posito $k = 0,05445$. Hinc autem erit $\frac{1}{2}\delta - \gamma = -0,02302$. Cum autem iam ante inuentum esset $\frac{1}{2}\delta - \gamma = -0,01742$ erit reuera $\frac{1}{2}\delta - \gamma = -0,00560$. Tum vero inueniemus: $\gamma = 1,40673$, vnde erit $\frac{1}{2}\delta = 1,40113$, et $\delta = 2,80226$ hincque $2\gamma - \frac{3}{2}\delta = -1,38993$ et $\frac{2\gamma - \frac{3}{2}\delta}{nn} = -0,00794$. Verum ex cognita ratione motus medii ad motum anomaliae est $\mathrm{O} = 1,0085272$, vnde $\varkappa = 1,0089562$. Verum esse debet $\varkappa = 1 + \frac{3 + 4\mu + \delta}{4\,nn}$; vnde foret $0,0089562 = 0,008289 + \frac{\mu}{nn}$; ideoque $\frac{\mu}{nn} = 0,000667$ qui valor cum sit tam exiguus, merito dubitamus, num μ non prorsus sit $= 0$.

CAPUT VII.

CORRECTIO INAEQUALITATUM LUNAE, ANTE INUENTARUM.

§. 100.

Quoniam nunc quidem valores litterarum γ et δ ita inuenimus, vt eos pro proxime veris habere queamus, ex iis coefficientes terminorum, quibus inaequalitates lunae continentur, accuratius definire poterimus. Cum enim sit $\gamma = 1,40673$ et $\delta = 2,80226$, colligamus hic in vnum omnes formulas, quas hactenus pro inueniendis coefficientibus assumtis elicuimus. Posueramus autem:

$$\begin{aligned}\int R\,dr = {} & \mathfrak{A}\cos 2\eta + \mathfrak{a}\,kk\cos 2\eta + \mathfrak{B}\cos 4\eta + \mathfrak{b}\,kk\cos 4\eta \\ & + \mathfrak{C}\,k\cos r + \mathfrak{D}\,k\cos(2\eta - r) + \mathfrak{F}\,k\cos(4\eta - r) \\ & + \mathfrak{E}\,k\cos(2\eta + r) + \mathfrak{G}\,k\cos(4\eta + r) \\ & + \mathfrak{H}\,kk\cos 2r + \mathfrak{J}\,k^2\cos(2\eta - 2r) + \mathfrak{L}\,k^2\cos(4\eta - 2r) \\ & + \mathfrak{K}\,k^2\cos(2\eta + 2r) + \mathfrak{M}\,k^2\cos(4\eta + 2r)\end{aligned}$$

$$\begin{aligned}v = {} & \mathrm{A}\cos 2\eta + a\,kk\cos 2\eta + \mathrm{B}\cos 4\eta + b\,kk\cos 4\eta \\ & + \mathrm{D}\,k\cos(2\eta - r) + \mathrm{F}\,k\cos(4\eta - r) \\ & + \mathrm{E}\,k\cos(2\eta + r) + \mathrm{G}\,k\cos(4\eta + r) \\ & + \mathrm{H}\,kk\cos 2r + \mathrm{J}\,kk\cos(2\eta - 2r) + \mathrm{L}\,kk\cos(4\eta - 2r) \\ & + \mathrm{K}\,kk\cos(2\eta + 2r) + \mathrm{M}\,kk\cos(4\eta + 2r)\end{aligned}$$

§. 101. Hinc posito $\varkappa = \sqrt{\left(1 + \frac{3 + 4\mu + \delta}{2nn}\right)}$ collegimus fore

$$\frac{d\Phi}{dr} = \varkappa + \frac{A(3\varkappa A + 2\mathfrak{A})}{2n^4} + \frac{D(3\varkappa D + 2\mathfrak{D})}{2n^4}kk + \frac{A(3\varkappa a + \mathfrak{a})}{2n^4}kk$$
$$+ \frac{a(3\varkappa A + 2\mathfrak{A})}{2n^4}kk - \frac{(2\varkappa A + \mathfrak{A})}{nn}\operatorname{cof} 2\eta - \frac{(2\varkappa B + \mathfrak{B})}{nn}\operatorname{cof} 4\eta$$
$$+ \frac{A(3\varkappa A + 2\mathfrak{A})}{2n^4}\operatorname{cof} 4\eta - \frac{\mathfrak{C}}{nn}k\operatorname{cof} r - \frac{(2\varkappa a + \mathfrak{a})}{nn}k^2\operatorname{cof} 2\eta$$
$$- \frac{(2\varkappa b + \mathfrak{b})}{nn}k^2\operatorname{cof} 4\eta + \frac{D(3\varkappa A + 2\mathfrak{A}) + A(3\varkappa D + 2\mathfrak{D})}{2n^4}k\operatorname{cof} r$$
$$- \frac{(2\varkappa D + \mathfrak{D})}{nn}k\operatorname{cof}(2\eta - r) - \frac{(2\varkappa F + \mathfrak{F})}{nn}k\operatorname{cof}(4\eta - r)$$
$$+ \frac{D(3\varkappa A + 2\mathfrak{A}) + A(3\varkappa D + 2\mathfrak{D})}{2n^4}k\operatorname{cof}(4\eta - r)$$
$$- \frac{(2\varkappa E + \mathfrak{E})}{nn}k\operatorname{cof}(2\eta + r) - \frac{(2\varkappa G + \mathfrak{G})}{nn}k\operatorname{cof}(4\eta + r)$$
$$- \frac{(2\varkappa J + \mathfrak{J})}{nn}kk\operatorname{cof}(2\eta - 2r) - \frac{(2\varkappa H + \mathfrak{H})}{nn}kk\operatorname{cof} 2r$$
$$+ \frac{J(3\varkappa A + 2\mathfrak{A}) + A(3\varkappa J + 2\mathfrak{J})}{2n^4}k^2\operatorname{cof} 2r$$
$$- \frac{(2\varkappa K + \mathfrak{K})}{nn}kk\operatorname{cof}(2\eta + 2r) - \frac{(2\varkappa L + \mathfrak{L})}{nn}k^2\operatorname{cof}(4\eta - 2r)$$
$$- \frac{D(3\varkappa D + 2\mathfrak{D}) + J(3\varkappa A + 2\mathfrak{A}) + A(3\varkappa J + 2\mathfrak{J})}{2n^4}k^2\operatorname{cof}(4\eta - 2r)$$
$$- \frac{(2\varkappa M + \mathfrak{M})}{nn}k^2\operatorname{cof}(4\eta + 2r)$$

§. 102. Si iam ponamus

$$\varkappa + \frac{A(3\varkappa A + 2\mathfrak{A})}{2n^4} + \frac{D(3\varkappa D + 2\mathfrak{D})}{2n^4}kk$$
$$+ \frac{A(3\varkappa a + 2\mathfrak{a})}{2n^4}kk + \frac{a(3\varkappa A + 2\mathfrak{A})}{2n^4}kk$$

