CAPVT I.

DE DIFFVSIONE IMAGINIS PER VNICAM LENTEM REPRAESENTATAE.

Definitio I.

1.

Imago principalis vocatur ea, quae a radiis axi lentis proximis per lentem refractis repraeſentatur. Tab. I. Fig. 1.

Scilicet ſi lentis PP axis ſit EF in eaque tantum ſpatium minimum aAa ſit apertum, per quod radiis tranſitus concedatur, radii a puncto lucido E emiſſi in vno puncto F colligentur, quod punctum imago principalis vocatur.

Coroll. 1.

2. Cum nempe apertura aAa ſit minima, omnes radii, qui a puncto quocunque in eam incidunt, ita aequabilem patiuntur refractionem, vt omnes iterum in vnum punctum colligantur, id quod non ſolum de puncto lucido E in axe lentis ſito, ſed etiam de quibusuis aliis extra axem poſitis eſt intelligendum.

Coroll. 2.

3. Quodſi ergo lentis apertura fuerit minima, ſingula cuiusuis obiecti puncta poſt refractionem iterum ſingulis punctis referentur, ſicque imago principalis erit diſtincta et confuſione carebit; ſiquidem confuſio tum demum oritur, quando radii ex vno puncto emiſſi non iterum in vno puncto colliguntur.

Coroll. 3.

4. Et quamdiu apertura lentis aAa eſt minima, nihil intereſt, ſecundum quamnam figuram facies lentis ſint elaboratae; quaecumque enim earum figura fuerit, quoniam tantum portiuncula minima in computum venit, ea ſemper vt ſphaerica ſpectari poterit.

Coroll. 4.

5. Pendet ergo locus imaginis principalis primum a loco puncti lucidi E, ſiue id in axe ſiue extra axem fuerit ſitum; deinde a ſphaericitate

vtrius-

vtriusque faciei aAa et bBb refringentis; tertio ab earum distantia AB seu lentis crassitie, et quarto a ratione refractionis, quam radii in transitu per lentem patiuntur.

Scholion I.

6. Refractio radiorum per huiusmodi lentium aperturas minimas transmissorum, ideoque determinatio imaginum principalium satis accurate in elementis Dioptricis tradi solet, quod igitur negotium hic fusius non prosequar; sed potius in eam refractionis rationem hic inquirere constitui, quando lentium apertura est modicae quantitatis, in quo imprimis ad vtriusque faciei figuram est spectandum. Hic autem perpetuo lentium figuras sphaericas assumo, propterea quod haec figura vulgo lentibus induci, vel saltem, nisi forte accurate successerit, intendi solet. In praxi certe nulla adhuc alia figura lentibus commode et accurate tribui poterit, atque adeo a sphaerica figura, etsi ad praxin maxime est accommodata, ab artificibus frequenter aberrari solet. A sollertioribus autem talia vitia iam plerumque satis feliciter euitantur, vnde non adeo erit verendum, ne ea, quae per calculum ex hypothesi sphaericae figurae elicientur, experientiae non consentanea sint futura. Hanc ob rem hic perpetuo postulo, vt ambae facies lentium exactissime secundum sphaericam figuram sint elaboratae.

Scholion 2.

7. Sphaerica autem figura hoc laborat incommodo, quod statim ac lenti maior apertura tribuatur, non amplius omnes radii, qui quidem ab vno obiecti puncto sunt profecti, post refractionem ad vnum punctum dirigantur radiique EM longius ab axe lentis transmissi non amplius in puncto F concurrant. Vnde confusionen eo maiorem nasci necesse est, quo magis hi radii remotiores ab iis, qui prope axem transeunt, declinauerint; et quoniam talis declinatio a figura sphaerica originem ducit, eo maior euadet, quo maior lenti apertura tribuatur. Quanta igitur quouis casu futura sit haec confusio, hoc capite definire constitui; quae cum ceteris paribus a quantitate aperturae pendeat, hic in perpetuum moneo, me cuiusuis lentis aperturam circularem assumere, per cuius centrum axis lentis transeat: ita vt semidiameter huius circuli simul mensuram aperturae exhibeat. Ita si in superficie lentis PP spatium MAM apertum relinquatur, reliqua parte MP velamine opaco obducta, punctorum extremorum MM distantia diametrum aperturae eiusque semissis semidiametrum aperturae praebebit.

Definitio 2.

8. *Imago extrema est ea, quam radii per extremitatem aperturae transmissi exhibent.*

Ita

Ita si MM sit apertura lentis, radiique EM, EM a puncto lucido E circa oram aperturae transmissi conueniant in puncto f, in hoc ipso puncto f erit imago extrema.

Coroll. 1.

9. Si punctum lucidum E est in ipso axe lentis, nullum est dubium, quin radii inde per marginem circularem MM transeuntes iterum in vno axis puncto f concurrant, imaginemque distinctam repraesentent, quae imago extrema vocatur.

Coroll. 2.

10. Verum si punctum lucidum non esset in axe lentis, hoc neutiquam eueniet, radiique per marginem illum circularem transmissi non amplius in vno puncto colligentur; vnde hoc casu imago extrema eo magis erit confusa, quo magis punctum lucidum ab axe fuerit remotum.

Scholion.

11. Quomodo se habeat refractio radiorum, quando punctum lucidum extra axem lentis fuerit constitutum, quaestio est non solum difficillima, sed etiam ita prolixis calculis inuoluitur, vt inde vix quicquam concludi possit. Ceterum in vsu, ad quem lentes accommodantur, nunquam obiecta ab axe remotiora spectari solent, atque contentos esse nos oportet, dummodo obiecta in ipso axe lentis sita distin-

cte

cte repraesententur, neque etiam confusio, qua obiecta axi proxima afficiuntur, sensibilis esse potest; nam cum imago extrema puncti E in ipso lentis axe sita sit punctum f, nulla confusione inquinatum; etiamsi id parumper esset remotum ab axe, vix sensibilis confusio se immiscere poterit. Quam ob causam investigationes sequentes tantum ad obiecta in ipso lentis axe sita adstringam.

Definitio 3.

12. *Spatium diffusionis vocatur interuallum, inter imaginem principalem et extremam interceptum.*

Ita si imago principalis sit in F, extrema vero in f, interuallum Ff appellatur spatium diffusionis.

Coroll. 1.

13. Si ergo apertura lentis MM euanescat, spatium simul diffusionis euanescit, tum enim tantum radii axi proximi transmittuntur, quibus imago distincta in F effingitur. Ex quo intelligi licet, quo maior fuerit apertura lentis, eo maius fore spatium diffusionis Ff.

Coroll. 2.

14. Cum in F imago a radiis axi proximis, in f autem imago a radiis circa marginem circularem MM transmissis formetur; si totam aperturam lentis infinitis circulis concentricis diuisam concipiamus, radii

radii per ſingulos circulos transmiſſi imagines intermedias exhibebunt, quibus interuallum Ff replebitur.

Coroll. 3.

15. Si enim apertura primum nulla, tum vero continuo increſcens ſtatuatur, imago extrema primum cum principali congruet; tum vero continuo magis ab ea diſcedet, ſicque cum vsque ad MM fuerit aucta, omnes illae imagines etiam nunc ſubſiſtent, ſpatiumque Ff implebunt.

Scholion I.

16. Spatium hoc diffuſionis cauſam continet confuſionis, qua repraeſentatio imaginis perturbatur; cum enim eiusdem puncti lucidi E infinitae imagines per interuallum Ff diſpoſitae exhibeantur; earum commiſtio confuſionem pariat neceſſe eſt, quae eo maior erit, quo maius fuerit ſpatium diffuſionis Ff. Quemadmodum enim ad repraeſentationem diſtinctam requiritur, vt omnes radii ex eodem obiecti puncto emiſſi iterum in vnico puncto colligantur; ita ſi hi radii in plura puncta coeant, pluresque eiusdem puncti imagines referant, prout hae magis minusue inter ſe diſcrepant, maior inde minorue confuſio naſcetur. Quemadmodum autem hanc confuſionem aeſtimari oporteat, deinceps demum explicare licebit, cum ante ſpatium diffuſionis accurate definire docuerimus: quo circa in hoc capite, cum propoſita fuerit lens quaecunque vitrea faciebus ſphaericis terminata

pro quauis puncti lucidi E ab ea distantia et quauis apertura spatium diffusionis Ff inuestigare constitui. Quo facto eandem inuestigationem pro duabus pluribusue lentibus inter se coniunctis suscipi conueniet, vt tandem inde confusionem in quibusuis instrumentis dioptricis assignare valeamus.

Scholion 2.

17. Quaestio igitur principalis huius capitis in hoc versatur, vt proposita lente PP in eiusque axe puncto lucido E, radius quicunque incidens EM consideretur, eiusque per lentem refractio definiatur, vnde punctum f, vbi iterum in axem incidat, assignari queat. Cum enim in eodem puncto f omnes radii per totam peripheriam circularem MM transmissi concurrant; ab his imago quaepiam puncti E in f exprimetur, quae erit extrema, si circulus MM in lente eius aperturam determinet; sin autem capiatur minor, imago quaedam intermedia habebitur. Cum autem hic duplex refractio eueniat, altera in ingressu radii EM in vitrum, altera in egressu eiusdem e vitro, quemadmodum eius directio in vtraque inflectatur, seorsim est inuestigandum; vnde duo nascuntur problemata quasi praeliminaria, ex quorum combinatione deinceps negotium conficietur. Verum quo haec problemata commodius calculo expediri queant, quaedam lemmata ex doctrina angulorum petita praemitti oportet.

Lem-

Lemma I.

