CAPVT V.

DE CAMPO APPARENTE OCVLIQVE LOCO MAXIME IDONEO.

Problema I.

222.

Tab. II. Fig. 12. Si ex obiecti puncto extra axem sumto ε radius quicunque εM per lentem PP transmittatur, definire eius concursum cum axe O.

Solutio.

Sit $F\zeta$ imago principalis per lentem proiecta, a diffusione enim hic mentem abstrahimus, ac ponamus pro lente eius distantias determinatrices $AE = a$, $aF = \alpha$, eius crassitiem $Aa = v$, et quantitatem arbitrariam $= k$, sitque breuitatis ergo $\frac{k-v}{k+v} = i$. His positis si puncti ε imago cadat in ζ, voceturque $E\varepsilon = z$, erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha z}{a}$ at pro puncto lentis M statuatur eius distantia ab axe $AM = x$; ac supra ostenditur eius distantia ab axe $AM = x$; ac supra ostendimus si radius a puncto E per punctum M transmitteretur, eum ita per m ad punctum F progressurum esse, vt foret $am = ix$. Radius igitur εM per punctum N ad ζ feretur, ita vt, cum obliquitas radiorum

rum in facies lentis incidentium perpetuo valde parva ſtatuatur, ſit proxime angulus $EM\varepsilon$ ad NMm vt n ad 1 poſito $n = \frac{31}{20}$. Hinc erit $E\varepsilon$ ad mN in ratione compoſita iſtorum angulorum, et diſtantiarum AE ad Aa, ſeu $E\varepsilon : mN = na : v$, vnde fit $mN = \frac{vz}{na}$, ideoque $aN = ix - \frac{vz}{na}$. Iam vero radius a puncto N recta ad ζ pergit, et propterea axem ita in O ſecabit, vt ſit $aN + F\zeta : aF = aN : aO$ ſicque

$$aO = \left(iax - \frac{\alpha vz}{na}\right) : \left(ix - \frac{vz}{na} + \frac{\alpha z}{ia}\right) = \frac{niia\alpha x - i\alpha vz}{niiax - ivz + n\alpha z}$$

et $FO = \frac{n\alpha\alpha z}{niiax - ivz + n\alpha z}$

Si x ſit ſemidiameter aperturae lentis in facie anteriori, haec interſectio O reſpondet caſui, quo radius ab ε per lentis terminum ſummum tranſeat: ſin autem is per terminum imum tranſmittatur, ſumto x negatiuo fiet

$$FO = \frac{n\alpha\alpha z}{-niiax - ivz + n\alpha z}$$

At ſi radius ex ε per centrum lentis A tranſeat, punctum interſectionis ita cadet in O vt ſit

$$FO = \frac{n\alpha\alpha}{n\alpha - iv}$$

Coroll. I.

223. Si igitur x denotet ſemidiametrum aperturae lentis in facie anteriori PMAP, vt omnes radii a puncto ε in hanc faciem incidentes per lentem tranſmittantur, neceſſe eſt, vt faciei poſterioris

PNaP semidiameter sit maior quam $\pm ix - \frac{vz}{na}$, sumto x tam negatiuo quam positiuo. Vnde hic semidiameter minor esse nequit, quam $ix + \frac{vz}{na}$.

Coroll. 2.

224. Si apertura in facie anteriori euanescat, vt sit $x = 0$ radii a puncto ε, cuius ab axe distantia E$\varepsilon = z$, per lentem non transmittentur, nisi in facie posteriori semidiameter aperturae sit $= \frac{vz}{na}$ vel maior. Vnde patet quo maior fuerit lentis crassities v eo maiori apertura in facie posteriori esse opus.

Coroll. 3.

225. Vicissim ergo si detur lentis apertura in facie posteriori, cuius semidiameter sit $= \frac{vz}{na}$; inde simul in obiecto extremum punctum ε determinatur, a quo radius in centrum lentis A incidens per eam transmittatur.

Coroll. 4.

226. Hic autem radius per A immissus post transitum cum axe in O occurret, vt ob $x = 0$ sit interuallum FO $= \frac{n\alpha\alpha}{n\alpha - iv}$. seu aO $= \frac{-i\alpha v}{n\alpha - iv}$. Nisi ergo oculus in hoc axis loco teneatur, radius transmissus non in oculum ingredietur; si quidem apertura pupillae vt infinite parua spectetur.

Coroll.

Coroll. 5.

227. At si semidiameter pupillae ponatur $= \omega$, oculus etiam in o positus illum radium excipiet, si fuerit $o\,\omega = \omega$. Cum autem casu $x = 0$ fit

$$a\,N\left(\frac{vz}{na}\right) : a\,O\left(\frac{i\alpha v}{n\alpha - iv}\right) = o\,\omega\,(\omega) : O\,o \text{ erit}$$

$$O\,o = \frac{n i a \alpha \omega}{z(n\alpha - iv)}.$$

Quod interuallum cum aeque positiuum ac negatiuum accipi possit, pro loco oculi o habebimus

$$a\,o = -\frac{i\alpha vz \pm n i a \alpha \omega}{z(n\alpha - iv)} = \frac{-i\alpha(vz \pm n a \omega)}{z(n\alpha - iv)}.$$

Scholion.

228. In hac tractatione, vbi campum apparentem et locum oculi idoneum inuestigamus, tam aperturam lentis obiectiuae in facie anteriori quam pupillae amplitudinem pro nihilo habebimus, vt quaestiones obtineamus determinatas. Quare horum elementorum, quae in instrumentis dioptricis ad visionem accommodatis maximi sunt momenti, sequentes definitiones constituemus.

Definitio I.

229. Campus apparens est spatium in obiecto, ex cuius singulis punctis radii in centrum lentis obiectiuae incidentes per reliquas lentes omnes transmittuntur. Quod spatium cum sit circulare, eius radius vocatur semidiameter campi apparentis.

Coroll.

Coroll. 1.

230. Si ergo $E\varepsilon = z$ fuerit ſemidiameter campi apparentis, erit ε punctum obiecti extremum, ſed ab axe maxime remotum, ex quo adhuc radii in centrum A lentis obiectiuae incidentes per omnes lentes transmittuntur.

Coroll. 2.

231. Determinatur igitur magnitudo campi apparentis per aperturam ſequentium facierum refringentium, ac fortaſſe per aperturam vnius, ſi ſcilicet radii a puncto quodam magis remoto quam ε venientes nullum transitum per eam inuenirent, etiamſi per reliquas facies transmitterentur.

Scholion.

232. Si radii ex ſolo puncto E, quod eſt centrum campi apparentis, conſiderentur, ii quidem qui in A incidunt, quoniam perpetuo ſecundum axem progrediuntur, per omnes reliquas facies refringentes, quantumuis parua fuerit earum apertura, certe transmittentur; hincque campus apparens nunquam penitus euaneſcere poteſt. At quo magis punctum ε ab axe diſtans accipitur, vt radii ab eo per omnes facies transmittantur, eo maior earum apertura requiritur, quae cum ab earum curuatura pendeat, neque certum limitem ſuperare debeat, hinc vltimum punctum ε, vnde radii etiam nunc transmittuntur, ac propterea ſemidiameter campi apparentis determinatur.

natur. In ſequentibus quidem propoſitionibus campum apparentem ſeu eius ſemidiametrum $E\varepsilon = z$ vt datum aſſumam, et quanta eſſe debeat cuiusque faciei apertura, inueſtigabo: hinc enim facile viciſſim, ſi quaeque apertura fuerit cognita, campum apparentem ipſum definire licebit. Ceterum in hac definitione aſſumſi aperturam primae faciei eſſe euaneſcentem: ex quo facile intelligitur ea aucta etiam campum aliquantum extendi oportere; verum hoc augmentum poſteá in lucrum cedet, quod cum nunquam ſoleat eſſe notabile, eius rationem hic non habendam cenſui; quemadmodum etiam in ſequente definitione aperturae pupillae rationem non habebo.

Definitio 2.

233. Locus oculi idoneus eſt id punctum in axe, in quo radii ab extremitate campi apparentis per lentes transmiſſi axem interſecant. Oculus ſcilicet in hoc loco conſtitutus totum campum apparentem conſpiciet.

Coroll. 1.

234. Hinc igitur idoneus locus oculo aſſignabitur, ſi illa interſectio radiorum extremorum cum axe poſt lentem vltimam cadat, ſin autem haec interſectio ante lentem vltimam reperiatur, fieri nequit vt oculus in eo loco teneatur neque propterea totum campum contueri poterit.

Coroll. 2.

235. At si ista intersectio pone lentem vltimam cadat, oculus totum campum apparentem perspiciet, etiamsi pupilla maxime esset contracta: neque tamen ob maiorem pupillae amplitudinem maiorem campum percipere valet.

Coroll. 3.

236. Verum ob amplitudinem pupillae hoc commodi assequimur, vt oculus etiamsi extra locum idoneum constituatur, dummodo distantia non sit nimis magna, tamen totum campum apparentem conspicere possit: id quod egregie vsu veniet iis casibus, quibus locus idoneus oculi ante faciem refringentem extremam cadit. Tum enim fieri poterit, vt oculus huic faciei immediate applicatus tamen totum campum percipiat.

Scholion.

237. Quando hic de visione loquor, id ita in genere est interpretandum, vt a puncto viso radius in oculum ingrediatur, neque hic curo, vtrum visio sit distincta nec ne? in sequentibus enim docebitur, quomodo lentes disponi conueniat, vt oculus in loco idoneo positus etiam in iusta ab vltima imagine distantia reperiatur, quo visio distincta reddatur. Hic igitur sine vllo respectu ad visionem distinctam habito, eum oculo locum assigno, vbi ab omnibus punctis

ctis in campo apparente contentis radios recipiat, et quoniam singula momenta, quae ad visionem pertinent, seorsim expediri conuenit; hic etiam non ad spatium diffusionis respicio, quod quidem semper per se euanescit, si apertura faciei primae euanescens statuatur.