$-\frac{1}{n} = \alpha$; vt fit neglectis terminis admodum exiguis

$$\frac{d\eta}{dr} = \alpha - \frac{(2\varkappa A + \mathfrak{A})}{nn}\operatorname{cof} 2\eta - \frac{(2n + \mathfrak{C})}{nn}k\operatorname{cof} r$$
$$- \frac{(2\varkappa D + \mathfrak{D})}{nn}k\operatorname{cof}(2\eta - r) - \frac{(2\varkappa E + \mathfrak{E})}{nn}k^2\operatorname{cof}(2\eta + r)$$

ex superioribus capitibus repetimus has determinationes:

$$2\alpha\mathfrak{A} = -\tfrac{3}{2}$$
$$4\alpha\mathfrak{B} = -\frac{3A}{2nn} + \frac{\mathfrak{A}(3\varkappa A + \mathfrak{A})}{nn}$$
$$\mathfrak{C} = \frac{\mathfrak{A}(2\varkappa D + \mathfrak{D})}{nn} - \frac{(\mathfrak{D} - \mathfrak{E})(2\varkappa A + \mathfrak{A})}{nn} - \frac{\mathfrak{A}(2\varkappa E + \mathfrak{E})}{nn} - \frac{3(D - E)}{2nn}$$
$$(2\alpha - 1)\mathfrak{D} = -3 + \frac{(2n + \mathfrak{C})}{nn}\mathfrak{A}$$
$$(2\alpha + 1)\mathfrak{E} = -3 + \frac{(2n + \mathfrak{C})}{nn}\mathfrak{A}$$
$$(4\alpha - 1)\mathfrak{F} = \frac{2(2n + \mathfrak{C})}{nn}\mathfrak{B} + \frac{\mathfrak{A}(2\varkappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{D}(2\varkappa A + \mathfrak{A})}{nn} - \frac{3(2A + D)}{2nn}$$
$$(4\alpha + 1)\mathfrak{G} = \frac{2(2n + \mathfrak{C})}{nn}\mathfrak{B} + \frac{\mathfrak{A}(2\varkappa E + \mathfrak{E})}{nn} + \frac{\mathfrak{E}(2\varkappa A + \mathfrak{A})}{nn} - \frac{3(2A + E)}{2nn}$$
$$2\alpha\mathfrak{a} = -\tfrac{15}{4} + \frac{(\mathfrak{D} + \mathfrak{E})(2n + \mathfrak{C})}{nn} + \frac{2\mathfrak{b}(2\varkappa A + \mathfrak{A})}{nn}$$
$$+ \frac{2\mathfrak{F}(2\varkappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{D}(2\varkappa F + \mathfrak{F})}{nn}$$

$$4\alpha\mathfrak{b} = -\frac{3D}{nn} + \frac{2(\mathfrak{F}+\mathfrak{G})(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{E}(2\kappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{a}(2\kappa A+\mathfrak{A})}{nn} + \frac{\mathfrak{D}(2\kappa E+\mathfrak{C})}{nn}$$

$$2\mathfrak{H} = -\frac{3(2D+J)}{2nn} + \frac{\mathfrak{E}(2\kappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn} - \frac{(\mathfrak{J}-\mathfrak{K})(2\kappa A+\mathfrak{A})}{nn}$$

$$2(\alpha-1)\mathfrak{J} = -\frac{15}{4} - \frac{3(H-L)}{2nn} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn} + \frac{\mathfrak{D}(2n+\mathfrak{C})}{nn} + \frac{2\mathfrak{L}(2\kappa A+\mathfrak{A})}{nn}$$

$$2(\alpha+1)\mathfrak{K} = -\frac{15}{4} - \frac{3H}{2nn} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn} + \frac{\mathfrak{E}(2n+\mathfrak{C})}{nn} + \frac{2\mathfrak{G}(2\kappa D+\mathfrak{D})}{nn}$$

$$2(2\alpha-1)\mathfrak{L} = -\frac{3(2D+J)}{2nn} + \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\kappa H+\mathfrak{H})}{nn} + \frac{2\mathfrak{F}(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn} + \frac{\mathfrak{D}(2\kappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{J}(2\kappa A+\mathfrak{A})}{nn}$$

$$2(2\alpha+1)\mathfrak{M} = \ldots \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\kappa H+\mathfrak{H})}{nn} + \frac{2\mathfrak{G}(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{K}(2\kappa A+\mathfrak{A})}{nn}$$

§. 103. Antequam vlterius progrediamur, sequentes notandae sunt nouae denominationes

$$A' = 2\alpha A + \frac{A(2\kappa B+\mathfrak{B})}{nn} - \frac{B(2\kappa A+\mathfrak{A})}{nn}$$

$$B' = 4\alpha B - \frac{A(2\kappa A+\mathfrak{A})}{nn}$$

$$C' = \frac{\mathfrak{A}(D-E)-A(\mathfrak{D}-\mathfrak{E})}{nn}$$

$$D' = (2\alpha-1)D - \frac{(2n+\mathfrak{C})}{nn}A$$

E' =

(2κD+𝔇)/nn κE+ℭ) 𝔎)(2κA+𝔄) 2κH+ℌ) κA+𝔄) κH+ℌ) κD+𝔇) 2𝔉(2n+ℭ) 𝔍(2κA+𝔄) 𝔎(2κA+𝔄) , sequen- +𝔄) E' =

$$E' = (2\alpha+1)E - \frac{(2n+\mathfrak{C})}{nn}A$$

$$F' = (4\alpha-1)F - \frac{4B}{n} - \frac{A(2\kappa D+\mathfrak{D})-D(2\kappa A+\mathfrak{A})}{nn}$$

$$G' = (4\alpha+1)G - \frac{4B}{n} - \frac{A(2\kappa E+\mathfrak{C})-E(2\kappa A+\mathfrak{A})}{nn}$$

$$a' = 2\alpha a - \frac{2b(2\kappa A+\mathfrak{A})}{nn} - \frac{(D+E)(2n+\mathfrak{C})}{nn} - \frac{2F(2\kappa D+\mathfrak{D})}{nn} + \frac{D(2\kappa F+\mathfrak{F})}{nn}$$

$$b' = 4\alpha b - \frac{a(2\kappa A+\mathfrak{A})}{nn} - \frac{A(2\kappa a+\mathfrak{a})}{nn} - \frac{D(2\kappa E+\mathfrak{E})}{nn} - \frac{E(2\kappa D+\mathfrak{D})}{nn} - \frac{2(F+G)(2n+\mathfrak{C})}{nn}$$

$$H' = 2H + \frac{D\mathfrak{E}-\mathfrak{D}E-A(\mathfrak{J}-\mathfrak{K})+\mathfrak{A}(J-K)}{nn}$$

$$J' = 2(\alpha-1)J + \frac{3A}{2n} - \frac{A(2\kappa H+\mathfrak{H})}{nn} - \frac{D(2n+\mathfrak{C})}{nn} - \frac{2\mathfrak{L}(2\kappa A+\mathfrak{A})}{nn} + \frac{A(2\kappa L+\mathfrak{L})}{nn}$$

$$K' = 2(\alpha+1)K - \frac{3A}{n} - \frac{A(2\kappa H+\mathfrak{H})}{nn} - \frac{E(2n+\mathfrak{C})}{nn} - \frac{2G(2\kappa D+\mathfrak{D})}{nn}$$

$$L' = (2\alpha-1)L - \frac{3B}{n} - \frac{2B(2\kappa H+\mathfrak{H})}{nn} - \frac{2F(2n+\mathfrak{C})}{nn} - \frac{D(2\kappa D+\mathfrak{D})}{nn} - \frac{A(2\kappa J+\mathfrak{J})}{nn} - \frac{J(2\kappa A+\mathfrak{A})}{nn}$$

$$M' = 2(2\alpha+1)M - \frac{3B}{n} - \frac{2B(2\kappa H+\mathfrak{H})}{nn} - \frac{2G(2n+\mathfrak{C})}{nn} - \frac{E(2\kappa E+\mathfrak{E})}{nn}$$