18. *Si angulus Φ triginta gradus non excedat eius finus fatis accurate erit* fin. $\Phi = \Phi - \frac{1}{6}\Phi^3$, *fi quidem in circulo, cuius radius eft* $= 1$, *arcus* Φ, *qui pro illius anguli menfura habetur, in partibus radii exprimatur.*

Demonftratio.

Quantuscunque fuerit angulus Φ, notum eft, eius finum hac ferie infinita exprimi:

$$\text{fin. } \Phi = \Phi - \tfrac{1}{6}\Phi^3 + \tfrac{1}{120}\Phi^5 - \tfrac{1}{5040}\Phi^7 + \text{etc.}$$

fumtis igitur tantum binis primis terminis, error committitur reliquis neglectis aequalis, fi ergo ftatuamus fin. $\Phi = \Phi - \frac{1}{6}\Phi^3$, hincque pro variis angulis Φ finus colligamus, eorum comparatio cum tabulis finuum errores manifeftabit. Ita fi fumatur $\Phi = 30°$, quia arcus 180° valet 3, 14159265, erit in partibus radii $\Phi = 0{,}52359877$, et $\frac{1}{6}\Phi^3 = 0{,}0239246$, hincque

$$\Phi - \tfrac{1}{6}\Phi^3 = 0{,}\ 4996741. \text{ at eft reuera}$$
$$\text{fin. } \Phi = \underline{0{,}\ 5000000} \text{ vnde habetur}$$

error $= 0{,}\ 0003259$, qui ergo ne ad $\frac{1}{3000}$ partem radii quidem affurgit. At fi angulus Φ caperetur duplo minor, fcilicet $\Phi = 15°$, reperiretur

$$\Phi - \tfrac{1}{6}\Phi^3 = 0{,}\ 2588088$$
$$\text{cum tamen fit} \quad \text{fin. } \Phi = 0{,}\ 2588190$$
$$\text{errore exiftente} = 0{,}\ 0000102$$

qui tricies bis minor eft quam cafu praecedente. Cum ergo in praxi error partem adeo termillefimam radii adaequans facile tolerari poffit, multo magis, fi angulus Φ fuerit 30° minor, expreſſio $\Phi - \frac{1}{6}\Phi^{3}$ verum eius ſinum exhibere cenſenda erit.

Lemma 2.

19. *Viciſſim ſi anguli triginta gradibus minoris detur ſinus* $= s$, *ex eo ipſe angulus ita proxime definitur, vt ſit in circulo, cuius radius* $= 1$, *arcus eum metiens* $= s + \frac{1}{6}s^{3}$.

Demonſtratio.

Si enim Φ deſignet iſtum angulum, cuius ſinus proponitur $= s$; modo vidimus, eſſe ſatis exacte $s = \Phi - \frac{1}{6}\Phi^{3}$, hinc autem per conuerſionem oritur proxime $\Phi = s + \frac{1}{6}s^{3}$, quae expreſſio quantum peccet in angulo triginta graduum, vt videamus, ſit $s = \frac{1}{2}$ eritque $s + \frac{1}{6}s^{3} = 0,5208333$. Cum autem numerus $3,14159265$ reſpondeat angulo 180°, hic numerus $0,5208333$ praebet angulum $29^{\circ}\ 50'\ 30''$: verus autem angulus eſt 30°, ita vt error ſit $9'\ 30''$. Sit autem $s = \frac{1}{4}$ cui ſinui reſpondet angulus $= 14^{\circ}\ 28'\ 39''$, erit $s + \frac{1}{6}s^{3} = 0,2526041 = 14^{\circ}\ 28'\ 23''$, ita vt hoc caſu error non excedat $16'''$. Patet ergo dum angulus minor ſit 30°, hoc modo ſatis exacte ex dato ſinu reperiri angulum.

Problema 1.

Tab. I. Fig. 2. 20. Si ex puncto lucido E in ſuperficiem ſphaericam reuolutione arcus circularis AM circa axem EC genitam incidat radius EM, definire eius, poſtquam fuerit refractus, concurſum cum axe EC.

Solutio.

Solutio.

Sit C centrum ſuperficiei ſphaericae, eiusque radius $CA = CM = f$, et diſtantia puncti lucidi E a ſuperficie refringente $EA = a$, tum vero ſit ratio refractionis radiorum ex aere in vitrum $= n:1$. Vocetur pro radio incidente CM angulus $ACM = \Phi$, eritque huius anguli ſinus $= \Phi - \frac{1}{6}\Phi^3$. Ponatur iam breuitatis gratia diſtantia $EC = a + f = c$, et quia in triangulo ECM dantur latera $CM = f$, $EC = c$ cum angulo $CEM = \Phi$, fiet producta CM in c

$$CM\ (f) : \text{ſin. } CEM = CE\ (c) : \text{ſin. } EMc$$

ideoque

$$\text{ſin. } EMc = \frac{c}{f}\text{ ſin. }\Phi = \frac{c}{f}(\Phi - \tfrac{1}{6}\Phi^3);$$

vnde ipſe angulus EMc elicitur, neglectis tertia altioribus poteſtatibus ipſius Φ:

$$EMc = \frac{c}{f}\Phi - \frac{c}{6f}\Phi^3 + \frac{c^3}{6f^3}\Phi^3 = \frac{c}{f}\Phi + \frac{c(cc-ff)}{6f^3}\Phi^3$$

qui cum ſit $= ECM + CEM$, erit

$$ECM = \frac{c-f}{f}\Phi + \frac{c(cc-ff)}{6f^3}\Phi^3$$

Verum angulus EMc eſt angulus incidentiae, ac ſi MO referat radium refractum, erit CMO angulus refractionis, ſicque per hypotheſin

$$\text{ſin. } EMc : \text{ſin. } CMO = n:1.$$

vnde colligitur

$$\text{ſin. } CMO = \frac{c}{nf}(\Phi - \tfrac{1}{6}\Phi^3)$$

hincque ipſe angulus

$$CMO = \frac{c}{nf}\Phi + \frac{c(cc-nnff)}{6n^3f^3}\Phi^3$$

 quo

quo ablato ab angulo ECM relinquitur angulus

$$COM = \frac{(n-1)c-nf}{nf}\Phi + \frac{c((n^3-1)cc-nn(n-1)ff)}{6n^3f^3}\Phi^3$$

cuius finus propterea erit

$$\text{fin. } COM = \frac{(n-1)c-nf}{nf}\Phi + \frac{3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3}{6nnf^3}\Phi^3$$

Iamuero ob

fin. $COM : CM\,(f) =$ fin. $CMO : CO$, obtinebitur

$$CO = \frac{\frac{c}{n} - \frac{c}{6n}\Phi\Phi.}{\frac{(n-1)c-nf}{nf} + \frac{3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3}{6nnf^3}\Phi^2}$$

cuius formulae euolutio praebet:

$$CO = \frac{cf}{(n-1)c-nf} - \frac{cf\Phi\Phi}{6((n-1)c-nf)} - \frac{c(3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3}{6nf((n-1)c-nf)^2}\Phi\Phi$$

quae porro reducitur ad hanc formam:

$$CO = \frac{cf}{(n-1)c-af} - \frac{(n-1)cc(c-f)(c+nf)}{2nf((n-1)c-nf)^2}\Phi\Phi$$

cui fi addamus $CA = f$ prodibit

$$AO = \frac{nf(c-f)}{(n-1)c-nf} - \frac{(n-1)cc(c-f)(c+nf)}{2nf((n-1)c-nf)^2}\Phi\Phi$$

atque hinc pofitio radii refracti MO primo per interuallum AO modo inuentum definitur, tum vero infuper ex angulo AOM, qui erit:

$$AOM = \frac{(n-1)c-nf}{nf}\Phi + \frac{c((n^3-1)cc-nn(n-1)ff)}{6n^3f^3}\Phi^3$$

fiue

$$AOM = \frac{(n-1)c-nf}{nf}\Phi + \frac{(n-1)c((nn+n+1)cc-nnff)}{6n^3f^3}\Phi^3$$

Coroll. I.

21. Cum pofuerimus $c = a+f$ erit

$c-f = a$, et $(n-1)c-nf = (n-1)a-f$,

atque

atque

$$(nn+n+1)cc-nnff=(nn+n+1)aa+2(nn+n+1)af+(n+1)ff$$

quibus valoribus ſubſtitutis habebimus

$$AO=\frac{naf}{(n-1)a-f}-\frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\Phi\Phi \text{ et}$$

$$AOM=\frac{(n-1)a-f}{nfa}\Phi+\frac{(n-1)(a+f)((nn+n+1)a(a+f)+(n+1)ff)}{6n^3f^3}\Phi^3$$

Coroll. 2.

22. Si M ſit in extremitate aperturae, erit ſemidiameter aperturae $=f$ ſin. ECM $=a\Phi$ ſatis exacte: neque enim neceſſe eſt, aperturam tam exacte noſſe. Vnde ſi ſemidiameter aperturae ponatur $=x$, erit prope $x=a\Phi$; ideoque $\Phi=\frac{x}{a}$.

Coroll. 3.

23. Si accuratius rem definire velimus, quia eſt

$$\text{ſin. } ECM=\frac{c-f}{f}\Phi+\frac{(c-f)(3c-f)}{6ff}\Phi^3=\frac{a}{f}\Phi+\frac{a(3a+2f)}{6ff}\Phi^3$$

erit $x=a\Phi+\frac{c(3a+2f)}{6f}\Phi^3$

ideoque $\Phi=\frac{x}{a}-\frac{(3a+2f)x^3}{6a^3f}$

ſed quia in valore ipſius AO non vltra ſecundam dimenſionem ipſius Φ aſcendimus, hac expreſſione non indigemus.

