & - n\alpha\delta D & - n\alpha\delta E \\
\quad - (m+n)\gamma\delta A & - (m+n)\gamma\delta B & - (m+n)\gamma\delta C & - (m+n)\gamma\delta D
\end{array}$$

unde quilibet coefficiens ex praecedentibus ita definitur

$$A = \alpha^m \beta^n,$$

$$B = \frac{m\beta\gamma + n\alpha\delta}{\alpha\beta} A,$$

$$C = \frac{(m-1)\beta\gamma + (n-1)\alpha\delta}{2\alpha\beta} B + \frac{(m+n)\gamma\delta}{2\alpha\beta} A,$$

$$D = \frac{(m-2)\beta\gamma + (n-2)\alpha\delta}{3\alpha\beta} C + \frac{(m+n-1)\gamma\delta}{3\alpha\beta} B,$$

$$E = \frac{(m-3)\beta\gamma + (n-3)\alpha\delta}{4\alpha\beta} D + \frac{(m+n-2)\gamma\delta}{4\alpha\beta} C,$$

etc.

His igitur coefficientibus inventis, si ponatur

$$t = \alpha x + \beta y \text{ et } \gamma x + \delta y,$$

transformatio formulae differentialis cujuscunque ita se habebit, ut sit

$$\left(\frac{\partial^{m+n}z}{\partial x^m \partial y^n}\right) = A\left(\frac{\partial^{m+n}z}{\partial t^{m+n}}\right) + B\left(\frac{\partial^{m+n}z}{\partial t^{m+n-1}\partial u}\right)$$
$$+ C\left(\frac{\partial t^{m+n}z}{\partial t^{m+n-2}\partial u^2}\right) + \text{etc.}$$

Problema 62.

385. Indolem functionis binarum variabilium x et y investigare, si ejus formula differentialis cujuscunque gradus evanescat.

Solutio.

Ex iis quae de formulis differentialibus tertii gradus nihilo aequatis ostendimus in praecedente problemate, satis perspicuum est solutionem hujus problematis pro formulis differentialibus quarti gradus ita se habere.

I. Si sit $\left(\frac{\partial^4 z}{\partial x^4}\right) = 0$, erit

$$z = x^3\,\Gamma : y + x^2\,\Delta : y + x\,\Sigma : y + \Theta : y.$$

II. Si sit $\left(\frac{\partial^4 z}{\partial x^3 \partial y}\right) = 0$, erit

$$z = x^2\,\Gamma : y + x\,\Delta : y + \Sigma : y + \Theta : x.$$

III. Si sit $\left(\frac{\partial^4 z}{\partial x^2 \partial y^2}\right) = 0$, erit

$$z = x\,\Gamma : y + \Delta : y + y\,\Sigma : x + \Theta : x.$$

IV. Si sit $\left(\frac{\partial^4 z}{\partial x \partial y^3}\right) = 0$, erit

$$z = \Gamma : y + y^2\,\Delta : x + y\,\Sigma : x + \Theta : x.$$

V. Si sit $\left(\frac{\partial^4 z}{\partial y^4}\right) = 0$, erit

$$z = y^3 \Gamma : x + y^2 \Delta : x + y \Sigma : x + \Theta : x,$$

unde simul progressus ad altiores gradus est manifestus.

Corollarium 1.

386. Cum hic quatuor functiones arbitrariae occurrunt, totidem scilicet quot integrationes institui oportet, in hoc ipso criterium integrationis completae continetur.

Corollarium 2.

387. Quin etiam vicissim facile ostenditur, formas inventas aequationi propositae satisfacere. Sic cum pro casu tertio invenerimus:

$$z = x \Gamma : y + \Delta : y + y \Sigma : x + \Theta : x,$$

differentiando hinc colligimus

Primo $\left(\frac{\partial z}{\partial x}\right) = \Gamma : y + y \Sigma' : x + \Theta' : x$,

deinde $\left(\frac{\partial \partial z}{\partial x^2}\right) = y \Sigma'' : x + \Theta'' : x$,

tertio $\left(\frac{\partial^3 z}{\partial x^2 \partial y}\right) = \Sigma'' : x$ et

quarto $\left(\frac{\partial^4 z}{\partial x^2 \partial y^2}\right) = 0$,

eodemque pervenitur, quocunque ordine differentiationes, vel solam x vel solam y variabilem sumendo, instituantur.

