CAPVT VI.

DE CONFVSIONE A DIVERSA RADIORVM INDOLE ORIVNDA.

Problema I.

287.

Si a puncto dato E radii per lentem PP transmittantur definire variationem in loco imaginis F, quae a diuersa radiorum refrangibilitate oritur.

Solutio.

Sit distantia puncti E ante lentem $AE = a$, facierum autem lentis radius anterioris $= f$ posterioris $= g$ et crassities $Aa = v$, quae quantitates sunt constantes. Posita nunc refractionis ratione ex aere in vitrum $= n : 1$, ob diuersam radiorum naturam numerus n erit variabilis, ideoque etiam locus imaginis F post lentem expressae, cuius distantia si ponatur $aF = \alpha$, erit ex supra inuentis

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+v} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-v}$$

vbi

vbi quantitas k etiam pro variabili eſt habenda, quia tantum a, f, g et v ſunt conſtantes. Quaeſtio ergo huc redit, vt ſi numerus n differentiali ſuo dn creſcere ſumatur, definiatur differentiale diſtantiae α. Quare differentientur ambae aequationes illae:

$$\frac{dn}{f} = \frac{2dn}{k+v} - \frac{2n\,dk}{(k+v)^2}$$

$$\frac{dn}{g} = \frac{-d\alpha}{\alpha\alpha} - \frac{2dn}{k-v} + \frac{2n\,dk}{(k-v)^2}.$$

indeque eliminato dk habebitur

$$\frac{dn(k+v)^2}{f} + \frac{dn(k-v)^2}{g} = 2\,dn(k+v) - 2dn(k-v) - \frac{d\alpha(k-v)^2}{\alpha\alpha}$$

Reſtituantur pro f et g valores initio poſiti, ac peruenietur ad hanc aequationem:

$$\frac{dn(k+v)^2}{a} + \frac{dn(k-v)^2}{\alpha} + 4v\,dn + \frac{(n-1)d\alpha}{\alpha\alpha}(k-v)^2 = 0$$

vnde reperitur:

$$\frac{d\alpha}{\alpha\alpha} = \frac{-dn}{(n-1)a}\left(\frac{k+v}{k-v}\right)^2 - \frac{dn}{(n-1)\alpha} - \frac{4v\,dn}{(n-1)(k-v)^2} \text{ ſeu}$$

$$d\alpha = \frac{-\alpha\,dn}{n-1}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right)$$

Tum vero cum etiam k ſit quantitas variabilis erit

$$dk = \frac{-(k+v)dn}{n(n-1)}\left(1 + \frac{k+v}{2a}\right)$$

Cum ergo poſuerimus $\frac{k-v}{k+v} = i$, ob $di = \frac{2v\,dk}{(k+v)^2}$ erit

$$di = \frac{-v\,dn}{n(n-1)}\left(\frac{1}{a} + \frac{2}{k+v}\right).$$

Coroll. I.

288. Si $n:1$ denotet rationem refractionis radiorum mediae naturae, vt ſit $n = \frac{31}{20} = 1,55$. erit

pro radiis rubris ſeu minime refractis $n = 1,54$, et violaceis $n = 1,56$. quorum valorum diſcrimen a medio, cum ſit $= \frac{1}{100}$, pro differentiali dn haberi poterit.

Coroll. 2.

289. Quare ſi α denotet diſtantiam imaginis a radiis mediis formatae, pro ea quae a rubris formatur, erit $dn = -\frac{1}{100}$ et $\frac{dn}{n-1} = \frac{-1}{55}$. Hinc diſtantia imaginis rubrae poſt lentem erit $= \alpha + \frac{\alpha}{55}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right)$ Diſtantia autem imaginis violaceae poſt lentem erit

$$\alpha - \frac{\alpha}{55}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right).$$

Coroll. 3.

290. Si craſſities lentis euaneſcat, vt ſit $v = 0$, ob variabilitatem numeri n erit

$$d\alpha = \frac{-\alpha\, dn}{n-1}\left(1 + \frac{\alpha}{a}\right) = \frac{-\alpha\alpha\, dn}{n-1}\left(\frac{1}{a} + \frac{1}{\alpha}\right)$$

Ac ſi diſtantia focalis lentis ponatur $= p$, cum ſit $\frac{1}{a} + \frac{1}{\alpha} = \frac{1}{p}$, erit $d\alpha = \frac{-\alpha\alpha\, dn}{(n-1)p} = \frac{-20\,\alpha\alpha dn}{11p}$.

Scholion.

291. Hoc ergo modo ob diuerſam radiorum naturam valor diſtantiae α immutatur, vnde ſi ab ea tanquam ab obiecto radii porro ad lentem ſecundam emittantur, fiet etiam reſpectu huius lentis diſtantia obiecti variabilis. Quam ob cauſam in loco imaginis ab ea formatae duplex variatio orietur: id quod deinceps etiam in lentibus ſequentibus multo

magis

magis eueniet. Hanc igitur variationem, quae pro quauis lente in loco imaginis nascitur, in sequente problemate determinemus.

Problema 2.

292. Si locus imaginis F, quae respectu lentis QQ vicem obiecti gerit, ob diuersam radiorum naturam ipse sit variabilis, determinare variationem, quam ob eandem causam imago sequens in G patietur.

Solutio.

Sit pro radiis mediae naturae, quibus respondet numerus n distantia obiecti F ante lentem $BF = b$; imaginisque inde proiectae distantia post lentem $bG = \beta$; dum autem n abit in $n + dn$, hae distantiae ambae b et β capiant sua incrementa differentialia db et $d\beta$. Ad quae inuenienda sit lentis QQ radius faciei anterioris $= f$, posterioris $= g$, et crassities $Bb = v$, eritque vt ante:

$$\frac{n-1}{f} = \frac{1}{b} + \frac{2n}{k+v} \quad \text{et} \quad \frac{n-1}{g} = \frac{1}{\beta} - \frac{2n}{k-v}$$

vbi k cum b et β pro variabili est habenda. Differentiatione ergo instituta habebitur.

$$\frac{dn}{f} = \frac{-db}{bb} + \frac{2dn}{k+v} - \frac{2n\,dk}{(k+v)^2}$$

$$\frac{dn}{g} = -\frac{d\beta}{\beta\beta} - \frac{2dn}{k-v} + \frac{2n\,dk}{(k-v)^2}$$

vnde eliminato dk, fit

$$\frac{dn(k+v)^2}{f} + \frac{dn(k-v)^2}{g} = \frac{-db}{bb}(k+v)^2 - \frac{d\beta}{\beta\beta}(k-v)^2 + 4v\,dn$$

 quae

quae multiplicata per $n-1$, ſi pro f et g valores dati ſubſtituantur prodit

$$\frac{dn(k+v)^2}{b}+2ndn(k+v)+\frac{dn(k-v)^2}{\mathfrak{b}}-2ndn(k-v)$$
$$=-\frac{(n-1)db}{bb}(k+v)^2-\frac{(n-1)d\mathfrak{b}}{\mathfrak{b}\mathfrak{b}}(k-v)^2+4(n-1)vdn$$

ſeu

$$\frac{dn(k+v)^2}{(n-1)b}+\frac{dn(k-v)^2}{(n-1)\mathfrak{b}}+\frac{4vdn}{n-1}+\frac{db}{bb}(k+v)^2+\frac{d\mathfrak{b}}{\mathfrak{b}\mathfrak{b}}(k-v)^2=0$$

Atque hinc elicitur:

$$d\mathfrak{b}=-\frac{\mathfrak{b}\mathfrak{b}db}{bb}\left(\frac{k+v}{k-v}\right)^2-\frac{\mathfrak{b}dn}{n-1}\left(1+\frac{\mathfrak{b}}{b}\left(\frac{k+v}{k-v}\right)^2+\frac{4\mathfrak{b}v}{(k-v)^2}\right)$$

Pro variabilitate autem ipſius k reperietur

$$\frac{dk}{(k+v)^2}=-\frac{db}{2nbb}-\frac{dn}{2n(n-1)b}-\frac{dn}{n(n-1)(k+v)}$$

Quare ſi ponatur $\frac{k-v}{k+v}=i$, ob $di=\frac{2vdk}{(k+v)^2}$ erit

$$di=\frac{-vdb}{nbb}-\frac{vdn}{n(n-1)b}-\frac{2vdn}{n(n-1)(k+v)}$$

ac ſi loco k numerus i introducatur, erit

$$d\mathfrak{b}=\frac{-\mathfrak{b}\mathfrak{b}db}{iibb}-\frac{\mathfrak{b}dn}{n-1}\left(1+\frac{\mathfrak{b}}{iib}+\frac{(1-i)^2\mathfrak{b}}{iiv}\right) \text{ et}$$

$$di=\frac{-vdb}{nbb}-\frac{vdn}{n(n-1)}\left(\frac{1}{b}+\frac{1-i}{v}\right).$$

Coroll. 1.

293. Inuenta aequatio differentialis etiam hac forma repraeſentari poteſt vt ſit

$$\frac{id\mathfrak{b}}{\mathfrak{b}\mathfrak{b}}+\frac{db}{ibb}=\frac{-dn}{n-1}\left(\frac{i}{\mathfrak{b}}+\frac{1}{ib}+\frac{(1-i)^2}{iv}\right)$$

ſiue

fiue reftituendo k

$$\frac{d\mathfrak{b}}{\mathfrak{b}\mathfrak{b}}\left(\frac{k-v}{k+v}\right)+\frac{db}{bb}\left(\frac{k+v}{k-v}\right)=\frac{-dn}{n-1}\left(\frac{1}{\mathfrak{b}}\left(\frac{k-v}{k+v}\right)+\frac{1}{b}\left(\frac{k+v}{k-v}\right)+\frac{4v}{kk-vv}\right)$$

vbi haec obferuanda eft analogia, vt quemadmodum ad b refertur $\frac{k+v}{k-v}$, ita ad $\mathfrak{b}$ referatur $\frac{k-v}{k+v}$.

Coroll. 2.

294. Si lentis huius craffities euanefcat, fit $v=0$, et $i=1$ vbi figura lentis non amplius in computum ingreditur fed fola diftantia focalis, vnde variatio in loco imaginis G ita erit comparata, vt fit

$$\frac{d\mathfrak{b}}{\mathfrak{b}\mathfrak{b}}+\frac{db}{bb}=\frac{-dn}{n-1}\left(\frac{1}{\mathfrak{b}}+\frac{1}{b}\right) \text{ ideoque}$$

$$d\mathfrak{b}=\frac{-\mathfrak{b}\mathfrak{b}}{bb}db-\frac{\mathfrak{b}\mathfrak{b}dn}{n-1}\left(\frac{1}{\mathfrak{b}}+\frac{1}{b}\right).$$

Coroll. 3.

