CAPVT III.

DE LENTIBVS COMPOSITIS SEV MVLTIPLICATIS.

Definitio I.

96.

Lens duplicata oritur, si duae lentes super communi axe sibi immediate iungantur.

Craſſitiem hic vtriusque lentis tanquam nullam aſſumo et quia diſtantia inter lentes nulla ponitur, craſſities etiam lentis duplicatae pro nulla haberi poterit.

Coroll. I.

97. Binae ergo lentes PP et QQ ſibi ad contactum fere coniunctae lentem duplicatam conſtituunt; de qua tamen notandum eſt, eius craſſitiem minus tuto negligi poſſe quam vtriusque lentis ſimplicis ſeorſim ſumtae. Si enim lentes immediate ſe in puncto contingerent; phaenomena colorum a *Newtono* obſeruata eſſent metuenda tum vero etiam oſtendemus, quomodo ratio diſtantiae inter binas lentes haberi poſſit. Tab. II. Fig. 7.

Coroll. 2.

98. Si lentis anterioris PP distantiae determinatrices sint a et α, lentis posterioris vero QQ, b et $\mathfrak{E}$, necesse est vt sit $\alpha + b = 0$ seu $\alpha = -b$. Tum vero obiecti ante lentem ad distantiam $AE = a$ positi imago principalis repraesentabitur post lentem ad distantiam $G = \mathfrak{E}$.

Coroll. 3.

99. Erunt ergo a et $\mathfrak{E}$ quasi distantiae determinatrices lentis duplicatae; ac sumendo α vel b ad libitum infinita paria lentium pro his distantiis exhiberi possunt. Cum deinde vtraque lens praeterea numerum indefinitum λ recipiat insuper infinita varietas locum habet.

Coroll. 4.

100. Quia crassities pro nihilo reputatur aperturae in singulis faciebus eadem est ratio; scilicet si in prima facie semidiameter aperturae sit $= x$, in reliquis quoque faciebus apertura eadem vel saltem non minor esse debet.

Scholion.

101. Non opus est, vt lentes plane ad contactum coniungantur quoniam forte refractionis lex turbari posset: hoc autem vel minima interposita distantia euitabitur id quod ad institutum nostrum sufficit, cum etiam crassities vtriusque non omnino sit nulla.

Proble-

Problema I.

102. Omnes lentes duplicatas describere, quibus obiectum $E\epsilon$ in data ante lentem distantia AE propositum post lentem in data distantia bG repraesentetur, simulque diffusionem imaginis Gg pro data lentis apertura definire.

Solutio.

Sit distantia obiecti $AE = a$, imaginis principalis $bG = \mathfrak{b}$, tum vero lentis primae PP distantiae determinatrices a, et α; lentis posterioris vero QQ, b et $\mathfrak{b}$; iam quia distantia lentium est nulla, erit $\alpha + b = 0$, seu $\alpha = -b$; Hinc si obiecti magnitudo sit $E\epsilon = z$, erit imaginis principalis magnitudo $G\eta = \frac{\mathfrak{b}}{a} z$ pro situ inuerso. Cum porro vtraque lens infinitis modis formari possit, sit λ numerus arbitrarius pro prima PP et λ' pro secunda QQ, quarum ergo constructio ita se habebit:

Pro Lente

$$PP \text{ radius faciei } \begin{cases} \text{anterioris} = \dfrac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda - 1)}} \\ \text{posterioris} = \dfrac{a\alpha}{\rho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda - 1)}} \end{cases}$$

$$QQ \text{ radius faciei } \begin{cases} \text{anterioris} = \dfrac{b\mathfrak{b}}{\rho\mathfrak{b} + \sigma b \pm \tau(b+\mathfrak{b})\sqrt{(\lambda' - 1)}} \\ \text{posterioris} = \dfrac{b\mathfrak{b}}{\rho b + \sigma\mathfrak{b} \mp \tau(b+\mathfrak{b})\sqrt{(\lambda' - 1)}}. \end{cases}$$

Si ratio refractionis sit diuersa; pro secunda lente scribi debet ρ', σ' et τ', loco ρ, σ et τ.

Pro

Pro ſpatio autem diffuſionis Gg inueniendo, ſit ſemidiameter aperturae lentis duplicatae $= x$, eritque

$$Gg = \mu \beta \beta x x . \left\{ \begin{array}{l} +\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \end{array} \right.$$

et radiorum in g concurrentium inclinatio ad axem $= \frac{x}{\beta}$. Si ratio refractionis diſcrepet, in parte ex lente ſecunda orta ſcribatur μ', ν', loco μ, ν.

Coroll. 1.

103. Huius ergo lentis duplicatae diſtantiae determinatrices ſunt a et β, praeterea vero duo numeri arbitrarii λ et λ', vna cum diſtantia α vel b eius perfectam determinationem conſtituunt, vnde in huiusmodi lentibus multo maior varietas locum habet, quam in lentibus ſimplicibus.

Coroll. 2.

104. Si ex eisdem diſtantiis determinatricibus a et β adiungendo numero arbitrario λ° lens ſimplex conſtruatur, ea imaginem eadem magnitudine $G\eta = \frac{\beta}{a} z$ referet; ſed pro eadem apertura, cuius ſemidiameter $= x$ habebitur ſpatium diffuſionis

$$Gg = \mu \beta \beta x x \left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\lambda^{\circ}\left(\frac{1}{a}+\frac{1}{\beta}\right)^2+\frac{\nu}{a\beta}\right)$$

Coroll. 3.

105. Fieri ergo poterit vt lens duplicata modo maiorem modo minorem diffuſionem gignat. Lens autem

autem ſimplex minimam parit diffuſionem, ſi $\lambda^\circ = 1$; ergo tum demum lentes duplicatae ſimplicibus erunt praeferendae, cum adhuc minorem diffuſionem producent.

Scholion.

106. Concipi quidem ſemper poterit lens ſimplex eandem diffuſionem gignens ac lens duplicata, ſi pro numero λ° omnes valores admittamus; quomodocunque enim lens duplicata fuerit comparata, ſi ſpatium diffuſionis inde productum huic ex lente ſimplici nato aequale ſtatuatur, determinatus valor pro numero λ° elicitur: qui ſi fuerit poſitiuus et vnitate maior, realis lens ſimplex aequiualens exhiberi poterit, ſin autem prodeat vnitate minor, vel adeo negatiuus, lens ſimplex inter imaginaria erit referenda. Quando autem fit $\lambda^\circ > 1$, euidens eſt lentem ſimplicem eundem plane effectum eſſe ituram ac duplicatam ideoque ſemper expediet lente ſimplici potius vti quam duplicata; quodſi vero prodierit $\lambda^\circ < 1$, quo caſu lens ſimplex fit imaginaria, tum lentes duplicatae effectum praeſtabunt a ſimplicibus non expectandum, qui adeo cum inſigni hoc commodo, quod ſpatium diffuſionis futurum ſit minus, erit coniunctus. Talibus ergo lentibus duplicatis maximo cum ſucceſſu vti poterimus, eoque magis eae ſimplicibus erunt anteponendae, quo minor fuerit valor numeri λ° iis reſpondens.

Problema 2.

107. Data lente duplicata ad binas distantias determinatrices $AE = a$ et $bG = \beta$ relata pro iisdem distantiis definire lentem simplicem, quae pro eadem apertura, eandem diffusionem imaginis producat.

Solutio.

Totum ergo negotium huc redit, vt spatium diffusionis Gg a lente simplici productum (104) aequale ponatur spatio diffusionis a lente duplicata orto, cuius expressio in problemate praecedente (101) est inuenta, indeque valor numeri λ° pro constructione lentis simplicis eliciatur. Quae inuestigatio quo commodius institui possit, ponamus:

$$\frac{1}{\alpha} = \frac{f-1}{a} + \frac{f}{\beta} \text{ eritque } \frac{1}{b} = \frac{-f+1}{a} - \frac{f}{\beta}$$

ita vt loco quantitatis α vel b numerum f introducamus eritque

$$\frac{1}{a} + \frac{1}{\alpha} = f\left(\frac{1}{a} + \frac{1}{\beta}\right) \text{ et } \frac{1}{b} + \frac{1}{b} = (1-f)\left(\frac{1}{a} + \frac{1}{\beta}\right)$$

vnde spatium diffusionis a lente duplicata ortum prodit:

$$Gg = \mu\beta\beta xx \begin{cases} +f\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\lambda ff\left(\frac{1}{a}+\frac{1}{\beta}\right)^2 + \nu\left(\frac{f-1}{aa}+\frac{f}{a\beta}\right)\right) \\ +(1-f)\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\lambda'(1-f)^2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2 + \nu\left(\frac{1-f}{\alpha\beta}-\frac{f}{\beta\beta}\right)\right) \end{cases}$$

quae expressio reducitur ad hanc formam:

$$Gg = \mu\beta\beta xx\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\left(\lambda f^3 + \lambda'(1-f)^3\right)\left(\frac{1}{a}+\frac{1}{\beta}\right)^2 + \nu\left(\frac{f(f-1)}{aa} + \frac{1-2f+2ff}{a\beta} + \frac{f(f-1)}{\beta\beta}\right)\right)$$

Verum postremum membrum $\frac{f(f-1)}{aa} + \frac{1-2f+2ff}{a\beta} + \frac{f(f-1)}{\beta\beta}$ mutatur

mutatur in $f(f-1)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2+\frac{1}{\alpha\beta}$, ſicque habebimus pro lente duplicata:

$$Gg=\mu\beta\beta xx\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\left(\left(\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2+\frac{\nu}{\alpha\beta}\right)$$

quae forma iam facillime cum ſpatio diffuſionis lentis ſimplicis comparatur indeque manifeſto colligitur:

$$\lambda^\circ=\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)$$

Cum autem loco quantitatum α et b numerum f introduxerimus conſtructio lentis duplicatae ita ſe habebit

Pro lente radius faciei

prima PP $\begin{cases} \text{anterioris} = \dfrac{\alpha\beta}{(\rho-\sigma(1-f))\beta+\sigma f\alpha\pm\tau f(\alpha+\beta)\sqrt{(\lambda-1)}} \\ \text{poſterioris} = \dfrac{\alpha\beta}{(\sigma-\rho(1-f))\beta+\rho f\alpha\mp\tau f(\alpha+\beta)\sqrt{(\lambda-1)}} \end{cases}$

ſecunda QQ′ $\begin{cases} \text{anterioris} = \dfrac{\alpha\beta}{(\sigma-\rho f)\alpha+\rho(1-f)\beta\pm\tau(1-f)(\alpha+\beta)\sqrt{(\lambda'-1)}} \\ \text{poſterioris} = \dfrac{\alpha\beta}{(\rho-\sigma f)\alpha+\sigma(1-f)\beta\mp\tau(1-f)(\alpha+\beta)\sqrt{(\lambda'-1)}} \end{cases}$

Tum vero inuento numero λ° conſtructio lentis ſimplicis aequiualentis erit

Radius faciei $\begin{cases} \text{anterioris} = \dfrac{\alpha\beta}{\rho\beta+\sigma\alpha\pm\tau(\alpha+\beta)\sqrt{(\lambda^\circ-1)}} \\ \text{poſterioris} = \dfrac{\alpha\beta}{\rho\alpha+\sigma\beta\mp\tau(\alpha+\beta)\sqrt{(\lambda^\circ-1)}} \end{cases}$

Coroll. I.

108. Quoties ergo fuerit

$$\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)>1$$

ſemper lens ſimplex parari poteſt duplicatae aequiualens

lens iisque ergo casibus praestabit lente simplici vti potius quam lente duplicata.

Coroll. 2.

109. Verum si fuerit

$$\lambda f^3 + \lambda'(1-f)^3 - \nu f(1-f) < 1$$

ob $\lambda^\circ < 1$, constructio lentis simplicis fit impossibilis, ac lens duplicata minorem pariet diffusionem quam per vllam lentem simplicem obtineri potest.

Coroll. 3

110. Si esset $f=0$, prodiret $\lambda^\circ = \lambda'$, et in lente duplicata anteriori nullam refractionem produceret ob facies parallelas, resque eodem rediret, ac si posterior sola adesset. Sin autem sumatur $f=1$, fit $\lambda^\circ = \lambda$, et lens posterior superflua; vtroque ergo casu nullum lucrum impetratur.

Coroll. 4.

111. Cum autem numeri λ et λ' vnitate nequeant esse minores, sitque $\nu = 0,232692$, patet pro f nullum valorem inter limites 0 et 1 assumi posse, vnde fiat $\lambda^\circ = 0$. At si f extra hos limites capiatur, vtique pro λ et λ' eiusmodi numeri vnitate maiores assignari poterunt, vt fiat $\lambda^\circ = 0$.

Scholion 1.

112. Ratione ergo numeri f tres casus lentium duplicatarum considerari conueniet, prout vel f intra limi-

limites o et 1 continetur, vel fuerit $f > 1$ vel $f < o$. Primo casu euenire non potest, vt fiat $\lambda^{\circ} = o$; sed plurimum intererit eam determinasse lentem duplicatam, pro qua λ° minimum obtineat valorem qui quo magis infra vnitatem cadat, eo perfectior lens erit censenda, maioreque iure simplicibus anteferenda. Binis reliquis vero casibus $f > 1$ et $f < o$ eiusmodi adeo lentes duplicatae parari poterunt, quae praebeant $\lambda^{\circ} = o$, quae ergo pro perfectissimis essent habendae. Verum hic quoque ad praxin est respiciendum, quae cum semper a praeceptis Theoriae aberrare soleat, euenire potest, vt leui errore commisso numerus λ° non solum non euanescat, sed adeo vnitatem excedat quo casu vtique expediret lente vti simplici.

Scholion 2.