Scholion 1.

388. Hactenus unam formulam differentialem nihilo esse aequalem assumsimus, calculus autem perinde succedit, si hujusmodi formula functioni cuicunque ipsarum x et y aequalis statuatur: quemadmodum in sequentibus problematibus sum ostensurus. Hoc tantum inculcandum censeo, si V fuerit functio quaecunque binarum variabilium x et y, tum $\int V \partial x$ id denotare integrale, quod obti-

netur si sola x pro variabili habeatur, in hac vero formula $\int V \partial y$ solam y pro variabili haberi: quod idem tenendum est de integrationibus repetitis veluti $\int \partial x \int V \partial x$, ubi in utraque sola x variabilis assumitur, in hac vero $\int \partial y \int V \partial x$, postquam integrale $\int V \partial x$ ex sola ipsius x variabilitate fuerit erutum, tum in altera integratione $\int \partial y \int V \partial x$ solam y variabilem accipiendam esse. Et cum perinde utra integratio prior instituatur, etiam hoc discrimen e modo signandi tolli potest, hocque integrale geminatum ita $\int\int V \partial x \partial y$ exhiberi: hincque intelligitur, quomodo has formulas

$$\int\int\int V \partial x^2 \partial y, \text{ seu } \int^3 V \partial x^2 \partial y \text{ et } \int^{m+n} V \partial x^m \partial y^n,$$

interpretari oporteat; hic scilicet signo integrationis $\int$ indices suffigimus, prorsus uti signo differentiationis ∂ suffigi solent, quippe qui indicant, quoties integratio sit repetenda.

Scholion 2.

389. Singulas has integrationes repetendas ita institui hic assumimus, ut nulla relatio inter binas variabiles x et y in subsidium vocetur, quae circumstantia eo diligentius est animadvertenda, cum vulgo, ubi talibus integrationibus opus est, calculus prorsus diverso modo institui debeat. Quodsi enim proposito quopiam corpore geometrico, ejus soliditas seu superficies sit investiganda per duplicem integrationem hujusmodi formula $\int\int V \partial x \partial y$ evolvi debet, existente V certa functione ipsarum x et y; ubi quidem primo quaeritur integrale $\int V \partial y$ spectata x ut constante; at absoluta integratione ad terminos integrationi praescriptos respici oportet, dum scilicet altero praescribitur, ut hoc integrale $\int V \partial y$ evanescat posito $y = 0$, altero vero id eo usque extendendum est, donec y datae cuipiam functioni ipsius x aequetur. Tum vero postquam hoc integrale $\int V \partial y$ isto modo fuerit determinatum, altera demum integratio formulae $\partial x \int V \partial y$ suscipitur, in qua quantitas y non amplius inest, dum ejus loco certa quaepiam functio ipsius x est sub-

stituta, eaque formula jam revera unicam variabilem x complectitur. Hic ergo prima integratione absoluta, variabilis y in functionem ipsius x abire est censenda, quam propterea in altera integratione, ubi x est variabilis, minime ut constantem spectare licebit. Ex quo patet hunc casum toto coelo esse diversum ab iis integrationibus repetendis, quas hic contemplamur, ad quem propterea hic eo minus respicimus, cum ista peculiaris ratio tantum in formula $\int\int V\,\partial x\,\partial y$ locum habere possit; reliquis vero ubi alterum differentiale ∂x vel ∂y saepius repetitur, adeo adversetur. Quam ob causam hinc omnem relationem, quae forte peracta una integratione inter binas variabiles x et y statui posset, merito removemus.

Problema 63.

390. Si formula quaepiam differentialis tertii altiorisve gradus aequetur functioni cuicunque binarum variabilium x et y, indolem functionis z definire.

Solutio.