295. Si cafum a radiis mediae naturae, ad quos formulae hactenus traditae funt accommodatae, ad radios rubros transferre velimus, poni oportet $dn=+\frac{1}{100}$ $dn=-\frac{1}{100}$, fin autem ad radios violaceos

Problema 3.

296. Si radii ab obiecto E per lentes quotcunque transmittantur, determinare variationem in locis fingularum imaginum, quae a diuerfa radiorum refrangibilitate proficifcitur.

 Solutio.

Solutio.

Retineantur omnes denominationes, quibus in ſuperioribus capitibus ſumus vſi, ac ſint $a, \alpha, b, \mathfrak{b}, c, \gamma$ etc. diſtantiae determinatrices lentium pro radiis mediae naturae. Variata ergo ratione refractionis etiam hae diſtantiae variabuntur, quarum variationes differentialibus indicemus. Cum autem diſtantiae inter binas lentes maneant conſtantes, illae variationes ita erunt comparatae vt ſit

$$d\alpha + db = 0,\ d\mathfrak{b} + dc = 0,\ d\gamma + d.d = 0 \text{ etc.}$$

Cum iam diſtantia obiecti $AE = a$ ſit inuariabilis erit ex problemate primo ſi ponatur $\frac{k-v}{k+v} = i$

$$d\alpha = -db = \frac{-\alpha\alpha dn}{i(n-1)}\left(\frac{1}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right) = \frac{-\alpha\alpha dn}{n-1} \cdot \frac{k+v}{k-v}\left(\frac{k-v}{\alpha(k+v)} + \frac{k+v}{a(k-v)} + \frac{4v}{kk-vv}\right)$$

$$\text{et } di = \frac{-v\,dn}{n(n-1)}\left(\frac{1}{a} + \frac{1-i}{v}\right)$$

Deinde pro ſecunda lente, ad quam referuntur diſtantiae determinatrices b, et $\mathfrak{b}$ cum arbitraria k' et craſſitie v', vnde fecimus $\frac{k'-v'}{k'+v'} = i'$, habebimus

$$d\mathfrak{b} = -dc = \frac{-\mathfrak{b}\mathfrak{b}\,db}{i'i'bb} - \frac{\mathfrak{b}\mathfrak{b}\,dn}{i'(n-1)}\left(\frac{i'}{\mathfrak{b}} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)$$

$$\text{et } di' = \frac{-v'\,db}{nbb} - \frac{v'\,dn}{n(n-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

Simili modo pro tertia lente, ad quam referuntur diſtantiae determinatrices c et γ cum arbitraria k'' et craſſitie v'' poſito $\frac{k''-v''}{k''+v''} = i''$ adipiſcemur:

$$d\gamma = -dd = \frac{-\gamma\gamma\,dc}{i''i''cc} - \frac{\gamma\gamma\,dn}{i''(n-1)}\left(\frac{i''}{\gamma} + \frac{1}{i''c} + \frac{(1-i'')^2}{i''v''}\right)$$

$$\text{et } di'' = \frac{-v''\,dc}{ncc} - \frac{v''\,dn}{n(n-1)}\left(\frac{1}{c} + \frac{1-i''}{v''}\right)$$

atque

atque vlterius progrediendo obtinebimus ſequentes formulas:

$$d\delta = -de = \frac{-\delta\delta\, dd}{i''' i''' dd} - \frac{\delta\delta\, dn}{i'''(n-1)}\left(\frac{i'''}{\delta} + \frac{1}{i''' d} + \frac{(1-i''')^2}{i''' v'''}\right)$$

$$\text{et } di''' = \frac{-v''' dd}{n\, dd} - \frac{v''' dn}{n(n-1)}\left(\frac{1}{d} + \frac{1-i'''}{v'''}\right)$$

vnde haec differentialia facile ad quotcunque lentes extenduntur. Atque ſi hic ſucceſſiue valores differentialium dd, dc, db, iam ante definiti ſubſtituantur omnia haec differentialia tam diſtantiarum determinatricium, quam numerorum i, i', i'', i''' etc. per differentiale dn exprimentur.

Si ratio refractionis pro ſingulis lentibus ſit diuerſa pro iisque ordine exprimatur numeris, n, n', n'' etc. perſpicuum eſt, differentialia hic inuenta ſequenti modo expreſſum iri:

$$\text{I. } d\alpha = -db = \frac{-\alpha\alpha\, dn}{i(n-1)}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right)$$

$$di = \frac{-v\, dn}{n(n-1)}\left(\frac{1}{a} + \frac{1-i}{v}\right)$$

$$\text{II. } d\beta = -dc = \frac{-\beta\beta\, db}{i' i'.\, bb} - \frac{\beta\beta\, dn'}{i'(n'-1)}\left(\frac{i'}{\beta} + \frac{1}{i'b} + \frac{(1-i')^2}{i' v'}\right)$$

$$di' = \frac{-v' db}{n' bb} - \frac{v' dn'}{n'(n'-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

$$\text{III. } d\gamma = -dc = \frac{-\gamma\gamma\, dc}{i''.\, i''.\, cc} - \frac{\gamma\gamma\, dn''}{i''(n''-1)}\left(\frac{i''}{\gamma} + \frac{1}{i'' c} + \frac{(1-i'')^2}{i''.\, v''}\right)$$

$$di'' = \frac{-v'' dc}{n'' cc} - \frac{v'' dn''}{n''(n''-1)}\left(\frac{1}{c} + \frac{1-i''}{v''}\right)$$

etc.

ex quibus formulis etiam mutationes ſingularum imaginum ac proinde etiam tandem vltimae imaginis

facile

facile definiri poterunt, seu potius loco imaginum angulos, sub quibus eae oculo ad iustam distantiam l positae sint adpariturae, consideremus.

Pro vna lente $\frac{1}{i} \cdot \frac{\alpha}{a} \cdot \frac{z}{l}$

Pro duabus lentibus $\frac{1}{i.i'} \cdot \frac{\alpha \beta}{a b} \cdot \frac{z}{l}$

Pro tribus lentibus $\frac{1}{i.i'.i''} \cdot \frac{\alpha \beta \gamma}{a b c} \cdot \frac{z}{l}$

etc.

quatenus scilicet praeter distantias $a, \alpha; b, \beta; c, \gamma$ etc. etiam litterae i, i', i'', i''' etc. sunt variabiles.

Coroll. 1.

297. Hinc igitur per differentiationem definire licet, quanta mutatio in loco vltimae imaginis, quae obiectum visionis constituit, ob diuersam radiorum refrangibilitatem oriri debeat.

Coroll. 2.

298. Deinde cum etiam magnitudinem cuiusque imaginis supra per distantias determinatrices et numeros i, i', i'' etc. definiuerimus, pro magnitudine imaginis habetur $\frac{1}{i\,i'\,i''\,i'''} \cdot \frac{\alpha\beta\gamma\delta}{a\,b\,c\,d} \cdot \frac{z}{l}$ (189). simili modo mutatio assignari potest, quam magnitudo vltimae imaginis ob diuersam refrangibilitatem radiorum patietur.

Coroll. 3.

299. Cognita autem vtraque mutatione, quam vltima imago tam respectu loci, quam magnitudinis subit,

fubit, non difficulter colligetur, quanta confufione ipfa vifio ob diuerfam radiorum refrangibilitatem perturbetur.

Coroll. 4.

300. Si craffities lentium euanefcat fiet

$$d\alpha = -db = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

$$d\mathfrak{E} = -dc = -\frac{\mathfrak{E}\mathfrak{E}db}{bb} - \frac{\mathfrak{E}\mathfrak{E}dn}{n-1}\left(\frac{1}{b}+\frac{1}{\mathfrak{E}}\right)$$

$$d\gamma = -dd = \frac{-\gamma\gamma dc}{cc} - \frac{\gamma\gamma dn}{n-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

$$d\delta = -de = \frac{-\delta\delta dd}{dd} - \frac{\delta\delta dn}{n-1}\left(\frac{1}{d}+\frac{1}{\delta}\right)$$

etc.

numeri autem i, i', i'' etc. abeunt in vnitatem nullique mutationi amplius funt obnoxii.

Si ergo craffities lentium euanefcat, pro diuerfa refractione fingularum lentium formulae fuperiores abibunt in fequentes:

I. $d\alpha = -db = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$

II. $d\mathfrak{E} = -dc = -\frac{\mathfrak{E}\mathfrak{E}db}{bb} - \frac{\mathfrak{E}\mathfrak{E}dn'}{n'-1}\left(\frac{1}{\mathfrak{E}}+\frac{1}{b}\right)$

III. $d\gamma = -dd = \frac{-\gamma\gamma dc}{cc} - \frac{\gamma\gamma dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$

etc.

Ceterum per fe manifeftum eft, quando §. 288 dn vel $+\frac{1}{100}$ vel $-\frac{1}{100}$ fignificare dicitur, id tantum de illa vitri fpecie, pro qua eft refractio radiorum me-

mediorum $n = \frac{31}{20}$ esse intelligendum; et pro aliis vitri speciebus differentialia dn', dn'', dn''', etc. haud mediocriter ab $\frac{1}{100}$ discrepare posse. Quanta autem futura sit haec diuersitas, optandum esset, vt ea potius experimentis, quam ex theoria quapiam definiretur.

Scholion

301. Cum igitur ob diuersam radiorum refrangibilitatem cuique imagini duplex alteratio inducatur, quarum altera eius magnitudinem, altera vero eius locum afficit, duplex inde confusio in visionem infertur. Si enim formulae in superioribus capitibus exhibitae ad radios mediae naturae restringantur, pro quibus est $n = \frac{31}{20}$, posito $dn = -\frac{1}{100}$, ex formulis hic traditis differentialibus locus et magnitudo imaginis a radiis rubris formatae definietur, posito autem $dn = +\frac{1}{100}$, locus et magnitudo imaginis violaceae declarabitur.

