LETTERS

OF

EULER

ON DIFFERENT SUBJECTS

IN

NATURAL PHILOSOPHY.

ADDRESSED TO

A GERMAN PRINCESS.

WITH

NOTES, AND A LIFE OF EULER,

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PREFACE.

It has long been a reproach against English Literature, that the composition of elementary and popular works has been left almost exclusively in the hands of inferior writers, who possess only a general and superficial knowledge of the subjects of which they treat. The influence of this practice upon the diffusion of general and correct knowledge, has been deeply felt by those who are desirous of introducing a system of Education, which embraces a wider range than the ordinary routine of classical instruction. The popular writings of those who acquire a knowledge of science for the purpose of teaching it, differ in the most essential manner from those of a Philosopher, who devotes himself to the task of perspicuous illustration. However correct may be the principles, and however copious the details of a compiled work, it must always be defective in

the selection of its topics, in the clearness of its reasoning, in the generality of its views, and in the suitableness of its illustrations. A mind like that of EULER, which from its infancy has been devoted to the study of Nature, selects at once the prominent features of the science which it is proposed to explain; excludes all perplexing and extraneous facts, and combines under general views the important truths which it is the object of the pupil to seize and retain. The justness of this remark cannot fail to be admitted by those who read the following Letters, which may be justly characterized as the most popular work that ever was written, and as the production of the profoundest Philosopher that ever wrote.

EULER'S Letters to a German Princess were first made known in Europe by an edition published by the Marquis CONDORCET and M. LACROIX, who enriched it with a variety of notes, and whose opinion of the work for the purposes of public instruction may be acceptable to the English reader :—

" The Letters of EULER to a German Princess," says M. CONDORCET, " have acquired, over all Europe, a celebrity, to which the reputation of the Author, the choice and importance of the several subjects, and the clearness of elucidation, justly entitle them. They have deservedly been considered as a treasury of science, adapted to the purposes of every common seminary of learning. They may be studied to advantage without much previous elementary knowledge; they convey accurate ideas respecting a variety of objects, highly interesting in themselves, or calculated to excite a laudable curiosity; they inspire a proper taste for the sciences, and for that sound philosophy which, supported by science, and never losing sight of her cautious, steady, methodical advances, runs no risk of perplexing or misleading the attentive student."

The English Reader is indebted to the late Rev. Dr. HENRY HUNTER, for the following Translation, which has gone through two Editions. In this Third Edition the Tranlation has received very essential improvements. The Plates have been re-engraved, and much improved, and a Life of the Author has been added, together with various Notes, which, the Editor trusts, will be both interesting and useful to the Reader.

Independently of the great popularity of this Work, it possesses a particular interest at the present time, in consequence of its containing a popular view of the Doctrine, that Light consists in the indulations of an Ethereal Medium, which is now generally adopted, in consequence of recent discoveries in Optics.

EDINBURGH, 22*d July* 1823.

THE LIFE OF EULER.

LEONARD EULER, one of the most distinguished mathematicians of the 18th century, was the son of Paul Euler, and Margaret Brucker, and was born at Bâsle on the 15th of April 1707.

His father, who had been instructed in mathematics by the celebrated James Bernoulli, became pastor of the village of Riechen, near Bâsle, in the year 1708; and as soon as his son had arrived at the proper age, he instilled into him a fondness for mathematical learning, although he had destined him for the study of theology. He was afterwards sent to the university of Bâsle, where he was found worthy to receive lessons from John Bernoulli, who was at that time regarded as the first mathematician in Europe. The assiduity and amiable disposition of Euler soon gained him the particular esteem of that great master, and the friendship of his two sons, Daniel and Nicolas Bernoulli, who had already become the disciples and the rivals of their father. John Bernoulli even condescended to give him once every week a particular lesson, for the purpose of explaining the difficulties which he encountered in the course of his studies. Euler had not the good fortune to enjoy long this inestimable advantage. In 1723, he received the degree of Master of Arts;

and on this occasion he obtained great applause, by the Latin discourse which he delivered, containing a comparison between the Newtonian and Cartesian philosophy. At the request of his father, he now began the study of theology; but his attachment to the mathematics was so strong, that his father at last consented to allow him to follow the bent of his own genius.