Problema 2.

238. Si obiectum per vnicam lentem aspiciatur, determinare tam campum apparentem, quam locum idoneum oculi. Tab. III. Fig. 12.

Solutio.

Sint vt in problemate superiori distantiae determinatrices huius lentis, scilicet distantia obiecti ante lentem $AE = a$, et imaginis post lentem $aF = \alpha$, tum vero lentis crassities $Aa = v$, et distantia arbitraria constructionem lentis plene determinans $= k$. Deinde ponamus semidiametrum campi apparentis $E\varepsilon = z$, ita vt posita faciei anterioris PAP apertura infinite parua etiamnum a puncto ε radii per lentem transmittantur. Sit porro breuitatis gratia $\frac{k - v}{k + v} = i$, atque supra demonstrauimus campum apparentem ad ε vsque extendi, si pro facie posteriori PaP fuerit semidiameter aperturae $aN = \frac{vz}{na}$ existente $n = \frac{31}{20}$; perinde enim est, siue hic semidiameter affirmatiue accipiatur siue negatiue. Hinc ergo vicissim si semidiameter huius aperturae ponatur $= \alpha$, erit semidiameter campi apparentis $E\varepsilon = z = \frac{na}{v}\alpha$.

Quod ad locum idoneum oculi attinet, qui ſit in O, quoniam inuenimus $FO = \frac{n\alpha\alpha}{n\alpha - iv}$, erit diſtantia $aO = \frac{-i\alpha v}{n\alpha - iv}$; ideoque negatiua, niſi ſit vel i numerus negatiuus, vel $iv > n\alpha$. Sin autem haec diſtantia aO fuerit poſitiua, oculus in O poſitus totum campum apparentem perſpiciet.

Coroll. 1.

239. Si craſſities lentis v euaneſcat, ob $z = \frac{na}{v}\alpha$, campus apparens euadet infinitus, ſeu potius indeterminatus; diſtantia vero aO euaneſcet. Oculus igitur lenti immediate applicatus tantum ſpatium conſpiciet, quantum per propriam indolem complecti valebit.

Coroll. 2.

240. Sin autem ob lentis craſſitiem $Aa = v$ diſtantia $aO = \frac{-i\alpha v}{n\alpha - iv}$ prodeat poſitiua, oculus in O poſitus totum campum apparentem aſpicere valebit, ſeu in obiecto ſpatium circulare ſpectabit, cuius ſemidiameter $E\varepsilon = z = \frac{na}{v}\alpha = \frac{3 i a a}{20 v}$, quod ergo eo erit maius, quo tenuior fuerit lens.

Coroll. 3.

241. At ſi diſtantia aO reſultet negatiua, oculum in loco idoneo conſtitui non licet, quoniam is neceſſario poſt lentem teneri debet. Vbicunque autem is poſt lentem collocetur, non vniuerſum campum apparentem contuebitur, ſed tantum eius partem, et quidem

quidem eo minorem, quo magis post lentem removeatur, propterea quod hoc modo magis a loco idoneo recedit.

Coroll. 4.

242. Hoc igitur casu conueniet oculum immediate ad faciem lentis posteriorem applicare, quo situ eatenus tantum radios accipiet, quatenus pupilla patet; sicque campus visus ab apertura pupillae pendebit; quae si esset nulla etiam campus apparens euanesceret.

Coroll. 5.

243. Hinc patet si pupilla excedit aperturam faciei PaP seu si sit $\omega > a$, denotante ω semidiametrum pupillae, quia tum oculus huic faciei applicatus omnes radios transmissos recipit, eum totum campum apparentem esse visurum. Sin autem sit $\omega < a$, partem tantum totius campi perspiciet, cuius semidiameter erit $= \frac{na}{v}\omega = \frac{31 a\omega}{20 v}$, scilicet non maiorem quam si apertura faciei PaP aequalis esset pupillae.

Scholion.

244. Interim tamen si hoc casu postremo apertura faciei lentis PaP, cui oculus est applicatus maior sit quam pupilla, nihil obstat quominus ea successiue totam aperturam peragret, sicque pedetentim totum campum apparentem con-

spicere poterit, etiamsi eum simul contueri non valeat. Ceterum notandum est, si etiam faciei anteriori apertura tribuatur, inde campum apparentem aliquantum augeri, sed partes obiecti ulteriores, quia non per medium lentis radios transmittunt, obscuriores apparebunt, vnde merito a campo apparente excluduntur. At si faciei anteriori tribuatur apertura, cuius semidiameter $= x$, vt facies posterior omnes radios in illam incidentes transmittat, eius apertura tanto maior esse debet, secundum regulas supra traditas; scilicet eius semidiametrum esse oportet $= ix + a$ seu $= ix + \frac{vz}{na}$.

Problema. 3.

245. Si instrumentum dioptricum duabus constet lentibus, definire campum apparentem, et locum oculi idoneum.

Solutio.

Tab. III. Fig. 13. Sit pro his lentibus vt hactenus:

pro PP: $AE = a, aF = \alpha, Aa = v$; dist. arb: $= k$, et $\frac{k-v}{k+v} = i$

pro QQ: $BF = b, bG = \beta, Bb = v'$; dist. arb: $= k'$ et $\frac{k'-v'}{k'+v'} = i'$

ac posito semidiametro campi apparentis $E\varepsilon = z$, in imaginibus erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z$ et $G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z$. Consideretur iam radius a puncto ε in centrum lentis primae A incidens et per lentes transmissus qui vtique per extremitates imaginum ζ et η transibit. Iam

Iam ex problemate praecedente patet eſſe $a\mathrm{N} = \frac{vz}{na}$, vnde in altera lente punctum M' ita definietur, vt ſit

$$\mathrm{F}\zeta - a\mathrm{N} : a\mathrm{F} = \mathrm{BM}' - \mathrm{F}\zeta : \mathrm{BF} \text{ ſiue}$$

$$\mathrm{BM}' = \mathrm{F}\zeta + \frac{\mathrm{BF}(\mathrm{F}\zeta - a\mathrm{N})}{a\mathrm{F}} = \frac{a\mathrm{B}.\mathrm{F}\zeta - \mathrm{BF}.a\mathrm{N}}{a\mathrm{F}}$$

vnde ſit

$$\mathrm{BM}' = \tfrac{1}{i}.\frac{\alpha+b}{a}z - \frac{bvz}{na\alpha}.$$

Nunc punctum N' hinc perinde definietur, atque ex problemate primo ex puncto M determinabatur punctum N; erit quippe

$$b\mathrm{N}' = i'.\mathrm{BM}' - \frac{v'}{n'b}.\mathrm{F}\zeta \text{ ideoque}$$

$$b\mathrm{N}' = \frac{i'}{i}.\frac{\alpha+b}{a}z - \frac{i'bv}{na\alpha}z - \tfrac{1}{i}.\frac{\alpha v'}{n'ab}z$$

Hinc autem punctum O, vbi eſt locus oculi idoneus facile aſſignabitur erit enim $b\mathrm{N}' + \mathrm{G}\eta : b\mathrm{G} = b\mathrm{N}' : b\mathrm{O}$, indeque

$$b\mathrm{O} = \frac{b\mathrm{G}.b\mathrm{N}'}{b\mathrm{N}' + \mathrm{G}\eta} = \frac{\frac{i'}{i}.\frac{\alpha+b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i}.\frac{\alpha v'}{nab}}{\frac{i'}{i}.\frac{\alpha+b}{a} - \frac{i'bv}{n'a\alpha} - \frac{1}{i}.\frac{\alpha v'}{nab} + \frac{1}{ii'}.\frac{\alpha\mathfrak{C}}{ab}}\mathfrak{C}$$

Quod ſi ergo ponamus ſemidiametrum aperturae

pro lente PP $\begin{cases} \text{faciei anterioris} = \mathfrak{A} = x \\ \text{faciei poſterioris} = \mathfrak{a} \end{cases}$

pro lente QQ $\begin{cases} \text{faciei anterioris} = \mathfrak{B} \\ \text{faciei poſterioris} = \mathfrak{b} \end{cases}$

habe-

habebimus

$$\mathfrak{A} = 0$$

$$\mathfrak{a} = \frac{v}{n\,a} z$$

$$\mathfrak{B} = \left(\frac{1}{i} \cdot \frac{\alpha+b}{a} - \frac{b\ v}{n\,a\,\alpha}\right) z$$

$$\mathfrak{b} = \left(\frac{i'}{i} \cdot \frac{\alpha+b}{a} - \frac{i' b v}{n\,a\,\alpha} - \frac{1}{i} \cdot \frac{\alpha\,v'}{n\,a\,b}\right) z$$

ac si diſtantia oculi poſt lentem QQ ponatur $bO = O$ erit

$$O = \frac{\mathfrak{b}}{\mathfrak{b} + \frac{1}{ii'} \cdot \frac{\alpha\,\beta}{a\,b} z} \cdot \beta$$

Coroll. 1.

246. Si ambarum lentium craſſities euaneſcat erit $v = 0.\ v' = 0$, et $i = i' = 1$; quo ergo caſu noſtrae formulae in ſequentes abibunt:

$$\mathfrak{A} = 0;\quad \mathfrak{a} = 0;\quad \mathfrak{B} = \frac{\alpha+b}{a} z \quad \text{et} \quad \mathfrak{b} = \frac{\alpha+b}{a} z$$

Coroll. 2.