§. 104. Nunc vt terminos completos obtineamus, saltem eos qui angulos 2η et r inuoluunt, notandum

est

est in nostris aequationibus $\sin 2\eta$ et $\cos 2\eta$ non per $\frac{3}{2}(1+2kk)$ sed per $\frac{3}{2}(1+2kk+\frac{3}{2}ee)$ esse multiplicatos. Hinc cum sit fere $\frac{3}{4}ee = \frac{1}{4}kk$, loco kk hic scribi oportebit $\frac{5}{4}kk$, vnde in valore ipsius a pro 3 scripsi $3\cdot\frac{5}{4}$ seu $\frac{15}{4}$. Deinde vt in his terminis quoque rationem habeamus inclinationis orbitae, cuius medius valor sit $= \epsilon$, ponamus $\frac{3}{4}(nn+2+3\mu+\gamma)\operatorname{tang}\epsilon^2 = f =$ $\frac{3}{4}(\frac{3}{2}\kappa\kappa nn - \frac{1}{2}nn - \frac{1}{4} + \gamma - \frac{3}{4}\delta)\operatorname{tang}\epsilon^2$ ob $\mu =$ $\frac{1}{2}(\kappa\kappa-1)nn - \frac{3}{4} - \frac{1}{4}\delta$, eritque nostra aequatio:

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta - \gamma + \tfrac{1}{4}kk - \gamma k\cos r + \tfrac{3}{2}kk\cos 2r + \tfrac{3}{2}\cos 2\eta + \tfrac{15}{4}kk\cos 2\eta$$
$$+ f + \tfrac{1}{2}fkk + fk\cos r + \tfrac{1}{2}fkk\cos 2r$$
$$+ 3k\cos(2\eta - r) + 3k\cos(2\eta + r) + \tfrac{15}{4}kk\cos(2\eta - 2r) + \tfrac{15}{4}kk\cos(2\eta + 2r)$$
$$-2\kappa\int R\,dr - v\left(1 - \tfrac{3}{2}kk + \frac{2f}{nn} + \frac{fkk}{nn} - \frac{(2\gamma - \frac{3}{2}\delta)}{nn}\right)$$
$$+ v\left(3\kappa\kappa + \frac{3}{2nn} - \frac{2f}{nn} + \frac{(2\gamma - \frac{3}{2}\delta)}{nn}\right)k\cos r$$
$$+ v\left(\tfrac{3}{2} - \frac{f}{nn}\right)k^2\cos 2r$$
$$+ v\left(\frac{3}{2nn}\cos 2\eta + \frac{3k}{nn}\cos(2\eta - r) + \frac{3k}{nn}\cos(2\eta + r)\right)$$
$$+ \frac{1}{nn}\left(\int R\,dr\right)^2 + \frac{6v}{nn}\int R\,dr + \frac{3vv}{nn} - \frac{3vv}{nn}k\cos r$$

§. 105. Sit breuitatis gratia:

$$1 + \frac{2f}{nn} - \frac{(2\gamma - \frac{3}{2}\delta)}{nn} = g;\quad 3\kappa\kappa + \frac{3}{2nn} - \frac{2f}{nn} + \frac{(2\gamma - \frac{3}{2}\delta)}{nn} = b$$

et $1 - \frac{3}{2}kk + \frac{2f}{nn} + \frac{fkk}{nn} - \frac{(2\gamma - \frac{3}{2}\delta)}{nn} = \beta$, quo termino

non per iplicatos. bi opor- 3·5/4 seu 1 habea- it = ε, f = μ = o: kcos2η (2η+2r) −3/2δ) nn)kcosr η+r) v/ kcosr δ) = b , quo rmino

termino in angulis ex 2η et r compositis vtemur: eritque $\frac{ddv}{dr^2} =$

$$\tfrac{1}{2}\delta - \gamma + \tfrac{1}{4}kk + f + \tfrac{1}{2}fkk + \frac{3A}{4nn} + \frac{3akk}{4nn}$$
$$+ \frac{3Dkk}{2nn} + \frac{3Ekk}{2nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{\mathfrak{C}\mathfrak{C}kk}{2nn} + \frac{\mathfrak{D}\mathfrak{D}kk}{2nn} + \frac{\mathfrak{E}\mathfrak{E}kk}{2nn} - \frac{3ADkk}{2nn}$$
$$+ \frac{3A\mathfrak{A}}{nn} + \frac{3D\mathfrak{D}kk}{nn} + \frac{3AA}{2nn} + \frac{3DDkk}{2nn} + \frac{3EEkk}{2nn}$$
$$+ \cos 2\eta\,(\tfrac{3}{2} - 2\kappa\mathfrak{A} - \beta A)$$
$$+ kk\cos 2\eta\left\{\tfrac{15}{4} - 2\kappa\mathfrak{a} - \beta a + \tfrac{1}{2}bD + \tfrac{1}{2}bE + \frac{\mathfrak{C}D}{nn} + \frac{3\mathfrak{C}E}{nn}\right.$$
$$+ \cos 4\eta\left\{-2\kappa\mathfrak{B} - \beta B + \frac{3A}{4nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A\mathfrak{A}}{nn} + \frac{3AA}{2nn}\right.$$
$$+ kk\cos 4\eta\begin{cases} -2\kappa\mathfrak{b} - \beta b + \tfrac{1}{2}bF + \tfrac{1}{2}bG + \frac{3E}{2nn} + \frac{3D}{2nn} \\ + \frac{\mathfrak{A}a}{nn} + \frac{\mathfrak{D}\mathfrak{E}}{nn} + \frac{3DE}{nn} + \frac{3Aa}{nn} + \frac{3A\mathfrak{a}}{nn} + \frac{3\mathfrak{A}a}{nn} \end{cases}$$
$$+ k\cos r\begin{cases} -\gamma + f - 2\kappa\mathfrak{C} + \frac{3D}{4nn} + \frac{3E}{4nn} + \frac{3A}{nn} + \frac{\mathfrak{A}\mathfrak{D}}{nn} + \frac{\mathfrak{A}\mathfrak{E}}{nn} \\ + \frac{3A\mathfrak{D}}{nn} + \frac{3A\mathfrak{E}}{nn} + \frac{3\mathfrak{A}D}{nn} + \frac{3\mathfrak{A}E}{nn} + \frac{3AD}{nn} + \frac{3AE}{nn} - \frac{3AA}{2nn} \end{cases}$$
$$+ k\cos(2\eta - r)\left\{3 - 2\kappa\mathfrak{D} - \beta D + \tfrac{1}{2}bA + \frac{3F}{4nn} + \frac{\mathfrak{A}\mathfrak{C}}{nn} + \frac{3A\mathfrak{C}}{nn}\right.$$
$$+ k\cos(2\eta + r)\left\{3 - 2\kappa\mathfrak{E} - \beta E + \tfrac{1}{2}bA + \frac{3G}{4nn} + \frac{\mathfrak{A}\mathfrak{C}}{nn} + \frac{3A\mathfrak{C}}{nn}\right.$$