113. Cum tanti sit momenti rationem aberrationis a qua praxis vix liberari potest habere, eam etiam in lentibus simplicibus perpendi conueniet. Postulauimus autem pro datis distantiis determinatricibus lentem construi posse in qua numerus λ datum obtineat valorem dum ne sit vnitate minor; hic igitur obseruari oportet, quo maior fuerit λ, eo difficilius fore errorem euitare; si enim in constructione leuissimus error committatur alius eo magis diuersus valor pro λ orietur, quo maior fuerit λ. Verum e contrario cum vnitas sit minimus valor, quem λ recipere potest ex natura minimi liquet etiamsi in praxi a praescripta regula notabiliter recedatur tamen inde vix

L 3 sensi-

ſenſibile diſcrimen in valorem λ eſſe redundaturum. Ex quo concludimus feliciſſimo cum ſucceſſu eiusmodi lentes ſimplices parari poſſe, pro quibus futurum ſit $\lambda = 1$, neque hic errores praxeos, niſi fuerint enormes, admodum eſſe pertimeſcendos. Deinde quo minus numerus λ vnitatem ſuperare debeat, eo certiores eſſe poterimus de ſucceſſu, ſed non eo gradu, quo caſu, $\lambda = 1$; at ſi opus ſit eiusmodi lente, pro qua valor ipſius λ debeat eſſe numerus ſatis magnus, difficillime per praxin ſatisfiet ac fortaſſe ingentem lentium numerum parare oportebit, antequam vna obtineatur ſcopo ſatisfaciens. Quamobrem ſi praxi conſulere velimus, vix alias lentes exigere debemus, niſi pro quibus numerus λ vel ſit vnitas ipſa, vel parumper maior. Sin autem ad inſigne aliquod commodum aliae lentes requirantur, labori non erit parcendum, cum fortaſſe non niſi poſt plurimos conatus irritos voti tandem compotes reddi queamus.

Problema 3.

114. Definire eam lentem duplicatam, pro qua, ſi numerus f intra limites 0 et 1 accipiatur numerus λ° minimum adipiscatur valorem.

Solutio.

Poſitis iisdem, quae in praecedentibus problematibus ſunt conſtituta, inuenimus eſſe

$$\lambda^{\circ} = \lambda f^{3} + \lambda'(1-f)^{3} - \nu f(1-f)$$

vbi cum f intra limites 0 et 1 aſſumi debeat, ambo termini

termini λf^3 et $\lambda'(1-f)^3$ erunt positiui. Quare vt λ° omnium minimum valorem nanciscatur, necesse est vtrique numero λ et λ' minimum valorem cuius est capax tribui.

Sit ergo $\lambda = 1$ et $\lambda' = 1$, ac habebimus

$$\lambda^{\circ} = 1 - 3f + 3ff - \nu f + \nu ff = 1 - (3+\nu)f(1-f)$$

quae expressio vt minima reddatur, oportet fieri $f(1-f)$ maximum, id quod fit sumendo $f = \frac{1}{2}$; hincque oritur

$$\lambda^{\circ} = 1 - \tfrac{1}{4}(3+\nu) = \frac{1-\nu'}{4} = 0,191827$$

Quare constructio huius lentis duplicatae ita se habebit:

$$\text{Pro lente PP radius faciei} \begin{cases} \text{anterioris} = \dfrac{2\alpha\beta}{(2\varrho-\sigma)\beta+\sigma\alpha} \\ \text{posterioris} = \dfrac{2\alpha\beta}{(2\sigma-\varrho)\beta+\varrho\alpha} \end{cases}$$

$$\text{Pro lente QQ radius faciei} \begin{cases} \text{anterioris} = \dfrac{2\alpha\beta}{(2\sigma-\varrho)\alpha+\varrho\beta} \\ \text{posterioris} = \dfrac{2\alpha\beta}{(2\varrho-\sigma)\alpha+\sigma\beta} \end{cases}$$

et si aperturae semidiameter sit $= x$, erit spatium diffusionis

$$Gg = \mu\beta\beta xx\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)\left(0,191827\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)^2 + \frac{\nu'}{\alpha\beta}\right)$$

Si eiusmodi vitro vtamur, pro quo est $n = 1,60 = \frac{8}{5}$, tum ob $\nu = \frac{4}{15}$, prodiret $\lambda^{\circ} = 0,18333$ ideoque haec vitri species adhuc minorem confusionem pareret.

Coroll.

Coroll. 1.

115. Si pro iisdem distantiis determinatricibus α et β lens simplex minimam diffusionem pariens construatur quod fit sumendo

radium faciei $\begin{cases} \text{anterioris} = \frac{\alpha\beta}{\rho\beta + \sigma\alpha} \\ \text{posterioris} = \frac{\alpha\beta}{\rho\alpha + \sigma\beta} \end{cases}$

spatium diffusionis foret

$$\mu\beta\beta xx\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\left(\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2+\frac{\nu}{\alpha\beta}\right).$$

Coroll. 2.

116. Apparet ergo a lente duplicata descripta multo minorem oriri diffusionem, quam a lente simplici, etiamsi haec iam ad minimam diffusionem sit instructa: Cum enim ceterae partes sint pares coefficiens membri $\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2$ plus quam quintuplo minor est in duplicata quam in simplici.

Coroll. 3.

117. Si ponamus $\lambda = 1$ et $\lambda' = 1$, vel saltem $\lambda' = \lambda$, minor valor pro λ° obtineri nequit, quam inuenimus, etiamsi pro f alios valores admittere velimus. Vnde si vtraque lens per se iam minimam diffusionem pariat, pro lente duplicata, valor ipsius λ° minor quam $0,191827$ fieri nequit.

Scholion.

118. Huiusmodi ergo lentes duplicatae maxime sunt notatu dignae cum loco simplicium adhibitae multo

multo minorem diffusionem pariant, ex quo in constructione Telescopiorum et Microscopiorum earum amplissimus erit vsus. Neque vero hac insigni proprietate sunt praeditae, sed etiam earum constructio in praxi minimis difficultatibus est obnoxia: propterea quod etsi a praescriptis regulis parumper aberretur, effectus tamen inde vix vllam mutationem patiatur. Siue enim in constructione vtriusque seorsim leuis error committatur, valores numerorum λ et λ' vnitatem haud sensibiliter excedent, siue in quantitate f valor iustus $f = \frac{1}{2}$ non exacte obseruetur, error vix sentietur, quoniam hi numeri ex natura minimi sunt eruti. Quomodocunque autem a regulis praescriptis aberretur, valor ipsius λ° inde paulisper maior prodibit. Veluti si eiusmodi errores committantur, vt sit $\lambda = 1 + \frac{1}{10}$; $\lambda' = 1 + \frac{1}{10}$ et $f = \frac{1}{2} + \frac{1}{20}$, prodibit $\lambda^\circ = 0,191827 + 0,02558$ seu $\lambda^\circ = 0,2174$, ita vt discrimen partem tantum quadragesimam vnitatis conficiat. Facile autem intelligitur, dummodo λ° prodeat minus quam $\frac{1}{4}$, quod nunquam non, facile praestari posse videtur, ab his lentibus duplicatis insignem vtilitatem expectandam esse. Etsi ergo eiusmodi lentes duplicatae confici possunt, pro quibus numerus λ° plane euanescat, ob harum lubricam constructionem illae istis anteferendae videntur, vt mox clarius patebit.

Problema 4.

119. Pro datis distantiis determinatricibus $AE = a$ et $bG = \mathfrak{b}$ eas lentes duplicatas inuenire, in quibus sit $\lambda^\circ = 0$.

 Solutio.

Solutio.

Cum fieri nequeat $\lambda^{\circ}=0$, nisi numerus f extra limites 0 et 1 accipiatur, simulque numeri λ et λ' fuerint inaequales; ita vt alterutra saltem lens non debeat seorsim minimam diffusionem parere; ponamus esse vel $f>1$ vel $f<0$. Sit ergo primo $f=1+\xi$, et cum sit

$$\lambda^{\circ}=\lambda(1+\xi)^3-\lambda'\xi^3+\nu\xi(1+\xi)$$

vt fiat $\lambda^{\circ}=0$ oportet esse:

$$\lambda'=\lambda(1+\tfrac{1}{\xi})^3+\tfrac{\nu(1+\xi)}{\xi^2}$$

vnde λ' necessario vnitatem superabit, cuius valor ne prodeat nimis magnus, sumi conueniet $\lambda=1$, ita vt sit

$$\lambda'=(1+\tfrac{1}{\xi})^3+\tfrac{0,2326\,02(1+\xi)}{\xi^2} \text{ et } \lambda=1.$$

Quicunque ergo valor ipsi ξ tribuatur, lens duplicata habetur, pro qua sit $\lambda^{\circ}=0$, ac propterea spatium diffusionis $=\mu\,\mathfrak{G}\mathfrak{G}xx(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}})\tfrac{\nu}{a\,\mathfrak{G}}$. Videamus nonnullos casus speciales.

$f=\frac{3}{2}$; $\xi=\frac{1}{2}$; $\lambda=1$ et $\lambda'=28,396152$

$f=2$; $\xi=1$; $\lambda=1$ et $\lambda'=\ \ 8,465384$

$f=3$; $\xi=2$; $\lambda=1$ et $\lambda'=\ \ 3,549519$

$f=4$; $\xi=3$; $\lambda=1$ et $\lambda'=\ \ 2,473789$

$f=5$; $\xi=4$; $\lambda=1$ et $\lambda'=\ \ 2,025841$

$f=6$; $\xi=5$; $\lambda=1$ et $\lambda'=\ \ 1,783846$

etc.

Pro

Pro altero casu sit $f = -\xi$, ideoque

$$\lambda^{\circ} = -\lambda\xi^3 + \lambda'(1+\xi)^3 + \nu\xi(1+\xi)$$

vnde facto $\lambda^{\circ} = 0$ prodit

$$\lambda = \lambda'(1+\tfrac{1}{\xi})^3 + \tfrac{\nu(1+\xi)}{\xi^2}.$$

Statui ergo conueniet $\lambda' = 1$, ac pro λ notentur casus sequentes.

$f = -\tfrac{1}{2}$; $\xi = \tfrac{1}{2}$; $\lambda = 28,396152$ et $\lambda' = 1$
$f = -1$; $\xi = 1$; $\lambda = 8,465384$ et $\lambda' = 1$
$f = -2$; $\xi = 2$; $\lambda = 3,549519$ et $\lambda' = 1$
$f = -3$; $\xi = 3$; $\lambda = 2,473789$ et $\lambda' = 1$
$f = -4$; $\xi = 4$; $\lambda = 2,025841$ et $\lambda' = 1$
$f = -5$; $\xi = 5$; $\lambda = 1,783846$ et $\lambda' = 1$

sicque patet infinitis modis huiusmodi lentes duplicatas parari posse, pro quibus sit $\lambda^{\circ} = 0$, et spatium diffusionis

$$Gg = \mu\mathfrak{b}\mathfrak{b}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\tfrac{\nu}{a\mathfrak{b}}.$$

Scholion 1.

120. Si huiusmodi lentes accuratissime parari possent, nullum est dubium, quin praecedentibus essent anteferendae propterea quod diffusio iis adhuc magis diminuitur. Verum dolendum est, quod minimus error in earum constructione commissus omnem

fere vſum deſtruat. Quo hoc facilius diiudicare queamus, examinemus eum caſum, quo eſt

$$f=5,\ \lambda=1 \text{ et } \lambda'=2,025841$$

hincque

$$\lambda^{\circ}=\lambda f^{3}-\lambda'(f-1)^{3}+\nu f(f-1)=0.$$

Ponamus autem in conſtructione errorem eſſe commiſſum vt reuera non ſit $f=5$ ſed $f=5\frac{1}{10}$, dum numeri λ et λ' ſuos iuſtos valores obtineant: ob hunc autem vix vitandum errorem non ſit $\lambda^{\circ}=0$, ſed adeo $\lambda^{\circ}=-2,011$; ſicque haec lens duplicata ſimplicibus longe eſt poſtponenda, ſimili modo ſi f eſſet $=5$, ſed vel λ vel λ' tantillum a praeſcripto valore aberraret, enorme ſtatim diſcrimen in valorem ipſius λ° redundaret. Minus quidem error metuendus videtur in ea ſpecie, qua $\lambda=1$, $\lambda'=28,396152$ et $f=\frac{3}{2}$; ſed praeterquam, quod lens ſimplex poſterior difficillime parari queat, pro qua λ' praeciſe valorem aſſignatum conſequatur, talis lens ad vſum dioptricum plane eſt inepta, ob ingentem alterius faciei curuaturam. Quae cum ita ſint, quia tam exiguus error in paratione huiusmodi lentium commiſſus facit, vt λ° adeo ſupra vnitatem excreſcat, vix ſperare poterimus, vt vnquam talis lens duplicata perficiatur, pro qua λ° vsque ad $\frac{1}{5}$ diminuatur. In lentibus autem praecedentis generis ſucceſſus vix fallere poterit, niſi in praxi enormiter a praeſcripta regula aberretur: ex quo his ſolis lentibus duplicatis cum fructu vti licebit, dum contra

contra eae, quas in praesente problemate descripsimus, penitus profligandae videntur.

Scholion 2.

121. Pro datis ergo binis distantiis determinatricibus a et $\mathfrak{b}$ semper eiusmodi lens duplicata parari potest, ex qua nascatur spatium diffusionis

$$Gg = \mu \mathfrak{b}\mathfrak{b}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\left(\lambda^{\circ}\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^{2}+\tfrac{\nu}{a\mathfrak{b}}\right)$$

ita vt λ° numerum quemcunque denotare possit. Ad hoc enim satisfieri oportet huic aequationi:

$$\lambda^{\circ} = \lambda f^{3} + \lambda'(1-f)^{3} - \nu f(1-f)$$

id quod semper fieri potest, cum f plane ab arbitrio nostro pendeat, et numeri λ et λ' tantum non vnitate minores accipi debeant. Definitis autem his tribus numeris λ, λ' et f ita, vt λ° datum valorem obtineat, binae lentes simplices, ex quibus duplicata est componenda, secundum formulas §. 107 datas confici debent. Vbi quidem tenenda sunt ea, quae modo obseruauimus, in praxi eas lentes facillime obtineri, quando numeri λ et λ' vnitatem parum superant, f vero propemodum $\frac{1}{2}$ denotat, cum contra quo magis hi numeri ab istis terminis recedant, eo maius sit periculum ne effectus enormiter fallat. Ceterum cum diffusionem a lente duplicata oriundam ad eandem formam reduxerimus, qua diffusio lentis simplicis exprimitur, inde id commodi consequimur, vt simili modo diffusionem a lentibus magis multiplicatis ortum definire valeamus.

