CAPVT II.

DE DIFFVSIONE IMAGINIS PER PLVRES LENTES REPRAESENTATAE.

Problema I.

68.

Tab. I. Fig. 4. Si loco obiecti adsit imago per spatium Ee diffusa, indeque radii per lentem PP aperturae indefinitae transmittantur, determinare spatium diffusionis Ff per hanc lentem productum.

Solutio.

Loco obiecti veri hic consideramus imaginem iam per aliam lentem repraesentatam, quae sit diffusa per spatium Ee, ita vt in E sit imago principalis per radios axi proximos formata in e autem imago extrema, radiis scilicet per marginem aperturae lentis praecedentis transmissis formata; qui radii cum axe constituant angulum $= \Phi$. Quare a puncto E nonnisi radii axi proximi in lentem PP emittuntur, a puncto e autem eiusmodi tantum radii, qui ad axem eA sub angulo $MeA = \Phi$ sint inclinati. Ponatur iam

iam diftantia $EA = a$, prae qua fpatium diffufionis Ee vt valde paruum fpectetur, lens autem PP fit eiusmodi vt obiecti in E exiftentis imaginem principalem referat in diftantia $BF = \alpha$, exiftente lentis craffitie $AB = d$. Hanc ob rem fi ponatur faciei anterioris AM radius $= f$, pofterioris $BN = g$, lente vt conuexa vtrinque fpectata oportet effe:

$$f = \frac{(n-1)a(k+d)}{k+d+2na} \text{ et } g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

vbi eft $n = \frac{31}{20}$ et k quantitas arbitraria. Hinc autem vti demonftrauimus, pro aperturae femidiametro $= x$ nafceretur fpatium diffufionis

$$\frac{n\alpha\alpha xx}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2 \end{array} \right\}$$

pro quo fcribamus breuitatis gratia $P\alpha\alpha xx$. Iam puncti E, quia inde nonnifi radii axi proximi ad lentem emittuntur imago exhibebitur in F, vt fit diftantia $BF = \alpha$: videamus ergo quorfum imago puncti e debeat cadere. Ac fi radii ex puncto e emisfi effent axi proximi, quia id a lente magis eft remotum quam E, eius imago propius ad lentem caderet puta in Φ, vt effet, ficut §. 60 definiuimus

$$F\Phi = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 \cdot Ee.$$

in Φ fcilicet caderet imago principalis, fi obiectum effet in e. Sed quia ab e tantum radii eM ad axem angulo $AeM = \Phi$ inclinati emittuntur, hi lenti in

 punctis

punctis M ab A interuallo $AO = e A . \Phi = a\Phi$ remotis occurrent, quandoquidem interuallum Ee prae distantia $Ae = a$ contemnimus, perinde igitur est ac si lenti apertura tribueretur, cuius semidiameter esset $= a\Phi$, et obiecti in e existentis imago extrema definiri deberet, quae cadat in f ita vt Φf sit spatium diffusionis obiecto in e existenti, et aperturae lentis, cuius semidiameter $= a\Phi$, conueniens. Hinc ergo erit interuallum $\Phi f = P a a \alpha \alpha \Phi\Phi$; et quia puncti E imago in F, puncti e autem imago in f exhibetur, erit spatium diffusionis per lentem PP productum

$$Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 . Ee + P a a \alpha\alpha \Phi\Phi.$$

In imagine autem extrema f radii Nf ita cum axe concurrent, vt sit

$$\text{angulus } BfN = \frac{k-d}{k+d} \cdot \frac{a\Phi}{\alpha}.$$

Coroll. 1.

69. Si ergo imago iam diffusa per spatium Ee locum obiecti teneat respectu lentis PP, per hanc spatium diffusionis nouum producitur Ff, ita vt imago principalis cadat in F extrema vero in f: fierique poterit, vt hoc nouum spatium diffusionis Ff maius sit vel minus proposito Ee.

Coroll. 2.

70. Aperturam lentis PP vt indefinitam assumsi, manifestum autem est sufficere, dum eius semidiameter non sit minor quam $a\Phi$. Si enim esset minor

minor, radii ex puncto e emissi plane non per lentem transitum inuenirent, neque imago puncti e exprimeretur, sed imaginis Ff punctum extremum responderet imagini cuipiam intermediae spatii Ee.

Coroll. 3.

71. Si diameter obiecti seu potius imaginis in E sitae sit $= z$, tum imaginis inde per lentem PP in F formatae diameter erit $= \frac{\alpha(k+d)}{a(k-d)} z$ quae expressio si sit positiua simul declarat situm imaginis in F esse inuersum respectu eius, quae est in E.

Scholion.

72. Cum in hoc capite plures lentes sim consideraturus, pro cuiuslibet determinatione, spectandae sunt primo binae distantiae determinatrices, altera obiecti seu imaginis, a qua lens radios accipit ante lentem, altera imaginis inde per lentem repraesentatae post lentem: quae distantiae ex imaginibus principalibus aestimentur. Deinde cuiusque lentis crassities in computum est ducenda. Tertio cum his lens nondum prorsus determinetur, insuper pro quaque lente accedit distantia quaedam arbitraria hactenus per litteram k indicata. Quarto vero imprimis ratio haberi deberet, spatii diffusionis, quod cuique lenti pro data apertura conueniat. Quemadmodum autem ex binis distantiis determinatricibus, crassitie lentis et quantitate illa arbitraria k, cum binae lentis facies tum

 etiam

etiam ſpatium diffuſionis pro apertura, cuius ſemidiameter ponitur $= x$ definiatur, in praecedente capite fuſius eſt expoſitum. Hic igitur cum plures lentes ſint conſiderandae, pro ſingulis haec elementa ſequentibus litteris indicabo.

Pro Lente	Diſtantiae determinatrices obiecti	Diſtantiae determinatrices imaginis	Craſſities lentis	Quantitas arbitraria	Spatium diffuſionis pro aperturae ſemidiametro x.
Prima	a	α	v	k	$P\alpha\alpha xx$
Secunda	b	β	v'	k'	$Q\beta\beta xx$
Tertia	c	γ	v''	k''	$R\gamma\gamma xx$
Quarta	d	δ	v'''	k'''	$S\delta\delta xx$
Quinta	e	ε	v''''	k''''	$T\varepsilon\varepsilon xx$

Sin autem ratio refractionis in ſingulis lentibus diſcrepet, pro prima lente eam ponamus $= n$; pro ſecunda $= n'$; pro tertia $= n''$ etc.