+

$$+kk\cos 2r\left\{\begin{array}{l}\frac{1}{2}+\frac{1}{2}f-2\varkappa\mathfrak{H}-\epsilon H+\frac{3J}{4nn}+\frac{3K}{4nn}+\frac{3E}{2nn}+\frac{3D}{2nn}\\ +\frac{\mathfrak{C}\mathfrak{C}}{nn}+\frac{\mathfrak{D}\mathfrak{C}}{nn}+\frac{3D\mathfrak{C}}{nn}+\frac{3\mathfrak{D}E}{nn}+\frac{3DE}{nn}-\frac{3AD}{2nn}\end{array}\right.$$

$$+kk\cos(2\eta-2r)\left\{\begin{array}{l}\frac{1}{4}\frac{5}{}-2\varkappa\mathfrak{J}-\epsilon J+\frac{1}{2}bD+\left(\frac{3}{4}-\frac{f}{2nn}\right)A\\ +\frac{3H}{4nn}+\frac{\mathfrak{A}\mathfrak{H}}{nn}+\frac{3A\mathfrak{H}}{nn}+\frac{3\mathfrak{C}D}{nn}\end{array}\right.$$

$$+kk\cos(2\eta+2r)\left\{\begin{array}{l}\frac{1}{4}\frac{5}{}-2\varkappa\mathfrak{K}-\epsilon K+\frac{1}{2}bE+\left(\frac{3}{4}-\frac{f}{2nn}\right)A\\ +\frac{3H}{4nn}+\frac{\mathfrak{A}\mathfrak{H}}{nn}+\frac{3A\mathfrak{H}}{nn}+\frac{3\mathfrak{C}E}{nn}\end{array}\right.$$

$$+kk\cos(4\eta-2r)\left\{\begin{array}{l}-2\varkappa\mathfrak{L}-\epsilon L+\left(\frac{3}{4}-\frac{f}{2nn}\right)B\\ +\frac{3J}{4nn}+\frac{3D}{2nn}+\frac{\mathfrak{D}\mathfrak{D}}{2nn}+\frac{3DD}{2nn}+\frac{1}{2}bF\end{array}\right.$$

$$+kk\cos(\eta 4+2r)\left\{\begin{array}{l}-2\varkappa\mathfrak{M}-\epsilon M+\left(\frac{3}{4}-\frac{f}{2nn}\right)B\\ +\frac{3K}{4nn}+\frac{3E}{2nn}+\frac{\mathfrak{C}\mathfrak{C}}{2nn}+\frac{3EE}{2nn}+\frac{1}{2}bG\end{array}\right.$$

$$+k\cos(4\eta-r)\left\{\begin{array}{l}-2\varkappa\mathfrak{F}-\epsilon F+\frac{1}{2}hB+\frac{3D}{4nn}+\frac{3A}{2nn}\\ +\frac{\mathfrak{A}\mathfrak{D}}{nn}+\frac{3A\mathfrak{D}}{nn}+\frac{3\mathfrak{A}D}{nn}+\frac{3AD}{nn}-\frac{3AA}{4nn}\end{array}\right.$$

$$+k\cos(4\eta+r)\left\{\begin{array}{l}-2\varkappa\mathfrak{G}-\epsilon G+\frac{1}{2}bB+\frac{3E}{4nn}+\frac{3A}{2nn}\\ +\frac{\mathfrak{A}\mathfrak{C}}{nn}+\frac{3A\mathfrak{C}}{nn}+\frac{3\mathfrak{A}E}{nn}+\frac{3AE}{nn}-\frac{3AA}{4nn}\end{array}\right.$$

§. 106.

lit ... I. ... II. ... III. ... IV. ... V. ... 106.

§. 106. Hinc denique nascentur sequentes aequalitates.

I. $\frac{1}{2}\delta-\gamma+\frac{1}{4}kk+\frac{1}{2}fkk+\frac{3A}{4nn}+\frac{\mathfrak{A}\mathfrak{A}}{2nn}+\frac{3A\mathfrak{A}}{nn}+\frac{3AA}{2nn}$

$+\frac{3(D+E)}{nn}kk-\frac{3ADkk}{2nn}+\frac{\mathfrak{C}\mathfrak{C}kk}{2nn}+\frac{(\mathfrak{D}\mathfrak{D}+\mathfrak{C}\mathfrak{C})}{2nn}kk$

$+\frac{3D\mathfrak{D}}{nn}kk+\frac{3(DD+EE)}{2nn}kk+\frac{\mathfrak{A}a}{nn}kk$

$+\frac{3Aa}{nn}kk+\frac{3\mathfrak{A}a}{nn}kk+\frac{3Aa}{nn}kk=\frac{A'(2\varkappa A+\mathfrak{A})}{nn}+\frac{a'(2\varkappa A+\mathfrak{A})}{nn}kk$

$+\frac{D'(2\varkappa D+\mathfrak{D})}{nn}kk+\frac{A'(2\varkappa a+a)}{nn}kk$

II. $\frac{3}{2}-2\varkappa\mathfrak{A}-\epsilon A=-2\alpha A'+\frac{A'(2\varkappa B+\mathfrak{B})}{nn}+\frac{2B'(2\varkappa A+\mathfrak{A})}{nn}$

III. $-2\varkappa\mathfrak{B}-\epsilon B+\frac{3A}{4nn}+\frac{\mathfrak{A}\mathfrak{A}}{2nn}+\frac{3A\mathfrak{A}}{nn}+\frac{3AA}{2nn}$

$=-4\alpha B'+\frac{A'(2\varkappa A+\mathfrak{A})}{nn}$

IV. $\frac{1}{4}\frac{5}{}-2\varkappa a-\epsilon a+\frac{1}{2}b(D+E)+\frac{3\mathfrak{C}(D+E)}{nn}=-2\alpha a'$

$+\frac{(D'+E')(2n+\mathfrak{C})}{nn}+\frac{2b'(2\varkappa A+\mathfrak{A})}{nn}+\frac{2F'(2\varkappa D+\mathfrak{D})}{nn}$

V. $-2\varkappa b-\epsilon b+\frac{1}{2}b(F+G)+\frac{3(D+E)}{nn}+\frac{3D(E+J)}{nn}+\frac{3Aa}{nn}=-4\alpha b'$

$+\frac{a'(2\varkappa A+\mathfrak{A})}{nn}+\frac{E'(2\varkappa D+\mathfrak{D})}{nn}+\frac{2(F'+G')(2n+\mathfrak{C})}{nn}+\frac{D'(2\varkappa E+\mathfrak{C})}{nn}$

VI.

