CAPUT III.

DE INVENTIONE DIFFERENTIARUM FINITARUM

44.

Quemadmodum ex functionum differentiis finitis earum differentialia facile inveniri queant, in initio fusius exposuimus, atque adeo ex hoc fonte principium differentialium derivavimus. Si enim differentiae, quae assumtae erant finitae, evanescant, in nihilumque abeant, oriuntur differentialia; & quia hoc casu plures & saepe innumeri termini, qui differentiam finitam constituunt, reiiciuntur, differentialia multo facilius inveniri, atque commodius succinctiusque exprimi possunt, quam differentiae finitae. Neque igitur hinc vicissim via patere videtur, a differentialibus ad differentias finitas ascendendi. Interim tamen eo modo, quo hic utemur, ex differentialibus omnium ordinum cuiuscunque functionis, eiusdem differentiae finitae omnes definiri poterunt.

45. Sit y functio quaecunque ipsius x, quae cum posito $x + dx$ loco x abeat in $y + dy$, si denuo loco x ponatur $x + dx$, valor $y + dy$ suo differentiali $dy + ddy$ augebitur, fietque $= y + 2dy + ddy$, qui ergo valor respondebit ipsius x valori $x + 2dx$. Simili modo si ponamus quantitatem x continuo suo differentiali dx augeri, ut successive valores

$$x + dx;\ x + 2dx;\ x + 3dx;\ x + 4dx;\ \&c.$$

induat, valores ipsius y respondentes erunt, quos haec tabella indicat:

Valores ipsius	Valores respondentes functionis
x	y
$x + dx$	$y + dy$
$x + 2dx$	$y + 2dy + ddy$
$x + 3dx$	$y + 3dy + 3ddy + d^3y$
$x + 4dx$	$y + 4dy + 6ddy + 4d^3y + d^4y$
$x + 5dx$	$y + 5dy + 10ddy + 10d^3y + 5d^4y + d^5y$
$x + 6dx$	$y + 6dy + 15ddy + 20d^3y + 15d^4y + 6d^5y + d^6y$
&c.	&c.

46. Generaliter ergo si x abeat in $x + ndx$, functio y recipiet hanc formam:

$$y + \frac{n}{1}dy + \frac{n(n-1)}{1.\ 2}ddy + \frac{n(n-1)(n-2)}{1.\ 2.\ 3}d^3y$$
$$+ \frac{n(n-1)(n-2)(n-3)}{1.\ 2.\ 3.\ 4}d^4y + \&c.$$

in qua expressione, etsi quilibet terminus infinities minor est quam praecedens, tamen nullum praetermisimus, quo ista formula ad praesens negotium apta redderetur. Statuemus enim pro n numerum infinite magnum, & quoniam notavimus, fieri posse ut productum ex quantitate infinite magna in infinite parvam aequetur quantitati finitae, terminus secundus utique homogeneus fieri poterit primo, seu ndy quantitatem finitam repraesentare poterit. Ob eandemque rationem terminus tertius $\frac{n(n-1)}{1.\ 2}ddy$, etsi ddy infinities minus est quam dy, tamen quia alter factor $\frac{n(n-1)}{1.\ 2}$ infinities maior est quam n, terminus quoque tertius quantitatem finitam exprimere poterit: sicque posito n numero infinito nullum illius expressionis terminum reiicere licebit.

47. Posito autem n numero infinito quocunque is nu-

mero finito ſive augeatur ſive diminuatur, numerus reſultans ad n habebit rationem aequalitatis, hincque pro ſingulis factoribus $n-1$, $n-2$, $n-3$, $n-4$, &c. ubique ſcribi poterit n. Cum enim ſit

$$\frac{n(n-1)}{1.2}ddy = \tfrac{1}{2}nnddy - \tfrac{1}{2}nddy$$

prior terminus $\frac{1}{2}nnddy$ ad poſteriorem $\frac{1}{2}nddy$ rationem tenebit ut n ad 1, ſicque hic reſpectu illius evaneſcet; loco $\frac{n(n-1)}{1.2}$ ergo ſcribi poterit $\frac{1}{2}nn$. Simili modo quarti termini coefficiens $\frac{n(n-1)(n-2)}{1.2.3}$ contrahi poterit in $\frac{n^3}{6}$ pariterque in ſequentibus numeri, quibus n in factoribus diminuitur, negligi poterunt. Hoc vero facto functio y, ſi loco x ponatur $x+ndx$, exiſtente numero n infinito, ſequentem valorem accipiet:

$$y + \frac{ndy}{1} + \frac{nnddy}{1.2} + \frac{n^3d^3y}{1.2.3} + \frac{n^4d^4y}{1.2.3.4} + \frac{n^5d^5y}{1.2.3.4.5} + \&c.$$

48. Cum igitur ſumto n numero infinite magno etiamſi dx ſit infinite parvum, productum ndx quantitatem finitam exprimere poſſit, ponamus $ndx = \omega$, ut ſit $n = \frac{\omega}{dx}$ erit utique n numerus infinitus, cum ſit quotus ex diviſione quantitatis finitae ω, per infinite parvam dx reſultans. Valore autem hoc loco n adhibito cognoſcemus, ſi quantitas variabilis x augeatur quavis quantitate finita ω, ſeu ſi loco x ponatur $x+\omega$, tum quamvis ipſius functionem y abituram eſſe in hanc formam:

$$y + \frac{\omega dy}{1dx} + \frac{\omega^2 ddy}{1.2dx^2} + \frac{\omega^3 d^3y}{1.2.3dx^3} + \frac{\omega^4 d^4y}{1.2.3.4dx^4} + \&c.$$

cuius expreſſionis ſinguli termini per continuam ipſius y differentiationem inveniri poterunt. Cum enim y ſit functio ipſius

x,

x, oſtendimus ſupra, has functiones omnes $\frac{dy}{dx}$; $\frac{ddy}{dx^2}$; $\frac{d^3y}{dx^3}$; &c. quantitates finitas exhibere.