Coroll. 4.

24. Etſi hae expreſſiones tantum ſunt prope verae; tamen in praxi ſine errore adhiberi poterunt, dummodo anguli, qui in calculum ſunt ingreſſi, infra 80°

30° ſubſiſtant. Non ſolum ergo neceſſe eſt, vt angulus $AEM = \Phi$ ſed etiam angulus EMx ſeu $\frac{c}{f}\Phi = \Phi + \frac{a}{f}\Phi$ minor ſit 30 gradibus.

Scholion.

25. Summo quidem rigore geometrico diſtantiam EO definire potuiſſemus, neque opus fuiſſet ad approximationes confugere: ſcilicet ſi poſuiſſemus angulum $ECM = \omega$, et diſtantiam $CO = u$; deueniſſemus ad hanc determinationem.

$$\frac{cf}{u} = -c\cos.\omega + \sqrt{(nn(c\cos.\omega - f)^2 + (nn-1)cc\sin.\omega^2)}$$

foretque

$$\cos.\omega = \frac{c}{f}\sin.\Phi^2 + \cos.\Phi\sqrt{(1 - \frac{cc}{ff}\sin.\Phi^2)} \text{ et}$$

$$\sin.\omega = \frac{c}{f}\sin.\Phi\cos.\Phi - \sin.\Phi\sqrt{(1 - \frac{cc}{ff}\sin.\Phi^2)}$$

hisque valoribus ſubſtitutis

$$\frac{cf}{u} = \frac{-cc\sin.\Phi^2}{f} - c\cos.\Phi\sqrt{(1 - \frac{cc}{ff}\sin.\Phi^2)}$$
$$+ (c\cos.\Phi - \sqrt{(ff - cc\sin.\Phi^2)})\sqrt{(nn - \frac{cc}{ff}\sin.\Phi^2)}$$

vnde ponendo angulo Φ valde paruo per aproximationem ſuperior expreſſio eliceretur. Verum praecedens analyſis magis ad praeſens inſtitutum videtur accommodata. Interim ſi quis propius ad veritatem accedere voluerit, ex formula vera hic exhibita adipiſcetur.

$$\frac{cf}{u} = (n-1)c\cos.\Phi - nf + \frac{(n-1)cc}{2nff}((n-1)f + c\cos.\Phi)\sin.\Phi^2$$
$$+ \frac{c^4}{8n^3f^4}((nn-1)^2 f + (n^3-1)c\cos.\Phi)\sin.\Phi^4$$

ſeu

seu

$$\frac{cf}{u}=(n-1)c-nf+\frac{(n-1)c(c-f)(c+nf)}{2nff}\Phi\Phi$$
$$+\frac{(n-1)(c-f)((nn+n+1)c^3+3nn(n+1)ccf+nn(3n+4)cff-n^3f^3)}{24n^3f^4}\Phi^4;$$

Problema 2.

26. Si prout in problemate praecedente inuenimus, radius MO in vitrum missus per superficiem sphaericam BN iterum in aerem erumpat, definire punctum V, vbi is cum axe concurret.

Solutio.

Ponatur interuallum $AB=d$; sitque superficiei sphaericae BN centrum in D eiusque radius $DB=DN=g$. Quoniam igitur positio radii incidentis MNO datur, ponamus $BO=b$, et angulum $BON=\psi$, et breuitatis ergo interuallum $DO=b+g=e$. Cum nunc in triangulo DON dentur latera $DN=g$, $DO=e$ cum angulo $BON=\psi$, reperitur

$$\text{sin. } DNM=\frac{e}{g}\text{ sin. }\psi=\frac{e}{g}\psi-\frac{e}{6g}\psi^3, \text{ hincque}$$
$$DNM=\frac{e}{g}\psi+\frac{e(ee-gg)}{6g^3}\psi^3 \text{ et}$$
$$ODN=\frac{e-g}{g}\psi+\frac{e(ee-gg)}{6g^3}\psi^3$$

Sed hic sin. DNM est sinus incidentiae, cui respondet angulus refractionis VNd, cuius sinus propterea est ad illum vt $n:1$, vnde fit

$$\text{sin. } VNd=\frac{ne}{g}\psi-\frac{ne}{6g}\psi^3.$$

hincque

$$\mathrm{VN}d = \frac{ne}{g}\psi + \frac{ne(nnee - gg)}{6g^3}\psi^3$$

a quo ablato angulo ODN relinquitur angulus

$$\mathrm{DVN} = \frac{(n-1)e+g}{g}\psi + \frac{e((n^3-1)ee-(n-1)gg)}{6g^3}\psi^3$$

ergo

$$\text{fin. DVN} = \frac{(n-1)e+g}{g}\psi + \frac{3n(n-1)e^3-3(n-1)^2eeg-4(n-1)egg-g^3}{6g^3}\psi^3$$

Cum nunc fit

fin. DVN : $g =$ fin. $\mathrm{VN}d$: DV erit

$$\frac{g}{\mathrm{DV}} = \frac{(n-1)e+g+\frac{1}{6gg}(3n(n-1)e^3-3(n-1)^2eeg-4(n-1)egg-g^3)\psi^2}{ne-\frac{1}{6}ne\psi^2}$$

quae expreffio reducitur ad hanc formam:

$$\frac{g}{\mathrm{DV}} = \frac{(n-1)e+g}{ne} + \frac{(n-1)(e-g)(ne+g)\psi^2}{2ngg}$$

vnde reciproce oritur

$$\mathrm{DV} = \frac{neg}{(n-1)e+g} - \frac{n(n-1)ee(e-g)(ne+g)}{2g((n-1)e+g)^2}\psi^2$$

et

$$\mathrm{BV} = \frac{g(e-g)}{(n-1)e+g} - \frac{n(n-1)ee(e-g)(ne+g)}{2g((n-1)e+g)^2}\psi^2$$

Puncto autem V inuento notandum eft effe angulum

$$\mathrm{BVN} = \frac{(n-1)e+g}{g}\psi + \frac{e((n^3-1)ee-(n-1)gg)}{6g^3}\psi^3$$

Coroll. I.

27. Cum fit

$e = b+g$ erit $(n-1)e+g = (n-1)b+ng$, quo

quo valore reſtituto habebimus.

$$BV = \frac{bg}{(n-1)b+ng} - \frac{n(n-1)b(b+g)^2(nb+(n+1)g)}{2g((n-1)b+ng)^2}\psi^2 \text{ et}$$

$$BVN = \frac{(n-1)b+ng}{g}\psi + \frac{(b+g)((n^3-1)(b+g)^2-(n-1)gg)}{6g^3}\psi^3$$

Coroll. 2.

28. Hae formulae etiam ex praecedentibus erui poſſunt, ſi pro litteris n, a, f ſcribantur $\frac{1}{n}$, $-b$, et $-g$, quoniam hoc modo caſus praecedentis problematis ad hunc reducitur. Praeterea autem, qui angulus ibi erat ϕ hic eſt ψ.

Coroll. 3.

29. Quia in praecedente problemate inuenimus tam lineam AO quam angulum AOM erit hoc problemate ad radium illum refractum MO accommodato

$$BO = b = \frac{naf}{(n-1)a-f} - d - \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\phi\phi$$

et

$$\psi = \frac{(n-1)a-f}{nf}\phi + \frac{(n-1)(a+f)((nn+n+1)a(a+2f)+(n+1)ff)}{6n^3f^3}\phi^3$$

Problema 3.

30. Propoſita lente vitrea MABN faciebus ſphaericis AM et BN terminata, ſi a puncto quocunque E in eius axe poſito incidat in eam radius EM, definire punctum V, in quo is poſt geminam refractionem iterum cum axe lentis ſit concurſurus. Tab. I. Fig. 2.

Solutio.

Solutio.

Confideremus lentem vt vtrinque conuexam, fitque faciei anterioris AM radius $AC = f$, pofterioris vero BN radius $BD = g$, ipfius lentis autem craffities $AB = d$. Lens porro fit vitrea, ita vt fi in eam radius lucis ex aere incidat, fit finus incidentiae ad finum refractionis vt n ad 1. Iam recta iungens centra vtriusque faciei C et D erit axis lentis, in quo reperiatur punctum lucidum E ante lentem in diftantia $AE = a$, vnde fub angulo $AEM = \Phi$ in lentem incidat radius EM, qui prima refractione ita inflectatur, vt productus cum axe concurrat in O. Quodfi iam ex iis, quae problemate primo funt inuenta, ponamus

$$BO = \frac{naf}{(n-1)a-f} - d - \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\Phi\Phi = b$$

$$\text{et ang. } BON = \frac{(n-1)a-f}{nf}\Phi = \psi$$

in valore enim anguli ψ negligere licet terminum Φ^3 inuoluentem, quoniam calculum tantum ad fecundam poteftatem ipfius ψ extendimus; his pofitis in problemate fecundo inuenimus fore.

$$BV = \frac{bg}{(n-1)b+ng} - \frac{n(n-1)b(b+g)^2(nb+(n+1)g)}{2g((n-1)b+ng)^2}\psi\psi$$

et

$$BVN = \frac{(n-1)b+ng}{g}\psi.$$

Totum ergo negotium huc redit, vt ifthic pro b et ψ valo-

valores aſſignatos ſubſtituamus, quod quo facilius fieri poſſit, ſtatuamus

$$b = P - Q\Phi\Phi \text{ et } \psi = R\Phi \text{ vt ſit}$$

$$P = \frac{naf}{(n-1)a-f} - d = \frac{naf-(n-1)ad+df}{(n-1)a-f}$$

$$Q = \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2} \text{ et } R = \frac{(n-1)a-f}{nf}$$