VI. $-\gamma + f - 2\varkappa\mathfrak{C} + \frac{3(D+E)}{4nn} + \frac{3A}{nn} - \frac{3AA}{2nn} + \frac{\mathfrak{A}(\mathfrak{D}+\mathfrak{E})}{nn}$

$+ \frac{3A(\mathfrak{D}+\mathfrak{E})}{nn} + \frac{3A(D+E)}{nn} + \frac{3\mathfrak{A}(D+E)}{nn} =$

$-C' + \frac{A'(2\varkappa D+\mathfrak{D})}{nn} + \frac{A'(2\varkappa E+\mathfrak{E})}{nn} + \frac{(D'+E')(2\varkappa A+\mathfrak{A})}{nn}$

VII. $\mathfrak{z} - 2\varkappa\mathfrak{D} - \mathfrak{C}D + \tfrac{1}{2}bA + \frac{3F}{4nn} + \frac{\mathfrak{C}(\mathfrak{A}+3A)}{nn} =$

$-(2\varkappa-1)D' + \frac{A'(2\varkappa+\mathfrak{C})}{nn}$

VIII. $\mathfrak{z}' - 2\varkappa\mathfrak{E} - \mathfrak{C}E + \tfrac{1}{2}bA + \frac{3G}{4nn} + \frac{\mathfrak{E}(\mathfrak{A}+3A)}{nn} =$

$-(2\varkappa+1)E' + \frac{A'(2\varkappa+\mathfrak{C})}{nn}$.

IX. $-3\varkappa\mathfrak{F} - \mathfrak{C}F + \tfrac{1}{2}bB + \frac{3A}{2nn} + \frac{3D}{4nn} + \frac{\mathfrak{A}\mathfrak{D}}{nn} + \frac{3A\mathfrak{D}}{nn}$

$+ \frac{3\mathfrak{A}D}{nn} + \frac{3AD}{nn} - \frac{3AA}{4nn} = -(4\alpha-1)F + \frac{4B'}{n}$

$+ \frac{A'(2\varkappa D+\mathfrak{D})}{nn} + \frac{D'(2\varkappa A+\mathfrak{A})}{nn}$

X. $-2\varkappa\mathfrak{G} - \mathfrak{C}G + \tfrac{1}{2}bB + \frac{3A}{2nn} + \frac{3E}{4nn} + \frac{\mathfrak{A}\mathfrak{E}}{nn}$

$+ \frac{3A\mathfrak{E}}{nn} + \frac{3\mathfrak{A}E}{nn} + \frac{3AE}{nn} - \frac{3AA}{4nn} = -(4\alpha+1)G' + \frac{4B'}{n}$

$+ \frac{A'(2\varkappa E+\mathfrak{E})}{nn} + \frac{E'(2\varkappa A+\mathfrak{A})}{nn}$

XI.

XI. $\tfrac{1}{2} + \tfrac{1}{2}f - 2\varkappa\mathfrak{H} - \mathfrak{C}H + \frac{3(D+E)}{2nn} + \frac{3(J+K)}{4nn}$

$-\frac{3AD}{2nn} + \frac{\mathfrak{C}\mathfrak{C}}{2nn} - \frac{3AE}{2nn} + \frac{\mathfrak{D}\mathfrak{E}}{nn} + \frac{3\mathfrak{D}E}{nn} + \frac{3D\mathfrak{E}}{nn} + \frac{3DE}{nn}$

$+ \frac{\mathfrak{A}\mathfrak{J}}{nn} + \frac{3\mathfrak{A}J}{nn} + \frac{3A\mathfrak{J}}{nn} + \frac{3AJ}{nn} = -2H' + \frac{A'(2\varkappa J+\mathfrak{J})}{nn}$

$+ \frac{E'(2\varkappa D+\mathfrak{D})}{nn} + \frac{D'(2\varkappa E+\mathfrak{E})}{nn} + \frac{(J'+K')(2\varkappa A+\mathfrak{A})}{nn}$

XII. $\tfrac{1}{4}\mathfrak{s} - 2\varkappa\mathfrak{J} - \mathfrak{C}J + \tfrac{1}{2}bD + \left(\tfrac{1}{4} - \frac{f}{2nn}\right)A + \frac{3H}{4nn}$

$+ \frac{\mathfrak{H}(\mathfrak{A}+3A)}{nn} + \frac{3\mathfrak{C}D}{nn} = -2(\alpha-1)J' + \frac{3A'}{2n}$

$+ \frac{D'(2n+\mathfrak{C})}{nn} + \frac{A'(2\varkappa H+\mathfrak{H})}{nn} + \frac{2L'(2\varkappa A+\mathfrak{A})}{nn}$

XIII. $\tfrac{1}{4}\mathfrak{s} - 2\varkappa\mathfrak{K} - \mathfrak{C}K + \tfrac{1}{2}bE + \left(\tfrac{1}{4} - \frac{f}{2nn}\right)A + \frac{3H}{4nn}$

$+ \frac{\mathfrak{H}(\mathfrak{A}+3A)}{nn} + \frac{3\mathfrak{C}E}{nn} = -2(\alpha+1)K' + \frac{3A'}{2n}$

$+ \frac{E'(2n+\mathfrak{C})}{nn} + \frac{A'(2\varkappa H+\mathfrak{H})}{nn} + \frac{2G'(2\varkappa D+\mathfrak{D})}{nn}$

XIV. $-2\varkappa\mathfrak{C} - \mathfrak{C}L + \tfrac{1}{2}bF + \left(\tfrac{1}{4} - \frac{f}{2nn}\right)B - \frac{3AD}{2nn}$

$+ \frac{3D}{2nn} + \frac{3J}{4nn} + \frac{\mathfrak{D}\mathfrak{D}}{2nn} + \frac{3\mathfrak{D}D}{nn} + \frac{3DD}{2nn} = -2(2\alpha-1)L' + \frac{3B'}{n}$

$+ \frac{2F'(2n+\mathfrak{C})}{nn} + \frac{A'(2\varkappa J+\mathfrak{J})}{nn} + \frac{J'(2\varkappa A+\mathfrak{A})}{nn} + \frac{D'(2\varkappa D+\mathfrak{D})}{nn}$

N XV.