Supplementum I.

De lentibus duplicatis.

Si pro lente anteriori ratio refractionis sit $n : 1$, pro lente posteriori vero alia ratio $n' : 1$ locum habeat; problemata hic tractata sequenti modo resolui poterunt

Pro Problemate I.

Si pro numeris ϱ, σ, τ, qui ex n oriuntur, quaerantur simili modo ex n' valores ϱ', σ', et τ' erunt

Pro Lente

$$PP \text{ radius faciei } \begin{cases} \text{anter.} = \dfrac{a\alpha}{\varrho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda - 1)}} \\ \text{poster.} = \dfrac{a\alpha}{\varrho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda - 1)}} \end{cases}$$

$$QQ \text{ radius faciei } \begin{cases} \text{anter.} = \dfrac{b\beta}{\varrho'\beta + \sigma' b \pm \tau'(b+\beta)\sqrt{(\lambda' - 1)}} \\ \text{poster.} = \dfrac{b\beta}{\varrho' b + \sigma'\beta \mp \tau'(b+\beta)\sqrt{(\lambda' - 1)}} \end{cases}$$

et definitis simili modo valoribus μ', ν' ex ratione $n' : 1$ reperietur spatium diffusionis

$$\mathfrak{E}\,\mathfrak{E}\,x\,x \begin{cases} +\mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\mu'\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right) \end{cases}$$

reliqua manent vt in problemate.

Pro Problemate II.

Quoniam hic duae vitri species occurrunt, ponamus lentem simplicem quaesitam ex alio quocunque vitri genere parari, cuius ratio refractionis sit $n^\circ : 1$, vnde prodeant numeri μ° et ν°; superfluum autem foret, numeros ϱ°, σ° et τ° computari, quoniam radii

dii facierum fiunt imaginarii, ita, vt eos exprimere non ſit opus, et quoniam pro hac lente ſimplici aequiualente numerus λ° eſt introductus, erit huius lentis ſpatium diffuſionis

$$\mathfrak{G}\mathfrak{G}xx.\mu^\circ\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)\left(\lambda^\circ\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\tfrac{\nu^\circ}{a\mathfrak{G}}\right)$$

quod vt cum ſpatio diffuſionis lentis duplicatae aequale fiat; ponatur, vti in problemate eſt factum,

$$\tfrac{1}{\alpha}=\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{G}} \text{ et } \tfrac{1}{b}=\tfrac{1-f}{a}-\tfrac{f}{\mathfrak{G}}$$

vt prodeat

$$\tfrac{1}{a}+\tfrac{1}{\alpha}=f\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right) \text{ et } \tfrac{1}{b}+\tfrac{1}{\mathfrak{G}}=(1-f)\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)$$

Vnde peruenietur ad hanc aequationem

$$\mu f\left(\lambda f^2\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\nu\left(\tfrac{f-1}{a^2}+\tfrac{f}{a\mathfrak{G}}\right)\right)$$
$$+\mu'(1-f)\left(\lambda'.(1-f)^2\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\nu'\left(\tfrac{1-f}{a\mathfrak{G}}-\tfrac{f}{\mathfrak{G}^2}\right)\right)$$
$$=\mu^\circ\left(\lambda^\circ\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\tfrac{\nu^\circ}{a\mathfrak{G}}\right)$$

quae ita repraeſentari poteſt

$$\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2\left(\mu\lambda f^3+\mu'\lambda'(1-f)^3\right)$$
$$+\left(\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{G}}\right)\left(\tfrac{\mu\nu f}{a}-\tfrac{\mu'\nu'(1-f)}{\mathfrak{G}}\right)$$
$$=\mu^\circ\lambda^\circ\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\tfrac{\nu^\circ\mu^\circ}{a\mathfrak{G}}$$

Vnde

$$\lambda^\circ=\tfrac{\mu\lambda f^3+\mu'\lambda'(1-f)^3}{\mu^\circ}+$$
$$\tfrac{aa\mathfrak{G}\mathfrak{G}}{\mu^\circ(a+\mathfrak{G})^2}\left(\left(\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{G}}\right)\left(\tfrac{\mu\nu f}{a}-\tfrac{\mu'\nu'(1-f)}{\mathfrak{G}}\right)-\tfrac{\nu^\circ\mu^\circ}{a\mathfrak{G}}\right)$$

Deni-

Denique si ex ratione refractionis $n' : 1$ computentur numeri ρ', σ' et τ', radii facierum lentis duplicatae erunt

Pro Lente

P P radius faciei $\begin{cases} \text{anterioris} = \frac{a\beta}{(\rho-\sigma(1-f))\beta+\sigma f a+\tau f(a+\beta)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \frac{a\beta}{(\sigma-\rho(1-f))\beta+\rho f a\mp\tau f(a+\beta)\sqrt{(\lambda-1)}} \end{cases}$

QQ radius faciei $\begin{cases} \text{anter.} = \frac{a\beta}{(\sigma'-\rho' f)a+\rho'(1-f)\beta\pm\tau'(1-f)(a+\beta)\sqrt{(\lambda'-1)}} \\ \text{poster.} = \frac{a\beta}{(\rho'-\sigma' f)a+\sigma'(1-f)\beta\mp\tau'(1-f)(a+\beta)\sqrt{\lambda'-1}} \end{cases}$

Ad Problema III.

Hoc problema non solum pro diuersa refractione n et n' hic generalius pertractabo, sed etiam rationem distantiae inter binas lentes habebo. Primum igitur vtramque lentem ad distantias determinatrices lentis duplicatae, a et β, reuocabo, ponendo

$$\tfrac{1}{a}+\tfrac{1}{\alpha}=f(\tfrac{1}{a}+\tfrac{1}{\beta}) \text{ et } \tfrac{1}{b}+\tfrac{1}{\beta}=g(\tfrac{1}{a}+\tfrac{1}{\beta})$$

vt sit

$$\tfrac{1}{\alpha}=\tfrac{f-1}{a}+\tfrac{f}{\beta} \text{ et } \tfrac{1}{b}=\tfrac{g}{a}+\tfrac{g-1}{\beta}$$

hincque

$$\alpha=\frac{a\beta}{fa+(f-1)\beta} \text{ et } b=\frac{a\beta}{(g-1)a+g\beta}$$

ideoque distantia lentium

$$\alpha+b=\frac{a\beta(f+g-1)(a+\beta)}{(fa+(f-1)\beta)((g-1)a+g\beta)}$$

quae si deberet esse $=0$, capi oporteret $g=1-f$ sed si distantiam aliquam inter lentes admittamus, statuamus

tuamus $f+g-1=\omega$, denotante ω fractionem quandam minimam, ſiue poſitiuam ſiue negatiuam vt diſtantia lentium prodeat poſitiua et valde parua. Cum hinc igitur ſit $g=1+\omega-f$; erit lentium diſtantia

$$a+b=\frac{\alpha\beta(\alpha+\beta)\omega}{(f\alpha+(f-1)\beta)((\omega-f)\alpha+(1+\omega-f)\beta)}$$

Spatium autem diffuſionis nunc ita exprimetur:

$$\beta\beta xx.\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\begin{cases}+\lambda\mu f^3\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2+\lambda'\mu' g^3\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2\\+\frac{\mu\nu f}{\alpha}\left(\frac{f-1}{\alpha}+\frac{f}{\beta}\right)\\+\frac{\mu'\nu' g}{\beta}\left(\frac{g}{\alpha}+\frac{g-1}{\beta}\right)\end{cases}$$

quae cum in hoc problemate ita tractari debeat, vt tam f, quam g poſitiue ſumantur, et haec formula minima reddatur; euidens eſt, litteris λ et λ' minimos valores tribui debere, ſcilicet $\lambda=1$ et $\lambda'=1$; deinde pro hoc caſu minimi f et g conuenienter definiantur, vbi quia $f+g-1=\omega$ ideoque conſtans in differentiatione, habebimus $dg=-df$; vndeobtinebimus hanc aequationem

$$(3\mu f^2-3\mu' g^2)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2+\frac{\mu\nu}{\alpha}\left(\frac{2f-1}{\alpha}+\frac{2f}{\beta}\right)$$
$$-\frac{\mu'\nu'}{\beta}\left(\frac{2g}{\alpha}+\frac{2g-1}{\beta}\right)=0$$

cui proxime ſatisfit, ponendo $f=g=\frac{1+\omega}{2}$ quibus valoribus ſubſtitutis ſpatium diffuſionis ipſum minimum, erit proxime

$$\frac{\beta^2x^2(1+\omega)}{8}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\begin{cases}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2(1+\omega)^2(\mu+\mu')\\+\frac{2\mu\nu}{\alpha}\left(\frac{\omega-1}{\alpha}+\frac{1+\omega}{\beta}\right)\\+\frac{2\mu'\nu'}{\beta}\left(\frac{1+\omega}{\alpha}+\frac{\omega-1}{\beta}\right)\end{cases}$$

 Tum

Tum vero diſtantia lentium erit

$$a+b=\frac{4\alpha\beta(\alpha+\beta)\omega}{((1+\omega)\alpha+(\omega-1)\beta)((\omega-1)\alpha+(1+\omega)\beta)}$$
$$=\frac{4\alpha\beta(\alpha+\beta)\omega}{-(1-\omega^2)(\alpha^2+\beta^2)+2(1+\omega)^2\alpha\beta}$$

cuius denominator cum ſit negatiuus ob ω minimum; neceſſe eſt, fractionem ω ſumi debere negatiuam; hincque adeo ſpatium diffuſionis minus reddetur.

Coroll.

Si ergo diſtantia obiecti a fuerit infinita ſeu $\alpha=\infty$; habebitur primo diſtantia lentium $=\frac{-4\beta\omega}{1-\omega^2}$; et ſecundo ſpatium diffuſionis

$$\frac{x^2(1+\omega)}{8\beta}\left\{\begin{array}{l}(1+\omega)^2(\mu+\mu')\\ -2\mu'\nu'(1-\omega)\end{array}\right.$$

quod cum ω debeat eſſe negatiuum, non mediocriter minus erit, quam ſi diſtantia lentium eſſet nulla.

Ad Problema IV.

In hoc problemate etiam diſtantiam lentium non negligamus; factaque reductione, vt ante, ſtatuamus ſpatium diffuſionis plane euaneſcens; id quod fieri nequit, niſi altera litterarum f et g ſit negatiua, quod cum etiam fiat, quando confuſio a diuerſa radiorum refrangibilitate oriunda ad nihilum redigi debet, vt infra videbimus; hic caſus multo magis euolutionem meretur. Ponatur igitur $g=-\zeta f$, vbi ζ ex illa

illa conditione determinatur, vt primo pro diſtantia lentium ſit

$$\alpha + b = \frac{-\alpha\beta(\alpha+\beta)(f-\zeta f-1)}{(f\alpha+(f-1)\beta)((\zeta f+1)\alpha+\zeta f\beta)}$$

et poſito $f+g-1=\omega$, prodeat $f-\zeta f=1+\omega$ ideoque

$f=\frac{1+\omega}{1-\zeta}$ et $g=\frac{-\zeta(1+\omega)}{1-\zeta}$ ſicque

$$\alpha + b = \frac{-\alpha\beta(\alpha+\beta)\omega(1-\zeta)^2}{((1+\omega)\alpha+(\omega+\zeta)\beta)((1+\omega\zeta)\alpha+\zeta(1+\omega)\beta)}$$

ac ſi in denominatore ω reiiciatur, erit

$$\alpha + b = \frac{-\alpha\beta(\alpha+\beta)(1-\zeta)^2\omega}{(\alpha+\zeta\beta)}$$

ſicque patet ω negatiue capi debere.

Poſito autem $g=-\zeta f$, fit ſpatium diffuſionis

$$\beta^2 x^2 \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\begin{cases}(\lambda\mu f^3-\lambda'\mu'\zeta^3 f^3)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2\\ +\frac{\mu\nu f}{\alpha}\left(\frac{f-1}{\alpha}+\frac{f}{\beta}\right)\\ -\frac{\mu'\nu'\zeta f}{\beta}\left(\frac{-\zeta f}{\alpha}-\frac{(\zeta f+1)}{\beta}\right)\end{cases}$$

ad nihilum redigendum; vnde ſequitur fore

$$\lambda' = \frac{\lambda\mu}{\mu'\zeta^3} + \frac{\mu\nu.\alpha.\beta^2}{\mu'\zeta^3 f^2(\alpha+\beta)^2}\left(\frac{f-1}{\alpha}+\frac{f}{\beta}\right)$$
$$+\frac{\nu'.\alpha^2\beta}{\zeta^2 f^2.(\alpha+\beta)^2}\left(+\frac{\zeta f}{\alpha}+\frac{\zeta f+1}{\beta}\right)$$

qui valor cum debeat eſſe maior vnitate, ſi forte eueniat, vt minor prodeat, tunc non λ' ſed λ definiri conueniet, vbi notandum eſt eſſe $f=\frac{1+\omega}{1-\zeta}$. Hincque per formulas ante datas facile eruuntur radii facierum vtriusque lentis.

Etſi formula ſuperius data pro ſpatio diffuſionis iam ad caſum, quo diſtantia lentium eſt nulla, eſt adcommodata; tamen quia hic distantiam minimam assumimus, nullus inde error est metuendus.

Definitio 2.

122. *Lens triplicata est, quae consistit ex tribus lentibus simplicibus sibi immediate iunctis ad communem axem.*

Hic quidem etiam crassitiem negligo, etiamsi necessario maior sit quam in lentibus duplicatis. In suplemento autem ostendetur, quomodo etiam distantiarum inter lentes ratio sit habenda.

Corollarium.

123. Potest ergo lens triplicata considerari, quasi composita ex lente duplicata et lente simplici, hocque duplici modo, prout vel binae anteriores, vel binae posteriores lentem duplicatam constituere concipiuntur.