Prima autem lens hic mihi perpetuo erit ea, quae obiecto eſt proxima indeque recedendo reliquas lentes ordine numero: ex quo ſimul habebuntur diſtantiae lentium, ſcilicet ſecundae a prima $= \alpha + b$, tertiae a ſecunda $= \beta + c$; quartae a tertia $= \gamma + d$; quintae a quarta $= \delta + e$ etc. quae diſtantiae ex ſua natura debent eſſe poſitiuae etiamſi ſingulae diſtantiae determinatrices praeter primam a, quippe quae ad ipſum obiectum neceſſario ante lentem primam conſtituendum refertur, ſint quandoque negatiuae. Craſſitiem lentium hic

hic littera v designo, quia littera d inter distantias determinatrices, si numerus lentium vltra ternarium assurgat, reperitur. Pro crassitie autem et quantitate arbitraria iisdem litteris vtor, commatibus inscriptis distinguendis ob penuriam litterarum diuersarum. Ceterum obseruandum est me omnes lentes tanquam super communi axe dispositas assumere.

Problema 2.

73. Si radii ab obiecto $E\varepsilon$ emissi per duas lentes PP et QQ transmittantur definire spatium diffusionis Gg, a data apertura primae lentis oriundum; vt et magnitudinem imaginis principalis in G exhibitae. Tab. I. Fig. 5.

Solutio.

Sit obiecti magnitudo $E\varepsilon = z$, eiusque imago principalis per primam lentem PP proiiciatur in $F\zeta$, at per ambas in $G\eta$ erunt distantiae determinatrices
pro lente prima PP, obiecti $EA = a$, imaginis $aF = \alpha$
pro lente secunda QQ, obiecti $FB = b$, imaginis $bG = \beta$
Deinde sit
pro lente PP, crassities $Aa = v$, quantitas arbitr: $= k$
pro lente QQ, crassities $Bb = v'$, quantitas arbitr: $= k'$
Denique pro apertura in anteriori facie vtriusque lentis cuius semidiameter sit $= x$

spatium diffusionis primae lentis $PP = P\alpha\alpha xx$
- - - - secundae lentis $QQ = Q\beta\beta xx$.

His positis pro imagine per primam lentem proiecta $F\zeta$ erit eius magnitudo $F\zeta = \frac{\alpha(k+v)}{a(k-v)}z$ pro inuersa habenda, si haec expressio fuerit positiua. Deinde si lentis primae PP semidiameter aperturae in anteriori facie ponatur $= x$, erit spatium diffusionis $Ff = P\alpha\alpha xx$ et inclinatio radiorum in f ad axem $= \frac{k-v}{k+v} \cdot \frac{x}{\alpha}$. tum vero in facie lentis PP posteriori semidiameter aperturae non minor esse debet quam $\frac{k-v}{k+v}x$. Tota iam haec imago diffusa per spatium Ff respectu alterius lentis QQ tanquam obiectum spectari debet, cuius proinde repraesentatio Gg per propositionem praecedentem determinabitur. Erit autem hic $\Phi = \frac{k-v}{k+v} \cdot \frac{x}{\alpha}$, et spatium ibi expressum $Ee = P\alpha\alpha xx$; tum vero pro a, α, k, d et P hic scribi oportet b, β, k', v' et Q, vnde fiet spatium diffusionis quaesitum:

$$Gg = \frac{\beta\beta}{bb}\left(\frac{k'+v'}{k'-v'}\right)^2 P\alpha\alpha xx + \frac{bb\beta\beta}{\alpha\alpha}\left(\frac{k-v}{k+v}\right)^2 Qxx$$

siue

$$Gg = \frac{\beta\beta}{bb}\left(\frac{k'+v'}{k'-v'}\right)^2 P\alpha\alpha xx + \frac{bb}{\alpha\alpha}\left(\frac{k-v}{k+v}\right)^2 Q\beta\beta xx$$

Radiorum porro in g incidentium inclinatio ad axem est $\left(\frac{k-v}{k+v}\right)\left(\frac{k'-v'}{k'+v'}\right)\frac{bx}{\alpha\beta}$. Denique cum sit $F\zeta = \frac{\alpha(k+v)}{a(k-v)}z$ erit magnitudo imaginis in G vt erutae consideratae:

$$G\eta = \frac{\alpha\beta}{ab}\left(\frac{k+v}{k-v}\right)\left(\frac{k'+v'}{k'-v'}\right)z.$$

Coroll. I.

74. Quod ad aperturam facierum lentis QQ attinet, ea maior esse debet spatio transitus radiorum; hinc ergo erit semidiameter aperturae

faciei

faciei anterioris $> \left(\frac{k-v}{k+v}\right)\frac{bx}{\alpha}$

faciei posterioris $> \left(\frac{k-v}{k+v}\right)\left(\frac{k'-v'}{k'+v'}\right)\frac{bx}{\alpha}$.

Coroll. 2.

75. Si ponatur pro lente prima PP' radius faciei anterioris $= f$, et posterioris $= g$ erit posito $n = \frac{31}{20}$.

$$f = \frac{(n-1)a(k+v)}{k+v+2na} \text{ et } g = \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$$

Similique modo si pro lente altera QQ radius faciei anterioris ponatur $= f'$, et posterioris $= g'$ erit

$$f' = \frac{(n-1)b(k'+v')}{k'+v'+2nb} \text{ et } g' = \frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$$

omnibus scilicet faciebus vt conuexis consideratis.

Coroll. 3.