247. Datis ergo viciſſim aperturis lentium ex aequationum traditarum ea, pro qua quantitas z minimum valorem adipiſcitur, definietur campus adparens.

Coroll. 3.

248. Si igitur craſſities lentium euaneſcat, campus apparens ex apertura lentis poſterioris facillime determinatur. Erit enim $z = \frac{a\ \mathfrak{B}}{\alpha+b}$; id quod intelligendum eſt, ſi diſtantia $bO = O$ fuerit poſitiua oculusque in O collocetur.

Coroll.

Coroll. 4.

249. Sin autem diſtantia $bO = O$ prodeat negatiua oculusque vltimae lenti immediate applicetur, tum eius apertura plus non praeſtat quam ſi amplitudini pupillae eſſet aequalis. Quare ſi $\mathfrak{b}$ maior fuerit ſemidiametro pupillae ω loco $\mathfrak{b}$ ſcribatur ω et ex vltima aequatione verus valor ipſius z elicietur niſi forte ex aliqua reliquarum aequationum adhuc minor valor pro z eſſet prodituros.

Problema 4.

250. Si inſtrumentum dioptricum ex tribus conſtet lentibus determinare cum campum apparentem, tum locum oculi idoneum. Tab. III. Fig. 14.

Solutio.

Exiſtentibus imaginibus per has lentes ſucceſſiue repraeſentatis in $F\zeta$, $G\eta$ et $H\theta$, obiecto vero ipſo in $E\varepsilon$, ponamus vt hactenus:

Pro Lente	Diſtantias		craſſitiem	diſt. arb: et
Prima PP..	$AE = a$;	$aF = \alpha$;	$Aa = v$;	k; $\frac{k-v}{k+v} = i$
Secunda QQ..	$BF = b$;	$bG = \mathfrak{E}$;	$Bb = v'$;	k'; $\frac{k'-v'}{k'+v'} = i'$
Tertia RR..	$CG = c$;	$cH = \gamma$;	$Cc = v''$;	k''; $\frac{k''-v''}{k''+v''} = i''$

Semidiametros vero aperturarum

Pro Lente

PP faciei $\begin{cases} \text{anterioris} = \mathfrak{A} = o \\ \text{poſterioris} = \mathfrak{a} \end{cases}$

$$\text{QQ faciei}\begin{cases}\text{anterioris} = \mathfrak{B}\\ \text{posterioris} = \mathfrak{b}\end{cases}$$

$$\text{RR faciei}\begin{cases}\text{anterioris} = \mathfrak{C}\\ \text{posterioris} = \mathfrak{c}\end{cases}$$

Tum vero ſit ſemidiameter campi apparentis $E\varepsilon = z$, ſupraque oſtendimus fore:

$$F\zeta = \frac{1}{i}\frac{\alpha}{a}z;\ G\eta = \frac{1}{i\,i'}\cdot\frac{\alpha\,\beta}{a\,b}z;\ H\theta = \frac{1}{i\,i'\,i''}\cdot\frac{\alpha\,\beta\,\gamma}{a\,b\,c}z.$$

His poſitis habebimus $aN = \alpha = \frac{vz}{na}$; vnde ſi per F recta ipſi NM' parallela ducta intelligatur erit

$$F\zeta - aN : aF = BM' - F\zeta : BF \text{ ſiue } BM' = \frac{aB.F\zeta - BF.aN}{a.F}$$

ideoque

$$BM' = \frac{\alpha + b}{\alpha}\cdot\frac{1}{i}\cdot\frac{\alpha}{a}z - \frac{bvz}{na\alpha} \text{ ſeu}$$

$$\mathfrak{B} = \frac{1}{i}\cdot\frac{\alpha + b}{a}z - \frac{b\,v}{n\,a\,\alpha}z.$$

Porro vero eſt ex problemate primo $bN' = i'.BM' - \frac{v'}{nb}.F\zeta$,

hincque $\mathfrak{b} = \frac{i'}{i}\cdot\frac{\alpha + b}{a}z - \frac{i'bv}{na\alpha}z - \frac{1}{i}\cdot\frac{\alpha v'}{nab}z.$

ſimili modo per G ducta intelligatur recta ipſi $N'M''$ parallela, eritque $bG : bN' + G\eta = CG : CM'' - G\eta$ ſiue

$$CM'' = \mathfrak{C} = G\eta + \frac{c}{\beta}(\mathfrak{b} + G\eta) = \frac{\beta + c}{\beta}.G\eta + \frac{c}{\beta}\mathfrak{b},$$

vnde fit

$$\mathfrak{C} = \frac{1}{i\,i'}\cdot\frac{\alpha(\beta + c)}{a\,b}z + \frac{i'}{i}\cdot\frac{c(\alpha + b)}{a\,\beta}z - \frac{i'bcv}{na\alpha\beta}z - \frac{1}{i}\cdot\frac{\alpha c v'}{n\,a\,b\,\beta}z$$

Deinde

Deinde ex CM'' ita definitur cN'' per problema primum, vt ſit

$$cN'' = i''.\,CM'' - \frac{v''}{nc}.\,G\eta \text{ ſeu } \mathfrak{c} = i''.\,\mathfrak{C} - \frac{1}{ii'}\cdot\frac{\alpha\beta v''}{nabc}z$$

hincque

$$\mathfrak{c} = \frac{i''}{ii'}\cdot\frac{\alpha(\beta+c)}{ab}z + \frac{i'i''}{i}\cdot\frac{c(\alpha+b)}{a\beta}z - i'i''.\frac{bcv'}{na\alpha\beta}z - \frac{i''}{i}.\frac{\alpha cv'}{nab\beta}z - \frac{1}{ii'}\cdot\frac{\alpha\beta v''}{nabc}z$$

Iſti ergo valores ſequenti modo determinantur:

$$\mathfrak{A} = 0$$

$$\mathfrak{a} = \frac{v}{na}.\,E\varepsilon$$

$$\mathfrak{B} = \frac{\alpha+b}{\alpha}.\,F\zeta - \frac{b}{\alpha}.\mathfrak{a}$$

$$\mathfrak{b} = i'.\mathfrak{B} - \frac{v'}{nb}.F\zeta$$

$$\mathfrak{C} = \frac{\beta+c}{\beta}.\,G\eta + \frac{c}{\beta}.\mathfrak{b}$$

$$\mathfrak{c} = i''.\,\mathfrak{C} - \frac{v''}{nc}.\,G\eta$$

Cum iam punctum O praebeat locum oculi iuſtum, ſi ponamus $cO = O$, erit $cN'' + H\theta : cH = cN'' : cO$ vnde reperitur

$$O = \frac{\gamma\mathfrak{c}}{\mathfrak{c}+H\theta}, \text{ vel } \frac{1}{O} = \frac{1}{\gamma} + \frac{1}{ii'i''}\cdot\frac{\alpha\beta}{ab}\cdot\frac{z}{c\mathfrak{c}}.$$

Coroll. I.

251. Ex his aequationibus ſequitur fore

$$\frac{\mathfrak{B}}{b} + \frac{\mathfrak{a}}{\alpha} = \frac{\alpha+b}{\alpha b}.\,F\zeta = \frac{1}{i}\left(\frac{\alpha}{b}+1\right)\frac{z}{a} \text{ et}$$

$$\frac{\mathfrak{C}}{c} - \frac{\mathfrak{b}}{\beta} = \frac{\beta+c}{\beta c}.\,G\eta = \frac{1}{ii'}\left(\frac{\alpha\beta}{bc}+\frac{\alpha}{b}\right)\frac{z}{a}.$$

hincque porro:

$$\frac{i\mathfrak{a}}{\alpha} + \frac{i\mathfrak{B}}{b} + \frac{ii'\mathfrak{b}}{\beta} - \frac{ii'\mathfrak{C}}{c} = \left(1 - \frac{\alpha\beta}{bc}\right)\frac{z}{a}.$$

Coroll. 2.

252. Si ergo craſſities lentium euaneſcat, cum ſit $\mathfrak{a} = 0$, $\mathfrak{b} = \mathfrak{B}$ et $\mathfrak{c} = \mathfrak{C}$, definitio campi apparentis reducitur ad has duas aequationes.

$$\text{I. } \mathfrak{B}\cdot\frac{1}{b} = \left(1 + \frac{\alpha}{b}\right)\frac{z}{a}$$

$$\text{II. } \mathfrak{B}\left(\frac{1}{b} + \frac{1}{\beta}\right) - \mathfrak{C}\cdot\frac{1}{c} = \left(1 - \frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

vnde minor valor ipſius z praebet ſemidiametrum campi apparentis.

Coroll. 3.

253. Deinde ſi diſtantia $cO = O$ prodeat negatiua, vt oculum cogamur lenti vltimae immediate applicare, pro c ſcribi oportet ſemidiametrum pupillae ω, et ex vltima aequatione definietur ſemidiameter campi z, niſi forte ex alia aequatione adhuc minor valor pro z prodeat.

Problema 5.

254. Si inſtrumentum Dioptricum ex quatuor lentibus ſuper eodem axe diſpoſitis conſtet, determinare cum campum apparentem, tum locum oculi idoneum.

Solutio.