49. Cum igitur, dum quantitas variabilis x quantitate finita ω augeri aſſumitur, functio eius quaecunque y augeatur ſua differentia prima, quam ſupra per Δy indicavimus, exiſtente $\omega = \Delta x$: differentia ipſius y per continuam differentiationem reperiri poterit; erit enim:

$$\Delta y = \frac{\omega dy}{dx} + \frac{\omega^2 ddy}{2dx^2} + \frac{\omega^3 d^3y}{6dx^3} + \frac{\omega^4 d^4y}{24dx^4} + \&c. \quad \text{ſeu}$$

$$\Delta y = \frac{\Delta x}{1} \cdot \frac{dy}{dx} + \frac{\Delta x^2}{2} \cdot \frac{ddy}{dx^2} + \frac{\Delta x^3}{6} \cdot \frac{d^3y}{dx^3} + \frac{\Delta x^4}{24} \cdot \frac{d^4y}{dx^4} + \&c.$$

Sicque differentia finita Δy exprimitur per progreſſionem, cuius ſinguli termini ſecundum poteſtates ipſius Δx procedunt. Atque hinc viciſſim patet, ſi quantitas x tantum quantitate infinite parva augeatur, ut Δx abeat in eius differentiale dx, omnes terminos prae primo evaneſcere, foreque $\Delta y = dy$; facto enim $\Delta x = dx$, differentia Δy abit per definitionem in differentiale dy.

50. Quoniam ſi loco x ponatur $x + \omega$, eius functio quaecunque y induit ſequentem valorem:

$$y + \frac{\omega dy}{dx} + \frac{\omega^2 ddy}{2dx^2} + \frac{\omega^3 d^3y}{6dx^3} + \frac{\omega^4 d^4y}{24dx^4} + \&c.$$

veritas huius expreſſionis comprobari poterit eiuſmodi exemplis, quibus differentialia altiora ipſius y tandem evaneſcunt: his enim caſibus numerus terminorum ſuperioris expreſſionis fiet finitus:

EXEMPLUM I.

Quaeratur valor expreſſionis xx − x *ſi loco* x *ponatur* x + 1.

Ponatur $y = xx - x$; & cum x in $x + 1$ abire ſtatuatur, fiet $\omega = 1$, ſumtis iam differentialibus erit:

dy

$\frac{dy}{dx} = 2x - 1$; $\frac{ddy}{dx^2} = 2$; $\frac{d^3y}{dx^3} = 0$; &c.

Hinc functio $y = xx - x$ posito $x + 1$ loco x abibit in: $xx - x + 1(2x - 1) + \frac{1}{2}.2 = xx + x$.

Quodsi autem in $xx - x$ loco x actu ponatur $x + 1$ abibit xx in $xx + 2x + 1$

x in $x + 1$

Ergo $xx - x$ in $xx + x$.

EXEMPLUM II.

Quaeratur valor expressionis $x^3 + xx + x$, *si loco* x *ponatur* $x + 2$.

Ponatur $y = x^3 + xx + x$, fietque $\omega = 2$; nunc cum sit $y = x^3 + xx + x$

erit $\frac{dy}{dx} = 3xx + 2x + 1$

$\frac{ddy}{dx^2} = 6x + 2$

$\frac{d^3y}{dx^3} = 6$

$\frac{d^4y}{dx^4} = 0$.

Ex his valor functionis $y = x^3 + xx + x$, si pro x statuatur $x + 2$, erit sequens:

$x^3 + xx + x + 2(3xx + 2x + 1) + \frac{4}{2}(6x + 2) + \frac{8}{6}.6 = x^3 + 7xx + 17x + 14$, qui idem prodit si actu loco x substituatur $x + 2$.

EXEMPLUM III.

Quaeratur valor expressionis $xx + 3x + 1$, *si loco* x *ponatur* $x - 3$.

Fiet ergo $\omega = -3$; & posito

$y = xx + 3x + 1$, erit $\frac{dy}{dx} = 2x + 3$; $\frac{ddy}{dx^2} = 2$:

unde posito $x-3$ loco x functio x^2+3x+1 abibit in
$$x^2+3x+1-\tfrac{3}{1}(xx+3)+\tfrac{9}{2}.2 = x^2-3x+1.$$

51. Si pro ω sumatur numerus negativus, reperietur valor, quem functio quaecunque ipsius x induit, dum ipsa quantitas x diminuitur data quantitate ω. Scilicet si loco x ponatur $x-\omega$, functio ipsius x quaecunque y accipiet istum valorem: $y-\frac{\omega dy}{dx}+\frac{\omega^2 ddy}{2dx^2}-\frac{\omega^3 d^3y}{6dx^3}+\frac{\omega^4 d^4y}{24dx^4}-$ &c. unde omnes variationes, quas functio y subire potest, dum quantitas x utrinque variatur, inveniri poterunt. Quodsi autem y fuerit functio rationalis integra ipsius x, quoniam tandem ad eius differentialia evanescentia devenitur, valor variatus per expressionem finitam exprimetur; sin autem y non fuerit huiusmodi functio, valor variatus per seriem infinitam exprimetur, cuius propterea summa, quoniam si substitutio actu instituatur, valor variatus facile assignatur, expressione finita exhiberi poterit.

52. Quemadmodum autem differentia prima est inventa, ita quoque differentiae sequentes similibus expressionibus exhiberi possunt. Induat enim x successive valores $x+\omega$, $x+2\omega$, $x+3\omega$, $x+4\omega$, &c. atque valores ipsius y respondentes indicentur per y^{I}, y^{II}, y^{III}, y^{IV}, &c. sicuti in initio huius libri posuimus. Quoniam ergo y^{I}, y^{II}, y^{III}, y^{IV}, &c. sunt valores, quos y nanciscitur, si loco x scribatur respective $x+\omega$, $x+2\omega$, $x+3\omega$, $x+4\omega$, &c. per modo demonstrata isti ipsius y valores ita exprimentur:

$$y^{I} = y+\frac{\omega dy}{dx}+\frac{\omega^2 ddy}{2dx^2}+\frac{\omega^3 d^3y}{6dx^3}+\frac{\omega^4 d^4y}{24dx^4}+\text{\&c.}$$
$$y^{II} = y+\frac{2\omega dy}{dx}+\frac{4\omega^2 ddy}{2dx^2}+\frac{8\omega^3 d^3y}{6dx^3}+\frac{16\omega^4 d^4y}{24dx^4}+\text{\&c.}$$
$$y^{III} = y+\frac{3\omega dy}{dx}+\frac{9\omega^2 ddy}{2dx^2}+\frac{27\omega^3 d^3y}{6dx^3}+\frac{81\omega^4 d^4y}{24dx^4}+\text{\&c.}$$
$$y^{IV} = y+\frac{4\omega dy}{dx}+\frac{16\omega^2 ddy}{2dx^2}+\frac{64\omega^3 d^3y}{6dx^3}+\frac{256\omega^4 d^4y}{24dx^4}+\text{\&c.}$$

&c.

53.