Tab. III. Fig. 16. Scilicet si Ii fuerit imago vltima visioni obiecta, quae a radiis mediae naturae formatur, per formulas modo inuentas, prout vel $dn = -\frac{1}{100}$ vel $dn = +\frac{1}{100}$ ponatur, definietur tam imago rubra $R\varrho$ quam violacea Vv; atque ex natura differentialium manifestum est cum interualla IR et IV inter se aequalia esse debere, tum etiam differentias $Ii - R\varrho$ et $Vv - Ii$, ita vt oculo series innumerabilium imaginum inter extremas $R\varrho$ et Vv sitarum simul cernenda offeratur, vnde eo maior confusio oriatur necesse est, quo maior fuerit differentia tam ratione loci quam magnitudinis. Quare haec confusio penitus tolleretur, si eiusmodi lentium dispositio definiri posset, vt tam interuallum RV, quam

quam differentia inter imagines $R\varrho$ et Vv ad nihilum redigeretur, quod vtrumque nisi simul praestari queat, confusionem perfecte tollere non licet. Verumtamen etiamsi neutri harum conditionum satisfieri possit, tamen dabitur pro oculo eiusmodi locus O, vbi confusio minime sensibilis percipiatur, qui erit in concursu rectae $v\varrho$ productae cum axe: ibi enim omnes extremitates $\varrho\iota v$ communibus radiis cernentur, neque propterea extremitas obiecti colore tincta apparebit. Quare si simul punctum O conueniat cum loco oculi idoneo alia confusio non percipietur, nisi quae inde originem trahit, quod forte imagines extremae $R\varrho$ et Vv nimis a distantia iusta discrepent, siquidem media $I\iota$ ad distantiam iustam ob oculo fuerit remota. Neque tamen hinc ora obiecti coloribus iridis cincta apparebit, cui confusionis speciei maxime est occurrendum; ideoque ea quae adhuc adfuerit confusio facile tolerari poterit, quae vero etiam omnino tolleretur si modo interuallum RV vel in nihilum redigi vel saltem satis paruum reddi posset. Hinc ergo intelligimus vitium illud, quo obiecta coloribus iridis circumdata saepe repraesentantur, non tam necessario cum instrumentis dioptricis esse coniunctum, vt nullo pacto ab iis separari queat, quamobrem eo magis operae erit pretium, vt investigemus quomodo haec instrumenta ab isto vitio liberari possint. Quae tota investigatio huc redit, vt determinetur punctum O, vbi recta per terminos imaginum v, ι, ϱ ducta cum axe concurrit hocque punctum cum loco oculi iam ſupra definito conueniens reddatur, ſi quidem fieri poteſt: vnde perſpicitur locum oculi O hac proprietate praeditum eſſe oportere, vt angulus, ſub quo vltima imago cernitur ob variabilitatem numero n tributam nullam mutationem patiatur. Tum vero inſuper videndum erit, num interualla IR et IV vel ad nihilum reduci, vel minima reddi queant.

Problema 4.

Tab. III. Fig. 12. 302. Propoſita vnica lente definire locum oculi, vnde obiectum ſine margine colorato cernatur.

Solutio.

Sit obiecti $E\varepsilon$ ante lentem diſtantia $EA = a$, imago vero per radios mediae naturae in $F\zeta$ repraeſentetur, ponaturque $aF = \alpha$. Pro lente vero ſit eius craſſities $Aa = v$, et quantitas arbitraria $= k$, vnde capiatur $\frac{k-v}{k+v} = i$. Hinc poſito $E\varepsilon = z$ erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z$ (86): quare ſi pro loco oculi ſtatuatur diſtantia $aO = O$, quae eſt fixa, erit $OF = \alpha - O$, et anguli $FO\zeta$ tangens $= \frac{1}{i} \frac{\alpha}{a} \frac{z}{\alpha - O}$, quae formula ob diuerſam radiorum refrangibilitatem nullam mutationem ſubire debet. Inde autem quantitates α et i tantum variantur, dum reliquae manent conſtantes. Quare iſtius formulae differentiale logarithmicum nihilo aequale poſitum praebet hanc aequationem

$$-\frac{di}{i} + \frac{d\alpha}{\alpha} - \frac{d\alpha}{\alpha - O} = 0 \quad \text{ſeu} \quad \frac{-di}{i} - \frac{O\,d\alpha}{\alpha(\alpha - O)} = 0$$

vbi

vbi si valores supra inuenti substituantur, prodit

$$\frac{v\,dn}{in(n-1)}\left(\frac{1}{a}+\frac{1-i}{v}\right)+\frac{O\alpha\,dn}{i(n-1)(\alpha-O)}\left(\frac{i}{\alpha}+\frac{1}{ia}+\frac{(1-i)^2}{iv}\right)=0$$

quae aequatio per $\frac{dn}{i(n-1)}$ diuisa praebet:

$$\frac{v}{na}+\frac{1-i}{n}+\frac{O\alpha}{\alpha-O}\left(\frac{i}{\alpha}+\frac{1}{ia}+\frac{(1-i)^2}{iv}\right)=0$$

vnde locus oculi definiri poterit: qui si debeat conuenire cum supra inuento (238), vbi inuenimus

$O=\frac{-i\alpha v}{n\alpha-iv}$, erit

$\alpha-O=\frac{n\alpha\alpha}{n\alpha-iv}$ et $\frac{O\alpha}{\alpha-O}=\frac{-iv}{n}$, hincque nostra aequatio per n multiplicata abit in:

$$\frac{v}{a}+1-i-\frac{iiv}{\alpha}-\frac{v}{a}-(1-i)^2=0$$

seu $i-ii-\frac{iiv}{\alpha}=0$, ideoque $i=\frac{\alpha}{\alpha+v}=\frac{k-v}{k+v}$. Quamobrem quantitatem arbitrariam k ita definiri conueniet vt sit $k=2\alpha+v$: et cum sit $i=\frac{\alpha}{\alpha+v}$ pro loco oculi habebimus $O=\frac{-\alpha v}{n\alpha+(n-1)v}=\frac{-20\,\alpha v}{31\alpha+11v}$ ob $n=\frac{31}{20}$.

Quod si porro hinc variationem in loco imaginis desideremus definiri oportet differentiale $d\alpha$, quod fiet:

$$d\alpha=\frac{-\alpha\alpha\,dn}{i(n-1)}\left(\frac{i}{\alpha}+\frac{1}{ia}+\frac{(1-i)^2}{iv}\right)$$

et pro i posito valore $\frac{\alpha}{\alpha+v}$

$$d\alpha=\frac{-(\alpha+v)dn}{n-1}\left(1+\frac{\alpha+v}{a}\right)$$

qui valor si ad nihilum redigi posset, confusio omnis a diuersa radiorum refrangibilitate oriunda perfecte tolleretur.

Coroll. 1.

303. Pro lentis ergo conſtructione quantitas arbitraria k ita accipi debet, vt ſit $k = 2\alpha + v$; atque tum oculus in eo loco conſtitutus, vbi totum campum apparentem percipiat, ſimul nullam confuſionem a diuerſa radiorum indole ſentiet.

Coroll. 2.

304. Vt autem oculus ſimul imaginem in diſtantia iuſta aſpiciat, oportet ſit $\alpha - O = -l$, ideoque $l = \frac{-11\alpha(\alpha+v)}{21\alpha+11v}$. Vnde colligitur: $\alpha = \frac{-l}{2} - \frac{v}{2} - \sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$, hincque
$O = \frac{1}{2}l - \frac{1}{2}v - \sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$ et $k = -l - 2\sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$

Coroll. 3.

305. Poteſt vero inſuper effici, vt etiam $d\alpha$ euaneſcat, quod euenit ſi $a + \alpha + v = 0$, hoc eſt $a = \frac{1}{2}l - \frac{1}{2}v + \sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$. Verum cum hoc caſu ob $\alpha = -a - v$ imago in ipſum obiectum cadet, ita, vt radii nullam refractionem pati ſint cenſendi, viſio per lentem perinde erit comparata atque nudis oculis.

Coroll. 4.

306. Si craſſities lentis v plane euaneſcat, tam ob campum apparentem, quam diuerſam radiorum refrangibilitatem fit $O = 0$ hoc ergo caſu oculus lenti immediate applicatus nullam confuſionem ob diuerſam radiorum naturam percipiet. Dum ergo fuerit $\alpha = -l$ viſio erit diſtincta.

Scho-

Scholion.

307. Hic scilicet penitus mentem abstrahimus a confusione iam supra determinata, quae a lentium apertura oritur, ideoque aperturam primae lentis vt euanescentem spectamus. Eam igitur hic tantum confusionis speciem contemplamur, quae a diuersa radiorum refrangibilitate originem ducit; quam plerumque tolli obseruauimus, si angulus ad O inuariabilis reddatur; tum enim ora obiecti satis bene terminata conspicietur, neque coloribus iridis cincta. Interim tamen adhuc aliqua confusio sentiri poterit inde oriunda, quod si imago media iustam ab oculo distantiam teneat, imaginum extremarum altera sit nimis propinqua alter animis remota, verum si earum interuallum non sit admodum magnum, confusio haec parum erit sensibilis. Ita hic inuenimus, quod experientia satis comprobatur, si obiecta per vnicam lentem spectemus, ea margine colorato destituta apparere dummodo oculus immediate applicetur quod si quando secus euenire videatur, causa aperturae lentis sine dubio erit tribuenda, cui conditioni rationes hic allegatae refragantur.

Problema 5.

308. Si instrumentum dioptricum duabus instructum sit lentibus, definire locum oculi, vnde obiectum sine margine colorato videatur. Tab. III. Fig. 13.

Solutio

Solutio.

Posita obiecti distantia $AE = a$, sint pro radiis mediae naturae reliquae distantiae determinatrices $aF = \alpha$, $BF = b$ et $bG = \beta$ crassities vero lentium $Aa = v$, $Bb = v'$, et distantiae arbitrariae k et k', ponaturque $\frac{k-v}{k+v} = i$ et $\frac{k'-v'}{k'+v'} = i'$. His positis si magnitudo obiecti $E\epsilon$ vocetur $= z$, erit imago $G\eta = \frac{1}{ii'}\frac{\alpha\beta}{ab}z$, vnde si oculi distantia ponatur $bO = O$,

ob $GO = \beta - O$, erit anguli $GO\eta$ tangens $\frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} \cdot \frac{z}{\beta - O}$ cuius differentiale logarithmicum nihilo aequatum praebet,

$$-\frac{di}{i} - \frac{di'}{i'} + \frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\beta}{\beta} - \frac{d\beta}{\beta - O} = 0$$

quae valoribus supra (296) inuentis substitutis abit in:

$$\frac{v\,dn}{in(n-i)}\left(\frac{1}{a} + \frac{1-i}{v}\right) + \frac{v'\,db}{n'i'bb} + \frac{v'\,dn'}{i'n'(n'-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

$$-\frac{db}{\alpha} - \frac{db}{b} + \frac{O}{\beta - O}\left(\frac{\beta\,db}{i'i'bb} + \frac{\beta\,dn'}{i'(n'-1)}\left(\frac{i'}{\beta} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)\right) = 0$$

Verum conditio campi exigebat $O = \frac{\beta\beta}{\beta + \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab}z}$, existente

$$\beta = \left(\frac{i'}{i} \cdot \frac{\alpha + b}{a} - \frac{i'bv}{n'a\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n'ab}\right)z, \quad (245), \text{ vnde fit}$$

$$\frac{O}{\beta - O} = \frac{\beta\beta}{\frac{1}{ii'} \cdot \frac{\alpha\beta\beta}{ab}z} = \frac{ii'ab}{\alpha\beta}\left(\frac{i'}{i} \cdot \frac{\alpha + b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n'ab}\right)$$