Scholion.

124. Lentibus triplicatis tum demum vsum in praxi concedi conueniet, cum eadem commoda per lentes simplices vel duplicatas consequi non licet. Haec autem commoda in paruitate numeri λ consistunt, qui quamdiu vnitate fuerit maior, semper lente simplici vti praestat, cum vero circumstantiae in expressione diffusionis minorem numerum λ requirunt, ad lentes multiplicatas erit confugiendum. Quoniam igitur eiusmodi lentes duplicatas conficere docuimus, in

in quibus valor ipſius λ non ſolum ad nihilum vsque, ſed adeo ad negatiua diminui queat, vſus lentium triplicatarum ſuperfluus videtur. Verum iam obſeruauimus in praxi aegre eiusmodi duplicatas lentes parari poſſe, pro quibus valor ipſius λ minor ſit quam 0, 191827, propterea quod ſi leuiſſimus error committatur, omnis labor irritus reddatur. His igitur caſibus imprimis, quando minori valore numeri λ opus eſt, lentes triplicatae in vſum erunt vocandae: et quia reſpectu ad praxin habito non omnes aequo ſucceſſu conſtruere licet, errores ineuitabiles hic quoque imprimis ſpectari oportet, vt pateat quousque numerus λ cum ſucceſſu diminui queat, ac ſi adhuc minore valore opus fuerit lentes adeo quadruplicate erunt inducendae.

Problema 5.

125. Datis binis diſtantiis determinatricibus omnes lentes triplicatas definire, ſimulque ſpatium diffuſionis ab iis ortum. Tab. II. Fig. 8.

Solutio.

Sit E*e* obiectum, cuius magnitudo $= z$, et diſtantia a lente AE $= a$: tum primae lentis PP diſtantiae determinatrices ſint a et α, ſecundae lentis QQ b et β; ac tertiae RR, c et γ. His poſitis quia lentes immediate iunctae ſumuntur, erit $\alpha + b = 0$, $\beta + c = 0$, et a et γ diſtantiae determinatrices lentis triplicatae, ita vt α et β arbitrio noſtro relinquantur.

Si igitur lens prima sola PP adesset, imago repraesentaretur in $F\zeta$, vt esset $AF = \alpha$ et $F\zeta = \frac{\alpha}{a} z$: si binae priores PP et QQ solae adessent imago exhiberetur in $G\eta$, vt esset $AG = \mathfrak{E}$ et $G\eta = \frac{\mathfrak{E}}{a} z$; at per lentem triplicatam referetur in $H\theta$ vt sit $cH = \gamma$ et $H\theta = \frac{\gamma}{a} z$, pro situ inuerso ob $\frac{\alpha \mathfrak{E}}{bc} = 1$. Hactenus scilicet res perinde se habet, ac si in A haberetur lens simplex ad distantias determinatrices a et γ accommodata.

At si ad spatium diffusionis Hh respiciamus figuram singularum lentium in computum ducere debemus, quatenus praeter distantias determinatrices numeri arbitrarii $\lambda, \lambda', \lambda''$ inuoluuntur; ex quibus facies singularum lentium supra §. 91 sunt definitae. Indidem autem colligitur pro apertura cuius semidiameter $= x$, spatium diffusionis ob $\frac{\alpha}{b} = -1$ et $\frac{\mathfrak{E}}{c} = -1$ fore:

$$Hh = \mu \gamma \gamma x x \begin{cases} + \left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{\nu}{\alpha a}\right) \\ + \left(\frac{1}{b} + \frac{1}{\mathfrak{E}}\right)\left(\lambda'\left(\frac{1}{b} + \frac{1}{\mathfrak{E}}\right)^2 + \frac{\nu}{b \mathfrak{E}}\right) \\ + \left(\frac{1}{c} + \frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c} + \frac{1}{\gamma}\right)^2 + \frac{\nu}{c \gamma}\right) \end{cases}$$

quae expressio, vt ad formam vni lenti respondentem reducatur statuamus:

$$\frac{1}{a} + \frac{1}{\alpha} = f\left(\frac{1}{a} + \frac{1}{\gamma}\right)$$

$$\frac{1}{b} + \frac{1}{\mathfrak{E}} = g\left(\frac{1}{a} + \frac{1}{\gamma}\right)$$

$$\frac{1}{c} + \frac{1}{\gamma} = h\left(\frac{1}{a} + \frac{1}{\gamma}\right)$$

et

et quia $\frac{1}{\alpha}+\frac{1}{b}=0$, et $\frac{1}{\beta}+\frac{1}{c}=0$, his aequationibus addendis adipiſcimur: $f+g+h=1$. Porro vero erit

$$\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)-\frac{1}{a};\quad \frac{1}{b}=\frac{1}{a}-f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{\beta}=(f+g)\left(\frac{1}{a}+\frac{1}{\gamma}\right)-\frac{1}{a};\quad \frac{1}{c}=\frac{1}{a}-(f+g)\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

ſiue ob $1=f+g+h$

$$\frac{1}{a}=\frac{f+g+h}{a};\quad \frac{1}{\alpha}=\frac{-g-h}{a}+\frac{f}{\gamma}$$

$$\frac{1}{b}=\frac{g+h}{a}-\frac{f}{\gamma};\quad \frac{1}{\beta}=-\frac{h}{a}+\frac{f+g}{\gamma}$$

$$\frac{1}{c}=\frac{h}{a}-\frac{f+g}{\gamma};\quad \frac{1}{\gamma}=\frac{f+g+h}{\gamma}$$

Vnde cum ſpatium diffuſionis fiat.

$$Hh=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left((\lambda f^3+\lambda' g^3+\lambda'' h^3)\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\nu\left(\frac{f}{a\alpha}+\frac{g}{b\beta}+\frac{h}{c\gamma}\right)\right)$$

reducetur id ad hanc formam

$$Hh=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left((\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h))\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Radiorum autem in h concurrentium inclinatio ad axem erit $=\frac{x}{\gamma}$.

Coroll. 1.

126. Haec igitur lens triplicata idem producit ſpatium diffuſionis quod produceret lens ſimplex ad easdem diſtantias determinatrices inſtructa, numero eius arbitrario (per litteram λ indicato) exiſtente

$$\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h);$$

vbi quidem eſt $f+g+h=1$.

Coroll.

Coroll. 2.

127. Quatenus ergo haec quantitas reddi poteſt minor non ſolum vnitate; ſed etiam fractione $0,191827$ ita ſcilicet, vt praxis non enormi aberrationi ſit expoſita, eatenus lentibus triplicatis vſus erit concedendus.

Coroll. 3.

128. Si ſit vel $f=0$, vel $g=0$, vel $h=0$, vna lentium habebit facies parallelas, et lens triplicata aequiualebit duplicatae ac ſi duae litterarum f, g, h ſimul euaneſcant, tertia in vnitatem abeunte, caſus habebitur lentis ſimplicis.

Coroll. 4.

129. Si ſit $f=1$, ideoque $h=-g$, valor numeri λ pro lente triplicata erit $\lambda+(\lambda'-\lambda'')g^3$, ideoque ſi $\lambda''=\lambda'$ lens triplicata ſimplici aequiualebit, quod idem euenit ſi fuerit vel $g=1$ vel $h=1$.

Coroll. 5.

130. Sumtis autem pro f, g, h numeris idoneis ob $f+g+h=1$, conſtructio lentis triplicatae ex formulis §. 91 exhibitis eſt petenda ſumendo.

$$\frac{1}{a}=\frac{-1+f}{a}+\frac{f}{\gamma}; \quad \frac{1}{b}=\frac{1-f}{a}-\frac{f}{\gamma}$$

$$\frac{1}{\beta}=\frac{-h}{a}+\frac{1-h}{\gamma}; \quad \frac{1}{c}=\frac{h}{a}-\frac{1+h}{\gamma}$$

Scholion

Scholion 1.

131. Quemadmodum hinc expressio inuenta prodeat, notandum est fore $\frac{f}{a\alpha}+\frac{g}{b\beta}+\frac{h}{c\gamma}=$

$$+\frac{1}{a\alpha}(-f(1-f)-gh(1-f))$$
$$+\frac{1}{a\gamma}(+ff+g(1-f)(1-h)+fgh+hh)$$
$$+\frac{1}{\gamma\gamma}(-fg(1-h)-h(1-h))$$

sed $-f(1-f)-gh(1-f)=-(1-f)(f+gh)=-(1-f)(1-g)(1-h)$ ob $f=1-g-h$ ideoque $f+gh=(1-g)(1-h)$. Simili modo pro $\frac{1}{\gamma\gamma}$ est $-fg(1-h)-h(1-h)=-(1-h)(h+fg)=-(1-f)(1-g)(1-h)$ ob $h=1-f-g$. Denique pro $\frac{1}{a\gamma}$, quia est $ff+hh=(f+h)^2-2fh=(1-g)^2-2fh=1-2g+gg-2fh$, hoc valore substituto coefficiens ipsius $\frac{1}{a\gamma}$ erit

$$1-2g-2fh+gg+fgh+g(1-f)(1-h)=$$
$$1+(g+fh)(g-2)-g(1-f)(1-h)=1-2(1-f)(1-g)(1-h)$$

ob $g+fh=(1-f)(1-h)$. Consequenter colligitur

$$\frac{f}{a\alpha}+\frac{g}{b\beta}+\frac{h}{c\gamma}=-(1-f)(1-g)(1-h)(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\frac{1}{a\gamma}.$$

Scholion 2.

132. Hoc problema etiam ope praecedentium facilius sequenti modo resolui potest. Considerentur scilicet binae lentes PP et QQ iunctim sumtae tanquam lens duplicata ad distantias determinatrices a et β per numeros arbitrarios λ, λ' et f instructa, ac posito $\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)=\lambda^{(2)}$, spatium diffusionis ex ea sola ortum erit

$$\mu\beta\beta xx(\tfrac{1}{a}+\tfrac{1}{\beta})(\lambda^{(2)}(\tfrac{1}{a}+\tfrac{1}{\beta})^2+\tfrac{\nu}{a\beta})$$

 quae

quae lens si iam in compositione cum tertia RR tanquam simplex tractetur exinde elicitur spatium diffusionis perinde atque ex coniunctione duarum simplicium.

$$\mathrm{H}b = \mu\gamma\gamma xx \begin{cases} +(\frac{1}{a}+\frac{1}{\mathfrak{b}})(\lambda^{(2)}(\frac{1}{a}+\frac{1}{\mathfrak{b}})^2+\frac{\nu}{a\mathfrak{b}}) \\ +(\frac{1}{c}+\frac{1}{\gamma})(\lambda''(\frac{1}{c}+\frac{1}{\gamma})^2+\frac{\nu}{c\gamma}) \end{cases}$$

vbi notandum est esse $\mathfrak{b}+c=0$, et constructio lentis duplicatae ex §. 107 erit petenda lentis vero simplicis RR ex distantiis determinatricibus $c=-\mathfrak{b}$ et γ vna cum numero arbitrario λ''. Ponatur iam $\frac{1}{\mathfrak{b}}=\frac{g-1}{a}+\frac{g}{\gamma}$, vt sit $\frac{1}{c}=\frac{1-g}{a}-\frac{g}{\gamma}$, eritque spatium diffusionis huius lentis triplicatae,

$$\mathrm{H}b=\mu\gamma\gamma xx(\tfrac{1}{a}+\tfrac{1}{\gamma})((\lambda^{(2)}g^3+\lambda''(1-g)^3-\nu g(1-g))(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\tfrac{\nu}{a\gamma})$$

Quare si pro lente triplicata ponatur

$$\lambda^{(2)}g^3+\lambda''(1-g)^3-\nu g(1-g)=\lambda^{(3)}$$

ita vt iam numerus g insuper arbitrio nostro relinquatur, habebitur spatium diffusionis more hactenus recepto expressum

$$\mathrm{H}b=\mu\gamma\gamma xx(\tfrac{1}{a}+\tfrac{1}{\gamma})(\lambda^{(3)}(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\tfrac{\nu}{a\gamma})$$

Hinc iam id intelligitur, quod ex praecedente solutione minus patet; si numerus $\lambda^{(2)}$ fuerit vnitate maior loco binarum priorum lentium PP et QQ commodius vnicam simplicem adhiberi; ex quo eatenus tantum lentes triplicatae resultare censendae sunt, quatenus nume-

numerus $\lambda^{(2)}$ vnitate eſt minor. Vidimus autem ſucceſſum tuto ſperari non poſſe, niſi $\lambda^{(3)}$ aequalis ſit fractioni 0,191827, vel ea non multo maior; vnde ſi praxi conſulere velimus, ipſi $\lambda^{(2)}$ minorem valorem tribui non conuenit: atque ob eandem rationem numerus g intra terminos 0 et 1 accipi debebit; cuius valor imprimis ad praxin erit accommodatus ſi reddat numerum $\lambda^{(3)}$ minimum, quia tum leues errores negotium minime turbant. Quodſi vero pro $\lambda^{(2)}$ valorem aſſignatum ſubſtituamus, obtinebimus

$$\lambda^{(3)} = \lambda f^3 g^3 + \lambda' g^3 (1-f)^3 + \lambda'' (1-g)^3 - \nu g^3 f(1-f) - \nu g(1-g)$$

Conducet autem vtramque expreſſionem pro numero $\lambda^{(3)}$, quo ſpatium diffuſionis a lente triplicata ortum definitur, hic expoſuiſſe cum aliae concluſiones ex altera facilius deducantur. Etſi autem hinc omnes valores pro $\lambda^{(3)}$ obtineri poſſunt tamen eos tantum, qui prope minimum ſubſiſtunt ad praxin adhiberi conueniet.

Problema. 6.

133. Datis diſtantiis determinatricibus $AE = a$ et $cH = \gamma$ definire eam lentem triplicatam, quae minimum ſpatium diffuſionis producat.

Solutio I.