Tab. III. Fig. 15. Exiſtente obiecto $E\varepsilon = z$, ſint imagines per lentes ſucceſſiue repraeſentatae $F\zeta$, $G\eta$, $H\theta$, et $I\iota$, ponamusque

musque pro lentium ſingularum determinatione vt hactenus:

Pro lente	diſtantias	craſſitiem	diſt. arb.	et
Prima PP	$AE = a$; $aF = \alpha$;	$Aa = v$;	$..k$;	$\frac{k-v}{k+v} = i$
Secunda QQ	$BF = b$; $bG = \beta$;	$Bb = v'$;	$..k'$,	$\frac{k'-v'}{k'+v'} = i'$
Tertia RR	$CG = c$; $cH = \gamma$;	$Cc = v''$;	$..k''$;	$\frac{k''-v''}{k''+v''} = i''$
Quarta SS	$DH = d$; $dI = \delta$;	$Dd = v'''$;	$..k'''$;	$\frac{k'''-v'''}{k'''+v'''} = i'''$

Semidiametri vero aperturarum ſint:

Pro Lente

Prima PP faciei $\begin{cases} \text{anterioris} = \mathfrak{A} = 0 \\ \text{poſterioris} = \mathfrak{a} \end{cases}$

Secunda QQ faciei $\begin{cases} \text{anterioris} = \mathfrak{B} \\ \text{poſterioris} = \mathfrak{b} \end{cases}$

Tertia RR faciei $\begin{cases} \text{anterioris} = \mathfrak{C} \\ \text{poſterioris} = \mathfrak{c} \end{cases}$

Quarta SS faciei $\begin{cases} \text{anterioris} = \mathfrak{D} \\ \text{poſterioris} = \mathfrak{d} \end{cases}$

Si iam $E\varepsilon = z$ exhibeat ſemidiametrum campi apparentis erit, vt iam ſupra oſtendimus:

$$F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z; \; G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z; \; H\theta = \frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma}{abc} z$$

$$\text{et } I\iota = \frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma\delta}{abcd} z$$

 Quod

Quod si ratiocinium nunc vt ante instituamus, obtinebimus sequentes aequationes:

$$\mathfrak{A}=0;\qquad \mathfrak{a}=\frac{v}{na}\cdot E\varepsilon$$

$$\mathfrak{B}=\left(1+\frac{b}{\alpha}\right)F\zeta-\frac{b}{\alpha}\mathfrak{a}:\qquad \mathfrak{b}=i'\mathfrak{B}-\frac{v'}{nb}\cdot F\zeta$$

$$\mathfrak{C}=\left(1+\frac{c}{\beta}\right)G\eta+\frac{c}{\beta}\cdot\mathfrak{b}:\qquad \mathfrak{c}=i''\mathfrak{C}-\frac{v''}{nc}\cdot G\eta$$

$$\mathfrak{D}=\left(1+\frac{d}{\gamma}\right)H\theta+\frac{d}{\gamma}\cdot\mathfrak{c}:\qquad \mathfrak{d}=i'''\mathfrak{D}-\frac{v'''}{nd}\cdot H\theta$$

Ex quarum ordine priori consequimur:

$$\frac{\mathfrak{B}}{b}+\frac{\mathfrak{a}}{\alpha}=\left(\frac{1}{\alpha}+\frac{1}{b}\right)F\zeta=\frac{1}{i}\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\frac{\mathfrak{C}}{c}-\frac{\mathfrak{b}}{\beta}=\left(\frac{1}{\beta}+\frac{1}{c}\right)G\eta=\frac{1}{ii'}\left(\frac{\alpha}{b}+\frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\frac{\mathfrak{D}}{d}-\frac{\mathfrak{c}}{\gamma}=\left(\frac{1}{\gamma}+\frac{1}{d}\right)H\theta=\frac{1}{ii'i''}\left(\frac{\alpha\beta}{bc}+\frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a}$$

ex ordine vero posteriori

$$\mathfrak{a}=\frac{vz}{na}$$

$$\mathfrak{b}=i'\mathfrak{B}-\frac{1}{i}\cdot\frac{\alpha}{b}\cdot\frac{v'z}{na}$$

$$\mathfrak{c}=i''\mathfrak{C}-\frac{1}{ii'}\cdot\frac{\alpha\beta}{bc}\cdot\frac{v''z}{na}$$

$$\mathfrak{d}=i'''\mathfrak{D}-\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{bcd}\cdot\frac{v'''z}{na}$$

Si denique pro loco oculi idoneo ponamus $dO=O$ erit

$$O=\frac{\delta\mathfrak{d}}{\mathfrak{d}+\mathrm{I}i}\ \text{seu}\ \frac{1}{O}=\frac{1}{\delta}+\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma}{bcd}\cdot\frac{z}{a\mathfrak{d}}.$$

Coroll.

Coroll. 1.

255. Ex aequationibus prioribus deducimus sequentes:

$$\frac{i\mathfrak{a}}{\alpha}+\frac{i\mathfrak{B}}{b}=\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\frac{i\mathfrak{a}}{\alpha}+i\frac{\mathfrak{B}}{b}+\frac{ii'\mathfrak{b}}{\beta}-\frac{ii'\mathfrak{C}}{c}=\left(1-\frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\frac{i\mathfrak{a}}{\alpha}+\frac{i\mathfrak{B}}{b}+\frac{ii'\mathfrak{b}}{\beta}-\frac{ii'\mathfrak{C}}{c}-\frac{ii'i''\mathfrak{c}}{\gamma}+\frac{ii'i''\mathfrak{D}}{d}=\left(1+\frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a}.$$

quae quomodo ad plures lentes sint continuandae, facile perspicitur.

Coroll. 2.

256. Si lentium crassities euanescat, fiet $\mathfrak{a}=0$; $\mathfrak{b}=\mathfrak{B}$; $\mathfrak{c}=\mathfrak{C}$ et $\mathfrak{d}=\mathfrak{D}$, porro $i=i'=i''=i'''=1$, vnde hae aequationes in sequentes formas abibunt:

$$\mathfrak{B}.\frac{1}{b}=\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)-\mathfrak{C}\frac{1}{c}=\left(1-\frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)-\mathfrak{C}\left(\frac{1}{c}+\frac{1}{\gamma}\right)+\mathfrak{D}\frac{1}{d}=\left(1+\frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a}$$

ex quibus tribus aequationibus, vti in genere, valor ipsius z, qui prodierit minimus verum semidiametrum campi apparentis praebebit.

Coroll. 3.

257. Totus iste campus apparens reuera spectabitur ab oculo in puncto O constituto, dummodo distantia dO$=$O fuerit positiua. Sed si ea sit negatiua, oculusque lenti vltimo SS immediate applicetur, pona-

ponatur $\mathfrak{d} = \omega$, ſcilicet ſemidiametro pupillae, et ex vltima aequatione elicietur ſemidiameter ſpatii in obiecto reuera conſpicui.

Scholion I.

258. Hinc igitur perſpicitur, quomodo campus apparens a ſingularum lentium apertura pendeat; ſimulque patet quanta eſſe debeat cuiusque lentis apertura, vt campus apparens datae magnitudinis obtineatur. Si enim quantitas z cum quantitatibus ad lentium determinationem pertinentibus pro data aſſumatur, per noſtras formulas ſucceſſiue ſemidiametri aperturarum pro ſingulis lentibus definiuntur: vbi quidem deinceps eſt diſpiciendum, num lentes tantae aperturae ſint capaces. Hinc ſcilicet campo apparenti limites praefiniuntur, quos transgredi non liceat; vnde ſequitur campum apparentem maiorem aſſumi non poſſe, quam vt aperturae inde pro ſingulis lentibus oriundae admitti queant. His autem definitis perinde eſt ſiue cuique lenti ea ipſa, quae fuerit inuenta apertura tribuatur, ſiue maior, dum ne ſit minor quandoquidem hic aperturam lentis obiectiuae euaneſcentem aſſumſimus. Verum ſi inſuper claritatis ratio habeatur, neceſſe eſt vt vera apertura cuiusque lentis eam, quam hic aſſignauimus, aliquantum ſuperet, et quidem ea quantitate, quam ſupra pro limitibus ob claritatem requiſitis exhibuimus, niſi enim hoc augmentum acceſſerit, extremitas in campo apparente minore lumine praedita erit quam

quam medium. Tum autem campus apparens latius patebit oramque obſcuriorem complectetur; quamobrem ſi circa extremitates minori lumine contenti eſſe velimus, ne opus quidem eſt, vt lentibus maior apertura, quam quidem per formulas noſtras definitur, tribuatur; ſuperfluumque foret aperturas vltra hos limites augere, ita vt hinc cuique lenti conueniens apertura conſtituatur.

Scholion 2.