Nunc

Nunc vero eſt $db = \frac{\alpha\alpha\, dn}{i(n-1)}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right)$, quem valorem antequam ſubſtituamus, transformemus aequationem noſtram in hanc formam

$$\frac{dn}{n-1}\left(\frac{v}{ina} + \frac{1-i}{in} + \frac{v'}{i'nb} + \frac{1-i'}{i'n} + \frac{\mathfrak{C}O}{i'(\mathfrak{C}-O)}\left(\frac{i'}{\mathfrak{C}} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)\right.$$

$$\left. + db\left(\frac{v'}{ni'bb} - \frac{1}{\alpha} - \frac{1}{b} + \frac{\mathfrak{C}O}{i'i'bb(\mathfrak{C}-O)}\right)\right) = 0$$

vbi poſterius membrum abit in $-\frac{iv}{n\alpha\alpha}db$, tum vero erit

$$0 = \frac{dn}{n-1}\left\{\begin{matrix} 1 + \frac{b}{\alpha} + \frac{i'i'b}{\mathfrak{C}}\left(1 + \frac{b}{\alpha}\right) + \frac{(1-i')^2 b(\alpha+b)}{\alpha v'} + \frac{2-i-i'}{n} \\ -\frac{iv}{n\alpha} - \frac{i'v'}{n\mathfrak{C}} - \frac{ibv}{n\alpha\alpha} - \frac{ii'i'bbv}{n\alpha\alpha\mathfrak{C}} - \frac{i(1-i')^2 bbv}{n\alpha\alpha v'} \end{matrix}\right\}$$

diſtinguendo n' ab n erit

$$\frac{dn}{n-1}\left(\frac{iv}{\alpha n} - \frac{(1-i)}{n}\right) = \frac{dn'}{n'-1}\left\{\begin{matrix} 1 + \frac{b}{\alpha} + \frac{i'i'b}{\mathfrak{C}}\left(1 + \frac{b}{\alpha}\right) - \frac{ii'i'bbv}{n\alpha\alpha\mathfrak{C}} - \frac{i'v'}{n'\mathfrak{C}'} + \frac{1-i'}{n'} \\ -\frac{ibv}{n\alpha\alpha} + \frac{b(1-i')^2}{v'}\left(1 + \frac{b}{\alpha}\right) - \frac{i(1-i')^2 bbv}{n\alpha\alpha v'} \end{matrix}\right\}$$

cuius aequationis complicatio obſtat, quominus quicquam commode inde concludi poſſit.

Coroll. I.

309. Si ambae lentes craſſitie careant, vt ſit $v = 0$, $v' = 0$, et $i = i' = 1$, aequatio differentialis prima eſt

$$\frac{d\alpha}{\alpha} - \frac{db}{b} - \frac{O\, d\mathfrak{C}}{\mathfrak{C}(\mathfrak{C}-O)} = 0$$

tum vero:

$$d\alpha = -db = \frac{-\alpha\alpha\, dn}{n-1}\left(\frac{1}{\alpha} + \frac{1}{a}\right) \text{ et}$$

$$d\mathfrak{C} = \frac{-\mathfrak{C}\mathfrak{C}db}{bb} - \frac{\mathfrak{C}\mathfrak{C}dn}{n-1}\left(\frac{1}{\mathfrak{C}} + \frac{1}{b}\right) = \frac{-dn}{n-1}\left(\frac{\alpha\alpha\mathfrak{C}\mathfrak{C}}{bb}\left(\frac{1}{\alpha} + \frac{1}{a}\right) + \mathfrak{C}\mathfrak{C}\left(\frac{1}{\mathfrak{C}} + \frac{1}{b}\right)\right.$$

qui-

quibus valoribus substitutis et per $\frac{dn}{n-1}$ diuisione facta fit

$$-\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)\left(\frac{1}{\alpha}+\frac{1}{b}\right)+\frac{O\,\mathfrak{B}}{\mathfrak{B}-O}\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\mathfrak{B}}+\frac{1}{b}\right)=0.$$

Vnde si oculus lenti posteriori immediate applicaretur vt esset:

$O=0$, deberet esse $\left(\frac{1}{\alpha}+\frac{1}{a}\right)\left(\frac{1}{\alpha}+\frac{1}{b}\right)=0.$

Coroll. 2.

310. Verum in eadem hypothesi, vt locus oculi congruat cum eo, quem visio campi exigit, debet esse $O=\frac{b\,\mathfrak{B}(\alpha+b)}{b(\alpha+b)+\alpha\mathfrak{B}}$, vnde fit $\mathfrak{B}-O=\frac{\alpha\mathfrak{B}\mathfrak{B}}{b(\alpha+b)+\alpha\mathfrak{B}}$, ideoque $\frac{O}{\mathfrak{B}-O}=\frac{b(\alpha+b)}{\alpha\,\mathfrak{B}}=\frac{bb}{\mathfrak{B}}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$:

quo valore substituto nostra aequatio erit

$$0=\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(-\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)+bb\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\mathfrak{B}}+\frac{1}{b}\right)\right)$$

quae reducitur ad hanc formam

$$0=bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\mathfrak{B}}+\frac{1}{b}\right)$$

distinguendo n' ab n, membra a dn pendentia se destruunt et oritur

$$0=\frac{dn'}{n'-1}bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\mathfrak{B}}+\frac{1}{b}\right)$$

quod ita ostenditur:

Cum aequatio prima differentialis praebeat:

$$\frac{d\alpha}{\alpha}-\frac{db}{b}-\frac{O\,d\mathfrak{B}}{\mathfrak{B}(\mathfrak{B}-O)}=0 \text{ siue}$$

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{O\,d\mathfrak{B}}{\mathfrak{B}(\mathfrak{B}-O)}=0$$

Cum

Cum igitur ex conditione campi apparentis ſit

$$\frac{O}{\beta - O} = \frac{bb}{\beta}\left(\frac{1}{\alpha}+\frac{1}{b}\right) \text{ erit}$$

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right) - \frac{bb}{\beta^2}\left(\frac{1}{\alpha}+\frac{1}{b}\right)d\beta = 0$$

ideoque loco $d\beta$ ſuum valorem ſuſtituendo

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right) - \frac{bb}{\beta^2}\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{\beta\beta\, d\alpha}{b\, O} - \frac{\beta^2\, dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)\right) = 0$$

vbi membra, quae $d\alpha$ continent, manifeſto ſe deſtruunt, et tota quaeſtio ad hanc aequationem perducitur

$$\frac{dn'}{n'-1} \cdot bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\beta}+\frac{1}{b}\right) = 0.$$

Coroll. 3.

311. Quod ſi ergo huic conditioni ſatisfieri poſſit, obiectum ſine margine colorato apparebit: praeterea vero confuſio penitus tolleretur, ſi reddi liceret $d\beta = 0$, quod fit per hanc aequationem

$$\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\beta}+\frac{1}{b} = 0 \text{ ſiue}$$

$$+\frac{dn}{n-1} \cdot \frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)$$

$$+\frac{dn_{\prime}}{n'-1}\left(\frac{1}{\beta}+\frac{1}{b}\right) = 0$$

$$\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)+bb\left(\frac{1}{\beta}+\frac{1}{b}\right) = 0.$$

Coroll. 4.

312. Priori autem aequationi ſatisfieri nequit niſi fuerit vel $\alpha + b = 0$ vel $\frac{1}{\beta}+\frac{1}{b} = 0$. Illo caſu ambae lentes coniungerentur, vt vnicam conſtituerent, hoc vero poſterioris diſtantia focalis fieret infinita, qui

 caſus

casus iterum ad casum vnicae lentis rediret; foret enim $O = -\alpha - b$, ob $\beta = -b$, et oculus priori lenti immediate applicari deberet.

Scholion.

313. Si simili modo has inuestigationes ad plures lentes extendere vellemus, non neglecta earum crassitie in formulas plane inextricabiles delaberemur vnde vix quicquam concludi posset. Verum quia in omnibus fere instrumentis dioptricis, praecipue quae pluribus lentibus constant, iis tam exigua crassities tribui solet, vt sine notabili errore pro nihilo haberi possit, tam taediosae indagationi facile supersedere poterimus. Ad quod accedit, quod hic non de summo rigore geometrico agatur, sed contenti esse queamus, dummodo hanc confusionem satis prope cognouerimus: ex quo sufficiet in consideratione plurium lentium earum crassitiem prorsus neglexisse.

Supplementum IV.

Si ratio refractionis in singulis lentibus sit diuersa solutio sequenti modo absoluetur:

I. Prima aequatio differentialis prorsus se habebit, vt in problemate, ita, vt sit

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)+d\beta\left(\tfrac{1}{\beta}+\tfrac{1}{c}\right)-\frac{O\,d\gamma}{\gamma(\gamma-O)}=0$$

et quia etiam, vt ante, est

$$\frac{O}{\gamma-O}=\frac{bbcc}{\alpha\beta^2\gamma}\left(1+\tfrac{\alpha}{b}\right)+\frac{cc}{\beta\gamma}\left(1+\tfrac{\beta}{c}\right)$$

erit

erit noſtra aequatio

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(d\alpha-\frac{bbcc}{\beta^2\gamma^2}d\gamma\right)+\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(d\beta-\frac{cc}{\gamma\gamma}d\gamma\right)=0.$$

II. Nunc autem ratio diuerſae refractionis eſt habenda; vnde in ſuperioribus additamentis inuenimus eſſe

$$d\alpha=-\frac{\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$
$$d\beta=\frac{\beta\beta d\alpha}{bb}-\frac{\beta\beta dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)$$
$$d\gamma=\frac{\gamma\gamma d\beta}{cc}-\frac{\gamma\gamma dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

Hincque ergo fiet

$$d\alpha-\frac{bbcc}{\beta^2\gamma^2}d\gamma=d\alpha-\frac{bbd\beta}{\beta^2}+\frac{bbccdn''}{\beta^2(n''-1)}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$
$$=\frac{bb.dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{bbccdn''}{\beta^2(n''-1)}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

Deinde

$$d\beta-\frac{cc}{\gamma\gamma}d\gamma=\frac{ccdn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

III. Ex his ergo noſtra aequatio differentialis abibit in hanc formam:

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{bbdn'}{n'-1}\left[\frac{1}{b}+\frac{1}{\beta}\right]+\frac{bbc^2dn''}{\beta^2(n''-1)}\left[\frac{1}{c}+\frac{1}{\gamma}\right]\right)$$
$$+\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(\frac{ccdn''}{n''-1}\left[\frac{1}{c}+\frac{1}{\gamma}\right]\right)$$

ſiue

$$\frac{dn'}{n'-1}.\,bb.\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\beta}+\frac{1}{b}\right)$$
$$+\frac{dn''}{n''-1}.\,cc\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\frac{bb}{\beta\beta}\left(\frac{1}{\alpha}+\frac{1}{b}\right)+\frac{1}{\beta}+\frac{1}{c}\right)=0$$