XV. $-2\varkappa\mathfrak{M}-\mathfrak{C}M+\frac{1}{2}bG+\left(\frac{3}{4}-\frac{f}{2nn}\right)B-\frac{3AE}{2nn}$

$$+\frac{3E}{2nn}+\frac{3K}{4nn}+\frac{\mathfrak{C}\mathfrak{C}}{2nn}+\frac{3\mathfrak{C}E}{nn}+\frac{3EE}{2nn}=$$

$$-2(2\alpha+1)M'+\frac{3B'}{n}+\frac{2G'(2n+\mathfrak{C})}{nn}.$$

§. 107. Nunc antequam hos valores inuenire queamus, verus valor ipsius α inuestigari debet: quod fiet ex valore integrali ipsius Φ, qui si vti §. 98. ponatur

$\Phi = Or + \mathfrak{A}' \sin 2\eta + a'kk \sin 2\eta$ + etc. obtinebitur.

$$\varkappa+\frac{A(3\varkappa A+2\mathfrak{A})}{2n^4}+\frac{D(3\varkappa D+2\mathfrak{D})}{2n^4}kk+\frac{A(3\varkappa a+a)}{2n^4}kk+\frac{a(3\varkappa A+2\mathfrak{A})}{2n^4}kk=$$

$$=O-(\mathfrak{A}'+a'kk)\frac{(2\varkappa A+\mathfrak{A})}{nn}-\frac{\mathfrak{D}'(2\varkappa D+\mathfrak{D})}{nn}kk=\alpha+\frac{1}{n}$$

vbi ex obseruationibus constat esse $O = 1,0085272$

Proxime autem esse supra inuenimus esse:

$\mathfrak{A}' = 0,01$	$\mathfrak{A} = -0,80$	$A = -1,25$
$a' = 0,05$	$a = -2,05$	$a = -12,60$
$\mathfrak{D}' = -0,44$	$\mathfrak{D} = -3,60$	$D = 34,25$

atque $\varkappa = 1,0085$; $kk = 0,003$; $nn = 175,71795$

vnde inuenimus $O + 0,000649 = \alpha + \frac{1}{n} = \varkappa + 0,000285$

§. 108. Cum nunc sit $O = 1,0085272$, erit $\alpha + \frac{1}{n} =$ $1,009176$, et ob $\frac{1}{n} = 0,075438$, habebitur vrues valor:

$\alpha = 0,933738$ et $l\alpha = 9,9702255$

atque $\varkappa = 1,008991$ et $l\varkappa = 0,0038874$

Hinc

$-\frac{3AE}{2nn}$ $\frac{E}{n}=$ queamus od fiet atur $\mathfrak{A})kk=$ $\alpha+\frac{1}{n}$ 5272 25 60 25 71795 00285 $-\frac{1}{n}=$ valor: Hinc

Hinc iam primo obtinemus:

$\mathfrak{A} = -0,80313$ $\quad l\text{-}\mathfrak{A} = 9,9047898$

Deinde cum sit satis prope $\mathfrak{C} = -0,67465$, erit

$\frac{(2n+\mathfrak{C})}{nn} = 0,147037$ et $l\frac{2n+\mathfrak{C}}{nn} = 9,1674260$

$\mathfrak{D} = -3,593620$ . . . $l\text{-}\mathfrak{D} = 0,5555310$

$\mathfrak{C} = -1,087320$ . . . $l\text{-}\mathfrak{C} = 0,0363580$

atque porro ex valore ipsius A proxime cognito erit

$\mathfrak{B} = +0,006967$ . . . $l\,\mathfrak{B} = 7,8430540$

et quia est satis prope $B = 0,0128$, erit $A' = 2\alpha A - 0,000720$ et $B' = 4\alpha B - 0,023926$, vnde fit;

$$\tfrac{3}{4} + 1,62172 - \mathfrak{C}A = -4\alpha\alpha A + 0,00144\alpha + 0,000374\alpha A - 0,152\,\alpha B + 0,00091$$

$$+ 0,01247 - \mathfrak{C}B = -16\alpha\alpha B + 0,09570\alpha - 0,038032\alpha A + 0,000014$$

§. 109. Nunc primum quaeri debent valores litterarum f, b et b: et cum sit $\varepsilon = 5^{\circ}, 9'$ et $2\gamma - \frac{3}{2}\delta =$ $-1,3899$ proxime, reperietur

$f = 1,093757$ et $lf = 0,0389208$

$b = 3,0423$ . . . $lb = 0,4832020$

$b = 1,01591$ . . . $lb = 0,0068560$

hincque erit

$2,4720\ A = -3,11947 - 0,142\ B$

$12,9369\ B = +0,07684 - 0,0355\ A$

unde concluditur fore:

$A = -1,262463$ . . $l\text{-}A = 0,1012186$

$B = +0,009404$ . . $l\,B = 7,9733114$

N 2 Porro

Porro vero eſt

$$D' = 0,867676\ D + 0,185628$$
$$E' = 2,867676\ E + 0,185628$$

et $A' = -2,35859$ . . . $l\text{-}A' = 0,3726530$

§. 110. Ex his valoribus aequationes VII et VIII induent has formas,

$$3 + 7,25185 - 6D - 1,92040 - 0,00244 + 0,01704 =$$
$$- 0,75286\ D - 0,16106 - 0,34680$$
$$3 + 2,19420 - 6E - 1,92040 + 0,00006 + 0,01704 =$$
$$- 8,22357\ E - 0,53232 - 0,34680$$

vnde prodibit

$$D = +33,6600 \quad . \; . \; . \quad l\,D = 1,5271130$$
$$E = -\ 0,5785 \quad . \; . \; . \quad l\text{-}E = 9,7623410$$

ergo

$$D' = 29,39153$$
$$E' = -1,47347$$

Ex his nanciſcemur ſequentes formulas pro calculo ſequenti

$$\frac{2\kappa A + \mathfrak{A}}{nn} = -0,01884 \quad l\text{-}\frac{(2\kappa A + \mathfrak{A})}{nn} = 8,275051$$
$$\frac{2\kappa B + \mathfrak{B}}{nn} = +0,000148 \quad l\,\frac{2\kappa B + \mathfrak{B}}{nn} = 6,170262$$
$$\frac{2\kappa D + \mathfrak{D}}{nn} = +0,36611 \quad l\,\frac{2\kappa D + \mathfrak{D}}{nn} = 9,563604$$
$$\frac{2\kappa E + \mathfrak{C}}{nn} = -0,01283 \quad l\text{-}\frac{(2\kappa E + \mathfrak{C})}{nn} = 8,108292$$

§. 111.

§. 111. Ex his iam porro inuenitur

$$\mathfrak{C} = -0,64383 \quad . \; . \; . \quad l\text{-}\mathfrak{C} = 9,808771$$

atque $C' = -0,13847$

Porro valores litterarum $\mathfrak{F}$ et $\mathfrak{G}$ determinabuntur per has aequationes

$$(4\alpha - 1)\ \mathfrak{F} = 0,002049 - 0,294032 + 0,067699$$
$$+ 0,021554 - 0,287335$$
$$(4\alpha + 1)\ \mathfrak{G} = 0,002049 + 0,010306 + 0,020484$$
$$+ 0,021554 + 0,004939$$

ex quibus reperitur

$$\mathfrak{F} = -0,17957 \quad . \; . \; . \quad l\text{-}\mathfrak{F} = 9,254241$$
$$\mathfrak{G} = +0,01253 \quad . \; . \; . \quad l\,\mathfrak{G} = 8,097936$$

atque

$$F' = (4\alpha - 1)\ F - 0,00283 + 0,46219 + 0,63411 =$$
$$(4\alpha - 1)\ F + 1,09347$$
$$G' = (4\alpha + 1)\ G - 0,00283 - 0,01620 - 0,01090 =$$
$$(4\alpha + 1)\ G - 0,02993$$

vnde aequationes IX et X prodibunt.

$$+ 0,36238 - 1,01591\,F + 0,00353 - 0,00680 - 1,00349 =$$
$$- (4\alpha - 1)^2\ F - 2,98415 - 0,86349 + 0,00337 - 0,55370$$
$$- 0,02528 - 1,01591\,G + 0,00353 - 0,00680 + 0,04634 =$$
$$- (4\alpha + 1)^2\ G + 0,14473 + 0,03027 + 0,00337 + 0,02776$$