53. Cum igitur, si Δy, $\Delta^2 y$, $\Delta^3 y$, $\Delta^4 y$, &c. denotent differentias, primam, secundam, tertiam, quartam, &c. sit:

$$\Delta y = y^{\mathrm{I}} - y$$
$$\Delta^2 y = y^{\mathrm{II}} - 2y^{\mathrm{I}} + y$$
$$\Delta^3 y = y^{\mathrm{III}} - 3y^{\mathrm{II}} + 3y^{\mathrm{I}} - y$$
$$\Delta^4 y = y^{\mathrm{IV}} - 4y^{\mathrm{III}} + 6y^{\mathrm{II}} - 4y^{\mathrm{I}} + y$$

istae differentiae per differentialia hoc modo exprimentur:

$$\Delta y = \frac{\omega dy}{dx} + \frac{\omega^2 ddy}{2dx^2} + \frac{\omega^3 d^3y}{6dx^3} + \frac{\omega^4 d^4y}{24dx^4} + \&c.$$

$$\Delta^2 y = \frac{(2^2 - 2.1)\omega^2 ddy}{2dx^2} + \frac{(2^3 - 2.1)\omega^3 d^3y}{6dx^3} + \frac{(2^4 - 2.1)\omega^4 d^4y}{24dx^4} + \&c.$$

$$\Delta^3 y = \frac{(3^3 - 3.2^3 + 3.1)\omega^3 d^3y}{6dx^3} + \frac{(3^4 - 3.2^4 + 3.1)\omega^4 d^4y}{24dx^4} + \&c.$$

$$\Delta^4 y = \frac{(4^4 - 4.3^4 + 6.2^4 - 4.1)\omega^4 d^4y}{24dx^4} + \frac{(4^5 - 4.3^5 + 6.2^5 - 4.1)\omega^5 d^5y}{120dx^5} + \&c.$$

&c.

54. Quantam utilitatem afferant istae differentiarum expressiones in doctrina serierum & progressionum, cum sponte patet, tum in sequentibus uberius exponemus. Interim tamen in hoc capite usum, qui hinc ad serierum notitiam immediate redundat, perpendamus. Quamquam vulgo indices terminorum seriei cuiuscunque progressionem arithmeticam, cuius differentia est unitas, constituere assumuntur; tamen quo usus latius pateat, atque applicatio facilius fieri possit, differentiam statuamus $= \omega$, ita ut, si terminus generalis seu is qui indici x respondet, fuerit y; sequentes conveniant indicibus

$$x + \omega,\ x + 2\omega,\ x + 3\omega,\ \&c.$$

Quodsi ergo his indicibus respondeant sequentes seriei termini

$$x,\ x + \omega,\ x + 2\omega,\ x + 3\omega,\ x + 4\omega,\ \&c.$$
$$y,\ \mathrm{P},\ \mathrm{Q},\ \mathrm{R},\ \mathrm{S},\ \&c.$$

singuli ex y eiusque differentialibus definientur hoc modo:

$$P = y + \frac{\omega dy}{dx} + \frac{\omega^2 ddy}{2dx^2} + \frac{\omega^3 d^3y}{6dx^3} + \frac{\omega^4 d^4y}{24dx^4} + \&c.$$

$$Q = y + \frac{2\omega dy}{dx} + \frac{4\omega^2 ddy}{2dx^2} + \frac{8\omega^3 d^3y}{6dx^3} + \frac{16\omega^4 d^4y}{24dx^4} + \&c.$$

$$R = y + \frac{3\omega dy}{dx} + \frac{9\omega^2 ddy}{2dx^2} + \frac{27\omega^3 d^3y}{6dx^3} + \frac{81\omega^4 d^4y}{24dx^4} + \&c.$$

$$S = y + \frac{4\omega dy}{dx} + \frac{16\omega^2 ddy}{2dx^2} + \frac{64\omega^3 d^3y}{6dx^3} + \frac{256\omega^4 d^4y}{24dx^4} + \&c.$$

&c.

55. Si hae expressiones a se invicem subtrahantur, in differentias non amplius ingredietur y, eritque

$$P - y = \frac{\omega dy}{dx} + \frac{\omega^2 ddy}{2dx^2} + \frac{\omega^3 d^3y}{6dx^3} + \frac{\omega^4 d^4y}{24dx^4} + \&c.$$

$$Q - P = \frac{\omega dy}{dx} + \frac{3\omega^2 ddy}{2dx^2} + \frac{7\omega^3 d^3y}{6dx^3} + \frac{15\omega^4 d^4y}{24dx^4} + \&c.$$

$$R - Q = \frac{\omega dy}{dx} + \frac{5\omega^2 ddy}{2dx^2} + \frac{19\omega^3 d^3y}{6dx^3} + \frac{65\omega^4 d^4y}{24dx^4} + \&c.$$

$$S - R = \frac{\omega dy}{dx} + \frac{7\omega^2 ddy}{2dx^2} + \frac{37\omega^3 d^3y}{6dx^3} + \frac{175\omega^4 d^4y}{24dx^4} + \&c.$$

$$T - S = \frac{\omega dy}{dx} + \frac{9\omega^2 ddy}{2dx^2} + \frac{61\omega^3 d^3y}{6dx^3} + \frac{369\omega^4 d^4y}{24dx^4} + \&c.$$

&c.

Si hae expressiones denuo a se invicem subtrahantur, etiam differentialia prima se destruent; eritque

$$Q - 2P + y = \frac{2\omega^2 ddy}{2dx^2} + \frac{6\omega^3 d^3y}{6dx^3} + \frac{14\omega^4 d^4y}{24dx^4} + \&c.$$

$$R - 2Q + P = \frac{2\omega^2 ddy}{2dx^2} + \frac{12\omega^3 d^3y}{6dx^3} + \frac{50\omega^4 d^4y}{24dx^4} + \&c.$$

$$S - 2R + Q = \frac{2\omega^2 ddy}{2dx^2} + \frac{18\omega^3 d^3y}{6dx^3} + \frac{110\omega^4 d^4y}{24dx^4} + \&c.$$

$$T - 2S + R = \frac{2\omega^2 ddy}{2dx^2} + \frac{24\omega^3 d^3y}{6dx^3} + \frac{194\omega^4 d^4y}{24dx^4} + \&c.$$

&c.