Nicolas and Daniel Bernoulli having, in 1725, accepted the invitation of Catherine I. to become a member of the Academy of Sciences of St. Petersburg, promised at their departure to employ their influence to procure for Euler an appointment in that city. In the following year they announced that they had a situation in view for him, and strongly advised him to apply his mathematical knowledge to physiology. Euler immediately attended the lectures of the most eminent medical professors at Bâsle, and made rapid progress in the study of medicine. His attention, however, was still directed to his favourite pursuits, and he found leisure to compose a dissertation on the *Nature and Propagation of Sound*, and another on the *Masting of Ships*, which was written for the prize proposed by the Academy of Sciences in 1727. As this subject was actually suggested by several members of the Academy, with the view of bringing into notice the talents of M. Bouguer, who had paid particular attention to the subject, and who was then professor of hydrography in the seaport town of Croisic, it was not likely that Euler, who was destitute of all practical knowledge of naval affairs, should have succeeded in the competition. Bouguer, of course, carried off the first prize; but Euler obtained what is called the *accessit*, or second prize, an honour of no trivial magnitude, when we consider that he was then only twenty years of age. About this time

Euler was a candidate for the vacant professorship of natural philosophy in the university of Bâsle; but he had not the good fortune to be elected.

Daniel and Nicolas Bernoulli used all their influence to procure an appointment for their young friend; and having at last succeeded, they requested him to repair immediately to St. Petersburg. Euler lost no time in obeying this welcome summons; but, after he had begun his journey, he had the mortification to learn that Nicolas Bernoulli had fallen a victim to the severity of the climate; and the very day upon which he entered the Russian territory, was that of the death of the Empress Catherine I.; an event which at first threatened the dissolution of the Academy, of which she had laid the foundation. Having reached St. Petersburg at this unfortunate period, Euler resolved to enter into the Russian navy, and had actually received the promise of a lieutenancy, and rapid promotion, from Admiral Sievers; but fortunately for geometry, a change took place in the aspect of public affairs in 1730, and Euler obtained the situation of Professor of Natural Philosophy. In 1733 he succeeded Daniel Bernoulli, when that illustrious mathematician retired into the country; and in the same year he married Mademoiselle Gsell, a Swiss lady, and the daughter of a painter, whom Peter the Great had carried into Russia upon his return from his first tour. In 1735, a very intricate problem having been proposed by the Academy of St. Petersburg, Euler completed the solution of it in three days; but the exertion of his mind had been so violent, that it threw him into a fever, which endangered his life, and deprived him of the use of one of his eyes. In 1738, the Academy of Sciences at Paris crowned his memoir, entitled, *Sur la Nature et les Proprietés du Feu*; and in 1740, he divided with Daniel Bernoulli,

and our countryman Colin Maclaurin, the prize given by the same Academy for the best dissertation on the flux and reflux of the sea. Daniel Bernoulli had treated the subject with a sagacity and method which characterized all his labours. The dissertation of Maclaurin contained his celebrated theorem on the equilibrium of elliptical spheroids; and that of Euler was marked with an improvement on the integral calculus, which seemed to resolve the fundamental equation of almost all the great problems on the motions of the heavenly bodies.

In consequence of an invitation from the King of Prussia, through his minister the Count de Mardefield, Euler quitted St. Petersburg, and went to Berlin in the month of June 1741. Upon his arrival, he was honoured with a letter from the King of Prussia, written from his camp at Reichenbach; and he was soon after presented to the queen-mother, a princess who took great pleasure in the conversation of illustrious men. She treated Euler with the utmost familiarity; but never being able to draw him into any conversation but that of monosyllables, she one day asked him why he did not wish to speak to her? "Madam," replied Euler, "it is because I have just come from a country where every person who speaks is hanged."

The memoirs and works with which Euler enriched mathematics and physics, are so extremely numerous, that it would occupy many pages to give even the briefest account of them. In many of his physical memoirs, Euler has been justly reproached for having applied the calculus to the most unfounded physical hypotheses, or to metaphysical principles which had not been sufficiently examined; and on this account several of his memoirs have no value whatever, except in so far as they exhibit fine specimens of the resources of analysis. His Dissertations on Wind-Mills, on Achromatic Telescopes, on Naval Architecture, and on Gunnery, are among the number of those which are liable to this criticism.