Hinc erit:

$$\frac{bg}{(n-1)b+ng} = \frac{Pg-Qg\Phi\Phi}{(n-1)P+ng-(n-1)Q\Phi\Phi} = \frac{Pg}{(n-1)P+ng} - \frac{nQgg\Phi\Phi}{((n-1)P+ng)^2}$$

at in altero membro ſufficit pro b ſcribere P: ex quo obtinebimus

$$BV = \frac{Pg}{(n-1)P+ng} - \frac{nQgg\Phi\Phi}{((n-1)P+ng)^2} - \frac{n(n-1)PRR(P+g)^2(nP+(n+1)g)}{2g((n-1)P+ng)^2}\Phi\Phi$$

$$\text{et } BVN = \frac{(n-1)P+ng}{g}R\Phi$$

Pro his autem ſubſtitutionibus notandum eſt fore:

$$(n-1)P+ng = \frac{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}{(n-1)a-f}$$

$$P+g = \frac{naf+(n-1)ag-fg-(n-1)ad+df}{(n-1)a-f}$$

$$nP+(n+1)g = \frac{nnaf+(nn-1)ag-(n+1)fg-n(n-1)ad+ndf}{(n-1)a-f}$$

vnde concluditur:

$$BV = \frac{nafg-(n-1)adg+dfg}{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}$$

$$- \frac{(n-1)agg(a+f)^2(a+(n+1)f)}{2f(n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df)^2}\Phi\Phi$$

$$- \frac{(n-1)(naf-(n-1)ad+df)(naf+(n-1)ag-fg-(n-1)ad+df)^2(nnaf+(nn-1)ag-(n+1)fg-n(n-1)ad+ndf)}{2nffg(n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df)^2}\Phi\Phi$$

$$\text{et } BVN = \frac{n(n-1)a(f+g)-nfg+(n-1)df-(n-1)^2ad}{nfg}\Phi.$$

Coroll. 1.

31. Si angulus Φ prorſus euaneſcat, punctum V cadet in imaginem principalem, cuius ſi a lente diſtantia dicatur $= \alpha$ erit

$$\alpha = \frac{nafg - (n-1)adg + dfg}{n(n-1)a(f+g) - nfg - (n-1)^2 ad + (n-1)df}$$

quae ſi vt data ſpectetur, hac aequatione relatio inter f et g definitur, vt haec diſtantia principalis oriatur.

Coroll. 2.

32. Quare ſi diſtantia obiecti ante lentem ſit $= a$, eiusque imago principalis poſt lentem ad diſtantiam $= \alpha$ proiici debeat huic aequationi ſatisfieri oportet.

$$\begin{aligned} n(n-1)a\alpha(f+g) - nafg + (n-1)adg - (n-1)^2 a\alpha d &= 0 \\ -n\alpha fg + (n-1)\alpha df \quad - dfg & \end{aligned}$$

Coroll. 3.

33. Hanc autem imaginis principalis diſtantiam α in calculum introducendo expreſſiones noſtrae inuentae haud mediocriter contrahentur. Cum enim ſit $\alpha = \frac{Pg}{(n-1)P + ng}$, erit hinc $P = \frac{n\alpha g}{g-(n-1)\alpha}$

et porro

$$(n-1)P + ng = \frac{ngg}{g-(n-1)\alpha}; \; P + g = \frac{gg + \alpha g}{g-(n-1)\alpha} = \frac{g(\alpha+g)}{g-(n-1)\alpha}$$

$$nP + (n+1)g = \frac{g(\alpha + (n+1)g)}{g-(n-1)\alpha}$$

vide

vnde facta ſubſtitutione ſequentes determinationes ſimpliciores aſſequemur.

$$BV = \alpha - \frac{(n-1)a(a+f)^2(g-(n-1)\alpha)^2(\alpha+(n+1)f)}{2nnfgg((n-1)a-f)^2}\Phi\Phi$$
$$- \frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2}\Phi\Phi$$

et

$$BVN = \frac{((n-1)a-f)g}{(g-(n-1)\alpha)f}\Phi.$$

Scholion.

34. Non ſolum hae formulae multo ſunt breuiores et concinniores, quam primo inuentae, ſed etiam perſpicuus ordo in iis obſeruatur quo litterae α et g cum litteris a et f ita connexae ſunt, vt permutationem admittant. Nullum igitur eſt dubium, quin ſi ſtatim diſtantiam α in calculum introduxiſſemus, via breuiori ad eas peruenire licuiſſet. Ceterum quia hae formulae poſteriores non amplius craſſitiem lentis $AB = d$ inuoluunt, euidens eſt eas ex primo inventis naſci, ſi introducta diſtantia α craſſities d eliminetur, ſeu eius loco hic valor ſurrogetur.

$$d = \frac{n(n-1)a\alpha(f+g) - n(a+\alpha)fg}{(n-1)^2 a\alpha - (n-1)(ag+\alpha f) + fg} = n\frac{(n-1)a\alpha(f+g) - (a+\alpha)fg}{((n-1)a-f)((n-1)\alpha-g)}$$

qui labor autem ne ſuſcipi quidem mereretur, niſi iam ante de eximio eius vſu certiores eſſemus facti. In ſequentibus igitur his elegantioribus formulis vtamur, quoad negotium adhuc ſuccinctius expedire didicerimus.

Proble-

Problema 4.

35. Propoſita lente quacunque faciebus ſphaericis terminata, ſi obiectum in data ab ea diſtantia ſit conſtitutum, pro data lentis apertura ſpatium diffuſionis aſſignare.

Solutio.

Tab. I. Fig. 1. Concipiamus lentem vtrinque conuexam, ſitque faciei anterioris MAM radius $= f$, poſterioris $NBN = g$, lentisque craſſities $AB = d$. Sit porro MM lentis huius apertura, cuius ſemidiameter ſit $= x$; atque in axe lentis expoſitum ſit obiectum vel ſaltem punctum lucidum E, cuius a lente ponatur diſtantia $AE = a$. Iam primo quaeramus eius imaginem principalem, quae cadat in F atque ſupra (31) invenimus fore

$$BF = \frac{nafg - (n-1)adg + dfg}{n(n-1)a(f+g) - nfg - (n-1)^2 ad + (n-1)df}$$

Vocetur ergo haec diſtantia $= \alpha$, et ſi radii EM circa oram aperturae tranſeant, erit angulus $AEM = \Phi = \frac{x}{a}$, quo valore in ſuperioribus formulis ſubſtituto prodibit diſtantia imaginis extremae f a lente ſcilicet:

$$Bf = \alpha - \frac{(n-1)a(a+f)^2(g-(n-1)\alpha)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2} \cdot \frac{xx}{aa}$$
$$- \frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2} \cdot \frac{xx}{aa}$$

ex quo colligitur ſpatium diffuſionis quaeſitum:

$$Ff = + \frac{(n-1)a(a+f)^2(g-(n-1)\alpha)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2} \cdot \frac{xx}{aa}$$
$$+ \frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2} \cdot \frac{xx}{aa}$$

ac

ac praeterea angulus $Bf N$ erit

$$Bf N = \frac{((n-1)a-f)g}{(g-(n-1)\alpha)f} \cdot \frac{x}{a}.$$

Coroll. 1.

36. Quoties ergo ſpatium diffuſionis hoc modo expreſſum eſt poſitiuum, imago extrema propius ad lentem cadit quam principalis ſeu eſt $Bf < BF$. Contra autem ſi iſta expreſſio valorem obtineat n gatiuum, imago extrema a lente longius erit remota principali.

Coroll. 2.

37. Patet hinc etiam ſpatium diffuſionis cum apertura ita creſcere, vt ſit quadrato ſemidiametri aperturae proportionale, ſequetur ergo ipſam aperturae rationem.

Coroll. 3.

38. Spatium diffuſionis etiam hoc modo exprimi poteſt

$$Ff = \begin{cases} +\dfrac{(n-1)a\left(1+\frac{a}{f}\right)^2\left(1-\frac{(n-1)\alpha}{g}\right)^2\left(n+1+\frac{a}{f}\right)}{2nn\left(\frac{(n-1)a}{f}-1\right)^2} \cdot \dfrac{xx}{aa} \\[2ex] +\dfrac{(n-1)\alpha\left(1+\frac{\alpha}{g}\right)^2\left(\frac{(n-1)\alpha}{f}-1\right)^2\left(n+1+\frac{\alpha}{g}\right)}{2nn\left(1-\frac{(n-1)\alpha}{g}\right)^2} \cdot \dfrac{x\,x}{a\,a} \end{cases}$$

et angulus

$$Bf N = \frac{\frac{(n-1)a}{f}-1}{1-\frac{(n-1)\alpha}{g}} \cdot \frac{x}{a}.$$

Coroll. 4.

39. Quia hae formulae introducendis litterarum valoribus reciprocis redditae sunt simpliciores, in hunc modum etiam aequatio (§. 32) exhibita tractetur quae per $a\alpha dfg$ diuisa abit in hanc formam.

$$n(n-1)\frac{1}{d}\left(\frac{1}{f}+\frac{1}{g}\right)-n\frac{1}{d}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+(n-1)\left(\frac{1}{\alpha f}+\frac{1}{a g}\right)-(n-1)^2\frac{1}{fg}-\frac{1}{a\alpha}=0$$

seu

$$\frac{n}{d}\left((n-1)a\alpha\left(\frac{1}{f}+\frac{1}{g}\right)-a-\alpha\right)=\left(\frac{(n-1)a}{f}-1\right)\left(\frac{(n-1)\alpha}{g}-1\right)$$

quae commodior erit tam ad relationem inter a et α, quam inter f et g definiendam.

Scholion.