His autem denuo a se invicem subtractis differentialia quoque secunda ex computo egredientur:

$$R - 3Q + 3P - y = \frac{6\omega^3 d^3y}{6dx^3} + \frac{36\omega^4 d^4y}{24dx^4} + \&c.$$

$$S - 3R + 3Q - P = \frac{6\omega^3 d^3y}{6dx^3} + \frac{60\omega^4 d^4y}{24dx^4} + \&c.$$

$$T - 3S + 3R - Q = \frac{6\omega^3 d^3y}{6dx^3} + \frac{84\omega^4 d^4y}{24dx^4} + \&c.$$

subtractionem autem ulterius continuando fiet:

$$S - 4R + 6Q - 4P + y = \frac{24\omega^4 d^4y}{24dx^4} + \&c.$$

$$T - 4S + 6R - 4Q + P = \frac{24\omega^4 d^4y}{24dx^4} + \&c. \quad \text{atque}$$

$$T - 5S + 10R - 10Q + 5P - y = \frac{120\omega^5 d^5y}{120dx^5} + \&c.$$

56. Quodsi ergo y fuerit functio rationalis integra ipsius x, quia eius differentialia altiora tandem evanescent, hoc modo procedendo tandem ad expressiones evanescentes pervenietur. Cum igitur istae expressiones sint differentiae ipsius y, earum formas & coefficientes diligentius perpendamus:

$$y = y$$

$$\Delta y = \frac{\omega dy}{dx} + \frac{\omega^2 ddy}{2dx^2} + \frac{\omega^3 d^3y}{6dx^3} + \frac{\omega^4 d^4y}{24dx^4} + \frac{\omega^5 d^5y}{120dx^5} + \&c.$$

$$\Delta^2 y = \frac{\omega^2 ddy}{dx^2} + \frac{3\omega^3 d^3y}{3dx^3} + \frac{7\omega^4 d^4y}{3.4dx^4} + \frac{15\omega^5 d^5y}{3.4.5dx^5} + \frac{31\omega^6 d^6y}{3.4.5.6dx^6} + \&c.$$

$$\Delta^3 y = \frac{\omega^3 d^3y}{dx^3} + \frac{6\omega^4 d^4y}{4dx^4} + \frac{25\omega^5 d^5y}{4.5dx^5} + \frac{90\omega^6 d^6y}{4.5.6dx^6} + \frac{301\omega^7 d^7y}{4.5.6.7dx^7} + \&c.$$

$$\Delta^4 y = \frac{\omega^4 d^4 y}{dx^4} + \frac{10\omega^5 d^5 y}{5dx^5} + \frac{65\omega^6 d^6 y}{5.6dx^6} + \frac{350\omega^7 d^7 y}{5.6.7.dx^7} + \&c.$$

$$\Delta^5 y = \frac{\omega^5 d^5 y}{dx^5} + \frac{15\omega^6 d^6 y}{6dx^6} + \frac{140\omega^7 d^7 y}{6.7dx^7} + \frac{1050\omega^8 d^8 y}{6.7.8dx^8} + \&c.$$

$$\Delta^6 y = \frac{\omega^6 d^6 y}{dx^6} + \frac{21\omega^7 d^7 y}{7dx^7} + \frac{266\omega^8 d^8 y}{7.8dx^8} + \frac{2646\omega^9 d^9 y}{7.8.9dx^9} + \&c.$$

&c.

In quibus feriebus quemadmodum denominatores procedant, clarum eft; numeratorum autem coefficientes ita formantur, ut quivis coefficiens numeratoris fit aggregatum ex fupra ftante & praecedente per exponentem differentiae multiplicato. Sic in ferie differentiam $\Delta^6 y$ exprimente, eft $2646 = 1050 + 6.266$.

57. Confideremus quoque feriem eandem fimul retro continuatam, quae continet terminos indicibus

$x-\omega$; $x-2\omega$; $x-3\omega$; &c. refpondentes:

$x-4\omega$;	$x-3\omega$;	$x-2\omega$;	$x-\omega$;	x;	$x+\omega$;	$x+2\omega$;	$x+3\omega$;	$x+4\omega$;	&c.
s,	r,	q,	p,	y,	P,	Q,	R,	S,	&c.

Cum igitur fit:

$$p = y - \frac{\omega dy}{dx} + \frac{\omega^2 ddy}{2dx^2} - \frac{\omega^3 d^3 y}{6dx^3} + \frac{\omega^4 d^4 y}{24dx^4} - \&c.$$

$$q = y - \frac{2\omega dy}{dx} + \frac{4\omega^2 ddy}{2dx^2} - \frac{8\omega^3 d^3 y}{6dx^3} + \frac{16\omega^4 d^4 y}{24dx^4} - \&c.$$

$$r = y - \frac{3\omega dy}{dx} + \frac{9\omega^2 ddy}{2dx^2} - \frac{27\omega^3 d^3 y}{6dx^3} + \frac{81\omega^4 d^4 y}{24dx^4} - \&c.$$

$$s = y - \frac{4\omega dy}{dx} + \frac{16\omega^2 ddy}{2dx^2} - \frac{64\omega^3 d^3 y}{6dx^3} + \frac{256\omega^4 d^4 y}{24dx^4} - \&c.$$

&c.

erit his valoribus a fuperioribus P, Q, R, S, &c. fubtrahendis:

$$\frac{P-p}{2} = \frac{\omega dy}{dx} + \frac{\omega^3 d^3 y}{6dx^3} + \frac{\omega^5 d^5 y}{120dx^5} + \&c.$$

$$\frac{Q-q}{2} = \frac{2\omega dy}{dx} + \frac{8\omega^3 d^3 y}{6dx^3} + \frac{32\omega^5 d^5 y}{120dx^5} + \&c.$$

$$\frac{R-r}{2} = \frac{3\omega dy}{dx} + \frac{27\omega^3 d^3 y}{6dx^3} + \frac{243\omega^5 d^5 y}{120dx^5} + \&c.$$

$$\frac{S-s}{2} = \frac{4\omega dy}{dx} + \frac{64\omega^3 d^3 y}{6dx^3} + \frac{1024\omega^5 d^5 y}{120dx^5} + \&c.$$

&c.

ſin autem termini hi ad ſuperiores addantur, tum, quemadmodum hic differentialia parium ordinum deerant, differentialia imparia ex computo egredientur. Erit enim

$$\frac{P+p}{2} = y + \frac{\omega^2 ddy}{2dx^2} + \frac{\omega^4 d^4 y}{24dx^4} + \frac{\omega^6 d^6 y}{720dx^6} + \&c.$$

$$\frac{Q+q}{2} = y + \frac{4\omega^2 ddy}{2dx^2} + \frac{16\omega^4 d^4 y}{24dx^4} + \frac{64\omega^6 d^6 y}{720dx^6} + \&c.$$

$$\frac{R+r}{2} = y + \frac{9\omega^2 ddy}{2dx^2} + \frac{81\omega^4 d^4 y}{24dx^4} + \frac{729\omega^6 d^6 y}{720dx^6} + \&c.$$

$$\frac{S+s}{2} = y + \frac{16\omega^2 ddy}{2dx^2} + \frac{256\omega^4 d^4 y}{24dx^4} + \frac{4096\omega^6 d^6 y}{720dx^6} + \&c.$$

&c.