76. Pro spatio autem diffusionis ab vtraque lente productae erit quemadmodum inuenimus:

$$P = \frac{n}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k+v}{k-v}\right)^2\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2 \\ +\left(\frac{k-v}{k+v}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2 \end{array} \right\}$$

similique modo

$$Q = \frac{n}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k'+v'}{k'-v'}\right)^2\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2 \\ +\left(\frac{k'-v'}{k'+v'}\right)^2\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2 \end{array} \right\}$$

Non opus est vt omnibus lentibus eadem refractionis ratio $n:1$ tribuatur, sed simili modo pro pluribus lentibus poni potest n, n', n'', n''', etc. vt iam supra monuimus vnde nullum aliud discrimen nascitur, nisi

nisi quod in formulis pro f' et g' inuentis loco n scribi debeat n', et Q statui debeat.

$$=\frac{n'}{2(n'-1)^2}\left\{\begin{matrix}+\left(\frac{k'+v'}{k'-v'}\right)^2\left(\frac{n'}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2\\+\left(\frac{k'-v'}{k'+v'}\right)^2\left(\frac{n'}{\mathfrak{b}}-\frac{2}{k'-v'}\right)\left(\frac{1}{\mathfrak{b}}-\frac{2}{k'-v'}\right)^2\end{matrix}\right\}$$

Coroll. 4.

77. Sin autem crassities lentium euanescat, et pro quantitate arbitraria k, k' introducamus numerum λ, λ', erit

pro lente PP

$$f=\frac{a\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\surd(\lambda-1)};\ g=\frac{a\alpha}{\varrho a+\sigma\alpha\mp\tau(a+\alpha)\surd(\lambda-1)}$$

et

$$P=\mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

at pro lente QQ

$$f'=\frac{b\mathfrak{b}}{\varrho\mathfrak{b}+\sigma b\pm\tau(b+\mathfrak{b})\surd(\lambda'-1)};\ g'=\frac{b\mathfrak{b}}{\varrho b+\sigma\mathfrak{b}\mp\tau(b+\mathfrak{b})\surd(\lambda'-1)}$$

$$Q=\mu\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu}{b\mathfrak{b}}\right).$$

In expressionibus autem inuentis formulae $\left(\frac{k-v}{k+v}\right)$ et $\left(\frac{k'-v'}{k'+v'}\right)$ abeunt in vnitatem.

Numerorum vero $\mu, \nu, \varrho, \sigma, \tau$ indoles in §. 55 exposita est.

Si refractio lentium differat etiam litterae $\mu, \nu, \varrho, \sigma, \tau$ diuersos valores pro singulis lentibus sortientur, quae litte-

litterae ſi etiam commatibus a prioribus diſtinguantur, vt ſit .

$$\varrho' = \frac{1}{2(n'-1)} + \frac{1}{n'+2} - 1$$

$$\sigma' = 1 + \frac{1}{2(n'-1)} - \frac{1}{n'+2}$$

$$\tau' = \frac{1}{3}\left(\frac{1}{2(n'-1)} + \frac{1}{n'+2}\right)\sqrt{(4n'-1)}$$

pro ſecunda lente erit

$$f' = \frac{b\beta}{\varrho'\beta + \sigma' b \pm \tau(b+\beta)\sqrt{(\lambda'-1)}}$$

$$g' = \frac{b\beta}{\varrho' b + \sigma'\beta \mp \tau(b+\beta)\sqrt{(\lambda'-1)}}$$

et ob eandem rationem erit

$$\mu' = \frac{1}{4(n'+2)} + \frac{1}{4(n'-1)} + \frac{1}{8(n'+1)^2}$$

$$\text{et } \nu' = \frac{4(n'-1)^2}{4n'-1}$$

quod et de ſequentibus lentibus eſt intelligendum, ſi forte diuerſa refractionis lege fuerint praeditae.

Scholion.

78. Quo formulas in problemate inuentas magis contrahamus, vt cum ad plures lentes proceſſerimus, ſuccinctiores euadant, ponamus

$$\frac{k-v}{k+v} = i \text{ et } \frac{k'-v'}{k'+v'} = i'$$

ita vt hi numeri i et i' abeant in vnitatem euaneſcente lentium craſſitie v et v'. Tum autem erit ſpatium diffuſionis

$$Gg = \frac{1}{i'i'} \cdot \frac{\beta\beta}{bb} \cdot P\alpha\alpha xx + ii \cdot \frac{bb}{\alpha\alpha} \cdot Q\beta\beta xx$$

et radiorum in g incidentium inclinatio ad axem $= i i' . \frac{bx}{\alpha\beta}$: porro magnitudo imaginis $G\eta = \frac{1}{i i'} . \frac{\alpha\beta}{ab} z$
Ac pro apertura ſingularum facierum erit vt ſequitur

Semidiam. aperturae lentis	faciei anterioris	faciei poſterioris
primae PP	x	$i x$
ſecundae QQ	$i . \frac{bx}{\alpha}$	$i i' . \frac{bx}{\alpha}$

aperturae autem hae praeter primam maiores eſſe debent aſſignatis: valores enim aſſignati ſufficerent, ſi obiectum eſſet merum punctum in axe poſitum; quando autem habet magnitudinem, radii ab eius terminis per primam faciem ingreſſi latius diuagantur, et ad ſequentes facies maiorem aperturam exigunt.

Problema 3.

Tab. II. Fig. 6. 79. Si radii ab obiecto $E\varepsilon$ emiſſi per tres lentes PP, QQ et RR refringantur definire ſpatium diffuſionis Hh ob datam aperturam lentis primae oriundum, vt et magnitudinem imaginis principalis in H exhibitae.

Solutio.

Poſita obiecti magnitudine $E\varepsilon = z$, cadat eius imago principalis per lentem primam PP in $F\zeta$, dehinc per ſecundam QQ in $G\eta$, tum vero per tertiam RR in $H\theta$. Erunt ergo diſtantiae determinatrices.

pro lente PP, obiecti $EA = a$, imaginis $aF = \alpha$
pro lente QQ, obiecti $FB = b$, imaginis $bG = \beta$
pro lente RR, obiecti $GC = c$; imaginis $cH = \gamma$

Deinde

Deinde fit

pro lente PP, craffities $Aa = v$, quantitas arbitr: $= k$
pro lente QQ, craffities $Bb = v'$, quantitas arbitr: $= k'$
pro lente RR craffities $Cc = v''$, quantitas arbitr: $= k''$
ac ponatur breuitatis gratia:

$$\frac{k-v}{k+v} = i;\ \frac{k'-v'}{k'+v'} = i';\ \frac{k''-v''}{k''+v''} = i''$$