40. Scilicet si proposita lente obiectum in variis distantiis exponatur ac pro singulis distantiam imaginis principalis definire velimus, aequatio hoc modo referatur:

$$\frac{1}{a\alpha}-(n-1)\left(\frac{1}{\alpha f}+\frac{1}{a g}\right)+n\left(\frac{1}{d a}+\frac{1}{d\alpha}\right)-n(n-1)\left(\frac{1}{d f}+\frac{1}{d g}\right)+(n-1)^2\frac{1}{fg}=0$$

quae per factores ita adornari poterit

$$\left(\frac{1}{a}-\frac{(n-1)}{f}+\frac{n}{d}\right)\left(\frac{1}{\alpha}-\frac{(n-1)}{g}+\frac{n}{d}\right)=\frac{nn}{dd}$$

Vnde patet productum ex his duobus factoribus semper esse idem. Cuius resolutio quo facilius instituatur, capiantur numeri μ et ν ita, vt eorum summa sit $=1$, scilicet $\mu+\nu=1$ ac statuatur

$$\frac{1}{a}-\frac{(n-1)}{f}+\frac{n}{d}=\frac{-\mu n}{\nu d} \text{ erit } \frac{1}{a}=\frac{n-1}{f}-\frac{n}{\nu d}$$

$$\frac{1}{\alpha}-\frac{(n-1)}{g}+\frac{n}{d}=\frac{-\nu n}{\mu d} \text{ erit } \frac{1}{\alpha}=\frac{n-1}{g}-\frac{n}{\mu d}$$

Quare

Quare si distantia obiecti $AE = a$ ita capiatur, vt sit $\frac{1}{a} = \frac{n-1}{f} - \frac{n}{\nu d}$, distantia imaginis principalis $BF = \alpha$ ita se habebit, vt sit $\frac{1}{\alpha} = \frac{n-1}{g} - \frac{n}{\mu d}$.

Eodem modo si dentur binae distantiae $AE = a$ et $BF = \alpha$ cum crassitie lentis $AB = d$, radii facierum f et g ita debent esse comparati, vt sit

$$\frac{1}{f} = \frac{1}{(n-1)a} + \frac{n}{\nu(n-1)d} \text{ et } \frac{1}{g} = \frac{1}{(n-1)\alpha} + \frac{n}{\mu(n-1)d}$$

id quod infinitis modis praestari potest, cum numeri μ et ν pro arbitrio accipi queant, dummodo eorum summa $\mu + \nu$ aequetur vnitati. Si hoc modo etiam in formulis pro spatio diffusionis inuentis omnes litterae in denominatores detrudantur, reperietur:

$$Ff = \frac{(n-1)\alpha\alpha xx}{2nn}\left\{\frac{\left(\frac{1}{a}+\frac{1}{f}\right)^2\left(\frac{1}{\alpha}-\frac{(n-1)}{g}\right)^2\left(\frac{n+1}{a}+\frac{1}{f}\right)}{\left(\frac{n-1}{f}-\frac{1}{a}\right)^2} + \frac{\left(\frac{1}{\alpha}+\frac{1}{g}\right)^2\left(\frac{n-1}{f}-\frac{1}{a}\right)^2\left(\frac{n+1}{\alpha}+\frac{1}{g}\right)}{\left(\frac{1}{\alpha}-\frac{(n-1)}{g}\right)^2}\right\}$$

et

$$\text{angulus } BfN = \frac{\frac{n-1}{f}-\frac{1}{a}}{\frac{1}{\alpha}-\frac{(n-1)}{g}} \cdot \frac{x}{\alpha} = \frac{\frac{n}{\nu d}}{\frac{-n}{\mu d}} \frac{x}{\alpha} = -\frac{\mu}{\nu} \cdot \frac{x}{\alpha}$$

Cum autem sit

$$\frac{n-1}{f} - \frac{1}{a} = \frac{n}{\nu d} \text{ et } \frac{1}{\alpha} - \frac{(n-1)}{g} = \frac{-n}{\mu d} \text{ erit}$$

$$Ff = \frac{(n-1)\alpha\alpha xx}{2nn}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{1}{a}+\frac{1}{f}\right)^2\left(\frac{n+1}{a}+\frac{1}{f}\right)+\frac{\mu\mu}{\nu\nu}\left(\frac{1}{\alpha}+\frac{1}{g}\right)^2\left(\frac{n+1}{\alpha}+\frac{1}{g}\right)\right)$$

et

$$\text{angulus } BfN = -\frac{\mu x}{\nu\alpha} = \frac{\mu x}{(\mu-1)\alpha}.$$

 Proble-

Problema 5.

Tab. I. Fig. 1. 41. Datis diſtantiis obiecti ante lentem $AE = a$ et imaginis principalis poſt lentem $BF = \alpha$ vna cum lentis craſſitie $AB = d$, definire omnes lentes ſatisfacientes, ſimulque pro ſingulis ſpatium diffuſionis Ff.

Solutio.

Modo vidimus ſi lentis faciei anterioris AM radius ponatur $= f$, poſterioris $BN = g$, lente vt conuexa vtrinque ſpectata, hos duos radios ita comparatos eſſe debere, vt ſit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{\nu d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} + \frac{n}{\mu d}$$

ſumtis pro μ et ν numeris quibuscunque, vt ſit $\mu + \nu = 1$; vnde infinitae lentes quaeſito ſatisfacientes obtinentur. Deinde ſi huius lentis ſemidiameter aperturae ponatur $= x$, ſpatium diffuſionis Ff ita exprimi poteſt vt ſit

$$Ff = \frac{\alpha\alpha xx}{2nn(n-1)^2}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{n-1}{a} + \frac{n-1}{f}\right)^2\left(\frac{nn-1}{a} + \frac{n-1}{f}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{n-1}{\alpha} + \frac{n-1}{g}\right)^2\left(\frac{nn-1}{\alpha} + \frac{n-1}{g}\right)\right)$$

et angulus $BfN = \frac{\mu x}{(\mu - 1)\alpha}$.

Quod ſi iam hic pro $\frac{n-1}{f}$ et $\frac{n-1}{g}$ valores aſſignati ſubſtituantur ſpatium diffuſionis erit

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{1}{a} + \frac{1}{\nu d}\right)^2\left(\frac{n}{a} + \frac{1}{\nu d}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{1}{\alpha} + \frac{1}{\mu d}\right)^2\left(\frac{n}{\alpha} + \frac{1}{\mu d}\right)\right)$$

ſeu

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left\{\begin{array}{l} + \frac{\nu\nu}{\mu\mu}\left(\frac{n}{a^3} + \frac{2n+1}{\nu aad} + \frac{n+2}{\nu\nu add} + \frac{1}{\nu^3 d^3}\right) \\ + \frac{\mu\mu}{\nu\nu}\left(\frac{n}{\alpha^3} + \frac{2n+1}{\mu\alpha\alpha d} + \frac{n+2}{\mu\mu\alpha dd} + \frac{1}{\mu^3 d^3}\right) \end{array}\right\}$$

Si

Si craſſities lentis fuerit valde parua, ne confuſio fiat enormis, neceſſe eſt pro μ et ν ſumi numeros vehementer magnos, alterum ſcilicet poſitiuum alterum negatiuum. Cum igitur ſit $\mu d+\nu d=d$, ſtatuatur $\mu d=\frac{d-k}{2}$ et $\nu d=\frac{d+k}{2}$, vt ſit

$$\frac{n-1}{f}=\frac{1}{a}+\frac{2n}{k+d} \text{ et } \frac{n-1}{g}=\frac{1}{\alpha}-\frac{2n}{k-d} \text{ ſeu}$$

$$f=\frac{(n-1)a(k+d)}{2na+k+d} \text{ et } g=\frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

hincque obtinetur ſpatium diffuſionis

$$Ff=\frac{n\alpha\alpha xx}{2(n-1)^2}\left\{\begin{array}{l}+\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2\\+\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2\end{array}\right\}$$

in quibus formulis paruitas craſſitiei lentis $AB=d$ nullum negotium faceſſit: tum vero eſt

angulus $BfN=\frac{k-d}{k+d}\cdot\frac{x}{\alpha}$.

Coroll. 1.

42. Propoſitis ergo binis diſtantiis $AE=a$ et $BF=\alpha$ vna cum craſſitie lentis $AB=d$, infinitis modis lentes idoneae parari poſſunt cum pro k quantitates pro lubitu ſiue poſitiuae ſiue negatiuae aſſumi queant.

Coroll. 2.

43. Cum ſemidiameter aperturae x faciem anteriorem reſpiciat, et angulus BfN ſeu BFN ſit $=\frac{k-d}{k+d}\cdot\frac{x}{\alpha}$, exiſtente diſtantia $BF=\alpha$, manifeſtum eſt ſemidiame-

 trum

trum aperturae poſterioris faciei eſſe debere $= \frac{k-d}{k+d} \cdot x$ vel ſaltem non minorem.

Coroll. 3.

44. Quodſi lentis craſſities tam ſit parua, vt ea prae k contemni queat, formulae noſtrae fient ſimpliciores

$$f = \frac{(n-1)ak}{k+2na} \text{ et } g = \frac{(n-1)\alpha k}{k-2n\alpha}$$

et ſpatium diffuſionis

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k}\right)\left(\frac{1}{a}+\frac{2}{k}\right)^2 + \left(\frac{n}{\alpha}-\frac{2}{k}\right)\left(\frac{1}{\alpha}-\frac{2}{k}\right)^2\right)$$

angulusque $BfN = \frac{x}{\alpha}$.

Scholion.