IV. Pro caſu autem illo ſingulari, quo oculus lenti vltimae immediate, debet adplicari, ob $O=0$, habebitur ſimpliciter haec aequatio

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)+d\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)=0$$

 ſiue

fiue, fubftituto valore $d\beta$

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)+\frac{\beta\beta\, d\alpha}{bb}\left(\frac{1}{\beta}+\frac{1}{c}\right)$$

$$-\frac{\beta\beta\, dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)=0.$$

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}+\frac{\beta\beta}{bb}\left[\frac{1}{\beta}+\frac{1}{c}\right]\right)$$

$$-\frac{\beta^2\, dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)=0,$$

feu tandem

$$+\frac{dn}{n-1}\alpha\alpha\left(\frac{1}{b}+\frac{1}{\alpha}\right)\left(\frac{1}{\alpha}+\frac{1}{b}+\frac{\beta^2}{b^2}\left[\frac{1}{\beta}+\frac{1}{c}\right]\right)$$

$$+\frac{dn'}{n'-1}\beta\beta\left(\frac{1}{b}+\frac{1}{\beta}\right)=0.$$

V. Hoc modo tantum margo coloratus tollitur; vt autem tota confufio tollatur, quod fit fi $d\gamma=0$, infuper fatisfieri debet huic aequationi

$$0=\frac{dn}{n-1}\cdot\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

$$+\frac{dn'}{n'-1}\cdot\frac{\beta\beta\gamma\gamma}{cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)$$

$$+\frac{dn''}{n''-1}\cdot\gamma\gamma\cdot\left(\frac{1}{c}+\frac{1}{\gamma}\right).$$

Problema 6.

Tab. III. Fig. 14. 314. Si inftrumentum dioptricum tribus conftet lentibus, quarum craffities euanefcat, eam definire difpofitionem, vt oculus in eo loco, quem campus poftulat, conftitutus obiectum fine margine colorato confpiciat.

Solutio.

Pofita ergo diftantia obiecti ante lentem obiectiuam $AE=a$, eiusque magnitudine $E\varepsilon=z$, vocentur

centur diſtantiae imaginum a radiis mediae naturae formatarum, vt hactenus

$aF = \alpha$, $BF = b$; $bG = \beta$, $CG = c$, et $cH = \gamma$.

eritque imago vltima $H\theta = \frac{\alpha\beta\gamma}{abc}z$, et poſita oculi poſt lentem vltimam diſtantia $cO = O$, erit $OH = \gamma - O$ et anguli $HO\theta$ tangens $= \frac{\alpha\beta\gamma}{abc} \cdot \frac{z}{\gamma - O}$, quae debet eſſe invariabilis. Poſito ergo eius differentiali logarithmico $= 0$ habebimus hanc aequationem:

$$\frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\beta}{\beta} - \frac{dc}{c} + \frac{d\gamma}{\gamma} - \frac{d\gamma}{\gamma - O} = 0$$

At et formulis ſupra (300) erutis habemus

$$d\alpha = -db = \frac{-dn}{n-1} \cdot \alpha\alpha\left(\frac{1}{a} + \frac{1}{a}\right)$$

$$d\beta = -dc = -\frac{\beta\beta dn}{n-1}\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{a} + \frac{1}{\alpha}\right) + \frac{1}{b} + \frac{1}{\beta}\right)$$

$$d\gamma = -\frac{\gamma\gamma dn}{n-1}\left(\frac{\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a} + \frac{1}{\alpha}\right) + \frac{\beta\beta}{cc}\left(\frac{1}{b} + \frac{1}{\beta}\right) + \frac{1}{c} + \frac{1}{\gamma}\right)$$

vnde noſtra aequatio erit

$$d\alpha\left(\frac{1}{\alpha} + \frac{1}{b}\right) + d\beta\left(\frac{1}{\beta} + \frac{1}{c}\right) - \frac{O\,d\gamma}{\gamma(\gamma - O)} = 0$$

Sed ob campum apparentem ſupra 254 inuenimus:

$$\mathfrak{B} = \beta = b\left(1 + \frac{\alpha}{b}\right)\frac{z}{a}$$

$$\mathfrak{C} = c = \frac{bc}{\beta}\left(1 + \frac{\alpha}{b}\right)\frac{z}{a} + \frac{\alpha c}{b}\left(1 + \frac{\beta}{c}\right)\frac{z}{a}$$

hincque

$O = \frac{\gamma c}{c + H\theta}$ et $\frac{O}{\gamma - O} = \frac{c}{H\theta}$ vnde fit ob

$H\theta = \frac{\alpha\beta\gamma}{bc} \cdot \frac{z}{a}$

$$\frac{O}{\gamma - O} = \frac{bbcc}{\alpha\beta\beta\gamma}\left(1 + \frac{\alpha}{b}\right) + \frac{cc}{\beta\gamma}\left(1 + \frac{\beta}{c}\right) \text{ ſeu}$$

$$\frac{O}{\gamma(\gamma - O)} = \frac{bbcc}{\beta\beta\gamma\gamma}\left(\frac{1}{\alpha} + \frac{1}{b}\right) + \frac{cc}{\gamma\gamma}\left(\frac{1}{\beta} + \frac{1}{c}\right)$$

Valo-

Valoribus iam his substitutis habebimus:

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(d\alpha-\frac{bbcc}{\beta\beta\gamma\gamma}d\gamma\right)+\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(d\beta-\frac{cc}{\gamma\gamma}d\gamma\right)=0$$

At est

$$d\alpha-\frac{bbcc}{\beta\beta\gamma\gamma}d\gamma=\frac{dn}{n-1}\left(bb\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{bbcc}{\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)$$

$$d\beta-\frac{cc}{\gamma\gamma}d\gamma=\frac{dn}{n-1}\left(cc\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)$$

Quare facta diuisione per $\frac{dn}{n-1}$ nanciscemur hanc aequationem:

$$bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{b}+\frac{1}{\beta}+\frac{cc}{\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)+cc\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(\frac{1}{c}+\frac{1}{\gamma}\right)=0$$

Quodsi vero oculus lenti postremae immediate applicetur, seu sit $O=0$, conditio praescripta hanc postulat aequationem:

$$\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\beta\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{1}{b}+\frac{1}{\beta}\right)=0$$

Confusio vero a diuersa radiorum refrangibilitate oriunda perfecte tolletur si praeterea fuerit $d\gamma=0$ seu

$$\frac{\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{\beta\beta}{cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{1}{c}+\frac{1}{\gamma}=0.$$

Coroll. 1.

315. Si rationes aperturarum lentium in computum ducantur, eaque pro lente secunda ponatur $=\pi$, pro tertia $=\pi'$ erit $bN'=\frac{\pi b\beta}{b+\beta}$ et $CM''=cN''=\frac{\pi' c\gamma}{c+\gamma}$. Tum vero posito $\frac{z}{a}=\Phi$, erit $G\eta=\frac{\alpha\beta}{ab}a\Phi$, et $H\theta=\frac{\alpha\beta\gamma}{abc}\cdot a\Phi$. Hinc fiet $\frac{O}{\gamma-O}=\frac{cN''}{H\theta}=\frac{\pi' abcc}{\alpha\beta(c+\gamma)a\Phi}$ et $\frac{O}{\gamma(\gamma-O)}=\frac{\pi' abcc}{\alpha\beta\gamma(c+\gamma)}\cdot\frac{1}{a\Phi}$.

Coroll.

Coroll. 2.

316. Quodſi porro vt ſupra ponatur

$$\alpha = A a, \; \beta = B b, \; \gamma = C c \text{ erit } \frac{O}{\gamma(\gamma - c)} = \frac{\pi'}{ABC(C+1)a\Phi};$$

tum vero $b N' = \frac{\pi B b}{B+1}$ et $C M'' = \frac{\pi' C c}{C+1}$: ac $G\eta = A B a \Phi$;

Cum iam ſit

$$b N' + G\eta : b G = C M'' - G\eta : C G \text{ erit } \frac{bN' + G\eta}{bG} = \frac{CM'' - G\eta}{CG}$$

ideoque

$$\frac{1}{bG} + \frac{1}{CG} = \frac{CM''}{CG.G\eta} - \frac{bN'}{bG.G\eta} \text{ hoc eſt } \frac{1}{\beta} + \frac{1}{c} = \frac{\pi' C}{AB(C+1)a\Phi} - \frac{\pi}{AB(B+1)a\Phi}$$

ſimili vero modo eſt $\frac{1}{\alpha} + \frac{1}{b} = \frac{\pi B}{A(B+1)a\Phi}$.

Coroll. 3.

317. Per easdem ſubſtitutiones fit

$$\frac{1}{a} + \frac{1}{\alpha} = \frac{A+1}{Aa}; \; \frac{1}{b} + \frac{1}{\beta} = \frac{B+1}{Bb}; \; \frac{1}{c} + \frac{1}{\gamma} = \frac{C+1}{Cc} \text{ etc.}$$

hincque:

$$a\alpha \left(\frac{1}{a} + \frac{1}{\alpha}\right) = A(A+1)a$$

$$\beta\beta \left(\frac{1}{b} + \frac{1}{\beta}\right) = B(B+1)b$$

$$\gamma\gamma \left(\frac{1}{c} + \frac{1}{\gamma}\right) = C(C+1)c \text{ ſicque porro.}$$

Coroll. 4.

318. His ergo nouis denominationibus introductis differentialia ex §. 300 ita exprimentur

$$d\alpha = -db = -\frac{dn}{n-1} . A(A+1)a$$

$$d\beta = -dc = -BBdb - \frac{dn}{n-1} . B(B+1)b$$

$$d\gamma = -dd = -CCdc - \frac{dn}{n-1} . C(C+1)c$$

quae formulae commodius in calculum introducentur.

Quemadmodum hic nouae formae adhibentur in ſequentibus vſurpandae; ita et pro caſu diuerſae refractionis ſequentibus formulis in poſterum vti licebit:

$$\frac{O}{\gamma(\gamma-O)} = \frac{\pi'}{ABC(C+1)a\Phi}$$

$$\frac{1}{\alpha}+\frac{1}{b} = \frac{\pi B}{A(B+1)a\Phi}$$

$$\frac{1}{\beta}+\frac{1}{c} = \frac{\pi' C}{AB(C+1)a\Phi} - \frac{\pi}{AB(B+1)a\Phi}$$

$$\frac{1}{\gamma}+\frac{1}{d} = \frac{\pi'' D}{ABC(D+1)a\Phi} - \frac{\pi'}{ABC(C+1)a\Phi}$$

etc.

tum vero formulae differentiales erunt

$$d\alpha = -db = -\frac{dn}{n-1}.\ A(A+1)a$$

$$d\beta = -dc = B^2 d\alpha - \frac{dn'}{n'-1}.\ B(B+1)b$$

$$d\gamma = -dd = C^2 d\beta - \frac{dn''}{n''-1} C(C+1)c$$

$$d\delta = -de = D^2 d\gamma - \frac{dn'''}{n'''-1}.\ D(D+1)d$$

etc.