259. Etſi pro caſu, quo lentium craſſities negligitur formulae noſtrae non multo ſimpliciores euadunt, tamen in iis percommode vſu venit, vt litterarum $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, coefficientes, ſcilicet $\frac{1}{b}+\frac{1}{\beta}$; $\frac{1}{c}+\frac{1}{\gamma}$; $\frac{1}{d}+\frac{1}{\delta}$; ipſam diſtantiam focalem, inuoluant cuiuſque lentis; in praxi autem apertura ſatis tuto ex diſtantia focali colligi ſolet. Nam ſi lentis QQ diſtantia focalis ponatur $= q$, erit $\frac{1}{b}+\frac{1}{\beta}=\frac{1}{q}$, ſicque $\mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})=\frac{\mathfrak{B}}{q}$; ac ne arcus nimis magni in apertura comprehendantur, neceſſe eſt vt ſit $\mathfrak{B} < \frac{1}{2} q$; et pro varia lentis forma valor fractionis $\frac{\mathfrak{B}}{q}$ vſque ad $\frac{1}{4}$, vel $\frac{1}{6}$ diminui debet. Quare ſi ponamus

$$\mathfrak{B}(\tfrac{1}{b}+\tfrac{1}{\beta})=\pi;\ \mathfrak{C}(\tfrac{1}{c}+\tfrac{1}{\gamma})=\pi';\ \mathfrak{D}(\tfrac{1}{d}+\tfrac{1}{\delta})=\pi''$$

hae litterae π, π', π'' eiusmodi denotabunt fractiones, quarum valor vt plurimum erit vel $\frac{1}{3}$ vel $\frac{1}{4}$ vel $\frac{1}{5}$, id ſcilicet tantum cauendum eſt, ne his litteris nimis magnus valor tribuatur. Quo obſeruato cum arbitrio noſtro relinquantur, imprimis conueniet has ipſas litteras

teras in calculum introduci, ex iisque reliquas deter-
minari; earum enim beneficio campus apparens facil-
lime definitur. Quin etiam ipſe campus apparens
ſtatim quoque in calculum induci poterit, quippe
cuius determinatio deinceps per formulam ſimpliciſſi-
mam expedietur. Hunc in finem vt tota inueſtiga-
tio ad meros numeros redigatur, ponam $\frac{z}{a} = \Phi$ ita vt Φ
ſit angulus, ſub quo ſemidiameter campi apparentis
ab oculo ad lentem obiectiuam collocato ſpectaretur.
Videamus ergo quomodo per hos numeros π, π', π'', etc.
et Φ reliquae quantitates definiantur.

Definitio 3.

260. Ratio aperturae cuiuſque lentis mihi vo-
cabitur quotus, qui oritur, ſi ſemidiameter aperturae
diuidatur per diſtantiam focalem lentis; eius craſſitie
pro nihilo habita.

Coroll. 1.

261. Ita ſi b et $\mathfrak{C}$ ſint diſtantiae determina-
trices lentis, et $\mathfrak{B}$ ſemidiameter aperturae eiuſdem,
quia diſtantia focalis eſt $= \frac{b\mathfrak{C}}{b+\mathfrak{C}}$, ratio aperturae erit
$= \mathfrak{B}(\frac{1}{b} + \frac{1}{\mathfrak{C}})$

Coroll. 2.

262. Ratio igitur aperturae cuiuſuis lentis eſt
fractio minor quam $\frac{1}{2}$, quandoquidem hanc legem
ſanciuimus vt neutrius faciei arcus 60 gradibus ma-
ior in apertura contineatur.

Coroll.

Coroll. 3.

263. Si scilicet ambae facies fuerint aeque curuae ratio aperturae per hanc legem vsque ad $\frac{1}{2}$ augeri poterit; sin autem altera facies fuerit plana, ratio aperturae $\frac{1}{4}$ superare vix poterit: ac si lens sit meniscus, ea adhuc minor statui debebit.

Coroll. 4.

264. Cum autem nihil sit, quo apertura lentium accuratius definiatur, ea fere arbitrio nostro relinquitur, et quouis casu commodissime per experientiam determinatur, sufficietque notasse, eam fractioni siue $\frac{1}{3}$ siue $\frac{1}{4}$ siue etiam $\frac{1}{5}$ pro forma lentis aequalem statui debere.

Scholion.

265. Vt scilicet quouis casu ratio aperturae recte definiatur, radios vtriusque faciei lentis contemplari oportet, qui si fuerint f et g, erit distantia focalis $= \frac{fg}{(n-1)(f+g)} = \frac{20}{11} \cdot \frac{fg}{f+g}$. Iam semidiameter aperturae minor esse debet quam $\frac{1}{2}f$ vel quam $\frac{1}{2}g$, prout vel f vel g fuerit minor. Sit $g < f$, et cum semidiameter aperturae minor esse debeat quam $\frac{1}{2}g$, ratio aperturae minor accipienda est quam $\frac{11}{40}(1+\frac{g}{f})$. Vnde patet, si lens sit vtrinque aeque conuexa, seu $g=f$, rationem aperturae capi debere infra $\frac{11}{20}$; sin autem sit altera facies plana seu $f=\infty$, illum limitem esse $\frac{11}{40}$, qui adhuc minor fiet, si lens sit meni-

meniſcus, ſeu $\frac{g}{f}$ numerus negatiuus. Ceterum ſi ratio aperturae ſit $= \pi$, eique hoc modo idoneus valor tribuatur, perinde eſt ſiue is negatiue ſiue poſitiue accipiatur $\frac{g}{f}$ ſemper autem conducet rationi aperturae minorem valorem tribui, quam ſecundum hanc regulam; partim vt obliquitas radiorum incidentium diminuatur, partim vero potiſſimum, vt ob claritatem aperturas lentium adhuc vltra augere liceat.

Problema 6.

266. Si inſtrumentum dioptricum ex quotcunque lentibus, quarum craſſitiem vt nullam ſpectare liceat, ſit compoſitum, dataque ſit ratio aperturae pro ſingulis lentibus vna cum campo apparente, definire diſtantias determinatrices ſingularum lentium.

Solutio.

Sit diſtantia obiecti ante lentem primam $AE = a$, imaginisque per eam repraeſentatae $aF = \alpha$, ac pro ſequentibus lentibus ponatur.

Pro Lente	Diſt: determinatrices	ratio aperturae
Secunda . .	$BF = b$; $bG = \beta$;	π
Tertia . .	$CG = c$; $cH = \gamma$;	π'
Quarta . .	$DH = d$; $dI = \delta$;	π''
Quinta . .	$EI = e$; $eK = \varepsilon$;	π'''

etc.

Hinc

Hinc ergo si semidiametri aperturarum harum lentium vt ante indicentur literis $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, etc. erit $\pi = \mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})$; $\pi' = \mathfrak{C}(\frac{1}{c}+\frac{1}{\gamma})$; $\pi'' = \mathfrak{D}(\frac{1}{d}+\frac{1}{\delta})$; $\pi''' = \mathfrak{E}(\frac{1}{e}+\frac{1}{\varepsilon})$ etc. Tum vero si sit semidiameter campi apparentis $= z$ ponatur etiam $\frac{z}{a} = \Phi$. Cum igitur hinc sit:

$$\mathfrak{B} = \frac{\pi b \beta}{b+\beta};\ \mathfrak{C} = \frac{\pi' c \gamma}{c+\gamma};\ \mathfrak{D} = \frac{\pi'' d \delta}{d+\delta};\ \mathfrak{E} = \frac{\pi''' e \varepsilon}{e+\varepsilon} \text{ etc.}$$

habebimus ex §. 256 sequentes aequationes:

$$\frac{\pi \beta}{b+\beta} = (1+\frac{\alpha}{b})\Phi$$
$$\pi - \frac{\pi' \gamma}{c+\gamma} = (1-\frac{\alpha\beta}{bc})\Phi$$
$$\pi - \pi' + \frac{\pi'' \delta}{d+\delta} = (1+\frac{\alpha\beta\gamma}{bcd})\Phi$$
$$\pi - \pi' + \pi'' - \frac{\pi''' \varepsilon}{e+\varepsilon} = (1-\frac{\alpha\beta\gamma\delta}{bcde})\Phi$$
etc.

Quo iam facilius hinc per π, π', π'', π''' etc. et Φ distantiae determinatrices lentium definiri queant, ponatur $\alpha = \mathrm{A}a$; $\beta = \mathrm{B}b$; $\gamma = \mathrm{C}c$; $\delta = \mathrm{D}d$; $\varepsilon = \mathrm{E}e$ etc. Ita vt litterae A, B, C, D, E etc. denotent numeros absolutos, ac nostrae aequationes induent has formas:

$$\frac{\mathrm{B}\pi}{\mathrm{B}+1} = (1+\frac{\mathrm{A}a}{b})\Phi$$
$$\pi - \frac{\mathrm{C}\pi'}{\mathrm{C}+1} = (1-\frac{\mathrm{AB}a}{c})\Phi$$
$$\pi - \pi' + \frac{\mathrm{D}\pi''}{\mathrm{D}+1} = (1+\frac{\mathrm{ABC}a}{d})\Phi$$
$$\pi - \pi' + \pi'' - \frac{\mathrm{E}\pi'''}{\mathrm{E}+1} = (1-\frac{\mathrm{ABCD}a}{e})\Phi$$

etc.

vnde eliciuntur ſequentes determinationes:

$$b = \frac{A(B+1)a\Phi}{B\pi - (B+1)\Phi}$$

$$c = \frac{AB(C+1)a\Phi}{C\pi' - (C+1)(\pi-\Phi)}$$

$$d = \frac{ABC(D+1)a\Phi}{D\pi'' - (D+1)(\pi'-\pi+\Phi)}$$

$$e = \frac{ABCD(E+1)a\Phi}{E\pi''' - (E+1)(\pi''-\pi'+\pi-\Phi)}$$

etc.

Datis ergo praeter numeros $\Phi, \pi, \pi', \pi'', \pi'''$ etc. numeris A, B, C, D, E etc. cum diſtantia obiecti $AE = a$, per has formulas diſtantiae b, c, d, e etc. determinantur indeque inſuper alterae $\alpha, \beta, \gamma, \delta, \varepsilon$ etc. hoc modo:

$$\alpha = Aa$$

$$\beta = \frac{AB(B+1)a\Phi}{B\pi - (B+1)\Phi}$$

$$\gamma = \frac{ABC(C+1)a\Phi}{C\pi' - (C+1)(\pi-\Phi)}$$

$$\delta = \frac{ABCD(D+1)a\Phi}{D\pi'' - (D+1)(\pi'-\pi+\Phi)}$$

$$\varepsilon = \frac{ABCDE(E+1)a\Phi}{E\pi''' - (E+1)(\pi''-\pi'+\pi-\Phi)}$$

etc.