45. Sufficiat haec de ſpatio diffuſionis in genere quaecunque fuerit lentis craſſities, tradidiſſe, cum non obſtante his ſatis concinnis transformationibus calculus nimium fieret moleſtus, ſi in determinatione ſpatii diffuſionis rationem craſſitiei lentis habere vellemus. Licebit autem, vt modo vidimus, craſſitiem lentis negligere non ſolum, quando ipſa eſt per ſe valde exigua verum etiam dummodo prae quantitate k fuerit perparua. Atque hinc etiam in ſequentibus facile iudicare poterimus, vtrum craſſitiem lentis recte in calculo contemſerimus, nec ne? quouis enim caſu conſideretur quantitas pro k aſſumta, quae ſi multoties fuerit maior quam d error nullus erit pertimeſcendus;

ſcendus; contra vero ſi k non multum ſuperet d multum aberrabitur, quantumuis exigua fuerit craſſities ipſa per ſe.

Vt fiat Ff minimum, definiri poteſt conueniens valor ipſius k hoc modo. Ponatur $\frac{k-d}{k+d}=z$ et pro minimo peruenitur ad hanc aequationem:

$$\begin{aligned} 0 = {} & 2z^4\left(\frac{n}{\alpha}+\frac{1}{d}\right)\left(\frac{1}{\alpha}+\frac{1}{d}\right)^2 \\ & -\frac{2nz^3}{\alpha a}\left(\frac{1}{\alpha}+\frac{1}{d}\right)-\frac{z^3}{d}\left(\frac{1}{\alpha}+\frac{2}{d}\right)^2 \\ & +\frac{2nz}{ad}\left(\frac{1}{a}+\frac{1}{d}\right)+\frac{z}{d}\left(\frac{1}{a}+\frac{2}{d}\right)^2 \\ & -2\left(\frac{n}{a}+\frac{1}{d}\right)\left(\frac{1}{a}+\frac{1}{d}\right)^2 \end{aligned}$$

vnde valor ipſius z erui debet.

Problema 6.

46. Neglecta lentis craſſitie, ſi detur cum obiecti ante lentem diſtantia $AE=a$, tum imaginis principalis poſt lentem diſtantia $BF=\alpha$, eam definire lentem quae pro data apertura minimam pariat diffuſionem.

Solutio.

Poſitis radiis faciei anterioris $AM=f$ et poſterioris $BN=g$ vtraque vt conuexa ſpectata, vidimus omnes lentes datis diſtantiis a et α conuenientes his formulis determinari: ſi ſcilicet pro k ſcribamus $2k$

$$\frac{n-1}{f}=\frac{1}{a}+\frac{n}{k} \text{ et } \frac{n-1}{g}=\frac{1}{\alpha}-\frac{n}{k}$$

deno-

denotante k quantitatem quamcunque. Tum autem ſpatium diffuſionis ita exprimi eſt repertum

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{1}{k}\right)\left(\frac{1}{a}+\frac{1}{k}\right)^2 + \left(\frac{n}{\alpha}-\frac{1}{k}\right)\left(\frac{1}{\alpha}-\frac{1}{k}\right)^2\right.$$

quae expreſſio ad hanc reducitur formam:

$$Ff = \frac{n\alpha\alpha xx}{2(n-\ )^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{2n+1}{k}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{n+2}{kk}\right)$$

Quaeſtio igitur huc reducitur, vt definiatur quantitas k qua huic expreſſioni valor minimus concilietur; cui quidem requiſito ſatisfacit valor

$$\frac{1}{k} = \frac{-(2n+1)}{2(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right) \text{ ſeu } \frac{1}{k} = \frac{2n+1}{2(n+2)}\left(\frac{1}{\alpha}-\frac{1}{a}\right)$$

quo ſubſtituto habebitur:

$$\frac{n-1}{f} = \frac{4+n-2nn}{2(n+2)a} + \frac{n(2n+1)}{2(n+2)\alpha} \text{ et } \frac{n-1}{g} = \frac{4+n-2nn}{2(n+2)\alpha} + \frac{n(2n+1)}{2(n+2)a}.$$

et ſpatium diffuſionis quod eſt minimum:

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)-\frac{(2n+1)^2}{4(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2\right)$$

At eſt

$$n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right) = n\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{n}{a\alpha} \text{ ideoque}$$

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\frac{4n-1}{4(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{n}{a\alpha}\right)$$

quae expreſſio etiam in hanc formam transfunditur

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\frac{4n-1}{4(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{(n-1)^2}{(n+2)a\alpha}\right)$$

ſeu

$$Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

Coroll.

Coroll. I.

47. Vt igitur lens minimum ſpatium diffuſionis producat, eam ita formari neceſſe eſt vt ſit

$$f = \frac{2(n-1)(n+2)a\alpha}{n(2n+1)a+(4+n-2nn)\alpha} \text{ et}$$

$$g = \frac{2(n-1)(n+2)a\alpha}{n(2n+1)\alpha+(4+n-2nn)a}$$

neglecta ſcilicet lentis craſſitie, quae quidem recte negligitur, ſi modo fuerit vehementer parua prae quantitate $k = \frac{2(n+2)a\alpha}{(2n+1)(a-\alpha)}$.

Coroll. 2.

48. Si igitur ſit $a=\alpha$, ſeu $BF=AE$ craſſities lentis, quantacunque fuerit, nihil turbat in ſpatio diffuſionis. Pro hoc autem caſu erit $f=g=(n-1)a$ et ſpatium diffuſionis ipſum $Ff = \frac{nnxx}{(n-1)^2 a}$. At quo magis diſtantiae a et α a ſe inuicem diſcrepant, ſeu minor fuerit quantitas $\frac{a\alpha}{a-\alpha}$, eo magis haec determinatio ob lentis craſſitiem fiet erronea.

Coroll. 3.

49. Spatium autem hoc confuſionis minimum Ff pluribus modis exhiberi poteſt, inter quos commodiſſimum eligi conuenit; ſunt autem praecipui:

I. $Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)a\alpha}\right)$

II. $Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{4n(n+1)}{(4n-1)a\alpha}\right)$

III. $Ff = \frac{n(4n-1)\alpha\alpha xx}{6(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{aa}+\frac{1}{\alpha\alpha}\right)+\frac{2(2nn+1)}{(4n-1)a\alpha}\right)$

IV. $Ff = \frac{n(4n-1)\alpha\alpha xx}{6(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{(2n+1)^2}{(4n-1)a\alpha}\right)$

Coroll. 4.

50. Cum ergo hoc ſpatium diffuſionis ſit minimum, ſi lenti alia quaecunque figura tribuatur, ita tamen, vt obiecti ad diſtantiam $AE = a$ remoti imaginem principalem in diſtantia $BF = \alpha$ exhibeat, ſpatium diffuſionis erit maius, quam hic inuenimus.

Scholion.

51. Quia $n : 1$ denotat rationem refractionis ex aere in vitrum, haec autem pro radiorum natura eſt variabilis, conueniet pro n medium valorem aſſumi, qui eſt $n = \frac{31}{20}$; hinc ergo erit

$$n - 1 = \frac{11}{20};\ n + 2 = \frac{71}{20};\ 2n + 1 = \frac{41}{10};\ 4 + n - 2nn = \frac{149}{200},$$

hincque

$$\frac{n(2n+1)}{2(n-1)(n+2)} = \frac{1271}{781} = 1,627401$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = \frac{149}{781} = 0,190781$$

vnde lens minimam confuſionem pariens ita definietur

$$\frac{1}{f} = \frac{149}{781a} + \frac{1271}{781\alpha} = \frac{0,190781}{a} + \frac{1,627401}{\alpha}$$

$$\frac{1}{g} = \frac{149}{781\alpha} + \frac{1271}{781a} = \frac{0,190781}{\alpha} + \frac{1,627401}{a}$$

ſeu

$$f = \frac{781a\alpha}{145\alpha + 1271a} = \frac{a\alpha}{0,190781\alpha + 1,627401a}$$

et

$$g = \frac{781a\alpha}{149a + 1271\alpha} = \frac{a\alpha}{0,190781a + 1,627401\alpha}$$

Pro

Pro ſpatio autem diffuſionis ipſo definiendo ob $4n-1=\frac{26}{5}$

$$\text{erit } \frac{n(4n-1)}{8(n-1)^2(n+2)}=\frac{8060}{8591}=0,938191$$

$$\text{et } \frac{4(n-1)^2}{4n-1}=\frac{121}{520}=0,232692$$

hincque

$$Ff=0,938191\,\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{0,232692}{a\alpha}\right)$$

vnde reliquae formulàe facile deducuntur. Ceterum hic monendum eſt, quoniam valor $n=\frac{31}{20}$ ex experimentis eſt concluſus, neque ipſe pro omnibus radiorum generibus valet, ſuperfluum fore in praxi hos numeros inuentos nimis ſtudioſe obſeruare; quin etiam ipſa natura minimi aliquam aberrationem permittit. Nihilo vero minus has fractiones decimales ad tot figuras producere viſum eſt, quo facilius quantum ab hac hypotheſi aberretur, dignoſci queat.

Problema 7.

52. Neglecta lentis craſſitie, ſi detur cum obiecti ante lentem diſtantia $AE=a$, tum imaginis principalis poſt lentem diſtantia $BF=\alpha$, eam definire lentem, quae pro data apertura non minimum, ſed datum pariat ſpatium diffuſionis Ff.

Solutio.