58. Quoniam termini antecedentes omnes exprimi poſſunt, ſi ii in unam ſummam colligantur, prodibit ſeriei propoſitae terminus ſummatorius. Reſpondeat ſcilicet terminus primus indici $x - n\omega$, eritque ipſe terminus primus $=$

$$y - \frac{n\omega dy}{dx} + \frac{n^2\omega^2 ddy}{2dx^2} - \frac{n^3\omega^3 d^3 y}{6dx^3} + \frac{n^4\omega^4 d^4 y}{24dx^4} \quad \&c.$$

Cum igitur terminus indici x reſpondens ſit $= y$, terminorumque omnium numerus ſit $= n + 1$, erit ſumma omnium a primo ad ultimum y incluſive ſumtorum ſeu terminus ſummatorius $=$

$$(n+1)y - \frac{\omega dy}{dx}(1 + 2 + 3 + \ldots + n)$$

$$+ \frac{\omega^2 ddy}{2dx^2}(1 + 2^2 + 3^2 + \ldots + n^2)$$

—

$$-\frac{\omega^3 d^3y}{6dx^3}(1+2^3+3^3+\ldots+n^3)$$

$$+\frac{\omega^4 d^4y}{24dx^4}(1+2^4+3^4+\ldots+n^4)$$

$$-\frac{\omega^5 d^5y}{120dx^5}(1+2^5+3^5+\ldots+n^5)$$

&c.

59. Supra autem ſingularum harum ſerierum ſummas exhibuimus, quae ſi hic ſubſtituantur, erit ſumma ſeriei noſtrae propoſitae =

$$(n+1)y-\frac{\omega dy}{dx}\left(\tfrac{1}{2}nn+\tfrac{1}{2}n\right)$$

$$+\frac{\omega^2 ddy}{2dx^2}\left(\tfrac{1}{3}n^3+\tfrac{1}{2}nn+\tfrac{1}{6}n\right)$$

$$-\frac{\omega^3 d^3y}{6dx^3}\left(\tfrac{1}{4}n^4+\tfrac{1}{2}n^3+\tfrac{1}{4}n^2\right)$$

$$+\frac{\omega^4 d^4y}{24dx^4}\left(\tfrac{1}{5}n^5+\tfrac{1}{2}n^4+\tfrac{1}{3}n^3-\tfrac{1}{30}n\right)$$

$$-\frac{\omega^5 d^5y}{120dx^5}\left(\tfrac{1}{6}n^6+\tfrac{1}{2}n^5+\tfrac{5}{12}n^4-\tfrac{1}{12}n^2\right)$$

&c.

ubi n dabitur ex indice termini primi, a quo ſumma computatur. Ita ſi ponatur $\omega=1$, & index termini primi ſit $=1$, ſecundi $=2$, & ultimi $=x$, ita ut haec ſeries ſit propoſita:

$$\begin{matrix} 1, & 2, & 3, & 4, & \ldots\ldots & x \\ a, & b, & c, & d, & \ldots\ldots & y \end{matrix}$$

erit huius ſeriei ſumma (ob $x-n=1$ & $n=x-1$)

$$=xy-\frac{dy}{dx}\left(\tfrac{1}{2}xx-\tfrac{1}{2}x\right)$$

$$+\frac{ddy}{2dx^2}\left(\tfrac{1}{3}x^3-\tfrac{1}{2}xx+\tfrac{1}{6}x\right)$$

—

$$- \frac{d^3 y}{6dx^3}\left(\tfrac{1}{4}x^4 - \tfrac{1}{2}x^3 + \tfrac{1}{4}xx\right)$$
$$+ \frac{d^4 y}{24dx^4}\left(\tfrac{1}{5}x^5 - \tfrac{1}{2}x^4 + \tfrac{1}{3}x^3 - \tfrac{1}{30}x\right)$$
$$- \frac{d^5 y}{120dx^5}\left(\tfrac{1}{6}x^6 - \tfrac{1}{2}x^5 + \tfrac{5}{12}x^4 - \tfrac{1}{12}x^2\right)$$
$$+ \frac{d^6 y}{720dx^6}\left(\tfrac{1}{7}x^7 - \tfrac{1}{2}x^6 + \tfrac{1}{2}x^5 - \tfrac{1}{6}x^3 + \tfrac{1}{42}x\right)$$
&c.

60. Ex hac ſummae expreſſione, quia coefficientes vehementer augentur, ſi x fuerit numerus magnus, parum utilitatis ad doctrinam ſerierum redundat; interim tamen iuvabit aliquas proprietates inde fluentes commemoraſſe. Sit terminus generalis $y = x^n$, atque terminus ſummatorius per Sy ſeu $S.x^n$ indicetur. Qua deſignatione ubique adhibita erit:

$$\tfrac{1}{2}xx - \tfrac{1}{2}x = S.x - x$$
$$\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 + \tfrac{1}{6}x = S.x^2 - x^2$$
$$\tfrac{1}{4}x^4 - \tfrac{1}{2}x^3 + \tfrac{1}{4}xx = S.x^3 - x^3 \quad \&c.$$

Quamobrem ex ſuperiori expreſſione obtinebitur:

$$S.x^n = x^{n+1} - nx^{n-1}S.x + nx^n$$
$$+ \frac{n(n-1)}{1.\ 2}x^{n-2}S.x^2 - \frac{n(n-1)}{1.\ 2}x^n$$
$$- \frac{n(n-1)(n-2)}{1.\ 2.\ 3}x^{n-3}S.x^3 + \frac{n(n-1)(n-2)}{1.\ 2.\ 3}x^n$$
&c.