Duplici modo hoc problema ſolui poteſt, prout ſpatium diffuſionis vel ita exprimitur, vti in ſolutione problematis praecedentis, vel in Scholio 2. Priori

ori modo numeros f, g, h ita determinari oportet, vt minima reddatur haec expressio:

$$\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h)$$

vbi notandum est esse $f+g+h=1$. Eius ergo differentiali nihilo aequali posito habebimus:

$$3\lambda ffdf+3\lambda' ggdg+3\lambda'' hhdh$$
$$+\nu df(1-g)(1-h)+\nu dg(1-f)(1-h)+\nu dh(1-f)(1-g)=0.$$

Cum autem sit $dh=-df-dg$ erit

$$\left.\begin{aligned}&+3\lambda ffdf-3\lambda'' hhdf+\nu df(1-g)(f-h)\\&+3\lambda' ggdg-3\lambda'' hhdg+\nu dg(1-f)(g-h)\end{aligned}\right\}=0$$

quia vero bina differentialia df et dg a se inuicem non pendent ambo membra huius aequationis seorsim euanescere debent, vnde ob $1-g=f+h$ et $1-f=g+h$ has duas nanciscimur aequationes:

$$3\lambda ff-3\lambda'' hh+\nu ff-\nu hh=0$$
$$3\lambda' gg-3\lambda'' hh+\nu gg-\nu hh=0$$

ex quibus elicimus:

$$f=h\sqrt{\frac{3\lambda''+\nu}{3\lambda+\nu}} \text{ et } g=h\sqrt{\frac{3\lambda''+\nu}{3\lambda'+\nu}}.$$

Cum autem sit

$$f+g+h=1 \text{ seu } \frac{1}{h}=1+\frac{f}{h}+\frac{g}{h} \text{ erit}$$
$$\frac{1}{h}=1+\sqrt{\frac{3\lambda''+\nu}{3\lambda+\nu}}+\sqrt{\frac{3\lambda''+\nu}{3\lambda'+\nu}}$$
$$\frac{1}{g}=1+\sqrt{\frac{3\lambda'+\nu}{3\lambda+\nu}}+\sqrt{\frac{3\lambda'+\nu}{3\lambda''+\nu}}$$
$$\frac{1}{f}=1+\sqrt{\frac{3\lambda+\nu}{3\lambda'+\nu}}+\sqrt{\frac{3\lambda+\nu}{3\lambda''+\nu}}$$

Quicun-

Quicunque ergo numeri λ, λ', λ'' in conſtructione ſingularum lentium fuerint vſurpati, hinc numeri f, g, et h determinantur, ex quibus ſpatium diffuſionis minimum reſultet. Pro lentium autem conſtructione hinc diſtantiae α, b, $\mathfrak{b}$, c ita definiuntur; vt ſit

$$\frac{1}{\alpha} = \frac{-1+f}{a} + \frac{f}{\gamma}; \quad \frac{1}{b} = \frac{1-f}{a} - \frac{f}{\gamma}$$
$$\frac{1}{\mathfrak{b}} = \frac{-h}{a} + \frac{1-h}{\gamma}; \quad \frac{1}{c} = \frac{h}{a} - \frac{1+h}{\gamma}$$

eſt vero:

$$f = \frac{\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)} + \sqrt{(3\lambda+\nu)(3\lambda''+\nu)} + \sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$
$$g = \frac{\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)} + \sqrt{(3\lambda+\nu)(3\lambda''+\nu)} + \sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$
$$h = \frac{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)} + \sqrt{(3\lambda+\nu)(3\lambda''+\nu)} + \sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

Ex diſtantiis autem a, α, b, $\mathfrak{b}$, c, γ cum numeris λ, λ', λ'' lentes ipſae per formulas §. 91 exhibitas conſtruuntur.

Coroll. I.

134. Si pro hac lente triplicata ponatur

$$\lambda f^3 + \lambda' g^3 + \lambda'' h^3 - \nu(1-f)(1-g)(1-h) = \lambda^{(3)}$$

ſubſtituendis his valoribus pro f, g, et h reperietur

$$\lambda^{(3)} = \frac{1}{3\left(\frac{1}{\sqrt{(3\lambda+\nu)}} + \frac{1}{\sqrt{(3\lambda'+\nu)}} + \frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2} - \frac{1}{3}\nu$$

vnde ſpatium diffuſionis fit

$$Hh = \mu\gamma.\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2 + \frac{\nu}{a\gamma}\right)$$

O 3 Coroll.

Coroll. 2.

135. Hoc autem spatium diffusionis omnium fiet minimum si numeris λ, λ', λ'' minimi valores, quos accipere possunt, tribuantur. Sit ergo $\lambda = 1$, $\lambda' = 1$, $\lambda'' = 1$, eritque

$$f = \tfrac{1}{3};\quad g = \tfrac{1}{3},\quad b = \tfrac{1}{3}$$

$$\text{et } \lambda^{(3)} = \frac{\nu + 3}{27} - \frac{\nu}{3} = \frac{3 - 8\nu}{27} = 0{,}042165$$

ob 0, 232692; qui ergo valor multo est minor, quam casu lentium duplicatarum.

Coroll. 3.

136. Hoc porro casu erit $\frac{1}{a} = -\frac{2}{3a} + \frac{1}{3\gamma}$; $\frac{1}{b} = \frac{2}{3a} - \frac{1}{3\gamma}$; $\frac{1}{\mathfrak{b}} = \frac{-1}{3a} + \frac{2}{3\gamma}$ et $\frac{1}{c} = \frac{1}{3a} - \frac{2}{3\gamma}$; vnde constructio lentium ternarum simplicium ita se habebit.

Pro Lente

- Prima radius faciei
  - anterioris $= \frac{3a\gamma}{(3\rho - 2\sigma)\gamma + \sigma a}$
  - posterioris $= \frac{3a\gamma}{(3\sigma - 2\rho)\gamma + \rho a}$
- Secunda radius faciei
  - anterioris $= \frac{3a\gamma}{(2\rho - \sigma)\gamma + (2\sigma - \rho)a}$
  - posterioris $= \frac{3a\gamma}{(2\sigma - \rho)\gamma + (2\rho - \sigma)a}$
- Tertia radius faciei
  - anterioris $= \frac{3a\gamma}{\rho\gamma + (3\sigma - 2\rho)a}$
  - posterioris $= \frac{3a\gamma}{\sigma\gamma + (3\rho - 2\sigma)a}$

Solutio altera Problematis.

137. Consideremus binas lentes priores PP et QQ vt lentem duplicatam ad distantias determinatrices a et $\mathfrak{b}$

ita

ita instructam vt posito $\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)=\lambda^{(2)}$ spatium diffusionis inde oriundum sit

$$=\mu\mathfrak{b}\mathfrak{b}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\left(\lambda^{(2)}\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2+\tfrac{\nu}{a\mathfrak{b}}\right)$$

sed pro constructione binarum lentium simplicium, ex quibus haec lens est composita, recordandum est esse

$$\tfrac{1}{\alpha}=\tfrac{-1+f}{a}+\tfrac{f}{\mathfrak{b}} \text{ et } \tfrac{1}{b}=\tfrac{1-f}{a}-\tfrac{f}{\mathfrak{b}}$$

Adiuncta iam tertia lente RR ad distantias determinatrices $c=-\mathfrak{b}$ et γ per numerum arbitrarium λ'' instructa si ponamus $\tfrac{1}{\mathfrak{b}}=\tfrac{-1+g}{a}+\tfrac{g}{\gamma}=-\tfrac{1}{c}$, et

$$\lambda^{(2)}g^3+\lambda''(1-g)^3-\nu g(1-g)=\lambda^{(3)}$$

erit spatium diffusionis

$$Hh=\mu\gamma\gamma xx\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu}{a\gamma}\right)$$

quod vt fiat minimum, valor ipsius $\lambda^{(3)}$ minimus reddi debet, quaeratur ergo primo g dum $\lambda^{(2)}$ vt numerus datus spectatur et habebimus

$$3\lambda^{(2)}gg-3\lambda''(1-g)^2-\nu+2\nu g=0$$

vnde elicitur

$$g=\frac{-3\lambda''-\nu+\sqrt{(3\lambda^{(2)}+\nu)(3\lambda''+\nu)}}{3\lambda^{(2)}-3\lambda''} \quad \text{siue}$$

$$g=\frac{\sqrt{(3\lambda''+\nu)}}{\sqrt{(3\lambda^{(2)}+\nu)}+\sqrt{(3\lambda''+\nu)}}$$

et hinc valor ipsius $\lambda^{(3)}$ erit

$$\lambda^{(3)}=\frac{1}{3\left(\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2}-\frac{1}{3}\nu$$

vel

vel

$$\frac{1}{3\lambda^{(3)}+\nu}=\left(\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2$$

Sicque fit:

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Simili modo si f ita definiatur, vt $\lambda^{(2)}$ fiat minimum reperietur:

$$f=\frac{\sqrt{(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)}+\sqrt{(3\lambda'+\nu)}}$$

hocque valore substituto

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}$$

Quare si numeris $\lambda, \lambda', \lambda''$, arbitrio nostro relictis bini numeri f et g ita definiantur, vt $\lambda^{(3)}$ consequatur valorem minimum erit

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

vnde idem valor pro $\lambda^{(3)}$ reperitur, quem ante inuenimus.

Coroll. I.

138. Ex hac ergo solutione numeri f et g ita definiuntur vt fit

$$\frac{1}{f\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}} \text{ et}$$

$$\frac{1}{g\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

siue

fiue hoc modo

$$\left(\frac{1}{f}-1\right)\frac{1}{\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda'+\nu)}} \text{ et}$$

$$\left(\frac{1}{g}-1\right)\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}\right)=\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Coroll. 2.

139. Ex inuentis minimis valoribus numerorum $\lambda^{(2)}$ et $\lambda^{(3)}$ numeri f et g etiam ita definiuntur vt fit

$$\frac{1}{f\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}} \text{ seu } f=\sqrt{\left(\frac{3\lambda^{(\,)}+\nu}{3\lambda+\nu}\right)} \text{ et}$$

$$\frac{1}{g\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}} \text{ seu } g=\sqrt{\left(\frac{3\lambda^{(3)}+\nu}{3\lambda^{(\,)}+\nu}\right)}$$

Coroll. 3

140. Diftantiae autem determinatrices fingularum lentium ita per a et γ prodibunt expreffae:

$$\frac{1}{\alpha}=-\frac{1}{b}=\frac{-1+fg}{a}+\frac{fg}{\gamma}$$

$$\frac{1}{\mathfrak{G}}=-\frac{1}{c}=\frac{-1+g}{a}+\frac{g}{\gamma}$$

vbi cum fit

$$g=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}} \text{ notandum eft effe } fg=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda+\nu}}.$$

Coroll. 4.

141. Eliminando autem numero $\lambda^{(2)}$ erit

$$\frac{1}{\alpha}=-\frac{1}{b}=-\frac{1}{a}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda'+\nu}}-\frac{1}{a}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda''+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda+\nu}}$$

$$\frac{1}{\mathfrak{G}}=-\frac{1}{c}=-\frac{1}{a}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda''+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda'+\nu}}$$

ex quibus formulis, inuento iam valore minimo $\lambda^{(3)}$ fingulae lentes commodiffime determinantur.

Coroll. 5.

142. Si praeterea fingulae lentes ita fuerint comparatae vt per fe minimam confufionem pariant quod fit fi $\lambda = \lambda' = \lambda'' = 1$, erit

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}} = \frac{2}{\sqrt{(3+\nu)}} \text{ hincque } 3\lambda^{(2)}+\nu = \frac{3+\nu}{4}$$

vnde fit $\lambda^{(2)} = \frac{1-\nu}{4}$. Deinde vero habebitur:

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}} = \frac{3}{\sqrt{(3+\nu)}} \text{ hincque } 3\lambda^{(3)}+\nu = \frac{3+\nu}{9}$$

ac propterea

$\lambda^{(3)} = \frac{1-8\nu}{27}$. Sin autem fuerit tantum $\lambda = \lambda' = \lambda''$ reperietur:

$$\lambda^{(2)} = \frac{3\lambda - 3\nu}{12} \text{ et } \lambda^{(3)} = \frac{3\lambda - 8\nu}{27}.$$

Coroll. 6.

143. Eodem autem cafu, quo $\lambda = \lambda' = \lambda''$, ob $\sqrt{(3\lambda^{(3)}+\nu)} = \frac{1}{3}\sqrt{(3\lambda+\nu)}$, diftantiae determinatrices pro lentibus fimplicibus erunt:

$$\frac{1}{\mathfrak{a}} = \frac{-2}{3a} + \frac{1}{3\gamma}; \quad \frac{1}{\mathfrak{b}} = \frac{2}{3a} - \frac{1}{3\gamma}$$

$$\frac{1}{\mathfrak{c}} = \frac{-1}{3a} + \frac{2}{3\gamma}; \quad \frac{1}{c} = \frac{1}{3a} - \frac{2}{3\gamma}$$

vnde eaedem formulae pro earum conftructione nafcuntur quae fupra (136) funt allatae nifi quod iam in denominatoribus membra $\pm\tau(a+\gamma)\sqrt{(\lambda-1)}$ adiungi debeant. Scholi-

Scholion.

144. Ternarum ergo lentium ſimplicium idonea coniunctione effici poteſt, vt in expreſſione ſpatii diffuſionis $Hb = \mu\gamma\cdot\gamma xx(\frac{1}{a}+\frac{1}{\gamma})(\lambda^{(2)}(\frac{1}{a}+\frac{1}{\gamma})^2+\frac{\nu}{a\gamma})$ numerus $\lambda^{(3)}$ fiat $=0,042165$. In lentibus autem duplicatis vidimus minimum valorem numeri $\lambda^{(2)}$ eſſe $=0,191827$: ſicque in triplicatis hic numerus fere quinquies minor reddi poteſt at is fere vicies quater eſt minor, quam per lentes ſimplices obtineri poteſt. Loquor hic autem de lentibus, ex principio minimi petitis, quippe quae ad praxin maxime ſunt accommodatae, dum conſtructio leuibus erroribus non admodum turbatur. Quanquam enim lentes triplicatae perinde ac duplicatae parari poſſent, pro quibus numerus conueniens λ non ſolum nihilo aequalis, ſed etiam negatiuus reſultaret tamen earum conſtructio tam eſt lubrica, vt minimus error totum laborem irritum reddat. Interim tamen periculum in triplicatis non tantum eſt quam duplicatis, vnde ſequens problema ſoluiſſe operae erit pretium.