Denique pro quauis lente, fi effet fingularis, eiusque aperturae femidiameter effet $= x$, fit

fpatium diffufionis primae lentis $PP = P\alpha\alpha xx$
– – – – fecundae lentis $QQ = Q\beta\beta xx$
– – – – tertiae lentis $RR = R\gamma\gamma xx$

Iam in praecedente problemate inuenimus fore fpatium diffufionis per duas lentes priores productum

$$Gg = \frac{1}{i'i'}\cdot\frac{\beta\beta}{bb}P\alpha\alpha xx + ii.\frac{bb}{\alpha\alpha}Q\beta\beta xx$$

et radiorum in g concurrentium inclinationem ad axem $= ii'.\frac{bx}{\alpha\beta}$ imaginisque in G magnitudinem $G\eta = \frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}z$.
Haec igitur fi ad problema 1. transferantur, ibi loco

$$Ee,\ \frac{\alpha\alpha}{aa},\ \left(\frac{k+d}{k-d}\right)^2,\ Paa\alpha\alpha \text{ et } \Phi \text{ fcribi debebit}$$

$$Gg;\ \frac{\gamma\gamma}{cc};\ \frac{1}{i''i''},\ Rcc\gamma\gamma \text{ et } ii'.\frac{bx}{\alpha\beta},$$

hincque fpatium diffufionis per tres lentes productum orietur.

$$Hb = \frac{1}{i'i'.i''i''}\cdot\frac{\beta\beta\gamma\gamma}{bb\ cc}P\alpha\alpha xx + \frac{ii}{i''i''}\cdot\frac{bb\gamma\gamma}{\alpha\alpha cc}\cdot Q\beta\beta xx$$
$$+ ii.\ i'i'.\ \frac{bbcc}{\alpha\alpha\beta\beta}\cdot R\gamma\gamma xx$$

at radiorum in b concurrentium inclinatio ad axem

 erit

erit $= i i' i''. \frac{bcx}{\alpha\beta\gamma}$. Denique imaginis in H formatae magnitudo erit $H\theta = \frac{1}{i i' i''}. \frac{\alpha\beta\gamma}{abc} z$ ad ſitum inuerſum relata.

Coroll. 1.

80. Quod ad aperturas ſingularum facierum attinet, eas ſequenti modo comparatas eſſe conuenit.

Semid. aperturae Lentis	faciei anterioris	faciei poſterioris
Primae PP	x	$i x$
Secundae QQ	$i. \frac{bx}{\alpha}$	$i i'. \frac{bx}{\alpha}$
Tertiae RR	$i i'. \frac{bcx}{\alpha\beta}$	$i i' i''. \frac{bcx}{\alpha\beta}$

his ſcilicet valoribus non debent eſſe minores.

Coroll. 2.

81. Si inclinatio radiorum in b concurrentium ad axem vocetur $= \Phi$. Cum ſit $\Phi = i i' i''. \frac{bcx}{\alpha\beta\gamma}$ et $H\theta = \frac{1}{i i' i''}. \frac{\alpha\beta\gamma}{abc} z$ erit $\Phi. H\theta = \frac{xz}{a}$: quae proprietas pro lentium numero quantumuis magno locum habet.

Coroll. 3.

82. Quemadmodum radii ſingularum facierum determinandi ſint, ex praecedentibus facile liquet:

Erit nempe pro	Radius faciei anterioris.	Radius faciei poſterioris:
Lente prima PP	$\frac{(n-1)a(k+v)}{k+v+2n.a}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Lente ſecunda QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2n.b}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Lente tertia RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2n.c}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$

exiſtente pro vitro $n = \frac{31}{20}$.

Coroll.

Coroll. 4.

83. Tum vero valores literarum P, Q, R ita ſe habebunt

$$P = \frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2 + ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right)$$

$$Q = \frac{n}{2(n-)^2}\left(\frac{1}{i'i'}\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2 + i'i'\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2\right)$$

$$R = \frac{n}{2(n-1)^2}\left(\frac{1}{i''i''}\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{c}+\frac{2}{k''+v''}\right)^2 + i''i''\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{1}{\gamma}-\frac{2}{k''-v''}\right)^2\right)$$

quibus valoribus ſpatia diffuſionis definiuntur.

Si refractio differat:

litteris	tribuatur refr.
P	n
Q	n'
R	n''

Coroll. 5.

84. Iuuabit etiam ſpatia diffuſionis, prout per vnam, duas ac tres lentes producuntur, inter ſe comparaſſe, quod ita commodiſſime fieri videtur.

$$Ff = \alpha\alpha xx.\,P$$

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}.\frac{\alpha\alpha}{bb}P + ii.\frac{bb}{\alpha\alpha}Q\right)$$

$$Hb = \gamma\gamma xx\left(\frac{1}{i'i'.i''i''}.\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{ii}{i''i''}.\frac{bb\beta\beta}{\alpha\alpha cc}Q + ii.i'i'.\frac{bbcc}{\alpha\alpha\beta\beta}R\right)$$

Scholion.

85. Hinc facile perſpicitur, quomodo determinatio ſpatii diffuſionis, ad plures lentes extendi debeat;

vnde problema generale statim ad numerum lentium quemcunque accommodabo, idque pro quacunque lentium crassitie.

Problema 4.

86. Si radii ab obiecto $E\varepsilon$ emissi per lentes quotcunque PP, QQ, RR, SS etc. super communi axe dispositas refringantur, definire spatium diffusionis a data apertura lentis primae oriundum vt et magnitudinem imaginis repraesentatae.

Solutio.

Posita obiecti magnitudine $E\varepsilon = z$ imagines eius principales contemplemur: cadat igitur eius imago principalis per lentem primam PP in $F\zeta$ deinde per secundam QQ in $G\eta$, tum per tertiam RR in $H\theta$, porro per quartam SS in $I\iota$, per quintam TT in $K\kappa$ etc. Hinc pro singulis lentibus habebimus distantias determinatrices quas ita litteris indicemus.

crassitiem

Pro lente PP obiecti $EA = a$, imaginis $aF = \alpha$; $Aa = v$

Pro lente QQ obiecti $FB = b$; imaginis $bG = \beta$; $Bb = v'$

Pro lente RR obiecti $GC = c$; imaginis $cH = \gamma$; $Cc = v''$

Pro lente SS obiecti $HD = d$; imaginis $dI = \delta$; $Dd = v'''$

Pro lente TT obiecti $IE = e$; imaginis $eK = \varepsilon$; $Ee = v''''$

etc.