When Euler was at Berlin, the Princess of Anhalt-Dessau, the niece of the King of Prussia, was desirous to receive from him some instruction in the different branches of natural philosophy; and for her use he drew up the present work, which was translated into most of the languages of Europe, and which has always been much esteemed, particularly for the singular perspicuity with which its author has explained some of the most profound truths in physics. The King of Prussia often employed Euler in calculations relative to the mint, and other objects of finance,—in the conducting of the waters of San Souci,—and in the examination of canals, and other public works.

In 1744, Euler was appointed Director of the Mathematical Class of the Academy, and in the same year he obtained the prize offered by the Academy of Sciences of Paris, for the best work on the theory of magnetism.

About this time Robin's Treatise on Gunnery had appeared in England, and though our countryman had treated Euler with great severity, this act of injustice did not prevent him from recommending it to the King of Prussia, as the best book on the subject. He even translated it, and in the additions which he made, he gave a complete theory of the motion of projectiles. M. Turgot ordered this work to be translated into French, and introduced into the schools of artillery; and about the same time there appeared a splendid edition of it in England.

In 1746, he published his new Theory of Light and Colours; and in 1759, his memoir *Sur les effets*

du Roulis et du Tangage, gained the prize offered by the French Academy of Sciences.

In 1750, Euler went to Frankfort to receive his mother, who was then a widow, and to conduct her to Berlin, where she remained till the time of her death, in 1761; having enjoyed for eleven years the assiduous attention of a favourite son, and the high pleasure of seeing him universally esteemed and admired.

When Euler remained at Berlin, he formed an intimate acquaintance with M. De Maupertuis, the learned President of the Prussian Academy of Sciences, and he defended Maupertuis' celebrated and favourite principle of the least action, by resolving, by means of it, some of the most difficult problems in mechanics. In the dispute into which he was thus led with Kœnig, who had attacked Maupertuis in 1751, he lost for a while his usual serenity, and became one of the enemies of that unfortunate individual.

Although the number of foreign associates in the French Academy of Sciences was limited to eight, yet Euler was appointed to the ninth place in 1755, on the condition that no appointment should take place at the first vacancy.

In the year 1760, the Russian army under General Tottleben penetrated into the March of Brandenburg, and pillaged a farm which Euler possessed near Charlottenberg. As soon as the Russian general was informed of the event, he immediately repaired the loss by a very large sum; and upon giving notice of the circumstance to the Empress Elizabeth, she added to this indemnity a present of four thousand florins. This act of generosity, no doubt, had a powerful effect in attaching Euler to the Russian government, which, in spite of his absence, had always paid him the pension which it granted him in 1742. Having received an invitation from the Empress Catherine, he obtained permission from the King of Prussia to return to St. Petersburg to spend the remainder of his days; but his eldest son was not allowed to accompany him. When Euler was on the eve of his departure, Prince Czartorisky invited him, in the name of the King of Poland, to take the road of Warsaw, where, loaded with kindness, he spent ten days with Stanislaus, who afterwards honoured him with his correspondence.

Shortly after his arrival in St. Petersburg, on the 17th July 1766, he lost the sight of his other eye, having been for a considerable time obliged to perform his calculations with large characters, traced with chalk upon a slate. His pupils and his children copied his calculations, and wrote all his memoirs, while Euler dictated to them. To one of his servants, who was quite ignorant of mathematical knowledge, he dictated his *Elements of Algebra*, a work of very great merit, which has been translated into English and many other languages. Euler now acquired the rare faculty of carrying on in his mind the most complicated analytical and arithmetical calculations; and M. d'Alembert, when he saw him at Berlin, was astonished at some examples of this kind which occurred in their conversation. With the design of instructing his grandchildren in the extraction of roots, he formed a table of the six first powers of all numbers, from 1 to 100, and he recollected them with the utmost accuracy. Two of his pupils having computed to the 17th term, a complicated converging series, their results differed one unit in the fiftieth cypher; and an appeal being made to Euler, he went over the calculation in his mind, and his decision was found correct.