Problema 7.

145. Pro datis diſtantiis determinatricibus a et γ eas definire lentes triplicatas, pro quibus valor ipſius $\lambda^{(3)}$ prorſus in nihilum abeat.

Solutio.

Conſideretur numerus $\lambda^{(2)}$ ex duabus prioribus lentibus natus vt datus, et cum ſit ſecundum ſolutionem poſteriorem praecedentis problematis

$$\lambda^{(3)} = \lambda^{(2)} g^3 + \lambda^{II} (1-g)^3 - \nu g (1-g)$$

definiri debet g ita vt iſta quantitas euaneſcat. Verum cum $\lambda^{(2)}$ commode nequeat minor effici quam $\frac{1-\nu}{4}$, ſtatuamus $\lambda^{(2)} = \frac{1-\nu}{4}$, fierique oportet:

$$0 = \tfrac{1}{4} g^3 + \lambda^{II} (1-g)^3 - \nu g \left(1 - \tfrac{1}{2} g\right)^2$$

ſed quia λ^{II} vnitate minor eſſe nequit, neceſſe eſt vt g capiatur vnitate maior euoluantur ergo quidam caſus.

I. $g = \frac{5}{4}$; $0 = \frac{125}{4 \cdot 64} - \frac{\lambda^{II}}{64} - \frac{5 \cdot 9}{4 \cdot 64} \nu$ et $\lambda^{II} = \frac{125 - 45\nu}{4}$

II. $g = \frac{6}{4}$; $0 = \frac{216}{4 \cdot 64} - \frac{8 \lambda^{II}}{64} - \frac{6 \cdot 4}{4 \cdot 64} \nu$ et $\lambda^{II} = \frac{216 - 24\nu}{4 \cdot 8} = \frac{27 - 3\nu}{4}$

III. $g = \frac{7}{4}$; $0 = \frac{343}{4 \cdot 64} - \frac{27 \lambda^{II}}{64} - \frac{7 \cdot 1}{4 \cdot 64} \nu$ et $\lambda^{II} = \frac{343 - 7\nu}{4 \cdot 27}$

IV. $g = \frac{8}{4}$; $0 = \frac{512}{4 \cdot 64} - \frac{64 \lambda^{II}}{64}$ et $\lambda^{II} = 2$

V. $g = \frac{9}{4}$; $0 = \frac{729}{4 \cdot 64} - \frac{125 \lambda^{II}}{64} - \frac{9 \cdot 1}{4 \cdot 64} \nu$ et $\lambda^{II} = \frac{729 - 9\nu}{4 \cdot 125}$

VI. $g = \frac{10}{4}$; $0 = \frac{1000}{4 \cdot 64} - \frac{216 \lambda^{II}}{64} - \frac{10 \cdot 4}{4 \cdot 64} \nu$ et $\lambda^{II} = \frac{1000 - 40\nu}{4 \cdot 216} = \frac{125 - 5\nu}{4 \cdot 27}$

In primo et ſecundo caſu fit valor ipſius λ^{II} nimis magnus, quam vt iſta lens commode in praxin recipi queat; ac ſi ipſi g multo maior tribuatur valor, leuis error ingentem effectum producit. Ponamus enim loco g per errorem ſumi $g + \omega$, et cum λ^{II} ex g rite fuerit definitum, fiet

$$\lambda^{(3)} = \omega \left(\tfrac{3}{4} gg - 3 \lambda^{II} (1-g)^2 - \nu \left(1 - \tfrac{1}{2} g\right)\left(1 - \tfrac{3}{2} g\right) \right)$$

et

et pro λ'' ſubſtituto valore:

$$\lambda^{(3)} = \frac{\omega}{4(1-g)}(3gg - \nu(4-gg))$$

vnde patet, quo minus g vnitatem excedat, ac ſimul quo maius fuerit g, valorem ipſius $\lambda^{(3)}$ ob errorem ω eo fieri maiorem. Intelligitur autem hunc errorem fieri minimum ſi capiatur

$$g = 1 + \sqrt{\frac{3-3\nu}{\nu+3}},$$ ex hoc autem valore elicitur,

$$\lambda'' = \frac{(3+\nu)\sqrt{3(1-\nu)(3+\nu)} + 3(3+\nu\nu)}{9(1-\nu)}$$

Capi ergo debebit $g = 1,84384$ vnde colligitur:

$$\lambda'' = \frac{g^3 - \nu g(2-g)^2}{4(g-1)^3} = 2,60372$$

et ſi hae menſurae exacte obſeruentur, fiet $\lambda^{(3)} = 0$. At ſi in valore ipſius g particula ω aberretur, vt ſit $g = 1,84384 + \omega$, prodibit ob hunc errorem:

$$\lambda^{(3)} = -2,981\ \omega,$$

ita ſi eſſet error $\omega = \pm \frac{1}{10}$, loco $\lambda^{(3)} = 0$, prodiret:

$\lambda^{(3)} = \mp 0,2981$ ideoque lens triplicata poſtponenda duplicatae, longe tamen praeferenda foret ſimplici.

In genere igitur pro quouis valore alio ipſius $\lambda^{(2)}$ idem commodum inueſtigemus: ac primo cum ſit

$$\lambda'' = \frac{\lambda^{(2)} g^3 + \nu g(g-1)}{(g-1)^3}$$

ſi loco iuſti valoris g capiatur $g + \omega$ fiet

$$\lambda^{(3)} = \omega(3\lambda^{(2)} g^2 - 3\lambda''(g-1)^2 - \nu + 2\nu g)$$

vbi ſi pro λ'' valor ſubſtituatur, erit,

$$\lambda^{(3)} = \frac{\omega(\nu - (3\lambda^{(2)} + \nu)gg)}{g-1}$$

 qui

qui error vt minimus reddatur capi debet

$$g = 1 + \sqrt{\frac{3\lambda^{(2)}}{3\lambda^{(2)}+\nu}}$$

qui eſt valor maxime idoneus pro g ſumendus, ex quo elicitur

$$\lambda'' = \frac{1}{3\lambda^{(2)}}\left(\sqrt{(3\lambda^{(2)}+\nu)}+\sqrt{3\lambda^{(2)}}\right)\left(\tfrac{1}{3}(6\lambda^{(2)}+\nu)\sqrt{3\lambda^{(2)}}+(2\lambda^{(2)}+\nu)\sqrt{(3\lambda^{(2)}+\nu)}\right)$$

et ſumto per errorem $g+\omega$ pro g erit

$$\lambda^{(3)} = -2\omega\left(3\lambda^{(2)}+\nu+\sqrt{3\lambda^{(2)}(3\lambda^{(2)}+\nu)}\right) \text{ ſeu}$$

$$\lambda^{(3)} = -2\omega\left(\sqrt{3\lambda^{(2)}}+\sqrt{(3\lambda^{(2)}+\nu)}\right)\sqrt{(3\lambda^{(2)}+\nu)}$$

Quo minor igitur iam fuerit valor ipſius $\lambda^{(2)}$ eo minus erit error metuendus, vnde ſolutio ante ex valore $\lambda^{(2)} = \frac{1-\nu}{4}$ eruta prae ceteris eſt commendanda. Tum autem erit $f = \frac{1}{2}$; $\lambda = 1$, $\lambda' = 1$, $\lambda'' = 2,60372$ et $g = 1,84384$, vnde pro lentium conſtructione habemus,

$$\frac{1}{\alpha} = -\frac{1}{b} = \frac{-2+g}{2a} + \frac{g}{2\gamma}$$

$$\frac{1}{\beta} = -\frac{1}{c} = \frac{-1+g}{a} + \frac{g}{\gamma}$$

vnde ſingulae lentes per formulas §. 91 conſtruentur.

Coroll. I.

146. Lens ergo prima PP conſtrui debet ex diſtantiis determinatricibus a et $\frac{2a\gamma}{ga-(2-g)\gamma}$ cum numero $\lambda = 1$

Lens vero ſecunda QQ ex diſtantiis determinatricibus $\frac{-2a\gamma}{ga-(2-g)\gamma}$ cum numero $\lambda' = 1$

at

at lens tertia RR ex diſtantiis determinatricibus

$$\frac{-\alpha\gamma}{g\alpha-(1-g)\gamma} \text{ et } \gamma \text{ cum numero } \lambda''=2,60372$$

exiſtente $g=1,84384$.

Coroll. 2.

147. Quoniam in formula ſpatium diffuſionis exprimente quae ob $\lambda^{(3)}=0$ eſt $\mu\gamma\gamma xx(\frac{1}{\alpha}+\frac{1}{\gamma})\frac{\nu}{\alpha\gamma}$, reiecto factore primo $\mu\gamma\gamma xx$, cuius ratio in his inueſtigationibus non eſt habita, diſtantiae α et γ inter ſe permutari poſſunt; hinc etiam alia lens triplicata quaeſito aeque ſatisfaciens exhiberi poterit.

Coroll. 3.

148. Nempe pro hac altera lente triplicata lens prima PP conſtrui debet ex diſtantiis determinatricibus

$$\alpha \text{ et } \frac{+\alpha\gamma}{(1-g)\alpha-g\gamma} \text{ cum numero } \lambda=2,60372$$

Lens ſecunda QQ ex diſtantiis determinatricibus

$$\frac{-\alpha\gamma}{(1-g)\alpha-g\gamma} \text{ et } \frac{+2\alpha\gamma}{(2-g)\alpha-g\gamma} \text{ cum numero } \lambda'=1$$

at lens tertia ex diſtantiis determinatricibus

$$\frac{-2\alpha\gamma}{(2-g)\alpha-g\gamma} \text{ et } \gamma \text{ cum numero } \lambda''=1$$

exiſtente vt ante $g=1,84384$.

Coroll. 4.

149. Hinc ergo duas nacti ſumus lentes triplicatas pro diſtantiis α et γ, quae producunt ſpatium diffu-

diffuſionis $Hb = \mu\gamma\gamma xx(\frac{1}{a}+\frac{1}{\gamma})\frac{y}{a\gamma}$. Atque hae inter infinitas alias eundem effectum praeſtantes hac gaudent praerogatiua, vt leuis error in conſtructione commiſſus ſcopum minime perturbet.

Coroll. 5.

150. Si in conſtructione harum lentium per errorem numerus g parumper maior accipiatur, quam 1, 84384, tum pro lente triplicata numerus $\lambda^{(3)}$ prodit nihilo minor ſeu negatiuus. Sin autem numerus g in praxi aliquantillum maior ſumatur numerus $\lambda^{(3)}$ fit nihilo maior, ſicque lens triplicata ad naturam duplicatarum accedet.

Coroll. 6.

151. Si ergo opus fuerit lente, pro qua numerus λ valorem habeat negatiuum, huic ſcopo ſatisfieri commode poterit per lentes deſcriptas triplicatas, dummodo pro g numerus aliquanto maior quam 1, 84384 aſſumatur. Scilicet ſi ſumatur

$$g = 1,84384 + \omega \text{ fiet } \lambda^{(3)} = -2,981\,\omega.$$

Supplementum II.

De lentibus triplicatis

Si pro ſingulis lentibus refractio ſit diuerſa, pro prima $n:1$ pro ſecunda $n':1$, pro tertia n''

$n'' : 1$, radiique facierum lentium ſequenti modo definiantur

	Diſt. determinatrices	Refractio et litterae inde pendentes	
I.	a et α	$n : 1,\ \mu,\ \nu,\ \varrho,\ \sigma,\ \tau.$	λ
II.	b et β	$n' : 1,\ \mu',\ \nu',\ \varrho',\ \sigma',\ \tau'.$	λ'
III.	c et γ	$n'' : 1,\ \mu'',\ \nu'',\ \varrho'',\ \sigma'',\ \tau''.$	λ''

ſcilicet ſi pro lente prima vocetur radius faciei anterioris $=F$; poſterioris $=G$; erit

$$\frac{1}{F}=\frac{\varrho}{u}+\frac{\sigma}{\alpha}\mp\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{\lambda-1}$$

$$\text{et }\ \frac{1}{G}=\frac{\varrho}{\alpha}+\frac{\sigma}{a}\pm\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{\lambda-1}$$

ſimilique modo pro reliquis lentibus, nempe pro ſecunda

$$\frac{1}{F'}=\frac{\varrho'}{b}+\frac{\sigma'}{\beta}\mp\tau'\left(\frac{1}{b}+\frac{1}{\beta}\right)\sqrt{\lambda'-1}$$

$$\frac{1}{G'}=\frac{\varrho'}{\beta}+\frac{\sigma'}{b}\pm\tau'\left(\frac{1}{b}+\frac{1}{\beta}\right)\sqrt{\lambda'-1}$$

tum vero pro tertia

$$\frac{1}{F''}=\frac{\varrho''}{c}+\frac{{}''}{\gamma}\mp\tau''\left(\frac{1}{c}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$

$$\frac{1}{G''}=\frac{\varrho''}{\gamma}+\frac{\sigma''}{c}\pm\tau''\left(\frac{1}{c}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$