ſeu

$$6,43486\ F = -\ 3,75359$$
$$21,40769\ G = +\ 0,18534$$

§. 110. Hinc prodeunt ſequentes valores correcti pro F et G,

$$F = -0,58360 \quad . \; . \; . \quad l\text{-}F = 9,766112$$
$$G = +0,00866 \quad . \; . \; . \quad l\,G = 7,937400$$

Ex

Ex formula autem sexta hinc leui calculo colligitur fore:

$$\gamma - f = 1,58161 \quad \text{et} \quad \gamma = 2,67537$$

Valores autem ex F et G deriuati erunt

$$F' = -0,49919 \quad \text{et} \quad G' = 0,01107$$

$$\frac{2\varkappa F + \mathfrak{F}}{nn} = -0,00772 \quad l - \frac{(2\varkappa F + \mathfrak{F})}{nn} = 7,887828$$

$$\frac{2\varkappa G + \mathfrak{G}}{nn} = 0,00017 \quad l\,\frac{2\varkappa G + \mathfrak{G}}{nn} = 6,232305$$

§. 113. Nunc procedamus ad valores litterarum $\mathfrak{a}$ et $\mathfrak{b}$ qui erunt

$$1,867676\,\mathfrak{a} = -3,75000 - 0,68827 - 0,13148 - 0,03764\,\mathfrak{b} + 0,02776$$

$$3,735352\,\mathfrak{b} = -0,57467 - 0,04913 - 0,39807 - 0,01884\,\mathfrak{a} + 0,04611$$

vnde reperitur:

$$\mathfrak{a} = -2,42686 \quad \ldots \quad l - \mathfrak{a} = 0,385044$$

$$\mathfrak{b} = -0,24899 \quad \ldots \quad l - \mathfrak{b} = 9,396182$$

hincque porro

$$a' = 2\alpha a + 0,03768\,b - 4,86420 + 0,16048$$

$$b' = 4\alpha b + 0,01884\,a + 0,16887 + 0,64373 + 0,01450\,a - 0,01744$$

seu

$$a' = 2\alpha a + 0,03768\,b - 4,70372$$

$$b' = 4\alpha b + 0,03334\,a + 0,79516$$

§. 114. Aequationes IV et V hinc induent sequentes formas:

IV.

$$\text{IV.} + 3,75000 - 1,01591\,a + 50,32200 - 0,36363 = -4\alpha\alpha a + 4,89730)$$
$$- 0,07034\,b + 8,78034 + 4,10500 - 0,36551 - 0,00125\,a$$
$$- 0,14067\,b \qquad - 0,02996$$

$$\text{V.} + 0,50246 - 1,01591\,b - 0,87350 + 0,28239 - 0,95732 - 0,02155\,a$$
$$= -16\alpha\alpha b - 0,12447\,a - 2,96860 - 0,00142\,b - 0,03517\,a + 0,17723 - 0,14354 - 0,91659$$

Hinc fit

$$2,47355\,a = -0,21101\,b - 46,11580$$

$$12,93836\,b = -0,13809\,a - 2,80553$$

et

$$a = -18,64200 \quad \ldots \quad l - a = 1,270493$$

$$b = -0,01794 \quad \ldots \quad l - b = 8,253822$$

ex quibus oriuntur:

$$a' = -39,52164 \quad \ldots \quad b' = +0,10663$$

et

$$\frac{2\varkappa a + \mathfrak{a}}{nn} = -0,22790 \quad l - \frac{(2\varkappa a + \mathfrak{a})}{nn} = 9,357744$$

valor autem ipsius $\frac{2\varkappa b + \mathfrak{b}}{nn}$ nullius plane erit momenti, vnde eum praetermittimus.

§. 115. Ex prima autem aequatione §. 106. colligitur

$$\tfrac{1}{2}\,\delta = \gamma - f + 0,02285$$

supra autem inuenimus esse $\gamma - f = 1,58161$, sicque erit $\tfrac{1}{2}\,\delta = 1,60446$ atque

$$\delta = 3,20892 \quad \ldots \quad l\,\delta = 0,506358$$

Nunc cum sit proxime: $\mathfrak{H} = -0,123$; $H = -1,033$ ideoque $\frac{2\varkappa H + \mathfrak{H}}{nn} = -0,0126$; ob $\mathfrak{L} = -1,453$ et $L = +6,252$; habebimus

-0,

$$-0{,}132324\,\mathfrak{J} = -3{,}75000 + 0{,}00882 - 0{,}09088$$
$$+ 0{,}05336 - 0{,}01012$$
$$- 0{,}52839 + 0{,}05474$$
$$+ 3{,}867676\,\mathfrak{K} = -3{,}75000 + 0{,}00882 - 0{,}09088$$
$$+ 0{,}01012$$
$$- 0{,}15987 + 0{,}00917$$

Hinc reperitur

$$\mathfrak{J} = 32{,}05945 \;\ldots\; l\,\mathfrak{J} = 1{,}505956$$
$$\mathfrak{K} = -1{,}02714 \;\ldots\; l\text{-}\mathfrak{K} = 0{,}011629$$

§. 116. Hinc vlterius progrediendo habebimus.

$$J' = 2(\alpha-1)J + 0{,}28571 - 4{,}94924 + 0{,}23502 =$$
$$- 0{,}01591 \qquad - 0{,}08015$$
$$2(\alpha-1)J - 4{,}52457$$
$$K' = 2(\alpha+1)K + 0{,}28571 + 0{,}08507 - 0{,}00317 =$$
$$- 0{,}01591$$
$$2(\alpha+1)K + 0{,}35170$$

vnde aequationes XII et XIII fiunt

$$+ 3{,}75000 - 64{,}69550 - 1{,}01591\,J + 51{,}28180$$
$$- 0{,}94292 + 0{,}00321 - 0{,}36999 =$$
$$- 0{,}00441)$$
$$- 4(\alpha-1)^2 J - 0{,}59871 - 0{,}26690 + 4{,}32164$$
$$+ 0{,}02972 - 0{,}47004$$

$$+ 3{,}75000 + 2{,}07275 - 1{,}01591\,K - 0{,}88006$$
$$- 0{,}94292 + 0{,}00321 + 0{,}00636 =$$
$$- 0{,}00441)$$
$$- 4(\alpha+1)^2 K - 1{,}36026 - 0{,}26690 - 0{,}21665$$
$$+ 0{,}02972 + 0{,}00810$$

ex quibus colligitur fore

$$J = -14{,}09600 \;\ldots\; l\text{-}J = 1{,}149096$$
$$K = -0{,}41676 \;\ldots\; l\text{-}K = 9{,}619888$$

Hinc

Hinc $J' = -2{,}65933 \;\ldots\; K' = -1{,}26020$

atque $\dfrac{2uJ + \mathfrak{J}}{nn} = +0{,}02057 \;.\; l\,\dfrac{2uJ+\mathfrak{J}}{nn} = 8{,}313172$

$$\frac{2uK + \mathfrak{K}}{nn} = -0{,}01063 \;.\; l\text{-}\frac{2uK+\mathfrak{K}}{nn} = 8{,}026598$$