Atque ex his formulis vt margo coloratus euaneſcat, ſatisfieri debet huic aequationi

$$0 = \frac{dn'}{n'-1}.\ \frac{\pi b}{Aa\Phi} + \frac{dn''}{n''-1}.\ \frac{\pi' c}{ABa\Phi}$$

Vt autem haec confuſio penitus tollatur, fieri debet

$$d\gamma = -\frac{dn}{n-1}.\ A(A+1)B^2C^2a$$
$$-\frac{dn'}{n'-1}.\ B(B+1)C^2.b$$
$$-\frac{dn''}{n''-1}.\ C(C+1)c = 0.$$

ſiue

ſiue

$$\frac{dn}{n-1}\cdot\frac{(A+1)a}{A}$$
$$+\frac{dn'}{n'-1}\cdot\frac{(B+1)b}{AAB}=0$$
$$+\frac{dn''}{n''-1}\cdot\frac{(C+1)c}{A^2B^2C}.$$

Problema 7.

319. Si inſtrumentum dioptricum quatuor conſtet lentibus quarum craſſities negligi queat, eam definire diſpoſitionem vt oculus in eo loco quem campus apparens poſtulat, conſtitutus obiectum ſine confuſione a diuerſa radiorum indole oriunda conſpiciat. Tab. III. Fig. 15.

Solutio.

Poſita diſtantia obiecti ante inſtrumentum $AE=a$, eiusque magnitudine $E\varepsilon=z$, vocentur diſtantiae imaginum a radiis mediae naturae formatarum vt hactenus.

$aF=\alpha$, $BF=b$; $bG=\beta$, $CG=c$; $cH=\gamma$; $DG=d$; $dI=\delta$

Tum vero ponamus praeterea

$\alpha=Aa$, $\beta=Bb$; $\gamma=Cc$; $\delta=Dd$

atque introducantur rationes aperturarum pro ſingulis lentibus poſt primam, quae ſint π pro ſecunda QQ, π' pro tertia RR et π'' pro quarta SS, ſumto pro campo $\frac{z}{a}=\Phi$. His poſitis erunt imagines $F\zeta=Aa\Phi$; $G\eta=ABa\Phi$; $H\theta=ABCa\Phi$ et $I\iota=ABCDa\Phi=\frac{\alpha\beta\gamma\delta}{abcd}z$.

 Iam

Iam poſita oculi diſtantia poſt inſtrumentum $d\text{O}=\text{O}$, vt ſit $\text{OI}=\delta-\text{O}$ erit anguli IOι tangens $\frac{\alpha\beta\gamma\delta}{abcd}\cdot\frac{z}{\delta-\text{O}}$, quae cum ob diuerſam radiorum refrangibilitatem immutata manere debeat, differentiata dabit hanc aequationem

$$\frac{d\alpha}{\alpha}-\frac{db}{b}+\frac{d\beta}{\beta}-\frac{dc}{c}+\frac{d\gamma}{\gamma}-\frac{dd}{d}-\frac{\text{O}\,d\,\delta}{\delta(\delta-\text{O})}=0$$

quae ob $db=-d\alpha$, $dc=-d\beta$ et $dd=-d\gamma$ abit in hanc

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)+d\beta\left(\tfrac{1}{\beta}+\tfrac{1}{c}\right)+d\gamma\left(\tfrac{1}{\gamma}+\tfrac{1}{d}\right)-\frac{\text{O}\,d\,\delta}{\delta(\delta-\text{O})}=0$$

Verum modo ante notauimus fore (318)

$$d\alpha=\frac{-dn}{n-1}.\ \text{A}(\text{A}+1)a$$
$$d\beta=\text{BB}\,d\alpha-\frac{dn}{n-1}.\ \text{B}(\text{B}+1)b$$
$$d\gamma=\text{CC}\,d\beta-\frac{dn}{n-1}.\ \text{C}(\text{C}+1)c$$
$$d\delta=\text{DD}\,d\gamma-\frac{dn}{n-1}.\ \text{D}(\text{D}+1)d$$

vbi pro b, c, d, valores §. 266 aſſignati ſubſtitui debent. Porro vero iam animaduertimus eſſe (316)

$$\frac{1}{\alpha}+\frac{1}{b}=\frac{1}{\text{A}a\Phi}\cdot\frac{\pi\,\text{B}}{\text{B}+1}$$
$$\frac{1}{\beta}+\frac{1}{c}=\frac{1}{\text{AB}a\Phi}\left(\frac{\pi'\,\text{C}}{\text{C}+1}-\frac{\pi}{\text{B}+1}\right)$$
$$\frac{1}{\gamma}+\frac{1}{d}=\frac{1}{\text{ABC}a\Phi}\left(\frac{\pi''\text{D}}{\text{D}+1}-\frac{\pi'}{\text{C}+1}\right)$$

atque

$$\frac{\text{O}}{\delta(\delta-\text{O})}=\frac{\pi''}{\text{ABCD}(\text{D}+1)a\Phi}.$$

Quod

Quodsi iam priores valores in posterioribus successiue substituantur, habebimus

$$d\alpha = \frac{-dn}{n-1} \cdot A(A+1)a$$

$$d\beta = \frac{-dn}{n-1} \cdot B(AB(A+1)a+(B+1)b$$

$$d\gamma = \frac{-dn}{n-1} \cdot C(AB^2C(A+1)a+BC(B+1)b+(C+1)c)$$

$$d\delta = \frac{-dn}{n-1} \cdot D(AB^2C^2D(A+1)a+BC^2D(B+1)b+CD(C+1)c+(D+1)d)$$

His igitur valoribus in aequatione differentiali substitutis, et diuisione per $\frac{dn}{(n-1)\Phi}$ facta aequatio nostra secundum singula membra distributa erit

$$-\frac{\pi B}{(B+1)} \cdot (A+1)a$$

$$-\frac{1}{A}\left(\frac{\pi' C}{C+1}-\frac{\pi}{B+1}\right)(AB(A+1)a+(B+1)b)$$

$$-\frac{1}{AB}\left(\frac{\pi'' D}{D+1}-\frac{\pi'}{C+1}\right)(AB^2C(A+1)a+BC(B+1)b+(C+1)c)$$

$$+\frac{1}{ABC} \cdot \frac{\pi''}{D+1}(AB^2C^2D(A+1)a+BC^2D(B+1)b+CD(C+1)c+(D+1)d)=0$$

Terna autem priora membra negatiua sola collecta praebent

$$-\frac{BCD(A+1)\pi''}{D+1}a-\frac{CD(B+1)\pi''}{A(D+1)}b-\frac{D(C+1)\pi''}{AB(D+1)}c$$

$$+\frac{\pi}{A}b+\frac{\pi'}{AB}c$$

quibus si quartum addatur prodit

$$\frac{\pi}{A}b+\frac{\pi'}{AB}c+\frac{\pi''}{ABC}d$$

Iam duo hic casus considerari oportet, alterum quo punctum O post lentem vltimam cadit, alterum vero

vero, quo ob diſtantiam O negatiuam oculus lenti vltimae immediate applicatur. Pro priori caſu, quo diſtantia $dO = O$ prodit poſitiua habetur iſta aequatio ſi quidem in aequatione modo inuenta pro b, c, d valores §. 266 inuenti ſubſtituantur

$$\frac{(B+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)\pi''}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

Pro caſu poſteriori, quo diſtantia O euaneſcens aſſumitur facta multiplicatione per $\frac{D+1}{a\ \Phi}$.

$$\frac{BCD(A+1)\pi''}{\Phi}+\frac{CD(B+1)^2\pi''}{B\pi-(B+1)\Phi}+\frac{D(C+1)^2\pi''}{C\pi'-(C+1)(\pi-\Phi)}$$
$$=\frac{(B+1)(D+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)(D+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}.$$

Hoc modo efficitur, vt obiectum ſine margine colorato appareat at omnis confuſio tolletur ſi praeterea fuerit

$$AB^2C^2D(A+1)+\frac{ABC^2D(B+1)^2\Phi}{B\pi-(B+1)\Phi}+\frac{ABCD(C+1)^2\Phi}{C\pi'-(C+1)(\pi-\Phi)}+\frac{ABC(D+1)^2\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

ſeu per ABC diuidendo:

$$BCD(A+1)+\frac{CD(B+1)^2\Phi}{B\pi-(B+1)\Phi}+\frac{D(C+1)^2\Phi}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)^2\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

$$BCD(A+1)+\frac{CD(B+1)b}{A\,a}+\frac{D(C+1)c}{ABa}+\frac{(D+1)\,d}{A\,B\,C\,a}=0$$

Coroll. I.

320. Si vtrique conditioni ſatisfieri poteſt, vt nec locus nec magnitudo imaginis vllam mutationem patiatur locus oculi non amplius in computum ingreditur ſed imago, vndecunque cernatur, ab omni confuſione prorſus libera erit.

Coroll.

Coroll. 2.

321. Vt ergo hunc summum perfectionis gradum consequamur, binis his aequationibus satisfieri oportet.

$$\frac{(B+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)\pi''}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

et

$$A+1+\frac{(B+1)^2\Phi}{B(B\pi-(B+1)\Phi)}+\frac{(C+1)^2\Phi}{BC(C\pi'-(C+1)(\pi-\Phi))}+\frac{(D+1)^2\Phi}{BCD(D\pi''-(D+1)(\pi'-\pi+\Phi))}=0.$$

Coroll. 3.

322. Si porro vt supra fecimus ponamus

$$\frac{A}{A+1}=\mathfrak{A},\ \frac{B}{B+1}=\mathfrak{B},\ \frac{C}{C+1}=\mathfrak{C}\ \text{et}\ \frac{D}{D+1}=\mathfrak{D}$$

istae aequationes sequenti modo simplicius exprimentur:

$$+\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0 \text{ et}$$

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0$$

hic prima membra multiplicari debent per $\frac{d\ n}{n-1}$

secunda per $\frac{d\ n'}{n'-1}$

tertia per $\frac{d\ n''}{n''-1}$

etc.

Coroll. 4.

323. Sin autem non liceat has ambas aequationes adimplere, curandum est, vt saltem priori, qua magnitudo apparens a confusione liberatur, satis-

fiat

fiat hoc enim modo obiectum fine margine colorato apparebit vbi quidem ad duos cafus refpici conuenit prout diftantia oculi dO prodierit pofitiua vel negatiua.

Scholion.