Deinde

Deinde cum determinatio cuiusuis lentis non solum has distantias determinatrices cum crassitie cuiusque sed etiam quantitatem quandam arbitrariam inuoluat, a qua spatium diffusionis cuiusque pendet, ponamus si quaelibet lens esset solitaria eiusque aperturae semidiameter $= x$.

Pro lente	quant: arbitr:	Spatium Diffusionis
Prima PP	k	$P\alpha\alpha xx$
Secunda QQ	k'	$Q\beta\beta xx$
Tertia RR	k''	$R\gamma\gamma xx$
Quarta SS	k'''	$S\delta\delta xx$
Quinta TT	k''''	$T\varepsilon\varepsilon xx$
	etc.	

Hinc ergo constructio cuiusque lentis ita se habebit posito $n = \frac{31}{20}$: vel si refractio differat, cuique lenti suus tribuatur valor primae n, secundae n', tertiae n'' etc.

Erit nempe pro	Radius faciei anterioris	Radius faciei posterioris
Lente prima PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Lente secunda QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Lente tertia RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$
Lente quarta SS	$\frac{(n-1)d(k'''+v''')}{k'''+v'''+2nd}$	$\frac{(n-1)\delta(k'''-v''')}{k'''-v'''-2n\delta}$
Lente quinta TT	$\frac{(n-1)e(k''''+v'''')}{k''''+v''''+2ne}$	$\frac{(n-1)\varepsilon(k''''-v'''')}{k''''-v''''-2n\varepsilon}$
	etc.	

at

at ſi ponamus breuitatis gratia:

$$\frac{k-v}{k+v}=i;\ \frac{k'-v'}{k'+v'}=i';\ \frac{k''-v''}{k''+v''}=i'';\ \frac{k'''-v'''}{k'''+v'''}=i''';\ \frac{k''''-v''''}{k''''+v''''}=i''''\ \text{etc.}$$

pro ſpatiis diffuſionis habebimus hos valores:

$$P=\frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2+ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right)$$

$$Q=\frac{n}{2(n-1)^2}\left(\frac{1}{i'i'}\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2+i'i'\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2\right)$$

$$R=\frac{n}{2(n-1)^2}\left(\frac{1}{i''i''}\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{c}+\frac{2}{k''+v''}\right)^2+i''i''\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{1}{\gamma}-\frac{2}{k''-v''}\right)^2\right)$$

$$S=\frac{n}{2(n-1)^2}\left(\frac{1}{i'''i'''}\left(\frac{n}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{d}+\frac{2}{k'''+v'''}\right)^2+i'''i'''\left(\frac{n}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{1}{\delta}-\frac{2}{k'''-v'''}\right)^2\right)$$

$$T=\frac{n}{2(n-1)^2}\left(\frac{1}{i''''i''''}\left(\frac{n}{e}+\frac{2}{k''''+v''''}\right)\left(\frac{1}{e}+\frac{2}{k''''+v''''}\right)^2+i''''i''''\left(\frac{n}{\varepsilon}-\frac{2}{k''''-v''''}\right)\left(\frac{1}{\varepsilon}-\frac{2}{k''''-v''''}\right)^2\right)$$

etc.

His conſtitutis pro magnitudine ſingularum imaginum habebimus:

ad ſitum

pro vna lente imaginem $F\zeta=\frac{1}{i}\cdot\frac{\alpha}{a}z$ inuerſum

pro duabus lentibus .... $G\eta=\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}z$ erectum

pro tribus lentibus .... $H\theta=\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}z$ inuerſum

pro quatuor lentibus .... $I\iota=\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta}{abcd}z$ erectum

pro quinque lentibus .... $K\varkappa=\frac{1}{ii'i''i'''i''''}\cdot\frac{\alpha\beta\gamma\delta\varepsilon}{abcde}z$ inuerſum

etc.

Deni-

Denique dum apertürae lentium non ſint minores, quam ſequentes formulae exhibent:

Semid. aperturae Lentis	faciei anterioris	faciei poſterioris
primae PP	x	ix
ſecundae QQ	$i.\frac{bx}{\alpha}$	$ii'.\frac{bx}{\alpha}$
tertiae RR	$ii'.\frac{bcx}{\alpha\beta}$	$ii'i''.\frac{bcx}{\alpha\beta}$
quartae SS	$ii'i''.\frac{bcdx}{\alpha\beta\gamma}$	$ii'i''i'''.\frac{bcdx}{\alpha\beta\gamma}$
quintae TT	$ii'i''i'''.\frac{bcdex}{\alpha\beta\gamma\delta}$	$ii'i''i'''i''''.\frac{bcdex}{\alpha\beta\gamma\delta}$

etc.

erit vt ſequitur pro quolibet lentium numero:

I. Pro vna lente

Spatium diffuſionis $Ff = \alpha\alpha xx.\ P$

inclinatio radiorum in f concurrentium ad axem $= i.\frac{x}{\alpha}$

II. Pro duabus lentibus

Spatium diffuſionis

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P + ii.\frac{bb}{\alpha\alpha}Q\right)$$

et radiorum in g concurrentium

inclinatio ad axem $= ii'.\frac{bx}{\alpha\beta}$

III. Pro tribus lentibus

Spatium diffuſionis:

$$Hh = \gamma\gamma xx\left(\frac{1}{i'i'.i''i''}\cdot\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{ii}{i''i''}\cdot\frac{bb\beta\beta}{\alpha\alpha cc}\cdot Q + ii.i'i'.\frac{bbcc}{\alpha\alpha\beta\beta}R\right)$$

et radiorum in h concurrentium

inclinatio ad axem $= ii'i''.\frac{bcx}{\alpha\beta\gamma}$.

IV. Pro quatuor lentibus.

Spatium diffusionis:

$$Ii = \delta\delta xx \left\{ \begin{array}{l} \frac{1}{i'i'.i''i''.i'''i'''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd} P + \frac{ii}{i''i''.i'''i'''} \cdot \frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd} Q \\ + \frac{ii.i'i'}{i'''i'''} \cdot \frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd} R + ii.i'i'.i''i'' \cdot \frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma} S \end{array} \right\}$$

et radiorum in i concurrentium

inclinatio ad axem $= ii'i''i''' \cdot \frac{bcdx}{\alpha\beta\gamma\delta}$.