At cum ſit

$$(1-1)^n = 0 = 1 - n + \frac{n(n-1)}{1.\ 2} - \frac{n(n-1)(n-2)}{1.\ 2.\ 3} + \&c.$$

erit $n - \frac{n(n-1)}{1.\ 2.} + \frac{n(n-1)(n-2)}{1.\ 2.\ 3} - \&c. = 1$,

ideoque excepto caſu $n = 0$, quo iſta expreſſio fit $= 0$.

$$S.x^n = x^{n+1} + x^n - nx^{n-1}\,S.x + \frac{n(n-1)}{1.\ 2}\,x^{n-2}\,S.x^2$$
$$-\frac{n(n-1)(n-2)}{1.\ 2.\ 3}\,x^{n-3}\,S.x^3 + \frac{n(n-1)(n-2)(n-3)}{1.\ 2.\ 3.\ 4}\,x^{n-4}\,S.x^4$$
&c.

61. Quo tam veritas quam vis huius formulae clarius perſpiciatur, evolvamus ſingulos caſus, ſitque primo $n = 1$, eritque: $S.x = x^2 + x - S.x$, ideoque $S.x = \frac{xx + x}{2}$, quemadmodum ſatis conſtat.

Ponamus ergo $n = 2$, & erit:
$$S.x^2 = x^3 + xx - 2x\,S.x + S.x^2,$$
quae aequatio, cum utrinque termini $S.x^2$ ſe tollant, idem dat, quod praecedens $S.x = \frac{xx + x}{2}$. Si ſit $n = 3$, erit
$$S.x^3 = x^4 + x^3 - 3x^2\,S.x + 3x\,S.x^2 - S.x^3,$$ ideoque
$$S.x^3 = \tfrac{3}{2}x\,S.x^2 - \tfrac{3}{2}x^2\,S.x + \tfrac{1}{2}x^3(x + 1),$$
ſi ponatur $n = 4$ prodibit:
$$S.x^4 = x^5 + x^4 - 4x^3\,S.x + 6x^2\,S.x^2 - 4x\,S.x^3 + S.x^4,$$
unde ob $S.x^4$ deſtructum erit:
$$S.x^3 = \tfrac{3}{2}x\,S.x^2 - x^2\,S.x + \tfrac{1}{4}x^3(x + 1)$$
a cuius triplo, ſi praecedentis duplum ſubtrahatur remanebit:
$$S.x^3 = \tfrac{3}{2}x\,S.x^2 - \tfrac{1}{4}x^3(x + 1).$$
Si ponatur $n = 5$ fiet:
$$S.x^5 = x^6 + x^5 - 5x^4\,S.x + 10x^3\,S.x^2 - 10x^2\,S.x^3 + 5x\,S.x^4 - S.x^5$$
ſeu
$$S.x^5 = \tfrac{5}{2}x\,S.x^4 - 5x^2\,S.x^3 + 5x^3\,S.x^2 - \tfrac{5}{2}x^4\,S.x + \tfrac{1}{2}x^5(x + 1)$$
atque ex $n = 6$ ſequitur:
$$S.x^6 = x^7 + x^6 - 6x^5\,S.x + 15x^4\,S.x^2 - 20x^3\,S.x^3 + 15x^2\,S.x^4 - 6x\,S.x^5 + S.x^6$$
ſeu

ſeu

$$S.x^5 = \tfrac{5}{2}xS.x^4 - \tfrac{10}{3}x^2S.x^3 + \tfrac{5}{2}x^3S.x^2 - x^4S.x + \tfrac{1}{6}x^5(x+1).$$

62. Ex his ergo generaliter concludimus, ſi fuerit $n = 2m+1$ fore:

$$S.x^{2m+1} = \frac{2m+1}{2}xS.x^{2m} - \frac{(2m+1)2m}{2.\ 1.\ 2}x^2S.x^{2m-1} + \frac{(2m+1)2m(2m-1)}{2.\ 1.\ 2.\ 3}x^3S.x^{2m-2}$$

$$- \ldots\ldots - \frac{(2m+1)}{2}x^{2m}S.x + \tfrac{1}{2}x^{2m+1}(x+1).$$

Sin autem ſit $n = 2m+2$, quia termini $S.x^{2m+2}$ ſe mutuo deſtruunt, reperietur:

$$S.x^{2m+1} = \frac{2m+1}{2}xS.x^{2m} - \frac{(2m+1)2m}{2.\ 3}x^2S.x^{2m-1} + \frac{(2m+1)2m(2m-1)}{2.\ 3.\ 4}x^3S.x^{2m-2}$$

$$- \ldots\ldots - x^{2m}S.x + \frac{1}{2m+2}x^{2m+1}(x+1).$$

Duplici ergo modo ſummae poteſtatum imparium ex ſummis poteſtatum inferiorum determinari poſſunt: atque ex varia combinatione harum duarum formularum infinitae aliae formari poſſunt.

63. Multo facilius autem ſummae poteſtatum imparium ex antecedentibus definiri poſſunt: atque ad hoc quidem ſufficit ſolam ſummam poteſtatis paris antecedentis noviſſe. Ex ſummis enim poteſtatum ſupra exhibitis conſtat, numerum terminorum ſummas conſtituentium, in paribus tantum poteſtatibus augeri, ita ut ſumma poteſtatis imparis totidem conſtet terminis, quot ſumma poteſtatis paris praecedentis. Sic ſi poteſtatis paris x^{2n} ſumma ſit:

$$S.x^{2n} = \alpha x^{2n+1} + \beta x^{2n} + \gamma x^{2n-1} - \delta x^{2n-3} + \varepsilon x^{2n-5} - \&c.$$

vidimus enim, poſt terminum tertium alternos terminos deficere, ſimulque ſigna alternari; hinc ſumma ſequentis poteſtatis x^{2n+1} invenietur, ſi ſinguli illius termini reſpective multiplicentur per hos numeros:

$$\frac{2n+1}{2n+2}x;\ \frac{2n+1}{2n+1}x;\ \frac{2n+1}{2n}x;\ \frac{2n+1}{2n-1}x;\ \frac{2n+1}{2n-2}x;\ \&c.$$

nou omittendo terminos deficientes; eritque ergo

$$S.x^{2n+1} = \frac{2n+1}{2n+2}\alpha x^{2n+2} + \frac{2n+1}{2n+1}\beta x^{2n+1} + \frac{2n+1}{2n}\gamma x^{2n}$$
$$- \frac{2n+1}{2n-1}\delta x^{2n-2} + \frac{2n+1}{2n-4}\varepsilon x^{2n-4} - \frac{2n+1}{2n-6}\zeta x^{2n-6} + \&c.$$

Quodſi ergo conſtet ſumma poteſtatis x^{2n}, ex ea expedite ſumma ſequentis poteſtatis x^{2n+1} formari poterit.