Hinc nanciſcimur diſtantias focales lentium

Primae $PP = \frac{Aa}{A+1}$

Secundae $QQ = \frac{ABa\Phi}{B\pi-(B+1)\Phi}$

Tertiae $RR = \frac{ABCa\Phi}{C\pi'-(C+1)(\pi-\Phi)}$

Quartae $SS = \frac{ABCDa\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$

Quintae $TT = \frac{ABCDEa\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$

etc.

Coroll.

Coroll. 1.

267. Ex angulo Φ cum distantia obiecti ante lentem primam $AE = a$, ita definitur semidiameter campi apparentis z, vt sit $z = a\Phi$: neque tamen campus apparens pro lubitu assumi potest, sed is per multiplicationem determinabitur, vt mox videbimus.

Coroll. 2.

268. Cum omnes numeri hic in calculum introducti aeque negatiue ac positiue accipi queant, obseruandum est eos perpetuo ita assumi debere, vt interualla lentium quae sunt $\alpha+b$; $\beta+c$; $\gamma+d$; $\delta+e$; etc. omnia prodeant positiua.

Coroll. 3.

269. Quod ad aperturam cuiusque lentis attinet eius semidiameter habebitur si eius distantia focalis multiplicetur per rationem aperturae littera π insignitam.

Scholion.

270. Quo formulas hic inuentas simpliciores reddamus quoniam litteris $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$; etc. non amplius indigebimus, ponamus ad abbreuiandum;

$$\frac{A}{A+1} = \mathfrak{A};\ \frac{B}{B+1} = \mathfrak{B};\ \frac{C}{C+1} = \mathfrak{C};\ \frac{D}{D+1} = \mathfrak{D};\ \frac{E}{E+1} = \mathfrak{E} \text{ etc.}$$

vt sit

$$A = \frac{\mathfrak{A}}{1-\mathfrak{A}};\ B = \frac{\mathfrak{B}}{1-\mathfrak{B}};\ C = \frac{\mathfrak{C}}{1-\mathfrak{C}};\ D = \frac{\mathfrak{D}}{1-\mathfrak{D}};\ E = \frac{\mathfrak{E}}{1-\mathfrak{E}} \text{ etc.}$$

at-

atque habebimus:

$$\alpha = \mathrm{A}a; \qquad b = \frac{\mathrm{A}a\Phi}{\mathfrak{B}\pi - \Phi}$$

$$\beta = \frac{\mathrm{AB}a\Phi}{\mathfrak{B}\pi - \Phi} \qquad c = \frac{\mathrm{AB}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$\gamma = \frac{\mathrm{ABC}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \qquad d = \frac{\mathrm{ABC}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$\delta = \frac{\mathrm{ABCD}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \qquad e = \frac{\mathrm{ABCD}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

$$\varepsilon = \frac{\mathrm{ABCDE}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \qquad \text{etc.}$$

Hincque porro definientur distantiae focales lentium:

$$\text{Primae } PP = \mathfrak{A}a$$

$$\text{Secundae } QQ = \frac{\mathrm{A}\mathfrak{B}a\Phi}{\mathfrak{B}\pi - \Phi}$$

$$\text{Tertiae } RR = \frac{\mathrm{AB}\mathfrak{C}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$\text{Quartae } SS = \frac{\mathrm{ABC}\mathfrak{D}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$\text{Quintae } TT = \frac{\mathrm{ABCD}\mathfrak{E}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

etc.

et lentium interualla:

$$\text{I et II} = \frac{\mathrm{A}\mathfrak{B}a\pi}{\mathfrak{B}\pi - \Phi}$$

$$\text{II et III} = \frac{\mathrm{AB}a\Phi(\mathfrak{C}\pi' - (1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$$

$$\text{III et IV} = \frac{\mathrm{ABC}a\Phi(\mathfrak{D}\pi'' - (1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi' - \pi + \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$$

$$\text{IV et V} = \frac{\mathrm{ABCD}a\Phi(\mathfrak{E}\pi''' - (1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$$

Quae interualla debent esse positiua.

Problema 7.

271. Positis iisdem, quae in problemate praecedente sunt assumta, definire locum idoneum oculi, vnde totus campus apparens conspici queat.

So-

Solutio.

Maneant omnes denominationes vt ante, et quia apertura lentis PP vt nulla ſpectatur, pro reliquis lentibus ex data aperturae ratione, ſemidiameter aperturae cuiusque ita ſe habebit

Lentis	ſemidiameter aperturae
Secundae QQ . .	$\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}\cdot\Phi$
Tertiae RR . .	$\frac{ABC\mathfrak{C}a\pi'}{\mathfrak{C}\pi'-\pi'+\Phi}\Phi$
Quartae SS . .	$\frac{ABC\mathfrak{D}a\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\Phi$
Quintae TT . .	$\frac{ABCD\mathfrak{E}a\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\Phi$

vbi litterae $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, etc. valores in praecedente Scholio aſſignatos obtinent.

Deinde magnitudines ſingularum imaginum conſiderari conuenit, quae ob $E\varepsilon = z = a\Phi$ et

$$\alpha = Aa,\ \beta = Bb;\ \gamma = Cc;\ \delta = Dd \text{ etc.}$$

erunt

$$F\zeta = \alpha\Phi = Aa\Phi$$

$$G\eta = ABa\Phi$$

$$H\theta = ABCa\Phi$$

$$I\iota = ABCDa\Phi$$

etc.

Iam pro quolibet lentium numero locus oculi idoneus ſeorſim definiri debet; denotante ergo O diſtantiam oculi poſt vltimam lentem

I. Pro vnica lente

Quia craſſities lentis vt nulla ſpectatur, euidens eſt pro loco oculi idoneo fore $O = o$.

II. Pro duabus Lentibus

Tab. III. Fig. 13. Cum hic ſit $bN' + G\eta : bG = bN' : bO$ erit $bO = O = \frac{bN'}{bN' + G\eta} \cdot \mathfrak{E}$. Sed eſt $bN' = \frac{A\mathfrak{B}\pi}{\mathfrak{B}\pi - \Phi} a\Phi$ et $G\eta = ABa\Phi$ vnde fit

$$bN' + G\eta = Aa\Phi \cdot \frac{(B+1)\mathfrak{B}\pi - B\Phi}{\mathfrak{B}\pi - \Phi} = \frac{ABa\Phi(\pi - \Phi)}{\mathfrak{B}\pi - \Phi}$$

ob $(B+1)\mathfrak{B} = B$. Erit ergo $\frac{bN'}{bN' + G\eta} = \frac{\mathfrak{B}\pi}{B(\pi - \Phi)}$, quae fractio per $\mathfrak{E} = \frac{ABa\Phi}{(\mathfrak{B}\pi - \Phi)}$ multiplicata dat locum oculi idoneum

$$O = \frac{A\mathfrak{B}a\pi\Phi}{(\pi - \Phi)(\mathfrak{B}\pi - \Phi)}$$

III. Pro tribus Lentibus

Fig. 14. Cum hic ſit $cN'' + H\theta : cH = cN'' : cO$, erit $cO = O = \frac{cN''}{cN'' + H\theta} \cdot \gamma$. Sed eſt

$$cN'' = \frac{AB\mathfrak{C}a\pi'\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \text{ et } H\theta = ABCa\Phi$$

hincque ob $(C+1)\mathfrak{C} = C$ fiet

$$cN'' + H\theta = \frac{ABCa\Phi(\pi' - \pi + \Phi)}{\mathfrak{C}\pi' - \pi + \Phi}$$

et $\frac{cN''}{cN'' + H\theta} = \frac{\mathfrak{C}\pi'}{C(\pi' - \pi + \Phi)}$. Nunc igitur ob $\gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ habebimus diſtantiam oculi idoneam:

$$O = \frac{AB\mathfrak{C}a\pi'\Phi}{(\pi' - \pi + \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$$

IV. Pro quatuor Lentibus

Cum fit $dN''' + I\iota : dI = dN''' : dO$, erit $dO = O = \frac{dN'''}{dN''' + I\iota} \cdot \delta$. Sed eft

Tab. III. Fig. 15.

$$dN''' = \frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \text{ et } I\iota = ABCDa\Phi$$

hincque ob $(D+1)\mathfrak{D} = D$ fiet

$$dN''' + I\iota = \frac{ABCDa\Phi(\pi'' - \pi' + \pi - \Phi)}{\mathfrak{D}\pi'' - \pi + \pi - \Phi} \text{ et}$$

$$\frac{dN'''}{dN''' + I\iota} = \frac{\mathfrak{D}\pi''}{D(\pi'' - \pi' + \pi - \Phi)}. \text{ Ergo ob } \delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

prodit diftantia oculi idonea

$$O = \frac{ABC\mathfrak{D}a\pi''\Phi}{(\pi'' - \pi' + \pi - \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}.$$

V. Pro quinque Lentibus.

Si ratiocinium fimili modo ad cafum quinque lentium extendatur, reperiemus diftantiam oculi idoneam

$$O = \frac{ABCD\mathfrak{E}a\pi'''\Phi}{(\pi''' - \pi'' + \pi' - \pi + \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}.$$

VI. Pro fex Lentibus

Eodemque modo progrediendo colligitur fore pro cafu fex lentium diftantiam oculi idoneam

$$O = \frac{ABCDE\mathfrak{F}a\pi''''\Phi}{(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi\,(\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \Phi)}$$

ficque vlterius, quousque libuerit progredi licet.

Coroll. 1.

272. Quouis ergo casu necesse est, vt distantia oculi idonea prodeat positiua: si enim fieret negatiua totus campus apparens nusquam conspici posset.