Sit V functio quaecunque binarum variabilium x et y, et incipientes a formulis tertii ordinis sit primo $\left(\frac{\partial^3 z}{\partial x^3}\right) = V$, et posita sola x variabili erit

$$\left(\frac{\partial\partial z}{\partial x^2}\right) = \int V\,\partial x + \Gamma : y:$$

tum vero porro

$$\left(\frac{\partial z}{\partial x}\right) = \int\partial x\int V\,\partial x + x\,\Gamma : y + \Delta : y = \int\int V\,\partial x^2 + x\,\Gamma : y + \Delta : y,$$

ac denique

$$z = \int^3 V\,\partial x^3 + \tfrac{1}{2}x^2\,\Gamma : y + x\,\Delta : y + \Sigma : y.$$

Simili modo patet, si fuerit $\left(\frac{\partial^3 z}{\partial x^2\,\partial y}\right) = V$ fore

$$z = \int^3 V\,\partial x^2\,\partial y + x\,\Gamma : y + \Delta : y + \Sigma : x;$$

ac si sit $\left(\frac{\partial^3 z}{\partial x\,\partial y^2}\right) = V$, erit

$$z = \int^3 V \partial x \partial y^2 + \Gamma : y + y \Delta : x + \Sigma : x;$$ denique

si sit $\left(\frac{\partial^3 z}{\partial y^3}\right) = V$, erit

$$z = \int^3 V \partial y^3 + y^2 \Gamma : x + y \Delta : x + \Sigma : x.$$

Eodem modo ad formulas altiorum graduum progredientes, reperiemus ut sequitur:

si sit $\left(\frac{\partial^4 z}{\partial x^4}\right) = V$, fore

$$z = \int^4 V \partial x^4 + x^3 \Gamma : y + x^2 \Delta : y + x \Sigma : y + \Theta : y,$$

si sit $\left(\frac{\partial^4 z}{\partial x^3 \partial y}\right) = V$, fore

$$z = \int^4 V \partial x^3 \partial y + x^2 \Gamma : y + x \Delta : y + \Sigma : y + \Theta : x,$$

si sit $\left(\frac{\partial^4 z}{\partial x^2 \partial y^2}\right) = V$, fore

$$z = \int^4 V \partial x^2 \partial y^2 + x \Gamma : y + \Delta : y + y \Sigma : x + \Theta : x,$$

si sit $\left(\frac{\partial^4 z}{\partial x \partial y^3}\right) = V$, fore

$$z = \int^4 V \partial x \partial y^3 + \Gamma : y + y^2 \Delta : x + y \Sigma : x + \Theta : x,$$

si sit $\left(\frac{\partial^4 z}{\partial y^4}\right) = V$, fore

$$z = \int^4 V \partial y^4 + y^3 \Gamma : x + y^2 \Delta : x + y \Sigma : x + \Theta : x,$$

neque pro altioribus gradibus res eget ulteriori explicatione.

Corollarium 1.

391. Quemadmodum signum integrationis in primo libro usitatum jam per se involvit constantem per integrationem ingredientem, ita quoque hic functiones arbitrariae per integrationem ingressae jam in formula integrali involvi sunt censendae, ita ut non sit opus eas exprimere.

Corollarium 2.

392. Sufficit ergo pro aequatione $\left(\frac{\partial^3 z}{\partial x^3}\right) = V$ integrale tri-

plicatum hoc modo dedisse $z = \int^3 V \partial x^3$, quae forma jam potestate complectitur partes supra adjectas

$$x x \Gamma : y + x \Delta : y + \Sigma : y,$$

quod idem de reliquis est tenendum.

Corollarium 3.

393. Si ergo in genere haec habeatur aequatio

$$\left(\frac{\partial^{m+n} z}{\partial x^m \partial y^n}\right) = V,$$

ejus integrale statim hoc modo exhibetur

$$z = \int^{m+n} V \partial x^m \partial y^n,$$

quae potestate jam involvit omnes illas functiones arbitrarias numero $m+n$ per totidem integrationes invectas.

Scholion.

394. Hi casus utique sunt simplicissimi, qui ad hoc revehendi videntur, pro magis autem complicatis vix certa praecepta tradere licet, cum ista calculi integralis pars vix adhuc coli sit coepta. Interim tamen jam intelligitur, si aequationes magis complicatae ope cujusdam transformationis ad has simplicissimas revocare liceat, etiam earum integrationem in promtu esse futuram, quod quidem negotium hic non copiosius persequendum videtur. Progredior igitur ad casus magis reconditos, eosque ita comparatos, ut ope aequationum inferiorum ordinum expediri queant, unde quidem insignis methodus satis late patens colligi poterit, qua saepius haud sine successu uti licebit. Neque tamen in hac pertractatione nimis diffusum esse convenit, sed sufficiet praecipuos fontes adhuc quidem cognitos patefecisse.