His principal amusement, after he lost his sight, was to make artificial loadstones, and to give lessons on mathematics to one of his grandchildren, who seemed to evince a taste for the science.

In 1771, a dreadful fire broke out in St. Petersburg, and reached the house of Euler. Peter Grimm, a native of Bâsle, having learned the danger in which his illustrious countryman was placed, threw himself among the flames, and, reaching Euler's apartment, brought him off on his shoulders, at the risk of his life. His library, however, and his furniture were consumed; but, by the activity of Count Orloff, his MSS. were saved.

Having revised the lunar theory with the aid of his son, and of his colleagues Krafft and Lexell, he constructed a set of new lunar tables, which appeared in 1772. These tables were, at the suggestion of Turgot, rewarded by the Board of Longitude in France; and when the more perfect tables of Mayer obtained the great premium of three thousand pounds offered by the British Parliament, the sum of three hundred pounds was given to Euler, for having furnished the theorems made use of by Mayer in his theory.

In the year 1773, Euler published, at St. Petersburg, his great work on the construction and management of vessels. A new edition soon afterwards appeared at Paris, and at the desire of the French King, it was introduced into the Schools of Marine, and a reward of 1000 rubles transmitted to the author, accompanied by a handsome letter from the celebrated Turgot. About the same time an Italian, an English, and a Russian translation of it appeared, and the Russian government presented Euler with a gift of 2000 rubles.

Three of Euler's memoirs on the Inequalities in the motions of the Planets, were crowned by the French Academy of Sciences; and he also gained the

prizes of 1770 and 1772, by his perfection of the lunar theory.

Having lost his first wife, by whom he had thirteen children, eight of whom died in early life, he was married a second time, in 1776, to Mademoiselle Gsell, the aunt of his first wife.

Euler underwent the operation of couching, which was attended with the happy result of restoring his sight; but whether from the negligence of his surgeon, or from his being too eager to avail himself of his new organs, he again lost his sight, and suffered much severe pain from the relapse. His love for science, however, continued unabated, and in the course of seven years he transmitted seventy memoirs to the Academy of St. Petersburg. On the 7th of September 1783, after having amused himself with calculating upon a slate the laws of the ascensional motion of balloons, which at that time occupied the attention of philosophers, he dined with his relation M. Lexell, and spoke of the planet Herschel, and of the calculations by which its orbit was determined. A short time afterwards, he was amusing himself with one of his grandchildren, when, on a sudden, his pipe fell from his hand, and he expired of an apoplectic stroke, in the 79th year of his age.

Euler left behind him three sons, having lost his two daughters in the latter years of his life. Twenty-six out of thirty of his grandchildren were alive at the time of his death.

After a long life, so successfully devoted to the sciences, Euler's reputation was very widely extended. Besides being a foreign member of the Academy of Sciences at Paris, he was a Fellow of the Royal Society of London, and he had received from most of the Princes of the North, with whom he was well acquainted, the most flattering marks of their esteem. When the Prince-Royal of Prussia

visited St. Petersburg, he anticipated the visit of Euler, and passed several hours at the bed-side of this great man, holding him all the time by the hand, and having, at the same time, upon his knee, one of Euler's grandchildren, who had displayed a premature attachment to geometry. The death of Euler was considered as a public loss even in the country where he lived: and the Academy of St. Petersburg decreed to him, at their own expense, a marble bust, which was placed in their public hall. In an allegorical picture which the Academy had put up during his life, Geometry was represented as placed upon a basement covered with calculations. These calculations were the formulæ of Euler's Theory of the Lunar Motions.

Euler's knowledge was not limited to mathematics and the physical sciences. He had carefully studied anatomy, chemistry, and botany, and he was also deeply versed in ancient literature. He could repeat the Æneid from the beginning to the end, and he could even tell the first and last lines in every page of the edition which he used. In one of his works there is a learned memoir on a question in mechanics, of which, as he himself informs us, a verse of the Æneid gave him the first idea.