Lente vtrinque vt conuexa ſpectata, ſit faciei anterioris radius $=f$, poſterioris $=g$; atque vt ex data diſtantia obiecti $AE = a$, data oriatur imaginis principalis diſtantia $= \alpha$ neceſſe eſt ſit in genere

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{k} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{n}{k}$$

vnde ſpatium diffuſionis ſit

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa} - \frac{1}{a\alpha} + \frac{1}{\alpha\alpha}\right) + \frac{2n+1}{k}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{n+2}{kk}\right)$$

Cum autem ſpatium diffuſionis minimum repertum ſit

$$\frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

neceſſe eſt vt illud ſit maius: ſtatuatur ergo:

$$Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha} + S\right)$$

et habebitur iſta aequatio:

$$4n(n+2)\left(\frac{1}{aa} - \frac{1}{a\alpha} + \frac{1}{\alpha\alpha}\right) + \frac{4(n+2)(2n+1)}{k}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{4(n+2)^2}{kk}$$
$$= (4n-1)\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{a\alpha} + (4n-1)S$$

quae in hanc formam redigitur.

$$(2n+1)^2\left(\frac{1}{a} - \frac{1}{\alpha}\right)^2 + \frac{4(n+2)(2n+1)}{k}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{4(n+2)^2}{kk} = (4n-1)S$$

vnde radice extracta ſit

$$(2n+1)\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{2(n+2)}{k} = \sqrt{(4n-1)S}$$

$$\text{et } \frac{1}{k} = \frac{-(2n+1)}{2(n+2)}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{\sqrt{(4n-1)S}}{2(n+2)}$$

Quare erit

$$\frac{n-1}{f} = \frac{1}{a} - \frac{n(2n+1)}{2(n+2)}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{n}{2(n+2)}\sqrt{(4n-1)S}$$

$$\frac{n-1}{g} = \frac{1}{\alpha} + \frac{n(2n+1)}{2(n+2)}\left(\frac{1}{a} - \frac{1}{\alpha}\right) - \frac{n}{2(n+2)}\sqrt{(4n-1)S}$$

ſiue

fiue

$$f = \frac{2(n-1)(n+2)a\alpha}{(4+n-2nn)\alpha + n(2n+1)a + na\alpha\sqrt{(4n-1)S}}$$

$$g = \frac{2(n-1)(n+2)a\alpha}{(4+n-2nn)a + n(2n+1)\alpha - na\alpha\sqrt{(4n-1)S}}$$

Oportet ergo pro S fumi quantitatem pofitiuam, atque vt cum reliqua parte, cui jungitur, fit homogenea, ponamus

$$S = (\lambda - 1)\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2$$

vbi λ denotat numerum vnitate maiorem; atque effici poterit, vt fpatium diffufionis ita exprimatur

$$Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

quod cum vt datum fpectetur, numerus λ pro dato affumi poterit, ac lens hunc effectum producens ita determinabitur.

$$f = \frac{2(n-1)(n+2)a\alpha}{(4+n-2nn)\alpha + n(2n+1)a + n(a+\alpha)\sqrt{(4n-1)(\lambda-1)}}$$

$$g = \frac{2(n-1)(n+2)a\alpha}{(4+n-2nn)a + n(2n+1)\alpha - n(a+\alpha)\sqrt{(4n-1)(\lambda-1)}}$$

quod ergo cum fignum radicale ambiguo illatum inuoluat, duplici modo fieri poterit.

Coroll. I.

53. Omnes igitur lentes, quae obiecti ad diftantiam $AE = a$ remoti, imaginem principalem in diftantia $BF = \alpha$ repraefentant, hanc habent proprietatem vt fit:

$$\frac{n-1}{f} + \frac{n-1}{g} = \frac{1}{a} + \frac{1}{\alpha} \text{ feu } \frac{fg}{f+g} = \frac{(n-1)a\alpha}{a+\alpha}.$$

 et

et cum ſit

$$n = \tfrac{31}{20} \text{ erit } \frac{fg}{f+g} = \frac{11 a\alpha}{20(a+\alpha)} = \frac{0,55 a\alpha}{a+\alpha}.$$

Coroll. 2.

54. Si ergo lens ſit vtrinque aeque conuexa ſeu $f = g$, oportet eſſe $f = g = \frac{2(n-1)a\alpha}{a+\alpha} = \frac{11 a\alpha}{10(a+\alpha)}$.

ſin autem lens deſideretur planó-conuexa vt ſit $f = \infty$, capi debet

$$g = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11 a\alpha}{20(a+\alpha)} = \frac{0,55 a\alpha}{a+\alpha}$$

At ſi lens debeat eſſe conuexo-plana ſeu $g = \infty$, capi oportet

$$f = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11 a\alpha}{20(a+\alpha)} = \frac{0,55 a\alpha}{a+\alpha}$$

Scholion I.

55. Subſtituamus pro n valorem ipſi conuenientem $\frac{31}{20}$, et iam vidimus fore:

$$\frac{n(4n-1)}{8(n-1)^2(n+2)} = \frac{8060}{8591} = 0,938191$$

$$\frac{4(n-1)^2}{4n-1} = \frac{121}{520} = 0,232692$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = \frac{149}{781} = 0,190781$$

$$\frac{n(2n+1)}{2(n-1)(n+2)} = \frac{1271}{781} = 1,627401$$

nunc vero notari oportet eſſe:

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)} = \frac{62\sqrt{130}}{781} = 0,905133$$

Quare

Quare si lens ita constructur vt sit

$$f = \frac{a\alpha}{0,190781\alpha + 1,627401a \pm 0,905133(a+\alpha)\sqrt{(\lambda-1)}}$$

$$g = \frac{a\alpha}{0,190781a + 1,627401\alpha \mp 0,905135(a+\alpha)\sqrt{(\lambda-1)}}$$

erit pro eius apertura, cuius semidiameter $= x$, spatium diffusionis

$$Ff = 0,938191\, a\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2 + \frac{0,232692}{a\alpha}\right)$$

Cum autem in posterum hi numeri frequentissime occurrant eorum loco ad abbreuiandum certis characteribus vtamur, ponamus ergo

$$\frac{n(4n-1)}{8(n-1)^2(n+2)} = 0,938191 = \mu$$

$$\frac{4(n-1)^2}{4n-1} = 0,232692 = \nu$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = 0,190781 = \varrho$$

$$\frac{n(2n+1)}{2(n-1)(n+2)} = 1,627401 = \sigma$$

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)} = 0,905133 = \tau$$

seu

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{x}{8(n+1)^2}$$

$$\varrho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1$$

$$\sigma = 1 + \frac{1}{2(n-1)} - \frac{1}{n+2}$$

$$\tau = \frac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right)\sqrt{(4n-1)}$$

vnde pro quouis alio medio pellucido hi valores supputari possunt sicque

ſicque vt prodeat ſpatium diffuſionis

$$Ff = \mu\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

lentis conſtructio erit

$$f = \frac{a\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

$$g = \frac{a\alpha}{\varrho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

Dummodo igitur λ fuerit numerus poſitiuus vnitate non minor talis lens duplici modo effici poterit. Caſu autem $\lambda = 1$, quo ſpatium diffuſionis eſt minimum, vnica lens propoſito ſatisfaciens conſtrui poteſt.

Coroll. 3.

56. Si igitur lens ſit vtrinque aequaliter conuexa, ideoque $f = g = \frac{2(n-\ )a\alpha}{a+\alpha} = \frac{11}{10}\cdot\frac{a\alpha}{a+\alpha}$, in expreſſione noſtra pro ſpatio diffuſionis inuenta valor ipſius λ ex aequalitate inter f et g ſtatuta definietur, vnde fit

$$(\sigma-\varrho)(a-\alpha) = 2\tau(a+\alpha)\sqrt{(\lambda-1)} \text{ hincque}$$

$$\lambda = 1 + \frac{0{,}629795\,\alpha\alpha - 1{,}259589\,a\alpha + 0{,}629795\,aa}{(a+\alpha)^2}.$$

Coroll. 4.

57. Si lens capiatur plano conuexa vt ſit:

$$f = \infty \quad \text{et} \quad g = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11}{20}\cdot\frac{a\alpha}{a+\alpha}$$

pro ſpatio diffuſionis habebitur

$$\varrho\alpha+\sigma a = \pm\tau(a+\alpha)\sqrt{(\lambda-1)}$$

vnde in numeris colligitur

$$\lambda = 1 + \frac{0{,}044427\,\alpha\alpha + 0{,}757940\,a\alpha + 3{,}232692\,aa}{(a+\alpha)^2}$$

Coroll.

Coroll. 5.

58. Si denique lens adhibeatur conuexo-plana, vt sit:

$$g = \infty \text{ et } f = \frac{(n-1)\alpha\alpha}{a+\alpha} = \frac{11}{20} \cdot \frac{\alpha\alpha}{a+\alpha}$$

pro ſpatio diffuſionis inueniendo poni oportet

$$\varrho a + \sigma\alpha = \pm \tau(a+\alpha)\sqrt{(\lambda - 1)}$$

vnde in numeris colligimus:

$$\lambda = 1 + \frac{3,232692\alpha\alpha + 0,757940 a\alpha + 0,044437 aa}{(a+\alpha)^2}$$

Scholion 2.

59. Quod ad aperturam attinet iam initio animaduertimus, in ea maiores arcus comprehendi non debere, quam qui ſint principiis ſtabilitis conformes. Scilicet vt nullus angulus ſupra 30° gradus occurrat, anguli ACM et BDN certe minores 30 gradibus eſſe debent, cum anguli EM*c* et VN*d* ipſis ſint maiores, quorum alter cum quandoque ad duplum aſſurgere poſſit, poterimus hanc regulam ſtatuere, vt aperturae ſemidiameter x neque $\frac{1}{4} f$ neque $\frac{1}{4} g$ ſuperet. Verum quouis caſu ad ipſos angulos EM*c* et VN*d*, qui ſunt maximi, attendi conueniet, atque tanta apertura admitti poterit, vnde neuter horum angulorum 30 gradus ſuperans oriatur, ſi cautius procedere velimus, etiam angulos 20 gradibus minores euitare poterimus, quo pacto apertura magis reſtringetur. Tab. I. Fig. 2.