Deinde quia diſtantiae lentium pro nihilo habentur ſcilicet $\alpha+b=0$ et $\beta+c=0$, erit ſpatium diffuſionis

$$=\gamma\gamma xx\left\{\begin{aligned}&+\mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)\\&+\mu'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu'}{b\beta}\right)\\&+\mu''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu''}{c\gamma}\right)\end{aligned}\right.$$

statuatur nunc

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{c}+\frac{1}{\gamma}=h\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

vt fiat

$$\frac{1}{\alpha}=\frac{f-1}{a}+\frac{f}{\gamma}=-\frac{1}{b} \quad \text{et}$$

$$\frac{1}{c}=\frac{h}{a}+\frac{h-1}{\gamma}=-\frac{1}{\beta}$$

vnde fit

$$\frac{1}{b}+\frac{1}{\beta}=\frac{1-f-h}{a}+\frac{1-f-h}{\gamma}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

hincque $f+g+h=1$; quibus valoribus substitutis erit primo pro radiis facierum

$$\frac{1}{F}=\frac{\rho+(f-1)\sigma}{a}+\frac{\sigma f}{\gamma}\mp\tau f\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda-1}$$

$$\frac{1}{G}=\frac{\sigma+(f-1)\rho}{a}+\frac{\rho f}{\gamma}\pm\tau f\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda-1}$$

$$\frac{1}{F'}=\frac{(1-f)\rho'-h\sigma'}{a}+\frac{(1-h)\sigma'-f\rho'}{\gamma}\mp\tau' g\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda'-1}$$

$$\frac{1}{G'}=\frac{(1-f)\sigma'-h\rho'}{a}+\frac{(1-h)\rho'-f\sigma'}{\gamma}\pm\tau' g\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda'-1}$$

et pro lente tertia

$$\frac{1}{F''}=\frac{h.\rho''}{a}+\frac{(h-1)\rho''+\sigma''}{\gamma}\mp\tau''.h\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$

$$\frac{1}{G''}=\frac{h.\sigma''}{a}+\frac{(h-1)\sigma''+\rho''}{\gamma}\pm\tau''.h\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$

Spatium

Spatium vero diffusionis iam ita exprimetur

$$\gamma\gamma\, xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left\{\begin{array}{l}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2(\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' h^3)\\ +\frac{\nu\mu f}{a}\left(\frac{f-1}{a}+\frac{f}{\gamma}\right)\\ +\nu'\mu' g\left(\frac{1-f}{a}-\frac{f}{\gamma}\right)\left(\frac{1-h}{\gamma}-\frac{h}{a}\right)\\ +\frac{\nu''\mu'' h}{\gamma}\left(\frac{h}{a}+\frac{h-1}{\gamma}\right)\end{array}\right.$$

Nunc igitur circa has lentes triplicatas sequentia sunt obseruanda

I. Diuersa media refringentia eum tantum in finem adhiberi solent, vt non solum spatium diffusionis hic determinatum ad nihilum redigatur, sed etiam confusio a diuersa radiorum refrangibilitate oriunda tollatur, quippe quod per lentes eiusdem refractionis obtineri nequit. Infra autem videbimus ad hanc conditionem implendam requiri, vt sit $\zeta f+\eta g+\vartheta h=0$, existente $\zeta=\frac{dn}{n-1}$; $\eta=\frac{dn'}{n'-1}$ et $\vartheta=\frac{dn''}{n''-1}$; ex quibus formis iam perspicitur, si haec differentialia dn, dn' et dn'' essent ipsis $n-1$; $n'-1$; $n''-1$ proportionalia, vti *Newtonus* statuerat; tum proditura esse $\zeta=\eta=\vartheta$ siue $f+g+h=0$; at iam vidimus, esse debere $f+g+h=1$; quare si *Newtoni* sententia esset vera; tum ne quidem diuersis refractionibus adhibendis diffusioni a diuersa refrangibilitate oriundae remedium adferri posset. Fatenus igitur tantum hoc incommodum vitari poterit, quatenus litterae ζ, η et ϑ sunt diuersae, ita, vt simul esse possit, et $f+g+h=1$ et $\zeta f+\eta g+\vartheta h=0$ ex quo perspicuum est, quantitatum f, g et h vnam vel adeo duas esse debere negatiuas sicque hac conditione superaddita casus ille principalis, quo omnes tres litterae f, g et h positiuae sunt assumtae hic locum inuenire nequit.

II. Vt ergo nostrum spatium diffusionis euanescat, satisfaciendum est huic aequationi

$$\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2 (\mu\lambda f^3 + \mu'\lambda' g^3 + \mu''\lambda'' h^3)$$
$$+\frac{\nu\mu f}{a}\left(\frac{f-1}{a}+\frac{f}{\gamma}\right)$$
$$+\nu'\mu' g\left(\frac{1-f}{a}-\frac{f}{\gamma}\right)\left(\frac{1-h}{\gamma}-\frac{h}{a}\right)$$
$$+\frac{\nu''\mu'' h}{\gamma}\left(\frac{h}{a}+\frac{h-1}{\gamma}\right)=0$$

Vnde vel λ vel λ' vel λ'' quaeri potest, dummodo caueatur, ne valor vel vnitate minor vel nimis magnus prodeat, quia prius naturae repugnat; alterum autem, quia constructio lentium fieret nimis lubrica. Hoc autem praestito circa binas reliquas litteras λ nihil amplius definitur neque etiam circa litteras f, g et h, praeterquam quod supra allatis conditionibus $f+g+h=1$ et $\zeta f+\eta g+\vartheta h=0$ continetur, vnde ex data vna harum litterarum duae reliquae sponte definiuntur sequenti scilicet modo:

$$g=\frac{(\vartheta-\zeta)f-\vartheta}{\eta-\vartheta}; \text{ et}$$
$$h=\frac{(\zeta-\eta)f+\eta}{\eta-\vartheta}$$

III. In praxi autem huiusmodi lentes triplicatae ideo potissimum quaeruntur, vt loco lentis obie-

ctiuae

ctiuae in telescopiis substitui queant, pro quibus est $a = \infty$. Statuamus ergo statim $a = \infty$ et pro radiis faciērum singularum lentium habebimus

$$\frac{1}{F} = \frac{\sigma f}{\gamma} \mp \frac{\tau f}{\gamma} \sqrt{\lambda - 1}; \quad \frac{1}{G} = \frac{\rho f}{\gamma} \pm \frac{\tau f}{\gamma} \sqrt{\lambda - 1}.$$

pro secunda lente

$$\frac{1}{F'} = \frac{(1-b)\sigma' - f\rho'}{\gamma} \mp \frac{\tau' g}{\gamma} \sqrt{\lambda' - 1}; \quad \frac{1}{G'} = \frac{(1-b)\rho' - f\sigma'}{\gamma} \pm \frac{\tau' g}{\gamma} \sqrt{\lambda' - 1}$$

pro tertia lente

$$\frac{1}{F''} = \frac{(b-1)\rho'' + \sigma''}{\gamma} \mp \frac{\tau'' b}{\gamma} \sqrt{\lambda'' - 1}; \quad \frac{1}{G''} = \frac{(b-1)\sigma'' + \rho''}{\gamma} \pm \frac{\tau'' b}{\gamma} \sqrt{\lambda'' - 1}$$

Spatium autem diffusionis tum ita exprimetur

$$\frac{x^2}{\gamma}(\mu \lambda f^3 + \mu' \lambda' g^3 + \mu'' \lambda'' b^3 - \nu' \mu'. fg(1-b) - \nu'' \mu''. b(1-b))$$

Ita vt satisfieri oporteat huic aequationi

$$\mu \lambda f^3 + \mu' \lambda' g^3 + \mu'' \lambda'' b^3 - \nu' \mu'. fg(1-b) - \nu'' \mu''. b(1-b) = 0.$$

Vnde vnus valorum λ, λ', λ'', qui ad vsum commodissimus videtur, determinari debet.

IV. Si tantum lentibus vitreis vti velimus, sufficiet duas tantum vitri species adhiberi; si igitur statuamus, lentem tertiam et primam ex eadem vitri specie parari, vt $\mu'' = \mu$; $\nu'' = \nu$; $\rho'' = \rho$; $\sigma'' = \sigma$; $\tau'' = \tau$ et $\zeta = \vartheta$ ob $n'' = n$; pro litteris autem f, g et b hae determinationes habebuntur, 1°. ex aequatione

$$\zeta f + \eta g + \zeta b = 0 \text{ fit } f + b = \frac{-\eta}{\zeta}. g,$$

quo valore substituto fiet

$$g\frac{(\zeta - \eta)}{\zeta} = 1; \quad g = \frac{\zeta}{\zeta - \eta} \text{ ideoque } f + b = \frac{-\eta}{\zeta - \eta};$$

 vnde

vnde patet, prouti littera g fuerit vel poſitiua vel negatiua, fore vic ſſim ſummam $f+b$ vel negatiuam vel poſitiuam et aequatio reſoluenda iam erit

$$\mu(\lambda f^3+\lambda'' b^3)+\mu'\lambda' g^3-\frac{\mu'\nu'.\zeta}{\zeta-\eta}.f(1-b)-\mu\nu.b(1-b)=0$$

Hinc igitur elicitur

$$\mu'\lambda' g^3=-\mu(\lambda f^3+\lambda'' b^3)+\frac{\mu'\nu'.\zeta}{\zeta-\eta}.f(1-b)+\mu\nu.b(1-b)$$

cui facile erit pro caſu quouis propoſito ſatisfacere.

IV. Dum lens prima et tertia ex eadem vitri ſpecie parantur, media conſtet ex aqua vel alia materia fluida, vt lens triplicata intra duas lentes vitreas contineat fluidum, et quia fluidum plerumque minorem refractionem patitur, quam vitrum; erit $n' < n$ indeque porro $\eta < \zeta$. Quia igitur pro hoc caſu fit g poſitiuum ſeu lens aquea conuexa, lentes vitreae vel ambae vel vna ſaltem debent eſſe concauae.

Inprimis autem praeter determinationes iam inuentas neceſſe eſt, vt radius faciei anterioris pro lente media aequalis et contrarius ſit radio faciei poſterioris lentis primae, eodemque modo radius faciei poſterioris aequalis et contrarius radio faciei anterioris lentis tertiae vnde hae aequalitates naſcentur

$$\frac{1}{F'}=\frac{-1}{G}, \text{ ſeu } F'+G=0; \text{ et } \frac{1}{G'}=\frac{-1}{F''} \text{ ſeu } F''+G'=0.$$

Ideoque ſatisfieri oportet iſtis aequationibus:

$$(1-b)\sigma'-f\varrho'\mp\tau' g.\sqrt{\lambda'-1}=-\varrho f\mp\tau f\sqrt{\lambda-1} \text{ et}$$

$$(1-b)\varrho'-f\sigma'\pm\tau'.g.\sqrt{\lambda'-1}=(1-b)\varrho-\sigma\mp\tau'' b\sqrt{\lambda''-1}$$

En

En igitur duas conditiones, quibus satisfieri oportet, vnde vel numeri λ et λ'' vel alter eorum cum alterutra litterarum f et b definiri debent; quem in finem probe obseruandum est, formulas $\sqrt{\lambda - 1}$ et $\sqrt{\lambda'' - 1}$ pro lubitu siue positiuas siue negatiuas assumi posse, neque a se inuicem pendere. Pro formula autem $\sqrt{\lambda' - 1}$ notandum est, si ea in priore aequatione positiue ponatur, in posteriore necessario negatiue sumi debere et viciſſim.

Problema 8.

152. Determinare eas lentes quadruplicatas ad datas distantias determinatrices $AE = a$ et $dI = \delta$ accommodatas, quae minimum spatium diffusionis Ii producant. Tab. II. Fig. 9.

Solutio.

Prima lens PP ad distantias determinatrices $AE = a$ et $AF = \alpha$ cum numero λ construatur, secunda QQ ad distantias $b = -\alpha$ et $AG = \beta$ cum numero λ', tertia RR ad distantias $c = -\beta$ et $AH = \gamma$ cum numero λ'': et quarta SS ad distantias $d = -\gamma$ et $dI = \delta$ cum numero λ''' construatur: vbi scilicet crassitiem lentium vt euanescentem spectamus. Posito iam semidiametro aperturae lentis x, sola prima lens PP produceret spatium diffusionis

$$Ff = \mu\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

Adiun-

Adiuncta autem secunda lente QQ, positoque

$$\frac{1}{\alpha}=\frac{-1+f}{a}+\frac{f}{\beta} \text{ et } \frac{1}{b}=\frac{1-f}{a}-\frac{f}{\beta}$$

si breuitatis gratia statuamus

$$\lambda^{(2)}=\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)$$

vidimus fore spatium diffusionis:

$$Gg=\mu\beta\beta xx\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\lambda^{(2)}\left(\frac{1}{a}+\frac{1}{\beta}\right)^2+\frac{\nu}{a\beta}\right)$$

Adiungatur insuper tertia lens RR, numerusque g ita sumatur vt sit

$$\frac{1}{\beta}=\frac{-1+g}{a}+\frac{g}{\gamma} \text{ et } \frac{1}{c}=\frac{1-g}{a}-\frac{g}{\gamma}$$

ac si breuitatis ergo ponamus:

$$\lambda^{(3)}=\lambda^{(2)}g^3+\lambda''(1-g)^3-\nu g(1-g)$$

erit spatium diffusionis:

$$Hh=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Nunc denique adiungatur lens quarta SS, et numero h ita in calculum introducto, vt sit

$$\frac{1}{\gamma}=\frac{-1+h}{a}+\frac{h}{\delta} \text{ et } \frac{1}{d}=\frac{1-h}{a}-\frac{h}{\delta}$$

si simili modo ponamus

$$\lambda^{(4)}=\lambda^{(3)}h^3+\lambda'''(1-h)^3-\nu h(1-h)$$

erit spatium diffusionis a lente quadruplicata productum:

$$Ii=\mu\delta\delta xx\left(\frac{1}{a}+\frac{1}{\delta}\right)\left(\lambda^{(4)}\left(\frac{1}{a}+\frac{1}{\delta}\right)^2+\frac{\nu}{a\delta}\right)$$

quod igitur minimum reddi debet. Hunc in finem considerentur numeri λ, λ', λ'' et λ''' vt dati, et quaerantur

rantur

rantur idonei valores pro numeris f, g, et h: atque vt valor $\lambda^{(4)}$ minimus euadat, necesse est quoque valores $\lambda^{(3)}$ et $\lambda^{(2)}$ minimos fieri. Incipiamus ergo a valore $\lambda^{(2)}$ qui minimus redditur sumendo

$$f=\frac{\sqrt{(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)}+\sqrt{(3\lambda'+\nu)}} \text{ seu } \frac{1}{f}=1+\sqrt{\frac{3\lambda+\nu}{3\lambda'+\nu}}$$

vnde fit

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}.$$

Deinde numerus $\lambda^{(3)}$ minimum induet valorem capiendo

$$g=\frac{\sqrt{(3\lambda''+\nu)}}{\sqrt{(3\lambda^{(2)}+\nu)}+\sqrt{(3\lambda''+\nu)}} \text{ seu } \frac{1}{g}=1+\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda''+\nu}}$$

hincque colligitur

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Denique numerus $\lambda^{(4)}$ ideoque et spatium diffusionis Iz minimum efficietur sumendo

$$h=\frac{\sqrt{(3\lambda'''+\nu)}}{\sqrt{(3\lambda^{(3)}+\nu)}+\sqrt{(3\lambda'''+\nu)}} \text{ seu } \frac{1}{h}=1+\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda'''+\nu}}$$

vnde obtinetur

$$\frac{1}{\sqrt{(3\lambda^{(4)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}.$$