Euler possessed naturally a strong constitution; and when we consider the nature of his studies, and the assiduity with which he pursued them, we cannot fail to be surprised at the great degree of health which he enjoyed. In all his habits he was sober and temperate,—in his manners unaffected and pleasing,—and in his temper lively and cheerful. In his moral and religious character there is much to admire. The high fame which he acquired, and the interruptions which he must have experienced, both at Berlin and St. Petersburg, never induced him to abandon the religious duties to which he had been

educated. As long as he preserved his sight, he assembled the whole of his family every evening, and read a chapter of the Bible, which he accompanied with an exhortation. Theology was one of his favourite studies, and the doctrines which he held were the most rigid doctrines of Calvanism.

The following is a list of the principal works which Euler published in a separate form. His papers, which appeared in the Memoirs of the Academies of Berlin and St. Petersburg, are extremely numerous; and he left behind him no fewer than *two hundred* ready for publication, in order to fulfil a promise which he had made to Count Orloff, to supply memoirs for the *Acta Petropolitana* for twenty years after his death.

Dissertatio Physica de Sono. Basle, 1727.

Mechanica, sive motus scientia analytice exposita. Petropoli, 1736, 2 vols.

Tentamen novæ theoriæ musicæ. Petrop. 1739. This work contains many new views; but as M. Fuss remarks, it had no great success, as it contained too much geometry for musicians, and too much music for geometers.

Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes. Lausannæ, 1744, 4to.

Introductio in Analysin Infinitorum. Lausannæ, 1744, 2 vols. 4to. This work, which had become very scarce, was reprinted at Lyons in 1797. It was translated into French in 1796, by J. B. Labey, and published at Paris.

Theoria motuum planetarum et cometarum. Berolini, 1744.

Opuscula varii argumenti. Berolini, 1746, 1750, 1751, 3 vols. in 4to. The tables of the sun and moon, which are sometimes to be found separately, form part of the 1st volume of this collection. As

the three volumes make only 600 pages, they are generally found in one.

Scientia navalis, seu tractatus de construendis ac dirigendis navibus. Petropoli, 1749, 2 vols. 4to.

Theoria motuum Lunæ exhibens omnes corporum inequalitates cum additamento. Berolini, 1753, 4to.

Dissertatio de principio minimæ actionis, una cum examine objectionum Cl. Koenigii, contra hoc principium factarum. Berolini, 1753, 4to.

Institutiones calculi differentialis, cum ejus usu in analysi infinitorum ac doctrina serierum. Berolini, 1755. Another edition of this work was published in 1787, in 2 vols. 4to, and another at St. Petersburg in 1804, in 2 vols. 4to.

Constructio lentium objectivarum ex duplici vitro. Petrop. 1762.

Meditationes de perturbatione motus cometarum ab attractione planetarum orta. Petrop. 1762, 4to.

Theoria motus corporum solidorum seu rigidorum. Rostochii, 1765, 4to.

Institutiones Calculi Integralis. Petrop. 1768-1770, 3 vols. 4to. Another edition, more correct, was published at Petersburg in 1792 and 1794, in 4 vols. 4to.

Dioptrica. Petrop. 1769, 1771, 3 vols. 4to.

Novæ Tabulæ Lunares singulari methodo constructæ. Petrop. 1772, 8vo.

Opuscula Analytica. Petrop. 1783, 1785, 2 vols. 4to.

Lettres a une Princesse d'Allemagne sur quelques sujets de Physique et de Philosophie. Petersburg, 1768, 1772, 3 vols. 8vo. Another edition was published at Berne in 1778, in 3 vols. 8vo. Another edition was published at Paris, with notes by Condorcet, and another in 1812, by J. B. Labey.

Elemens d'Algebre trad. de l'Allemand, par J. Bernoulli, avec des notes par Lagrange et Garnier. Paris, 1807, 2 vols. 8vo. Two editions of this work were published at Lyons in 1774 and 1796, and an edition appeared in London translated into English.

Theorie complete de la construction et de la manœuvre des vaisseaux, (le style retouché par Keralio.) Paris, 1776, 8vo. The original edition of this work appeared at St. Petersburg in 1773.

A collection of the best productions of Euler appeared at Brienne in 1797, in 18 volumes.

A more extended list of the writings of this illustrious mathematician will be found in his Eulogé by Nicholas Fuss, which was published at St. Petersburg, in 1783, in 4to.