Proble-

Problema 8.

Tab. I. Fig. 3. 60. Non neglecta lentis crassitie AB si pro distantia obiecti $AE = a$ detur distantia imaginis principalis $BF = \alpha$, obiectum autem parumper longius in e remoueatur, definire locum imaginis principalis f.

Solutio.

Posita lentis crassitie $AB = d$, radiisque faciei anterioris $AM = f$ et posterioris $BN = g$, supra vidimus hos radios ita a binis distantiis a et α atque crassitie lentis d pendere debere, vt sit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d}$$

denotante k quantitatem quamcunque. Hinc ergo cum sit

$$\frac{k+d}{2n} = \frac{af}{(n-1)a-f} \text{ et } \frac{k-d}{2n} = \frac{\alpha g}{g-(n-1)\alpha}$$

erit eliminando k.

$$\frac{d}{n} = \frac{af}{(n-1)a-f} - \frac{\alpha g}{g-(n-1)\alpha}.$$

Ponamus iam distantiam $AE = a$ crescere particula $Ee = da$ ac per differentiationem inueniemus, quantum inde distantia imaginis $BF = \alpha$ immutetur: habebimus scilicet:

$$\frac{-ffda}{((n-1)a-f)^2} - \frac{gg\,d\alpha}{(g-(n-1)\alpha)^2} = 0$$

unde elicimus

$$d\alpha = \frac{-ff(g-(n-1)\alpha)^2}{gg((n-1)a-f)^2}\,da = \frac{-\alpha\alpha}{aa}\,da\left(\frac{k+d}{k-d}\right)^2$$

Quare

Quare obiecto E per ſpatiolum minimum Ee longius a lente remoto, imago principalis ex F propius ad lentem admouebitur per ſpatiolum minimum Ff, ita vt ſit

$$Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 . Ee = \frac{ff(g-(n-1)\alpha)^2}{gg((n-1)a-f)^2} . Ee.$$

Coroll. 1.

61. Quia quantitas $\frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2$ neceſſario eſt poſitiua, euidens eſt ſi obiectum longius a lente remoueatur, imaginem ſemper propius ad lentem admoueri. Contra ergo etiam ſi obiectum propius ad lentem accedat, imago longius ab ea recedet.

Coroll. 2.

62. Si craſſities lentis d euaneſcat erit $Ff = \frac{\alpha\alpha}{aa} . Ee$, ſin autem ea non euaneſcat fieri poteſt vt ſit vel $Ff > \frac{\alpha\alpha}{aa} Ee$ vel $Ff < \frac{\alpha\alpha}{aa} . Ee$, prius eueniet ſi k ſit quantitas poſitiua, poſterius ſi negatiua. At ſi ſit vel $k = \infty$ vel $k = 0$, vtroque caſu erit $Ff = \frac{\alpha\alpha}{aa} . Ee$ etiamſi craſſities lentis non euaneſcat.

Coroll. 3.

63. Cum in diſtantiis a et α mutatio minima fieri concipiatur, ſpatium diffuſionis nullam inde variationem ſubire cenſendum eſt: ſiue ergo obiectum in E ſiue e reperiatur, ac ſemidiameter aperturae

lentis in facie anteriore fuerit $= x$, erit ſpatium diffuſionis:

$$\frac{n\alpha a x x}{2(n-1)^2}\left\{\begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2 \end{array}\right\}$$

in facie autem poſteriori ſemidiameter aperturae debet eſſe $=\frac{k-d}{k+d}\cdot x$.

Problema 9.

Tab. I. Fig. 3. 64. Rationem definire, quam habet magnitudo imaginis ad magnitudinem obiecti non neglecta lentis craſſitie.

Solutio.

Sit obiecti ante lentem diſtantia $AE = a$, imaginis vero $BF = \alpha$ exiſtente lentis craſſitie $= d$; vbi quidem tantum imaginem principalem ſpectamus neglecto ſpatio diffuſionis. Sit iam $E\varepsilon$ obiectum, cui tribuatur magnitudo quam minima $E\varepsilon = z$, normaliter axi lentis inſiſtens; eiusque imago in $F\zeta$ exhibebitur, cuius magnitudo $F\zeta$ quaeritur. Cum igitur punctum ζ a puncto ε oriatur, ita vt radii ab ε emiſſi in ζ colligantur, conſideretur radius quicunque εM, qui productus cum axe in e occurrat: et perinde eſt, ac ſi hic radius ex axis puncto e emanaret. Quare is poſt refractionem cum axe in f concurret, vt ſit $Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 \cdot Ee$, indeque ad ζ perget: ex quo magnitudinem $F\zeta$ definire licebit. Statuatur

in

in hunc finem $AM = x$, erit $BN = \frac{k-d}{k+d} \cdot x$; hincque colligemus has proportiones:

$$Ee : E\varepsilon = eA : AM = a : x$$

$$Ff : F\zeta = fB : BN = \alpha : \frac{k-d}{k+d} x$$

ob spatiola Ee et Ff quam minima: vnde habebimus

$$\frac{Ff}{Ee} : \frac{F\zeta}{E\varepsilon} = \frac{\alpha}{a} : \frac{k-d}{k+d} = \frac{\alpha\alpha}{aa} \left(\frac{k+d}{k-d}\right)^2 : \frac{F\zeta}{E\varepsilon}$$

Concluditur ergo $\frac{F\zeta}{E\varepsilon} = \frac{\alpha}{a} \cdot \frac{k+d}{k-d}$, ex quo cum magnitudo obiecti $E\varepsilon$ posita sit $= z$, erit magnitudo imaginis $F\zeta = \frac{\alpha(k+d)}{a(k-d)} z$.

Coroll. 1.

65. Secundum hanc ergo rationem diameter obiecti immutatur. Vbi notandum est, si expressio $\frac{\alpha(k+d)}{a(k-d)}$ habeat positiuum valorem obiecti imaginem situ inuerso repraesentari, contra autem situ erecto, si $\frac{\alpha(k+d)}{a(k-d)}$ negatiuum valorem adipiscatur.

Coroll. 2.

66. Si crassities lentis euanescat, fit $F\zeta = \frac{\alpha}{a} z$. quo ergo casu recta iungens puncta extrema ε et ζ transit per centrum lentis. At si crassities in computum ducatur, recta $\varepsilon\zeta$ modo intra lentem modo extra, axem lentis secare poterit.

Scholion.

67. Sic igitur omnia, quae circa vnam lentem quamcunque nosse oportet, expediuimus, vt etiam

craſſitiei rationem habuerimus. Ac primo quidem ad binas diſtantias lentis quaſi determinatrices ſpectari conuenit, quae ſunt obiecti diſtantia ante lentem $AE = a$ et imaginis principalis poſt lentem diſtantia $BF = \alpha$, quibus addi debet lentis craſſities $AB = d$. His autem conditionibus infinitae lentes ſatisfaciunt; ſi enim radius faciei anterioris AM dicatur $= f$, et poſterioris $BN = g$, vtraque tanquam conuexa conſiderata, conſtructio lentis his continetur formulis:

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d}$$

ſeu

$$f = \frac{(n-1)a(k+d)}{k+d+2na} \text{ et } g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

exiſtente $n = \frac{31}{20}$; vbi k eſt quantitas arbitraria, hincque lentes innumerabiles quaeſito ſatisfacientes obtinentur.

Quod ſi iam obiecti diameter ponatur $= z$, erit imaginis principalis repraeſentatae diameter $= \frac{\alpha(k+d)}{a(k-a)} z$, quatenus imago ſitu inuerſo exhibita conſideratur.

Deinde ſi aperturae in facie anterioris lentis ſemidiameter ſit $= x$, ſpatium diffuſionis, quatenus ab imagine principali ad lentem porrigitur, ita exprimetur vt ſit:

$$\frac{n\alpha\alpha xx}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2 \left(\frac{n}{a} + \frac{2}{k+d}\right)\left(\frac{1}{a} + \frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2 \left(\frac{n}{\alpha} - \frac{2}{k-d}\right)\left(\frac{1}{\alpha} - \frac{2}{k-d}\right)^2 \end{array} \right\}$$

in facie autem poſteriori aperturae ſemidiameter debet eſſe $= \frac{k-d}{k+d} \cdot x$ vel ſaltem non minor. Quia ſpatium diffu-

diffusionis factorem habet $\alpha\alpha xx$, breuitatis gratia id ita $P\alpha\alpha xx$ indicabimus. Haecque in genere teneantur etiam crassitiei lentis ratione habita. At si crassities lentis euanescat, formulas magis euoluere licuit, scilicet si breuitatis gratia ponatur:

$$\mu = 0,938191;\ \varrho = 0,190781;\ \tau = 0,905133$$

$$\nu = 0,232692;\ \sigma = 1,627401;\ \lambda > 1 \text{ arbitr.}:$$

sumaturque pro formatione lentis:

$$f = \frac{a\alpha}{\varrho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

$$g = \frac{a\alpha}{\varrho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

spatium diffusionis erit pro aperturae semidiametro x

$$\mu\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

et obiecti diametro existente $= z$ imaginis diameter erit $= \frac{\alpha}{a}z$. His ergo pro vna lente determinatis, videamus quomodo in combinatione duarum pluriumve lentium spatium diffusionis definiatur: vt deinceps in omnis generis instrumentis dioptricis siue Telescopiis siue microscopiis confusionem assignare, modumque eam diminuendi inuestigare possimus.

CAPVT II.