64. Haec ſequentium ſummarum inveſtigatio etiam ad poteſtates pares extenditur; quoniam autem harum ſummae novum terminum recipiunt, hic per iſtam methodum non invenitur, ex natura tamen ipſius feriei, qua conſtat, ſi ponatur $x = 1$, ſummam quoque fieri debere $= 1$, ſemper erui poterit. Viciſſim autem ſemper ex ſumma cuiuſvis poteſtatis cognita praecedentium poteſtatum ſummae inveniri poterunt. Si enim fuerit:

$$S.x^n = \alpha x^{n+1} + \beta x^n + \gamma x^{n-1} - \delta x^{n-3} + \varepsilon x^{n-5} - \zeta x^{n-7} + \&c.$$

erit pro poteſtate praecedente:

$$S.x^{n-1} = \frac{n+1}{n}\alpha x^n + \frac{n}{n}\beta x^{n-1} + \frac{(n-1)}{n}\gamma x^{n-2} - \frac{(n-3)}{n}\delta x^{n-4} + \&c.$$

hincque ulterius regredi licet, quouſque libuerit. Notandum autem eſt eſſe perpetuo $\alpha = \frac{1}{n+1}$ & $\beta = \frac{1}{2}$, uti ex formulis iam ſupra datis apparet.

65. Attendenti ſtatim patebit ſummam poteſtatum x^{n-1} prodire, ſi ſumma poteſtatum x^n differentietur, eiuſque differentiale per ndx dividatur; eritque adeo

$$d.Sx^n = ndx.Sx^{n-1} \quad \& \text{ quia eſt } d.x^n = nx^{n-1}dx;$$

$$\text{erit } d.Sx^n = S.nx^{n-1}dx = S.d.x^n;$$

ex quo intelligitur differentiale ſummae aequari ſummae differentialis: ita in genere ſi feriei cuiuſpiam terminus generalis fuerit $= y$ & Sy eius terminus ſummatorius; erit quoque $S.dy = d.Sy$: hoc eſt ſumma differentialium omnium terminorum aequatur differentiali ſummae ipſorum terminorum.

rum. Ratio autem huius aequalitatis facile perſpicitur ex iis, quae ſupra de ſerierum differentiatione attulimus. Cum enim ſit

$$S.x^n = x^n + (x-1)^n + (x-2)^n + (x-3)^n + (x-4)^n + \&c.$$

erit

$$\frac{d.Sx^n}{ndx} = x^{n-1} + (x-1)^{n-1} + (x-2)^{n-1} + (x-3)^{n-1} + \&c. = S.x^{n-1}$$

quae demonſtratio ad omnes alias ſeries patet.

66. Revertamur autem, unde digreſſi ſumus, ad differentias functionum, circa quas adhuc quaedam annotanda ſunt. Quoniam vidimus, ſi y fuerit functio quaecunque ipſius x, atque loco x ubique ponatur $x \pm \omega$, functionem y adepturam eſſe ſequentem valorem:

$$y \pm \frac{\omega dy}{1dx} + \frac{\omega^2 ddy}{1.2dx^2} \pm \frac{\omega^3 d^3y}{1.2.3.dx^3} + \frac{\omega^4 d^4y}{1.2.3.4dx^4} \pm \frac{\omega^5 d^5y}{1.2.3.4.5dx^5} + \&c.$$

haec expreſſio locum habebit, ſive pro ω quantitas quaecunque conſtans accipiatur, ſive etiam variabilis, ab ipſa x pendens. Inventis enim per differentiationem valoribus fractionum $\frac{dy}{dx}$; $\frac{ddy}{dx^2}$; $\frac{d^3y}{dx^3}$; &c. in factoribus ω, ω^2, ω^3, &c. variabilitas non ſpectatur, hincque perinde eſt ſive ω denotet quantitatem conſtantem, ſive variabilem ab x pendentem.

67. Ponamus ergo eſſe $\omega = x$, atque in functione y loco x ſcribi $x - x = 0$. Quamobrem ſi in functione ipſius x quacunque y loco x ubique ſcribatur 0, valor functionis erit hic:

$$y - \frac{xdy}{1dx} + \frac{x^2 ddy}{1.2dx^2} - \frac{x^3 d^3y}{1.2.3dx^3} + \frac{x^4 d^4y}{1.2.3.4dx^4} - \&c.$$

Haec ergo expreſſio ſemper indicat valorem, quem functio quaecunque y induit, ſi in ea ponatur $x = 0$, cuius veritatem ſequentia exempla illuſtrabunt:

EXEMPLUM I.

Sit $y = xx + ax + ab$, cuius valor, ſi ponatur $x = 0$, quaeratur, quem quidem conſtat fore $= ab$

Cum sit $y = xx + ax + ab$ erit

$$\frac{dy}{1dx} = 2x + a \quad ; \quad \frac{ddy}{1.2dx^2} = 1$$

ideoque prodibit valor quaesitus

$$= xx + ax + ab - x(2x + a) + xx . 1 = ab.$$

EXEMPLUM II.

Sit $y = x^3 - 2x + 3$, cuius valor, posito $x = 0$, quaeratur, quem constat fore $= 3$.

Cum sit $y = x^3 - 2x + 3$ erit

$$\frac{dy}{dx} = 3xx - 2 \quad ; \quad \frac{ddy}{1.2dx^2} = 3x \quad ; \quad \frac{d^3y}{1.2.3dx^3} = 1$$

obtinebitur valor quaesitus

$$= x^3 - 2x + 3 - x(3xx - 2) + xx . 3x - x^3 . 1 = 3.$$

EXEMPLUM III.

Sit $y = \frac{x}{1-x}$, cuius valor posito $x = 0$, quaeritur, quem constat fore $= 0$.

Cum sit $y = \frac{x}{1-x}$; erit $\frac{dy}{dx} = \frac{1}{(1-x)^2}$;

$$\frac{ddy}{1.2dx^2} = \frac{1}{(1-x)^3}; \frac{d^3y}{1.2.3dx^3} = \frac{1}{(1-x)^4}; \text{ \&c.}$$

Hinc erit valor quaesitus

$$= \frac{x}{1-x} - \frac{x}{(1-x)^2} + \frac{xx}{(1-x)^3} - \frac{x^3}{(1-x)^4} + \frac{x^4}{(1-x)^5} - \text{\&c.}$$

huiusque ergo seriei valor est $= 0$.

Quod etiam hinc patet, quod haec series primo termino truncata $\frac{x}{(1-)^2} - \frac{xx}{(1-x)^3} + \frac{x^3}{(1-x)^4} - $ &c. sit series geometrica, eiusque summa $= \frac{x}{(1-x)^2 + x(1-x)} = \frac{x}{1-x}$; unde valor inventus erit $= \frac{x}{1-x} - \frac{x}{1-x} = 0$.