Coroll. 2.

273. Iis autem casibus, quibus distantia O fit negatiua oculum immediate vltimae lenti applicari conueniet; Tum vero oculus plus non cernet, quam si vltimae lentis apertura aequalis esset amplitudini pupillae.

Coroll. 3.

274. Hoc ergo casu statuatur semidiameter aperturae vltimae lentis $= \omega$ semidiametro pupillae, ex eaque aequatione eliciatur valor ipsius Φ, quo invento erit $a\Phi$ semidiameter campi apparentis, qui in obiecto reuera conspicietur.

Problema 8.

275. Positis iisdem atque in problematibus praecedentibus, eam conditionem in lentium dispositione definire, vt oculus in loco idoneo positus obiectum simul distincte videat.

Solutio.

Quia aperturam lentis obiectiuae euanescentem assumimus, in visione alia confusio locum habere nequit nisi quatenus oculus non in distantia iusta ab vltima

vltima imagine, quam intuetur exiſtit, quae ergo tolletur, ſi lentes ita diſponantur, vt imago vltima ante oculum in O ſitum in diſtantia iuſta, quam littera l deſignauimus, verſetur. Cum igitur in figuris locus oculi ante imaginem vltimam cadat haec diſtantia negatiue ſumta ipſi l aequalis eſt ponenda; vnde pro quouis lentium numero ſequentes habebimus determinationes.

I. Pro vnica lente

Cum hic ſit $O = o$, et $OF = \alpha = \mathrm{A}a$, oportet eſſe $\mathrm{A}a = -l$, ideoque $\mathrm{A} = -\frac{l}{a}$: et $\alpha = -l$, vnde indoles huius lentis determinatur, ita vt eius diſtantia focalis eſſe debeat $= \frac{al}{l-a}$. Tab. III. Fig. 12.

II. Pro duabus lentibus

Ex inuenta diſtantia $bO = O$, erit $OG = \frac{O}{bN'}$. Fig. 13. $G\eta$. Eſt vero $\frac{O}{bN'} = \frac{1}{\pi - \Phi}$; vnde fit $\frac{\mathrm{AB}a\Phi}{\pi - \Phi} = -l$, hincque pro ſecunda lente $\mathrm{B} = -\frac{(\pi - \Phi)l}{\mathrm{A}a\Phi}$. et $\mathfrak{B} = \frac{\mathrm{B}}{\mathrm{B}+1}$. Vel cum ſit $\mathrm{A}a\Phi = -\frac{(\pi-\Phi)l}{\mathrm{B}}$, erit pro loco oculi

$$O = \frac{-\mathfrak{B}\pi}{\mathrm{B}(\mathfrak{B}\pi - \Phi)}\, l$$

III. Pro tribus lentibus

Hic eſt $OH = \frac{O}{cN''}$. $H\theta = \frac{H\theta}{\pi' - \pi + \Phi}$, vnde obtinetur: Fig. 14.

$$OH = \frac{\mathrm{ABC}a\Phi}{\pi' - \pi + \Phi} = -l$$

ſicque pro vltima lente habebitur:

$$\mathrm{C} = \frac{-(\pi' - \pi + \Phi)l}{\mathrm{AB}a\Phi}$$

At ſi pro determinatione primae lentis capiatur $Aa\Phi = \frac{-(\pi' - \pi + \Phi)l}{BC}$, erit diſtantia oculi

$$O = \frac{-\mathfrak{C}\pi'}{C(\mathfrak{C}\pi' - \pi + \Phi)} l$$

IV. Pro quatuor lentibus

Tab. III. Fig. 15.

Cum ſit $OI = \frac{O}{dN'''}$. $I\iota = \frac{I\iota}{\pi'' - \pi' + \pi - \Phi}$, habebitur

$$OI = \frac{ABCDa\Phi}{\pi'' - \pi' + \pi - \Phi} = -l$$

Vnde pro vltima lente

$$D = -\frac{(\pi'' - \pi' + \pi - \Phi)l}{ABCa\Phi}$$

Cum autem ſit $ABCa\Phi = -\frac{(\pi'' - \pi' + \pi - \Phi)l}{D}$ erit in loco oculi hoc valore ſurrogando

$$O = \frac{-\mathfrak{D}\pi''}{D(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} l.$$

V. Pro quinque Lentibus

Simili modo pro quinque lentibus vltima ita comparata eſſe debet, vt ſit

$$E = \frac{-(\pi''' - \pi'' + \pi' - \pi + \Phi)l}{ABCDa\Phi}$$

Prima autem inde definita ſit

$$O = \frac{-\mathfrak{E}\pi'''}{E(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)} l.$$

VI. Pro ſex Lentibus

Eodem modo patet pro ſex lentibus fore

$$F = \frac{-(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)l}{ABCDEa\Phi}$$

atque

atque ſi hinc $\mathrm{A}a$ definiatur:

$$O = \frac{-\mathfrak{F}\pi''''}{F(\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)}\, l$$

quas formulas quousque libuerit, continuare licet.

Coroll. 1.

276. Si diſtantia oculi iuſta l fuerit infinita erit vt ſequitur:

I. Pro vna lente $\mathrm{A} = \infty$ et $\mathfrak{A} = 1$

II. Pro duabus lentibus $\mathrm{B} = \infty$ et $\mathfrak{B} = 1$

III. Pro tribus lentibus $\mathrm{C} = \infty$ et $\mathfrak{C} = 1$

IV. Pro quatuor lentibus $\mathrm{D} = \infty$ et $\mathfrak{D} = 1$

etc.

Coroll. 2.

277. Caſu ergo, quo diſtantia oculi iuſta l eſt infinita, diſtantia oculi poſt lentem vltimam erit pro quouis lentium numero:

I. Pro vnica lente $O = 0$

II. Pro duabus lentibus $O = \frac{\mathrm{A}a\pi\Phi}{(\pi - \Phi)^2}$

III. Pro tribus lentibus $O = \frac{\mathrm{AB}a\pi'\Phi}{(\pi' - \pi + \Phi)^2}$

IV. Pro quatuor lentibus $O = \frac{\mathrm{ABC}a\pi''\Phi}{(\pi'' - \pi' + \pi - \pi)^2}$

etc.

Scholion.

278. Hactenus campum apparentem Φ vt datum conſideraui, ex eoque tam lentium indolem quam

quam earum dispositionem determinaui, vt campus datae amplitudinis appareat, nihilque obstare deprehendimus, quominus huic conditioni satisfiat; cum numeri A, B, C, D, etc. penitus arbitrio relinquantur, aperturarum vero rationes π, π', π'', etc. infra $\frac{1}{3}$ vel $\frac{1}{4}$ accipi debeant. Verum hic multiplicationis ratio nondum in computum est ducta qua simul campus apparens ita adstringitur, vt certum limitem excedere nequeat. Quoniam igitur in omnibus instrumentis dioptricis multiplicatio imprimis proposita esse solet, quemadmodum per eam campus apparens definiatur in sequente problemate exponamus.

Problema 9.

279. Si instrumentum dioptricum ex quotcunque lentibus fuerit compositum, quarum quidem crassities vt nulla spectetur, simul vero multiplicationis ratio sit proposita, determinare campum apparentem.

Solutio.

Manentibus omnibus denominationibus, quibus hactenus sumus vsi, ita vt $a\Phi$ semidiametrum campi apparentis denotet, sit b distantia, ad quam multiplicationem referamus. Magnitudo igitur $a\Phi$ in distantia hac$=b$ nudo oculo cerneretur sub angulo, cuius tangens est $=\frac{a\Phi}{b}$. Quare si multiplicationis ratio statuatur $=m$, necesse est vt eadem magnitudo $a\Phi$ per lentes spectetur sub angulo, cuius tangens sit $=\frac{ma\Phi}{b}$.

Iam

Iam vero ex iis, quae in problematibus praecedentibus ſunt tradita iſte angulus facile aſſignatur, ſicque obtinebitur angulus Φ, indeque ſemidiameter campi apparentis $a\Phi$. Cum autem haec multiplicatio non ad ipſos angulos ſed eorum tangentes referatur euidens eſt tantum partes obiecti minimas circa centrum E ſitas in ratione propoſita multiplicari, remotiores vero in ratione minore. Quo notato hanc multiplicationis rationem m pro quouis lentium numero contemplemur.

I. Pro vnica Lente

Tangens anguli, quo imago Fζ ab oculo in O conſtituto conſpicitur, eſt $\frac{F\zeta}{CF} = \frac{F\zeta}{aF}$ ob $aO = o$, Erit ergo $\frac{ma\Phi}{b} = \Phi$, ſeu $ma = b$. Hoc ergo caſu campus apparens non determinatur ſed multiplicationis ratio eſt $m = \frac{b}{a}$. Verum vt viſio ſit diſtincta, per ſuperius problema debet eſſe $A = -\frac{l}{a}$ et diſtantia oculi poſt lentem $O = o$. At obiectum ſitu erecto cernetur: Tab. III. Fig. 12.