324. Introducendis ergo rationibus aperturarum quibus fupra iam commode fumus vfi ad campum apparentem definiendum aequationes etiam iftae confufionem a diuerfa radiorum refrangibilitate oriundam tollentes fatis fiunt fimplices, vt fine moleftia tractari queant; fi quidem craffities lentium negligatur. Hoc ergo modo problema generale, quicunque fuerit lentium numerus, expediri conueniet.

Supplementum V.

Si ratio refractionis in fingulis lentibus difcrepet, prodit primo quidem eadem aequatio differentialis

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)+d\mathfrak{E}\left(\frac{1}{\mathfrak{E}}+\frac{1}{c}\right)+d\gamma\left(\frac{1}{\gamma}+\frac{1}{d}\right)\frac{-Od\delta}{\delta(\delta-O)}=0$$

in qua ergo cafu $O=0$ vltimum membrum abiici debet.

I) Si autem O habeat valorem pofitiuum, erit, vt ante,

$$\frac{O}{\delta(\delta-O)}=\frac{\pi''}{ABCD(D+1)a\Phi}$$

atque etiam valores $\frac{1}{\alpha}+\frac{1}{b}$; $\frac{1}{\mathfrak{E}}+\frac{1}{c}$; $\frac{1}{\gamma}+\frac{1}{d}$ manent iidem, vt ante.

At

At vero ob diuersitatem refractionis habebimus

$$d\alpha = \frac{-dn}{n-1}. A(A+1)a$$

$$d\beta = BBd\alpha - \frac{dn'}{n'-1}. B(B+1)b$$

$$d\gamma = CCd\beta - \frac{dn''}{n''-1} C.(C+1)c$$

$$d\delta = DDd\gamma - \frac{dn'''}{n'''-1}. D(D+1)d$$

Hinc ergo aequatio nostra successiue ita formetur:

$$\frac{-O d\delta}{\delta(\delta-O)} - \frac{-\pi'' d\delta}{ABCD(D+1)a\Phi}$$

$$= \frac{-\pi'' D d\gamma}{ABC(D+1)a\Phi} + \frac{dn'''}{n'''-1}. \frac{\pi''. d}{ABC. a\Phi}$$

Addatur $d\gamma(\frac{1}{\gamma}+\frac{1}{d}) = \frac{d\gamma.\pi'' D}{ABC(D+1)a\Phi} - \frac{\pi' d\gamma}{ABC(C+1)a\Phi}$

et prodibit $\frac{-\pi' d\gamma}{ABC(C+1)a\Phi} + \frac{dn'''}{n'''-1}. \frac{\pi''. d}{ABC. a\Phi}$

et pro $d\gamma$ substituto valore

$$\frac{-\pi' C d\beta}{AB(C+1)a\Phi} + \frac{dn''}{n''-1}. \frac{\pi' c}{AB a\Phi} + \frac{dn'''}{n'''-1}. \frac{\pi''. d}{ABC. a\Phi}$$

Iam addatur

$$d\beta(\frac{1}{\beta}+\frac{1}{c}) = \frac{\pi' C d\beta}{AB(C+1)a\Phi} - \frac{\pi d\beta}{AB(B+1)a\Phi}$$

proditque

$$\frac{-\pi d\beta}{AB(B+1)a\Phi} + \frac{dn''}{n''-1}. \frac{\pi' c}{AB. a\Phi} + \frac{dn'''}{n'''-1}. \frac{\pi'' d}{ABC a\Phi}$$

et substituto valore ipsius $d\beta$

$$\frac{-\pi B d\alpha}{A(B+1)a\Phi} + \frac{dn'}{n'-1}. \frac{\pi. b}{A. a\Phi} + \frac{dn''}{n''-1}. \frac{\pi' c}{AB a\Phi} + \frac{dn'''}{n'''-1}. \frac{\pi'' d}{ABC. a\Phi}$$

denique addatur $d\alpha(\frac{1}{\alpha}+\frac{1}{b}) = \frac{\pi B d\alpha}{A(B+1)a\Phi}$

ac aequatio quaesita, qua margo coloratus euanescit, erit

$$\frac{dn'}{n'-1}. \frac{\pi b}{A a\Phi} + \frac{dn''}{n''-1}. \frac{\pi' c}{AB a\Phi} + \frac{dn'''}{n'''-1}. \frac{\pi'' d}{ABC a\Phi} = 0 \qquad \text{II)}$$

II). Sin autem O habeat valorem negatiuum: tunc sumi debet $O=0$ et pro eodem scopo aequatio ita formabitur:

Cum sit

$$d\gamma\left(\frac{1}{\gamma}+\frac{1}{d}\right)=\frac{\pi'' D\, d\gamma}{ABC(D+1)a\Phi}-\frac{\pi'\, d\gamma}{ABC(C+1)a\Phi},$$

substituto valore $d\gamma$ fiet

$$CC\,d\beta\left(\frac{\pi''\ D}{ABC(D+1)a\Phi}-\frac{\pi'}{ABC(C+1)a\Phi}\right)$$
$$-\frac{dn''}{n''-1}.\ C(C+1)c\left(\frac{\pi''\ D}{ABC(D+1)a\Phi}-\frac{\pi'}{ABC(C+1)a\Phi}\right)$$

Addatur

$$d\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)=\frac{\pi' C d\beta}{AB(C+1)a\Phi}-\frac{\pi\, d\beta}{AB(B+1)a\Phi}$$

eritque

$$d\beta\left(\frac{CD\pi''}{AB(D+1)a\Phi}-\frac{\pi}{AB(B+1)a\Phi}\right)-\frac{dn''}{n''-1}(C+1)c\left(\frac{\pi''\ D}{AB(D+1)a\Phi}-\frac{\pi'}{AB(C+1)a\Phi}\right)$$

et substituto valore ipsius $d\beta$

$$d\alpha\left(\frac{BCD\pi''}{A(D+1\,a\Phi}-\frac{B\pi}{A(B+1)a\Phi}\right)$$
$$-\frac{dn'}{n'-1}\left(\frac{(B+1)CD.b\pi''}{A(D+1)a\Phi}-\frac{\pi b}{A.a\Phi}\right)-\frac{dn''}{n''-1}(C+1)c\left(\frac{\pi''\ D}{AB(D+1)a\Phi}-\frac{\pi'}{AB(C+1)a\Phi}\right)$$

Addatur

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)=\frac{\pi B d\alpha}{A(B+1)a\Phi}$$

eritque

$$-\frac{dn}{n-1}.\left(\frac{(A+1)BCDa,\pi''}{(D+1)a\Phi}\right)$$
$$-\frac{dn'}{n'-1}\left(\frac{(B+1)CDb\pi''-(D+1)b\pi}{A(D+1)a\Phi}\right)$$
$$-\frac{dn''}{n''-1}\left(\frac{(C+1)Dc\pi''-(D+1)c\pi'}{AB(D+1)a\Phi}\right).$$

Vnde

Vnde pro caſu $O = 0$ aequatio, qua margo coloratus deſtruitur, erit

$$0 = \frac{adn}{n-1} \cdot (A+1) B C D \pi''$$
$$+ \frac{bdn'}{n'-1} \cdot \left(\frac{(B+1) C D \pi'' - (D+1) \pi}{A}\right)$$
$$+ \frac{cdn''}{n''-1} \left(\frac{(C+1) D \pi'' - (D+1) \pi'}{A B}\right)$$

III) Vt autem praeterea omnis confuſio tollatur, inſuper reddi oportet $d\delta = 0$; vnde haec aequatio naſcitur

$$0 = \begin{cases} \frac{adn}{n-1} \cdot A(A+1) B^2 C^2 D^2 \\ + \frac{bdn'}{n'-1} \cdot B(B+1) C^2 D^2 \\ + \frac{cdn''}{n''-1} \cdot C(C+1) D^2 \\ + \frac{d.dn'''}{n'''-1} \cdot D(D+1) \end{cases}$$

quae per $A^2 B^2 C^2 D^2$ diuiſa dat

$$0 = \begin{cases} \frac{adn}{n-1} \cdot \frac{A+1}{A} \\ + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2 B} \\ + \frac{cdn''}{n''-1} \cdot \frac{C+1}{A^2 B^2 C} \\ + \frac{d.dn'''}{n'''-1} \cdot \frac{D+1}{A^2 B^2 C^2 D} \end{cases}$$

Circa hanc autem aequationem inprimis notandum eſt, ſi omnes lentes pari facultate refringente ſint praeditae, ei ſatisfieri haud poſſe; ex quo haec aequatio proprie pertinet ad caſum, quo diuerſae refractiones locum habent.

 Pro-

Problema 8.

325. Si instrumentum dioptricum ex quotcunque lentibus sit compositum, quarum crassitiem negligere liceat, eam determinare dispositionem, vt oculus in eo loco, quem campus apparens postulat positus nullam confusionem sentiat.

Solutio.

Sit obiecti ante lentem primam distantia $AE = a$, eiusque magnitudo $E\varepsilon = z$, quae quidem conspici queat, et statuatur $\frac{z}{a} = \Phi$. Deinde sint distantiae imaginum a radiis mediae naturae formatarum vt supra:

$$AE = a;\ BF = b;\ CG = c;\ DH = d;\ EI = e$$
$$aF = \alpha;\ bG = \beta;\ cH = \gamma;\ dI = \delta;\ eK = \varepsilon$$

etc.

ac vocemus breuitatis gratia

$$\alpha = Aa,\ \beta = Bb,\ \gamma = Cc,\ \delta = Dd,\ \varepsilon = Ee \text{ etc.}$$

tum vero etiam

$$\frac{A}{A+1} = \mathfrak{A};\ \frac{B}{B+1} = \mathfrak{B};\ \frac{C}{C+1} = \mathfrak{C};\ \frac{D}{D+1} = \mathfrak{D},\ \frac{E}{E+1} = \mathfrak{E} \text{ etc.}$$

Iam apertura primae lentis PP vt euanescente considerata sit ratio aperturae pro reliquis lentibus

$$QQ = \pi;\ RR = \pi';\ SS = \pi'';\ TT = \pi''' \text{ etc.}$$

His

His positis supra vidimus (266) fore praeter $\alpha = Aa$

$b = \frac{Aa\Phi}{\mathfrak{B}\pi - \Phi}$ $\quad$ $\mathfrak{b} = \frac{ABa\Phi}{\mathfrak{B}\pi - \Phi}$

$c = \frac{ABa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ $\quad$ $\gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$

$d = \frac{ABCa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$ $\quad$ $\delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$

$e = \frac{ABCDa\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$ $\quad$ $\varepsilon = \frac{ABCDEa\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$

etc. $\quad$ etc.