V. Pro quinque lentibus.

Spatium diffusionis:

$$Kk = \varepsilon\varepsilon xx \left\{ \begin{array}{l} + \frac{1}{i'i'.i''i''.i'''i'''.i''''i''''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee} P \\ + \frac{ii}{i''i''.i'''i'''.i''''i''''} \cdot \frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee} Q \\ + \frac{ii.i'i'}{i'''i'''.i''''i''''} \cdot \frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee} R \\ + \frac{ii.i'i'.i''i''}{i''''i''''} \cdot \frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee} S \\ + ii.i'i'.i''i''.i'''i''' \cdot \frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta} T \end{array} \right\}$$

et radiorum in k concurrentium

inclinatio ad axem $= ii'i''i'''i'''' \cdot \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$.

Vnde progressio harum formularum ad plures adhuc lentes satis est manifesta. Si in lentibus ratio refractionis sit diuersa atque ad singulas lentes ordine his litteris indicetur n, n', n'', n''', etc.; haec diuersitas facile ad formulas hic inuentas accommodabitur. Primum enim haec correctio occurrit in formulis pro radiis lentium,

lentium, ita, vt quemadmodum formulae f et g pro prima lente numerum n inuoluunt, ita pro secunda lente numerus n', pro tertia n'' et ita porro introducatur. Similem correctionem etiam requirunt valores litterarum P, Q, R, S etc. et loco litterae n, quae in valore P occurrit, in valoribus Q, R, S etc. scribi oportet n', n'', n''' etc.

Coroll. 1.

87. Si obiectum sit tantum punctum in axe positum, sufficit vt lentium aperturae sint tantae, quantas assignauimus sin autem obiectum habeat quandam magnitudinem, tum aperturae praeter primam eo magis mensuras assignatas superare debent, quo maior fuerit obiecti magnitudo z.

Coroll. 2.

88. In expressione spatii diffusionis quadratum semidiametri aperturae primae faciei xx primo multiplicatur per quadratum distantiae postremae imaginis ab vltima lentium: quae ergo si fuerit infinita, etiam spatium diffusionis fit infinitum.

Coroll. 3.

89. Ceteris ergo paribus, quotcunque fuerint lentes, spatium diffusionis semper est proportionale quadrato diametri aperturae primae faciei, hoc est ipsi huic aperturae. Vnde diametro aperturae primae

faciei ad femiffem redacto fpatium diffufionis quadruplo fiet minus.

Scholion.

90. Confiderauimus hic ftatim loca fingularum imaginum principalium tanquam data ex iifque ftructuram cuiusque lentis quantitatem arbitrariam introducendo determinauimus. Quod fi vero ipfae lentes fuerint datae ita vt tam radii ambarum facierum cuiusque quam craffities, vna cum earum interuallis cognofcantur tum ope formularum exhibitarum, viciffim diftantiae determinatrices innotefcunt. Sint fcilicet radii facierum anterioris et pofterioris primae lentis PP, f, g, fecundae lentis QQ, f', g'; tertiae lentis RR, f'', g'' etc. craffitie earum exiftente v, v' v'' etc. tum vero dentur diftantiae aB$=$F; bC$=$G; cD$=$H etc. Praeterea autem diftantia obiecti ante lentem primam fit AE$=a$, ac fequenti modo omnia elementa ad problema fuperius neceffaria elicientur;

$$\begin{cases} 1.\ \frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+v}, & \text{hinc reperitur } k \\ 2.\ \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-v}, & \text{hinc vero } \alpha \\ 3.\ F = \alpha + b & \text{vnde } b = F - \alpha \end{cases}$$

$$\begin{cases} 4.\ \frac{n-1}{f''} = \frac{1}{b} + \frac{2n}{k'+v'}, & \text{hinc reperitur } k' \\ 5.\ \frac{n-1}{g''} = \frac{1}{\beta} - \frac{2n}{k'-v'}, & \text{hinc vero porro } \beta \\ 6.\ G = \beta + c & \text{vnde } c = G - \beta \end{cases}$$

$$\begin{cases} 7.\ \frac{n-1}{f''} = \frac{1}{c} + \frac{2n}{k''+v''}, & \text{hinc reperitur } k''' \\ 8.\ \frac{n-1}{g''} = \frac{1}{\gamma} - \frac{2n}{k''-v''}, & \text{hinc vero } \gamma \\ 9.\ H = \gamma + d & \text{vnde } d = H - \gamma \end{cases}$$

etc.

Vtcunque ergo lentes datae fuerint dispositae super axe communi, si ante eas constituatur obiectum in data distantia $AE = a$, inde singulae distantiae determinatrices $\alpha, b, \mathfrak{b}, c, \gamma$ etc. cum arbitrariis k, k', k'', k''' etc. facile definiuntur ex iisque porro spatium diffusionis cum reliquis phaenomenis in solutione problematis commemoratis assignabitur. Operae pretium autem erit casum, quo crassities lentium vt euanescens spectatur, accuratius euoluisse.

Problema 5.

91. Si crassities lentium euanescat, et quotcunque huiusmodi lentes super communi axe fuerint dispositae ante quas exisstat obiectum $E\varepsilon$, definire spatium diffusionis, per quod imago erit dissipata, vt et magnitudinem imaginis.

Solutio.

Si obiecti magnitudo $E\varepsilon = z$, cuius imagines principales successiue cadant in $F\zeta$, $G\eta$, $H\theta$, $I\iota$, $K\kappa$ etc. hincque pro singulis lentibus sequentes habebimus distantias determinatrices, cum imago per quamuis lentem repraesentata respectu lentis sequentis vicem obiecti gerat.

 distan-

	distantia	distantia
Pro lente PP	obiecti $EA = a$;	imaginis $aF = \alpha$
Pro lente QQ	obiecti $FB = b$;	imaginis $bG = \beta$
Pro lente RR	obiecti $GC = c$;	imaginis $cH = \gamma$
Pro lente SS	obiecti $HD = d$;	imaginis $dI = \delta$
Pro lente TT	obiecti $IE = e$;	imaginis $eK = \varepsilon$

etc.