Quod si hic valores ante inuentos substituamus, nanciscemur

$$\frac{1}{\sqrt{(3\lambda^{(4)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}$$

Pro praecedentibus vero erit

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

et

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}.$$

Tum vero ex his porro consequimur:

$$f=\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}},\; g=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}},\; b=\sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(3)}+\nu}}.$$

Superest ergo vt constructionem singularum lentium luculentius exponamus: et earum distantias determinatrices per solas propositas a et δ exprimamus: erit igitur

$$\frac{1}{\gamma}=\frac{-1+b}{a}+\frac{b}{\delta}\;;\quad \frac{1}{d}=\frac{1-b}{a}-\frac{b}{\delta}$$

$$\frac{1}{\beta}=\frac{-1+gb}{a}+\frac{gb}{\delta}\;;\quad \frac{1}{c}=\frac{1-gb}{a}-\frac{gb}{\delta}$$

$$\frac{1}{\alpha}=\frac{-1+fgb}{a}+\frac{fgb}{\delta}\;;\quad \frac{1}{b}=\frac{1-fgb}{a}-\frac{fgb}{\delta}$$

Ex superioribus vero formulis colligitur:

$$b=\sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(3)}+\nu}};\; g=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}};\; f=\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}}$$

vnde

vnde fit

$$g\,h = \sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(2)}+\nu}} \text{ et } f g\,h = \sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda+\nu}}.$$

et superiores valores ita exprimi poterunt

$$\frac{1}{\alpha} = -\frac{1}{b} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\left(\frac{1}{\sqrt{(3\lambda^{I}+\nu)}}+\frac{1}{\sqrt{(3\lambda^{II}+\nu)}}+\frac{1}{\sqrt{(3\lambda^{III}+\nu)}}\right)+\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\cdot\frac{1}{\sqrt{(3\lambda+\nu)}}.$$

$$\frac{1}{\beta} = -\frac{1}{c} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\left(\frac{1}{\sqrt{(3\lambda^{II}+\nu)}}+\frac{1}{\sqrt{(3\lambda^{III}+\nu)}}\right)+\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda^{I}+\nu)}}\right)$$

$$\frac{1}{\gamma} = -\frac{1}{d} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\cdot\frac{1}{\sqrt{(3\lambda^{III}+\nu)}}+\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda^{I}+\nu)}}+\frac{1}{\sqrt{(3\lambda^{II}+\nu)}}\right).$$

Coroll. 1.

153. Si pro lentibus simplicibus numeri λ, λ^{I} λ^{II} et λ^{III} sumantur inter se aequales, fiet pro minimo spatio diffusionis:

$$\sqrt{(3\lambda^{(2)}+\nu)} = \tfrac{1}{2}\sqrt{(3\lambda+\nu)};\quad \lambda^{(2)} = \frac{3\lambda - 1.3\nu}{3.4}$$

$$\sqrt{(3\lambda^{(3)}+\nu)} = \tfrac{1}{3}\sqrt{(3\lambda+\nu)};\quad \lambda^{(3)} = \frac{3\lambda - 2.4\nu}{3.9}$$

$$\sqrt{(3\lambda^{(4)}+\nu)} = \tfrac{1}{4}\sqrt{(3\lambda+\nu)};\quad \lambda^{(4)} = \frac{3\lambda - 3.5\nu}{3.16}$$

hinc $f = \frac{1}{2}$; $g = \frac{2}{3}$; $h = \frac{3}{4}$, et pro constructione lentium simplicium:

$$\tfrac{1}{\alpha} = -\tfrac{3}{4a}+\tfrac{1}{4\delta};\quad \tfrac{1}{b} = +\tfrac{3}{4a}-\tfrac{1}{4\delta}$$

$$\tfrac{1}{\beta} = -\tfrac{2}{4a}+\tfrac{2}{4\delta};\quad \tfrac{1}{c} = +\tfrac{2}{4a}-\tfrac{2}{4\delta}$$

$$\tfrac{1}{\gamma} = -\tfrac{1}{4a}+\tfrac{3}{4\delta};\quad \tfrac{1}{d} = +\tfrac{1}{4a}-\tfrac{3}{4\delta}$$

 Coroll.

Coroll. 2.

154. Hinc ex §. 91 sequens quatuor lentium simplicium constructio obtinetur:

Pro lente radius faciei

Prima PP $\begin{cases} \text{anterioris} = \dfrac{4a\delta}{(4\varrho - 3\sigma)\delta + \sigma a \pm \tau(a+\delta)\surd(\lambda - 1)} \\ \text{posterioris} = \dfrac{4a\delta}{(4\sigma - 3\varrho)\delta + \varrho a \mp \tau(a+\delta)\surd(\lambda - 1)} \end{cases}$

Secunda QQ $\begin{cases} \text{anterioris} = \dfrac{4a\delta}{(3\varrho - 2\sigma)\delta + (2\sigma - \varrho)a \pm \tau(a+\delta)\surd(\lambda - 1)} \\ \text{posterioris} = \dfrac{4a\delta}{(3\sigma - 2\varrho)\delta + (2\varrho - \sigma)a \mp \tau(a+\delta)\surd(\lambda - 1)} \end{cases}$

Tertia RR $\begin{cases} \text{anterioris} = \dfrac{4a\delta}{(2\varrho - \sigma)\delta + (3\sigma - 2\varrho)a \pm \tau(a+\delta)\surd(\lambda - 1)} \\ \text{posterioris} = \dfrac{4a\delta}{(2\sigma - \varrho)\delta + (3\varrho - 2\sigma)a \pm \tau(a+\delta)\surd(\lambda - 1)} \end{cases}$

Quarta SS $\begin{cases} \text{anterioris} = \dfrac{4a\delta}{\varrho\delta + (4\sigma - 3\varrho)a \pm \tau(a+\delta)\surd(\lambda - 1)} \\ \text{posterioris} = \dfrac{4a\delta}{\sigma\delta + (4\varrho - 3\sigma)a \mp \tau(a+\delta)\surd(\lambda - 1)} \end{cases}$

Coroll. 3.

155. Si praeterea numeri λ, λ', λ'', λ''' vnitati aequales statuantur, qui est valor minimus, quem recipere possunt erit: ob $\nu = 0,232692$

$$\lambda^{(2)} = \frac{3 - 7\nu}{3 \cdot 4} = 0,191827$$

$$\lambda^{(3)} = \frac{3 - 8\nu}{3 \cdot 9} = 0,042165$$

$$\lambda^{(4)} = \frac{3 - 15\nu}{3 \cdot 16} = -0,010216$$

sicque pro lente quadruplicata valor numeri $\lambda^{(4)}$ adeo infra nihilum deprimitur.

Coroll.

Coroll. 4.

156. Maiorem ergo valorem numeris $\lambda, \lambda', \lambda'', \lambda'''$ tribuendo effici poterit, vt valor ipfius $\lambda^{(4)}$ praecife nihilo aequalis prodeat; quippe hoc fiet fumendo $\lambda = 5\nu = 1,163460$. Hinc habebitur $\lambda - 1 = 0,163460$ et $\tau\sqrt{(\lambda-1)} = 0,365947$, vnde fingulae lentes fimplices duplici modo per formulas exhibitas conftrui poterunt.

Scholion 1.

157. Si inter fe comparemus hos duos cafus, quibus eft vel $\lambda^{(4)} = -0,010216$ vel $\lambda^{(4)} = 0$, videmus, etiamfi difcrimen vix partem centefimam vnitatis fuperet, in conftructione tamen lentium fimplicium fatis magnum difcrimen deprehendi, cum denominatores formularum, (154) fiue augeri fiue diminui debeant quantitate: $0,365947\,(a+\delta)$; quae differentia maior eft, quam errores, qui forte ab artifice non nimis rudi committi queant. Ex quo viciffim colligimus, etiamfi in conftructione harum lentium quadruplicatarum ab artifice leues errores committantur, inde vix perceptibilem effectum in fpatio diffufionis vel valore ipfius $\lambda^{(4)}$ effe metuendum, quam ob caufam hae lentes imprimis ad praxin accommodatae videntur. Si fcilicet opus fuerit lente, pro qua valor ipfius λ in nihilum abeat, multo magis his lentibus quadruplicatis Coroll. 4 defcriptis erit vtendum, quam triplicatis, quas fupra definiui. Quin etiam eas lentes quadruplicatas adhibere licebit, in

in quibus eſt $\lambda^{(4)} = -0,010216$, quia hic numerus vix nihilo eſt minor; ac ſi in conſtructione a praeſcriptis menſuris aberretur, ille valor adhuc propius ad nihilum perducatur, ita vt hoc caſu adeo errores commiſſi ſcopo magis attingendo inſeruiant. Infra autem videbimus, plurimos dari caſus, quibus eiusmodi lentibus vti conueniat, pro quibus valor ipſius λ non ſolum ſit nihilo aequalis, ſed etiam tantillum infra nihilum deprimatur; tunc optimo cum ſucceſſu huiusmodi lentes quadruplicatas in vſum vocabimus. Sin autem eiusmodi lentes ſufficiant, pro quibus valor ipſius λ ſit $0,042165$ vel aliquantillum excedat, lentes triplicatae erunt commendandae, quarum conſtructionem ſupra (§. 136.) dedimus; quemadmodum duplicatae negotium conficient ſi non opus fuerit minore valore ipſius λ quam $0,191827$. Tota autem inſtrumentorum dioptricorum perfectio in hoc maxime conſiſtit, vt lentes habeantur, pro quibus valor ipſius λ ſit quam minimus, cum eae ſint aptiſſimae ad confuſionem penitus tollendam; ex quo lentium quadruplicatarum hic deſcriptarum vſus erit ampliſſimus.

Scholion 2.

158. Si haec, quae de lentibus quadruplicatis hic tradidimus, attente conſiderentur, facile patebit, quomodo lentes quintuplicatae magisque multiplicatae ad vſum ſint accommodandae. Eiusmodi ſcilicet ſemper conſtructione erit opus, quae a natura minimi parumper

rumper recedat, quoniam hoc modo errores in praxi commiſſi effectum propoſitum minime perturbant. Cum autem vix vnquam vſu veniat, vt lentibus opus ſit, pro quibus valor numeri λ magis vltra nihilum imminuatur, ſuperfluum foret, conſtructionem lentium quintuplicatarum magisue multiplicatarum vlterius proſequi. Interim tamen iuuabit, ſi pro huiusmodi lentibus numeri λ lentium numero conuenienter ſignis $\lambda^{(5)}$ $^{(6)}$ etc. indicentur, eorum valores, quos ex natura minimi recipiunt, expoſuiſſe. Sumamus omnes numeros λ, λ^{I}, λ^{II}, λ^{III}, λ^{IIII}, λ^{IIIII} etc. lentibus ſimplicibus reſpondentes vnitati aequales, et eorum qui lentibus multiplicatis conueniunt ex natura minimi ita ſe habebunt.

Pro Lente

Solitaria . . . $\lambda^{(1)} = \frac{3 - 0\nu}{3 \cdot 1} = 1{,}000000$

Duplicata . . . $\lambda^{(2)} = \frac{3 - 3\nu}{3 \cdot 4} = 0{,}191827$

Triplicata . . . $\lambda^{(3)} = \frac{3 - 8\nu}{3 \cdot 9} = 0{,}042165$

Quadruplicata $\lambda^{(4)} = \frac{3 - 15\nu}{3 \cdot 16} = -0{,}010216$

Quintuplicata $\lambda^{(5)} = \frac{3 - 24\nu}{3 \cdot 25} = -0{,}034461$

Sextuplicata $\lambda^{(6)} = \frac{3 - 35\nu}{3 \cdot 36} = -0{,}047632$

Septuplicata $\lambda^{(7)} = \frac{3 - 48\nu}{3 \cdot 49} = -0{,}055573$

Octuplicata $\lambda^{(8)} = \frac{3 - 63\nu}{3 \cdot 64} = -0{,}060727$

Noncuplicata $\lambda^{(9)} = \frac{3 - 80\nu}{3 \cdot 81} = -0{,}064261$

Decuplicata $\lambda^{(10)} = \frac{3 - 99\nu}{3 \cdot 100} = -0{,}066788$

ac

ac ſi huiusmodi lentes in infinitum multiplicentur, valor numeri reſpondentis $\lambda^{(\infty)}$ erit $= -\frac{\nu}{3} = -0,077564$ ita vt nunquam infra hunc numerum deprimi poſſit: ex quo patet vix vnquam caſum exiſtere poſſe, quo lente ſaltem quintuplicata opus eſſet. Ceterum etiam in genere, quicunque alii valores praeter vnitatem, numeris λ, λ', λ'', λ''' etc. tribuantur, ex formulis ſuperioribus numeri $\lambda^{(5)}$, $\lambda^{(6)}$ etc. facile deriuabuntur; quin etiam diſtantiae determinatrices ſingularum lentium ſimplicium indidem ſine difficultate definientur, cum lex progreſſionis ſatis ſit manifeſta. Verum per totum hoc caput probe tenendum eſt, vbique craſſitiem lentium tanquam euaneſcentem eſſe conſideratam, in ſequente autem capite iterum lentes in genere non neglecta craſſitie ſumus contemplaturi.

CAPVT