Deinde singularum imaginum magnitudo ita se habebit

$F\zeta = Aa\Phi$; $G\eta = ABa\Phi$; $H\theta = ABCa\Phi$; $I\iota = ABCDa\Phi$

etc.

et ipsi aperturarum semidiametri:

Lentis secundae $QQ = \frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi - \Phi}\pi$

Lentis tertiae $RR = \frac{AB\mathfrak{C}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}\pi'$

Lentis quartae $SS = \frac{ABC\mathfrak{D}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\pi''$

Lentis quintae $TT = \frac{ABCD\mathfrak{E}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}\pi'''$

Mutata iam refractionis lege $n:1$ infinite parum, distantiae $\alpha, b, \mathfrak{b}, c; \gamma, d, \delta$ etc. tales mutationes recipiunt.

$$d\alpha = -\frac{dn}{n-1}.\,Aa(A+1)$$

$$d\mathfrak{b} = \frac{-dn}{n-1}.\,ABa\left((A+1)B + \frac{(B+1)\Phi}{\mathfrak{B}\pi - \Phi}\right)$$

$$d\gamma = \frac{-dn}{n-1}.\,ABCa\left((A+1)BC + \frac{(B+1)C\Phi}{\mathfrak{B}\pi - \Phi} + \frac{(C+1)\Phi}{\mathfrak{C}\pi' - \pi + \Phi}\right)$$

$$d\delta = \frac{-dn}{n-1}\cdot ABCDa\left((A+1)BCD + \frac{(B+1)CD\Phi}{\mathfrak{B}\pi - \Phi} + \frac{(C+1)D\Phi}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{(D+1)\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\right)$$

etc.

 Porro

Porro vero habetur:

$$\frac{1}{\alpha}+\frac{1}{b}=\frac{1}{Aa\Phi}\mathfrak{B}\pi$$

$$\frac{1}{\beta}+\frac{1}{c}=\frac{1}{ABa\Phi}\left(\mathfrak{C}\pi'-\frac{\mathfrak{B}\pi}{B}\right)$$

$$\frac{1}{\gamma}+\frac{1}{d}=\frac{1}{ABCa\Phi}\left(\mathfrak{D}\pi''-\frac{\mathfrak{C}\pi'}{C}\right)$$

$$\frac{1}{\delta}+\frac{1}{e}=\frac{1}{ABCDa\Phi}\left(\mathfrak{E}\pi'''-\frac{\mathfrak{D}\pi''}{D}\right)$$

etc.

His positis pro quolibet lentium numero seorsim formulas quaesito satisfacientes expediamus: posita distantia oculi post lentem vltimam $=O$.

I. Pro vnica Lente

Habetur haec aequatio differentialis $\frac{-O\,d\alpha}{\alpha(\alpha-O)}=0$, pro quo casu tam magnitudo imaginis quam eius locus manebit inuariatus si fuerit $d\alpha=0$, hoc est $A(A+1)=0$, vnde deberet esse vel $A=0$ vel $A=-1$: quorum prius visio non admittit posterius autem lentem tollit. Tum vero ob campum esse debet $O=0$.

II. Pro duabus lentibus

Habetur haec aequatio differentialis, qua margo coloratus tollitur:

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{O\,d\beta}{\beta(\beta-O)}=0$$

at ob campum apparentem est $\frac{O}{\beta(\beta-O)}=\frac{1}{BB}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$, ita vt habeamus:

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{d\beta}{BB}\left(\frac{1}{\alpha}+\frac{1}{b}\right)=0$$

Verum

Verum est $d\mathfrak{C} = BB\,d\alpha - \frac{dn}{n-1}.\ AB\alpha.\ \frac{(B+1)\Phi}{\mathfrak{B}\pi - \Phi}$

Quare si O habeat valorem positiuum erit ob

$\mathfrak{B}(B+1) = B;\ \frac{\pi}{\mathfrak{B}\pi - \Phi} = 0$

Sin autem valor O prodeat negatiuus, quo casu capitur $O = 0$ erit $\frac{(A+1)\mathfrak{B}\pi}{\Phi} = 0.$

Omnis autem confusio penitus tolletur, si insuper fuerit

$$\frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} = 0$$

III. Pro tribus Lentibus

Si calculum eodem modo prosequamur, obiectum sine margine colorato conspicietur:

1. Si ex campo apparente distantia O prodeat positiua hanc aequationem adimplendo:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = 0.$$

2. Si ob distantiam O prodeuntem negatiuam capiatur $O = 0$ huic aequationi erit satisfaciendum:

$$\frac{\pi'}{\mathfrak{A}\Phi} + \frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} = \frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi - \Phi)}$$

Omnis autem confusio penitus tolletur si fuerit insuper

$$\frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} = 0.$$

IV. Pro quatuor Lentibus

Vt obiectum sine margine colorato spectetur:

1. Si

1. Si ex campo apparente diſtantia O prodeat poſitiua huic aequationi erit ſatisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0$$

2. Sin autem capiatur $O=0$, huic

$$\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}=\frac{\pi}{ABC\mathfrak{D}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'}{ABC\mathfrak{D}(\mathfrak{C}\pi'-\pi+\Phi)}$$

Omnis vero confuſio penitus tolletur ſi fuerit praeterea

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0$$

V. Pro quinque Lentibus

Vt obiectum tantum ſine margine colorato conſpiciatur

1. Si ex campo apparente diſtantia O prodeat poſitiua, huic aequationi erit ſatisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}+\frac{\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=0$$

2. Sin autem capiatur $O=0$ huic

$$\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

$$\frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right)$$

Omnis autem confuſio penitus tolletur, ſi praeterea ſatisfiat huic aequationi:

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}=0$$

Atque hinc manifeſta eſt progreſſio ad maiorem lentium numerum.

Coroll.

Coroll. 1.

326. In casu ergo vnicae lentis licet quidem obiectum a margine colorato liberare neque vero confusionem penitus tollere. Casu autem duarum lentium ne margo quidem coloratus tolli potest, siquidem oculus in eo loco, quem campi apparentis conditio postulat, teneatur.

Coroll. 2.

327. Quodsi vero plures duabus habeantur lentes sufficiens quantitatum numerus adest, quarum determinatione non solum margo coloratus deleri, sed etiam forte omnis confusio penitus auferri posse videtur praecipue si lentium numerus ternarium superet.

Scholion.

328. Quod ergo incommodum a diuersa radiorum natura oriundum adeo graue vel summo Neutono est visum, vt instrumenta dioptrica nullo modo ab eo liberari posse sit arbitratus, id quidem saltem quod ad marginem coloratum attinet ad quem Neutonus inprimis spectabat, iam satis feliciter tolli posse certum est; ita vt saltem ob hanc causam non opus sit ad Telescopia Catoptrica confugere. Hoc autem vitio sublato si praeterea alterum confusionis fontem obstruamus lentes scilicet nullam confusionem parientes adhibendo, nullum est dubium quin instrumenta dioptrica ad summum perfectionis gradum euehi queant.

Quae igitur hactenus particulatim circa ſingulas horum inſtrumentorum affectiones propoſuimus, ea colligi conueniet, vnde in capite ſequente praecepta generalia pro omnium inſtrumentorum dioptricorum conſtructione tradere eſt viſum.

Supplementum VI.

Ex iis, quae ante ſunt adiecta, poterimus etiam problematis ſolutionem pro caſu exhibere, quo ſingulae lentes peculiari refractione ſunt praeditae, vbi quidem tantum poſtremae aequationes pro confuſione vitanda mutationem quandam poſtulant; interim tamen etiam priores formulas, quibus locus oculi, quem campus apparens requirit, diſtinctius repraeſentemus.

I. *Diſtantia Oculi poſt vltimam lentem* pro quouis lentium numero ſe habebit, vt ſequitur.

Num. lentium	O id eſt, diſtantia oculi poſt lentem vltimam
I.	o
II.	$\frac{A\mathfrak{B}a\pi\Phi}{(\pi-\Phi)(\mathfrak{B}\pi-\Phi)}$ ſeu $\frac{\mathfrak{B}b\pi}{\pi-\Phi}$
III.	$\frac{AB.\mathfrak{C}a\pi'\Phi}{(\pi'-\pi+\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$ ſeu $\frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi}$
IV.	$\frac{ABC\mathfrak{D}.a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$ ſeu $\frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi}$
V.	$\frac{ABCD\mathfrak{E}.a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$ ſeu $\frac{\mathfrak{E}e\pi'''}{\pi'''-\pi''+\pi'-\pi+\Phi}$

etc.

II.

II. Si valor ipſius O ſit poſitiuus, ad marginem coloratum tollendum ſequentes aequationes ſunt adimplendae:

Num. lentium	
I.	$0 = 0$
II.	$0 = \frac{b\,d\,n'}{n'-1} \cdot \frac{\pi}{A\,a\,\Phi}$
III.	$0 = \frac{b\,d\,n'}{n'-1} \cdot \frac{\pi}{A\,a\,\Phi} + \frac{c\,d\,n''}{n''-1} \cdot \frac{\pi'}{AB\,a\,\Phi}$
IV.	$0 = \frac{b\,d\,n'}{n'-1} \cdot \frac{\pi}{A\,a\,\Phi} + \frac{c\,d\,n''}{n''-1} \cdot \frac{\pi'}{AB\,a\,\Phi} + \frac{d\,d\,n'''}{n'''-1} \cdot \frac{\pi''}{ABC\,a\,\Phi}$

etc.

III. Si valor ipſius O prodeat negatiuus, quo caſu capi debet $O = 0$, ad marginem coloratum tollendum ſequentes aequationes ſunt adimplendae.

Num. lentium	
I.	$0 = 0$
II.	$0 = \frac{a\,d\,n}{n-1} \cdot (A+1)\,B\,\pi$
III.	$0 = \frac{a\,d\,n}{n-1}(A+1)\,B\,C\,\pi' + \frac{b\,d\,n'}{n'-1} \cdot \frac{(B+1)\,C\,\pi' - (C+1)\,\pi}{A}$
IV.	$0 = \frac{a\,d\,n}{n-1} \cdot (A+1)BCD\pi'' + \frac{b\,d\,n'}{n'-1}\left(\frac{(B+1)CD\pi'' - (D+1)\pi}{A}\right) + \frac{c\,d\,n''}{n''-1}\left(\frac{(C+1)D\pi'' - (D+1)\pi'}{A\,B}\right)$

etc.

IV. Vt autem insuper omnis confusio huius generis tollatur, sequentes aequationes sunt adimplendae:

Num. lentium	
I.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A}$
II.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B}$
III.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B} + \frac{c\,dn''}{n''-1} \cdot \frac{C+1}{A^2 B^2 C}$
IV.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B} + \frac{c\,dn''}{n''-1} \cdot \frac{C+1}{A^2 B^2 C} + \frac{d\,dn'''}{n'''-1} \cdot \frac{D+1}{A^2 B^2 C^2 D}$

etc.

Quarum formularum ordo hinc distinctius perspicitur, quam in problemate.

CAPVT