Porro autem sint numeri arbitrarii vnitate maiores cuiusque lentis figuram determinantes, λ pro lente PP, λ' pro QQ, λ'' pro RR, λ''' pro SS, λ'''' pro TT, etc. ita vt ponendo breuitatis gratia

$$\rho = 0,190781, \quad \sigma = 1,627401, \quad \tau = 0,905133$$

Pro lente PP radius faciei $\begin{cases} \text{anter.} = \dfrac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\surd(\lambda - 1)} \\ \text{poster.} = \dfrac{a\alpha}{\rho a + \sigma\alpha \mp \tau(a+\alpha)\surd(\lambda - 1)} \end{cases}$

Pro lente QQ radius faciei $\begin{cases} \text{anter.} = \dfrac{b\beta}{\rho\beta + \sigma b \pm \tau(b+\beta)\surd(\lambda' - 1)} \\ \text{poster.} = \dfrac{b\beta}{\rho b + \sigma\beta \mp \tau(b+\beta)\surd(\lambda' - 1)} \end{cases}$

Pro lente RR radius faciei $\begin{cases} \text{anter.} = \dfrac{c\gamma}{\rho\gamma + \sigma c \pm \tau(c+\gamma)\surd(\lambda'' - 1)} \\ \text{poster.} = \dfrac{c\gamma}{\rho c + \sigma\gamma \mp \tau(c+\gamma)\surd(\lambda'' - 1)} \end{cases}$

Pro lente SS radius faciei $\begin{cases} \text{anter.} = \dfrac{d\delta}{\rho\delta + \sigma d \pm \tau(d+\delta)\surd(\lambda''' - 1)} \\ \text{poster.} = \dfrac{d\delta}{\rho d + \sigma\delta \mp \tau(d+\delta)\surd(\lambda''' - 1)} \end{cases}$

Pro lente TT radius faciei $\begin{cases} \text{anter.} = \dfrac{e\varepsilon}{\rho\varepsilon + \sigma e \pm \tau(e+\varepsilon)\surd(\lambda'''' - 1)} \\ \text{poster.} = \dfrac{e\varepsilon}{\rho e + \sigma\varepsilon \mp \tau(e+\varepsilon)\surd(\lambda'''' - 1)} \end{cases}$

etc.

Deinde

Deinde si quaelibet lens cum binis suis distantiis determinatricibus seorsim consideretur, eiusque aperturae semidiameter foret $= x$, posito $\mu = 0,93819$ et $\nu = 0,232692$ esset spatium diffusionis

$$\text{Lentis } PP' = \mu \alpha \alpha x x \left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)^2 + \tfrac{\nu}{a\alpha}\right)$$

$$\text{Lentis } QQ' = \mu \beta \beta x x \left(\tfrac{1}{b} + \tfrac{1}{\beta}\right)\left(\lambda'\left(\tfrac{1}{b} + \tfrac{1}{\beta}\right)^2 + \tfrac{\nu}{b\beta}\right)$$

$$\text{Lentis } RR' = \mu \gamma \gamma x x \left(\tfrac{1}{c} + \tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c} + \tfrac{1}{\gamma}\right)^2 + \tfrac{\nu}{c\gamma}\right)$$

$$\text{Lentis } SS' = \mu \delta \delta x x \left(\tfrac{1}{d} + \tfrac{1}{\delta}\right)\left(\lambda'''\left(\tfrac{1}{d} + \tfrac{1}{\delta}\right)^2 + \tfrac{\nu}{d\delta}\right)$$

$$\text{Lentis } TT' = \mu \varepsilon \varepsilon x x \left(\tfrac{1}{e} + \tfrac{1}{\varepsilon}\right)\left(\lambda''''\left(\tfrac{1}{e} + \tfrac{1}{\varepsilon}\right)^2 + \tfrac{\nu}{e\varepsilon}\right)$$

etc.

His constitutis pro magnitudine singularum imaginum habebimus

Pro vna lente $F\xi = \frac{\alpha}{a} z$ situ inuerso

Pro duabus lentibus $G\eta = \frac{\alpha\beta}{ab} z$ situ erecto

Pro tribus lentibus $H\theta = \frac{\alpha\beta\gamma}{abc} z$ situ inuerso

Pro quatuor lentibus $I\iota = \frac{\alpha\beta\gamma\delta}{abcd} z$ situ erecto

Pro quinque lentibus $K\varkappa = \frac{\alpha\beta\gamma\delta\varepsilon}{abcde} z$ situ inuerso

etc.

At si semidiameter aperturae primae lentis PP' ponatur $= x$, necesse est, vt reliquarum lentium aperturae superent sequentes valores

Semi-

Semidiameter aperturae

Lentis ſecundae $QQ > \frac{b}{\alpha}x$

Lentis tertiae $RR > \frac{bc}{\alpha\beta}x$

Lentis quartae $SS > \frac{bcd}{\alpha\beta\gamma}x$

Lentis quintae $TT > \frac{bcde}{\alpha\beta\gamma\delta}x$

etc.

Hinc ſpatium diffuſionis pro quolibet lentium numero ita ſe habebit.

I. Pro vna lente

ſpatium diffuſionis

$$Ff = \mu\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

radiorum in f concurrentium

inclinatio ad axem $= \frac{x}{\alpha}$

II. Pro duabus lentibns

ſpatium diffuſionis

$$Gg = \mu\beta\beta xx\begin{cases} +\frac{\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb}{\alpha\alpha}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \end{cases}$$

et radiorum in g concurrentium

inclinatio ad axem $= \frac{bx}{\alpha\beta}$

III. Pro tribus lentibus

ſpatium diffuſionis

$$Hh = \mu\gamma\gamma xx\begin{cases} +\frac{\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{\alpha a}\right) \\ +\frac{bb\beta\beta}{\alpha\alpha cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ +\frac{bbcc}{\alpha\alpha\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \end{cases}$$

et

et radiorum in b concurrentium
inclinatio ad axem $\frac{bcx}{\alpha\beta\gamma}$

IV. Pro quatuor lentibus

ſpatium diffuſionis

$$I\,i = \mu\,\delta\,\delta\,x\,x \begin{cases} +\frac{\alpha\alpha\beta\beta\gamma\gamma}{bb\,cc\,dd}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ +\frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \\ +\frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right) \end{cases}$$

et radiorum in i concurrentium
inclinatio ad axem $= \frac{bcdx}{\alpha\beta\gamma\delta}$

V. Pro quinque lentibus

ſpatium diffuſionis

$$K\,k = \mu\,\varepsilon\,\varepsilon\,x\,x \begin{cases} +\frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{boccddee}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccaaee}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ +\frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \\ +\frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right) \\ +\frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)\left(\lambda''''\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)^2+\frac{\nu}{e\varepsilon}\right) \end{cases}$$

et radiorum in k concurrentium
inclinatio ad axem $= \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$
neque ergo caſus, quibus plures occurrunt lentes, vlla amplius laborant difficultate.