II. Pro duabus Lentibus

Tangens anguli, quo imago Gη ab oculo in O conſtituto cernetur, eſt $= \frac{bN'}{bO} = \pi - \Phi = \frac{ma\Phi}{b}$; vnde ſequitur ſemidiameter campi apparentis Fig. 13.

$$\Phi = \frac{\pi b}{ma+b} \text{ pro ſitu inuerſo}$$

Quo inuento, vt viſio ſit diſtincta oportet eſſe

$$B = \frac{-ml}{Ab}, \text{ et } \mathfrak{B} = \frac{-ml}{Ab - ml}$$

hincque prodit diſtantia oculi poſt lentem ocularem

$$O = \frac{\mathrm{A}bl(ma+b)}{mm\alpha l + \mathrm{A}bb}.$$

III. Pro tribus Lentibus

Tab. III. Fig. 14. Tangens anguli, quo imago $H\theta$ ab oculo in O conſtituto cernitur eſt $\frac{c\mathrm{N}''}{c\mathrm{O}} = \pi' - \pi + \Phi = \frac{ma\Phi}{b}$, vnde fit ſemidiameter campi apparentis:

$$\Phi = \frac{(\pi'-\pi)b}{ma-b} \text{ pro ſitu erecto}$$

Deinde vt viſio ſit diſtincta, oportet eſſe

$$C = \frac{-ml}{\mathrm{AB}b} \text{ et } \mathfrak{C} = \frac{-ml}{\mathrm{AB}b - ml}$$

Cum igitur ſit $\pi' - \pi + \Phi = \frac{ma(\pi'-\pi)}{ma-b}$ erit

$$\mathfrak{C}\pi' - \pi + \Phi = \pi' - \pi + \Phi - \frac{\mathrm{AB}b\pi'}{\mathrm{AB}b - ml} = \frac{ma(\pi'-\pi)}{ma-b} - \frac{\mathrm{AB}b\pi'}{\mathrm{AB}b - ml}$$

hincque pro loco oculi

$$O = \frac{-\mathrm{AB}bl\pi'}{(\mathrm{AB}b - ml)(\mathfrak{C}\pi' - \pi + \Phi)} = \frac{\mathrm{AB}bl(ma-b)\pi'}{(mmal - \mathrm{AB}bb)\pi' + ma(\mathrm{AB}b - ml)\pi}$$

IV. Pro quatuor lentibus

Fig. 15. Tangens anguli, quo imago $I\iota$ ab oculo in O conſtituto cernitur eſt $= \frac{d\mathrm{N}'''}{d\mathrm{O}} = \pi'' - \pi' + \pi - \Phi = \frac{ma\Phi}{b}$;

vnde elicitur:

$$\Phi = \frac{(\pi''-\pi'+\pi)b}{ma+b} \text{ pro ſitu inuerſo}$$

Hinc porro pro viſione diſtincta eſſe debet

$$D = \frac{-ml}{\mathrm{ABC}b}, \text{ et } \mathfrak{D} = \frac{-ml}{\mathrm{ABC}b - ml}$$

vnde fit

$$\mathfrak{D}\pi'' - \pi' + \pi - \Phi = \frac{ma(\pi''-\pi'+\pi)}{ma+b} - \frac{\mathrm{ABC}b\pi''}{\mathrm{ABC}b - ml}$$

et

et pro loco oculi

$$O = \frac{ABCbl(ma+b)\pi''}{(mmal+ABCbb)\pi''+ma(ABCb-ml)(\pi'-\pi)}.$$

V. Pro quinque lentibus

Eodem modo progrediendo pro campo apparente reperitur:

$$\Phi = \frac{(\pi'''-\pi''+\pi'-\pi)b}{ma-b} \text{ pro ſitu erecto}$$

et vt viſio euadat diſtincta

$$E = \frac{-ml}{ABCDb} \text{ et } \mathfrak{E} = \frac{-ml}{ABCDb-ml}$$

vnde pro loco oculi idoneo concluditur

$$O = \frac{ABCDbl(ma-b)\pi'''}{(mmal-ABCDbb)\pi'''+ma(ABCDb-ml)(\pi''-\pi'+\pi)}.$$

VI. Pro ſex lentibus

Hic campus apparens ita definitur, vt ſit:

$$\Phi = \frac{(\pi''''-\pi'''+\pi''-\pi'+\pi)b}{ma+b} \text{ pro ſitu inuerſo}$$

viſio vero diſtincta exigit

$$F = \frac{-ml}{ABCDEb};\ \mathfrak{F} = \frac{-ml}{ABCDEb-ml}$$

vnde pro loco oculi idoneo

$$O = \frac{ABCDEbl(ma+b)\pi''''}{(mmal+ABCDEbb)\pi''''+ma(ABCDEb-ml)(\pi'''-\pi''+\pi'-\pi)}$$

ſicque progreſſio ad plures lentes eſt manifeſta.

Coroll. I.

280. Datis ergo rationibus aperturarum ſingularum lentium π, π', π'', etc. vna cum ratione multiplicationis m, diſtantia b, ad quam multiplicatio refer-

refertur, et diſtantia obiecti ante inſtrumentum a, determinatur campus apparens.

Coroll. 2.

281. Vt ergo campus apparens pro data multiplicatione maximus obtineatur litteris π, π', π'', etc. ita valores maximos tribui conueniet, vt alternatim ſint poſitiui et negatiui.

Coroll. 3.

282. Si igitur valores π, π', π'', etc. vsque ad $\frac{1}{3}$ augeri liceat, maximus valor ipſius Φ pro quouis lentium numero erit vt ſequitur:

Pro caſu duarum lentium $\Phi = \frac{b}{3(ma+b)}$

Pro caſu trium lentium $\Phi = \frac{2b}{3(ma-b)}$

Pro caſu quatuor lentium $\Phi = \frac{3b}{3(ma+b)}$

Pro caſu quinque lentium $\Phi = \frac{4b}{3(ma-b)}$

etc.

Coroll. 4.

283. Quo plures ergo lentes adhibentur eo magis campus apparens augeri poteſt ſimul vero patet, quo maior multiplicatio deſideretur, eo minorem fieri campum apparentem.

Coroll.

Coroll. 5.

284. Ratio multiplicationis m tam poſitiue quam negatiue capi poteſt. Si poſitiue accipitur pro lentium numero pari ſitum inuerſum, pro impari autem ſitum erectum declarat. Contrarium vero euenit, ſi m fuerit numerus negatiuus.

Scholion 1.

285. Hic autem imprimis notandum eſt, valorem ipſius Φ tum ſolum angulum $EA\varepsilon$ praebere, quando fuerit tam exiguus, vt aliquot gradus non ſuperet; ſi enim valor ipſius Φ prodeat multo maior, tum tangentem huius anguli $EA\varepsilon$ exprimit. Plerumque autem, ſi quidem multiplicatio ſit modica, iſte valor ipſius Φ tam paruus reperitur, vt ſine errore pro ipſo angulo $EA\varepsilon$ accipi poſſit. Hic igitur ob aliam cauſam amplitudo campi apparentis reſtringitur, vt certum limitem ſuperare nequeat; cum enim angulus, quo radii in oculum incidentes in O ad axem inclinantur, nunquam poſſit eſſe rectus, neque fortaſſe vix 60° ſuperare queat, quandoquidem ne nudo quidem oculo ſpatium in coelo maius quam 120° conſpicere valeamus; ſi illum angulum maximum, quem oculus capere valeat, circiter 63° ſtatuamus, vt eius tangens ſit $= 2$ pro quouis lentium numero habebimus $\frac{ma\Phi}{b} = 2$, vnde fit $\Phi = \frac{2b}{ma}$, et $a\Phi = \frac{2b}{m}$. Data ergo multiplicatione m et diſtantia b ad quam refertur, ſemidiameter ſpatii in obiecto

confpicui nunquam maior exiftere poteft quam $\frac{2b}{m}$ quotcunque etiam adhibeantur lentes, eaeque ita difponantur vt maximum campum patefaciant. In Telefcopiis ergo, vbi fumitur $b=a$, et femidiameter campi ex ipfo angulo Φ aeftimatur, eius tangens nunquam maior effe poteft quam $\frac{2}{m}$: vnde fequentem tabellam adiungo, quae pro quauis multiplicatione femidiametrum campi apparentis maximi oftendit, quem nunquam fuperare liceat.

Multiplicatio m	Semidiam: campi app: maximi	Multiplicatio m	Semidiam: campi app: maximi
5	21°, 48′	60	1°, 54′, 33″
10	11, 18	70	1, 38, 13
15	7, 35	80	1, 25, 57
20	5, 42	90	1, 16, 24
25	4, 34	100	1, 8, 45
30	3, 49	150	0, 45, 50
35	3, 16½	200	0, 34, 22
40	2, 52	250	0, 27, 30
45	2, 33	300	0, 22, 55
50	2, 17½	400	0, 17, 11
		500	0, 13, 45

Quod fi ergo numerum lentium multiplicando iam fere ad tantum campum apparentem pertigerimus, is vlterius nullo modo augeri poterit.

Scho-

Scholion 2.

286. Quod ad locum oculi idoneum attinet, eum ideo in O conſtituimus, vt omnes radios per lentes transmiſſos accipiat, etiamſi pupilla maxime eſſet conſtricta: ex quo patet ob amplitudinem pupillae oculum de hoc loco ſine vllo detrimento aliquantillum remoueri poſſe, ita vt ſuperfluum foret hunc locum nimis ſollicite obſeruare, niſi forte apertura vltimae lentis fuerit admodum magna. Sin ea autem pupillam non ſuperet, eaque adeo ſit minor, manifeſtum eſt oculum ei immediate applicatum aeque omnes radios excipere, et eundem campum contueri, ac ſi in loco idoneo eſſet conſtitutus. His igitur caſibus, ſi forte diſtantia O pro loco oculi prodeat negatiua, nihil de campo apparente perit, dummodo oculus lenti vltimae immediate applicetur. His itaque, quae ad viſionem per inſtrumenta dioptrica in genere pertinent, expeditis, ſupereſt, vt inueſtigemus, quantum viſio ob diuerſam radiorum refrangibilitatem turbetur, et quemadmodum hanc perturbationem euitare queamus.

CAPVT