EXEMPLUM IV.

Sit $y = e^x$, denotante e numerum, cuius logarithmus hyperbolicus est unitas, & quaeratur valor ipsius y si ponatur $x = 0$, quem quidem constat fore $= 1$.

Cum sit $y = e^x$; erit $\frac{dy}{dx} = e^x$; $\frac{ddy}{dx^2} = e^x$; &c.

ideoque valor quaesitus erit

$$= e^x - \frac{e^x x}{1} + \frac{e^x xx}{1.2} - \frac{e^x x^3}{1.2.3} + \frac{e^x x^4}{1.2.3.4} - \&c.$$

$$= e^x \left(1 - \frac{x}{1} + \frac{xx}{1.2} - \frac{x^3}{1.2.3} + \frac{x^4}{1.2.3.4} - \&c.\right)$$

At supra vidimus seriem $1 - \frac{x}{1} + \frac{xx}{1.2} - \frac{x^3}{1.2.3} + \&c.$ exprimere valorem e^{-x}, erit ergo valor quaesitus utique

$$= e^x . e^{-x} = \frac{e^x}{e^x} = 1.$$

EXEMPLUM V.

Sit $y = \sin x$, atque posito $x = 0$ manifestum est fore $y = 0$, id quod etiam formula generalis indicabit.

Cum enim sit $y = \sin x$; erit $\frac{dy}{dx} = \cos x$;

$\frac{ddy}{dx^2} = -\sin x$; $\frac{d^3 y}{dx^3} = -\cos x$; $\frac{d^4 y}{dx^4} = \sin x$; &c.

erit posito $x = 0$ valor ipsius y hic:

$$\sin x - \frac{x}{1}\cos x - \frac{xx}{1.2}\sin x + \frac{x^3}{1.2.3}\cos x + \frac{x^4}{1.2.3.4}\sin x - \&c.$$

qui est $= \sin x \left(1 - \frac{xx}{1.2} + \frac{x^4}{1.2.3.4} - \frac{x^6}{1.2.3\ldots6} + \&c.\right)$

$$- \cos x \left(\frac{x}{1} - \frac{x^3}{1.2.3} + \frac{x^5}{1.2.3.4.5} - \frac{x^7}{1.2.3\ldots7} + \&c.\right)$$

ha-

harum autem serierum superior exprimit $\cos x$, inferior autem $\sin x$, unde valor quaesitus erit

$$= \sin x . \cos x - \cos x . \sin x = 0.$$

68. Hinc igitur vicissim cognoscimus, si y eiusmodi fuerit functio ipsius x, ut ipsa evanescat, posito $x = 0$, tum fore

$$y - \frac{x\,dy}{1\,dx} + \frac{xx\,ddy}{1.2\,dx^2} - \frac{x^3 d^3y}{1.2.3\,dx^3} + \frac{x^4 d^4 y}{1.2.3.4\,dx^4} - \&c. = 0.$$

Unde haec est aequatio generalis omnium omnino functionum ipsius x, quae dum fit $x = 0$, simul ipsae evanescunt. Et hancobrem ista aequatio ita est comparata, ut quaecunque functio ipsius x, dummodo ea evanescat evanescente x, loco y substituatur, aequationi perpetuo satisfiat. Quodsi vero y eiusmodi fuerit functio ipsius x, quae posito $x = 0$, recipiat valorem datum $= A$, tum erit:

$$y - \frac{x\,dy}{1\,dx} + \frac{x^2\,ddy}{1.2\,dx^2} - \frac{x^3 d^3y}{1.2.3\,dx^3} + \frac{x^4 d^4 y}{1.2.3.4\,dx^4} - \&c. = A.$$

in qua aequatione omnes continentur functiones ipsius x, quae posito $x = 0$, abeunt in A.

69. Si loco x scribatur $2x$, seu $x + x$, functio quaecunque ipsius x, quae designetur per y hunc induet valorem

$$y + \frac{x\,dy}{1\,dx} + \frac{x^2\,ddy}{1.2\,dx^2} + \frac{x^3 d^3y}{1.2.3\,dx^3} + \frac{x^4 d^4 y}{1.2.3.4\,dx^4} + \&c.$$

Atque si loco x scribamus nx, hoc est $x + (n-1)x$ functio y accipiet valorem sequentem:

$$y + \frac{(n-1)x\,dy}{1\,dx} + \frac{(n-1)^2 xx\,ddy}{1.2\,dx^2} + \frac{(n-1)^3 x^3 d^3 y}{1.2.3\,dx^3} + \&c.$$

Sin autem generaliter pro x scribamus t, functio quaecunque y ipsius x, transmutabitur ob $t = x + t - x$ in formam sequentem:

$$y + \frac{(t-x)dy}{1\,dx} + \frac{(t-x)^2\,ddy}{1.2\,dx^2} + \frac{(t-x)^3 d^3y}{1.2.3\,dx^3} + \&c.$$

Si

Si igitur v fuerit talis functio ipsius t, qualis y est ipsius x, quia v ex y nascitur, ponendo t loco x, erit:

$$v = y + \frac{(t-x)dy}{1dx} + \frac{(t-x)^2 ddy}{1.2dx^2} + \frac{(t-x)^3 d^3y}{1.2.3dx^3} + \&c.$$

cuius veritas quibuscunque exemplis comprobari potest.

EXEMPLUM.

Sit enim $y = xx - x$: manifestum est posito t loco x fore $v = tt - t$, quod idem expressio inventa declarabit. Nam ob

$$y = xx - x; \quad \text{erit} \quad \frac{dy}{dx} = 2x - 1; \quad \& \quad \frac{ddy}{2dx^2} = 1;$$

unde fiet

$$v = xx - x + (t-x)(2x-1) + (t-x)^2 =$$
$$xx - x + 2tx - 2xx - t + x + tt - 2tx + xx = tt - t.$$

Si itaque y fuerit eiusmodi functio ipsius x, quae posito $x = a$ abeat in A; ob $t = a$ & $v = A$ fiet

$$A = y + \frac{(a-x)dy}{1dx} + \frac{(a-x)^2 ddy}{1.2dx^2} + \frac{(a-x)^3 d^3y}{1.2.3dx^3} + \&c.$$

huicque ergo aequationi omnes functiones ipsius x, quae facto $x = a$ abeunt in A, satisfaciunt.

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