Si lentes ratione refractionis diſcrepent, ad easque referendae ſint litterae n, n', n'', n''', etc. formu-

 lae

lae in hoc problemate inuentae sequenti modo facile ad hunc casum latius patentem adcommodabuntur. Primo scilicet in formulis pro radiis facierum inuentis litterae ρ, σ, et τ tantum ad primam lentem pertinent, earumque loco pro secunda lente scribi oportet ρ', σ', et τ'; pro tertia autem ρ'', σ'' et τ'' et ita porro. Praeterea vero spatia diffusionis hinc aliquam mutationem requirunt, eritque spatium diffusionis.

I. Pro vna lente

$$\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\mu\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right).$$

II. Pro duabus lentibus

$$\beta\beta xx\begin{cases}\frac{\mu\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb}{\alpha\alpha}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right).\end{cases}$$

III. Pro tribus lentibus

$$\gamma\gamma xx\begin{cases}+\frac{\mu\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb\beta\beta}{\alpha\alpha cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right)\\ +\frac{\mu'' bbcc}{\alpha\alpha\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right).\end{cases}$$

IV. Pro quatuor lentibus.

$$\delta\delta xx\begin{cases}+\frac{\mu\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right)\\ +\frac{\mu'' bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right)\\ +\frac{\mu''' bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu'''}{d\delta}\right).\end{cases}$$

valores autem harum litterarum μ', ν', μ'', ν'' etc. iam supra definiuimus §. 77.

Coroll.

Coroll. 1.

92. Si lentis primae PP ponatur radius faciei anterioris $=f$ et poſterioris $=g$ erit

$$\frac{1}{f}=\frac{\varrho}{a}+\frac{\sigma}{\alpha}\pm\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}$$

$$\frac{1}{g}=\frac{\varrho}{\alpha}+\frac{\sigma}{a}\mp\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}$$

vnde ſi detur diſtantia obiecti $EA=a$, primo inuenitur α ex hac aequatione

$$\frac{1}{f}+\frac{1}{g}=(\varrho+\sigma)\left(\frac{1}{a}+\frac{1}{\alpha}\right)=1,818182\left(\frac{1}{a}+\frac{1}{\alpha}\right)=\frac{20}{11}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

Inuenta autem diſtantia α numerus λ reperitur ex hac aequatione.

$$\frac{1}{f}-\frac{1}{g}+(\sigma-\varrho)\left(\frac{1}{a}-\frac{1}{\alpha}\right)=2\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}.$$

Coroll. 2.

93. Deinde ſi diſtantia ſecundae lentis a prima ſit $=F$, ob $F=\alpha+b$ habetur $b=F-\alpha$; qua diſtantia b cognita, ſi pro lente ſecunda datus ſit radius faciei anterioris $=f'$, et poſterioris $=g'$ habebuntur iterum duae aequationes

$$\frac{1}{f'}=\frac{\varrho}{b}+\frac{\sigma}{\beta}\pm\tau\left(\frac{1}{b}+\frac{1}{\beta}\right)\sqrt{(\lambda'-1)}$$

$$\frac{1}{g'}=\frac{\varrho}{\beta}+\frac{\sigma}{b}\mp\tau\left(\frac{1}{b}+\frac{1}{\beta}\right)\sqrt{(\lambda'-1)}$$

ex quibus cum diſtantiam β tum numerum λ' definire licet. Similique modo ex forma ſequentium lentium earumque diſtantia reliqua elementa innoteſcent.

Coroll. 3.

94. Si ſingulae lentes ad minimum ſpatium diffuſionis fuerint accommodatae, erit $\lambda=1, \lambda'=1, \lambda''=1, \lambda'''=1$ etc. ſin autem hae lentes alia forma fuerint praeditae, iſti numeri erunt vnitate maiores.

Scholion.

95. Quo plures fuerint lentes eo pluribus constabit membris spatium diffusionis ab iis productum; neque tamen propterea aucto lentium numero spatium diffusionis necessario augetur. Cum enim quantitates α, b, β, c, γ, d, δ, etc. valores quoque negatiuos recipere queant, dummodo binarum summae $\alpha+b$; $\beta+c$; $\gamma+d$; $\delta+e$ etc. vtpote lentium distantiae maneant positiuae, fieri potest, vt vnum vel aliquot membra fiant negatiua, hincque spatium diffusionis diminuatur, quin etiam interdum prorsus euanescat, quo casu repraesentatio sine dubio erit perfectissima. Verum in instrumentis dioptricis ad visionem instructis veluti Telescopiis ac Microscopiis non tam hoc, quod definiuimus, spatium diffusionis quam confusio in ipsa visione orta spectari debet; quae autem etsi a spatio diffusionis plurimum differt, tamen ex eo definiri potest vti mox explicabimus. Ante autem conueniet lentes compositas seu multiplicatas considerare, cuiusmodi oriuntur, si duae pluresue lentes, quarum crassities tam est parua vt negligi queat, immediate iungantur quo quidem pacto instar lentium simplicium spectari possunt; verum tali coniunctione effici potest, vt spatium diffusionis multo fiat minus, quam si lens simplex adhiberetur, atque adeo euanescat valorque numeri λ istiusmodi lenti compositae conveniens vnitate minor sit proditurus vnde maxima commoda ad confusionem diminuendam obtinebuntur.

CAPVT III.