§. 117. Quaeramus iam valorem ipsius $\mathfrak{H}$, ex aequatione

$$2\,\mathfrak{H} = -0{,}45434 - 0{,}39771 - 0{,}01652 + 0{,}62339$$

erit $\mathfrak{H} = -0{,}12264 \;\ldots\; l\text{-}\mathfrak{H} = 9{,}088632$

hincque reperitur: $H' = 2H + 0{,}08011$

vnde aequatio XI praebet:

$$2{,}04688 + 0{,}24748 - 1{,}01591\,H + 0{,}27805 - 3{,}06194$$
$$+ 0{,}36275 + 0{,}00118 - 0{,}00623 \quad + 0{,}02224 + 0{,}03552$$
$$- 0{,}62485 - 0{,}33247 - 0{,}83753 \quad + 0{,}49710 =$$
$$- 4\,H - 0{,}16022 - 0{,}04851 \quad - 0{,}53944 - 0{,}37802$$
$$+ 0{,}07384$$

seu $2{,}98409\,H = -2{,}68053$

Ergo $H = -0{,}89829 \;\ldots\; l\text{-}H = 9{,}953417$

$$H' = -1{,}71647\;;\quad \frac{2uH + \mathfrak{H}}{nn} = -0{,}01102$$

§. 118. Tandem superfunt litterae $\mathfrak{L}$ et $\mathfrak{M}$

$$2(2\alpha-1)\,\mathfrak{L} = -0{,}45434 - 0{,}05280 - 0{,}01652 - 1{,}31570$$
$$+ 0{,}00158 \qquad - 0{,}60396$$
$$- 0{,}00015$$
$$2(2\alpha+1)\,\mathfrak{M} = + 0{,}00158 + 0{,}00368 + 0{,}01935$$
$$- 0{,}00015$$

Hinc $\mathfrak{L} = +1{,}40715 \;\ldots\; l\text{-}\mathfrak{L} = 0{,}148340$

$$\mathfrak{M} = +0{,}00426 \;\ldots\; l\mathfrak{M} = 7{,}629896$$

Deinde

Deinde vero habebitur:

$L' = 2(2a-1)\,L - 0{,}00213 + 0{,}17162 - 12{,}31170 - 0{,}02596$
$\quad + 0{,}00013 \qquad\qquad\qquad\qquad - 0{,}26555$

$M' = 2(2a+1)\,M - 0{,}00213 - 000254 - 0{,}00742$
$\quad + 0{,}00013$

seu $\quad L' = 2(2a-1)\,L - 12{,}43359$
$\qquad M' = 2(2a+1)\,M - 0{,}01196$

§. 119. Nunc denique aggrediamur aequationes XIV et XV

XIV. $-2{,}83960 - 1{,}01591\,L - 0{,}88774 + 0{,}00702 + 0{,}36275$
$\quad + 0{,}28733 \qquad - 0{,}06016 + 0{,}03675 + 7{,}60650 =$
$\quad - 3{,}01144\,L + 21{,}57666 + 0{,}00255 - 0{,}14680 + 10{,}75272$

XV. $+0{,}00860 - 1{,}01591\,M + 0{,}01317 + 0{,}00702 - 0{,}00623$
$\quad - 0{,}00494 \qquad - 0{,}00176 + 0{,}00336 + 0{,}01360 =$
$\quad - 32{,}89415\,M + 0{,}06859 + 0{,}00255 + 0{,}00325$

ex quibus eruitur

$L = +\,13{,}86720 \;\cdots\; l\,L = 1{,}141988$

$M = +\,0{,}00131 \;\cdots\; l\,M = 7{,}117165$

hincque $L' = 11{,}3090$ et $M' = -\,0{,}00445$

$\frac{2\varkappa L + \mathfrak{L}}{nn} = 0{,}15125 \;\cdots\; l\frac{2\varkappa L + \mathfrak{L}}{nn} = 9{,}179684$

$\frac{2\varkappa M + \mathfrak{M}}{nn} = 0{,}00004 \qquad l\frac{2\varkappa M + \mathfrak{M}}{nn} = 5{,}592770$

Ex his valoribus nouae correctiones inueniri possent, sed differentiae prodirent tam exiguae, vt operae pretium non sit eas inuestigare.

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§. 120. His igitur valoribus inuentis, denotante iam a distantiam Lunae mediam a Terra, et eius distantia curtata $= x$, cum sit $x = \frac{(1-kk)\,a}{1-k\cos r}\,u$, erit:

	log. coefficient:
$u = 1 - 0{,}0074991\,\cos 2\eta$	7,875009
$+\,0{,}0000532\,\cos 4\eta$	5,725912
$+\,0{,}191557\,k\,\cos(2\eta - r)$	9,282297
$-\,0{,}003293\,k\,\cos(2\eta + r)$	7,517525
$-\,0{,}003321\,k\,\cos(4\eta - r)$	7,521296
$+\,0{,}000049\,k\,\cos(4\eta + r)$	5,692584
$-\,0{,}00511\,kk\,\cos 2r$	7,708601
$-\,0{,}08022\,kk\,\cos(2\eta - 2r)$	8,904280
$-\,0{,}00237\,kk\,\cos(2\eta + 2r)$	7,375072
$+\,0{,}07892\,kk\,\cos(4\eta - 2r)$	8,897172
$+\,0{,}00001\,kk\,\cos(4\eta + 2r)$	4,872349

vbi quidem in duobus primis terminis simul eos, qui per kk erant affecti, sumus complexi, posito $k = 0{,}05445$. Etiamsi enim hic valor non omnino esset iustus, tamen inde in his terminis minimis nullus error nasci poterit.

§. 121. Porro quoque hinc ex §. 116. valorem ipsius $\frac{d\Phi}{dr}$ determinabimus, quatenus a sola excentricitate orbitae lunaris pendet.

$\frac{d\Phi}{dr} =$		log. coeff.
	$1,009276$	$0,004010$
$+$	$0,0195144 \cos 2\eta$	$8,290355$
$-$	$0,0000322 \cos 4\eta$	$5,507856$
$-$	$0,001231 k \cos r$	$7,090258$
$-$	$0,366103 k \cos(2\eta - r)$	$9,563604$
$+$	$0,012832 k \cos(2\eta + r)$	$8,108292$
$+$	$0,002829 k \cos(4\eta - r)$	$7,451633$
$-$	$0,000171 k \cos(4\eta + r)$	$6,232305$
$+$	$0,01182 kk \cos 2r$	$8,072618$
$-$	$0,02057 kk \cos(2\eta - 2r)$	$8,313172$
$+$	$0,01063 kk \cos(2\eta + 2r)$	$8,026598$
$-$	$0,09883 kk \cos(4\eta - 2r)$	$8,994889$
$-$	$0,00004 kk \cos(4\eta + 2r)$	$5,592770$

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122.

§. 122. Cum nunc sit $\frac{d\theta}{dr} = \frac{ds}{dr} = \frac{1+2ee}{n} + \frac{2}{n} k \cos r + \frac{3}{2n} kk \cos 2r$, erit

$\frac{d\eta}{dr} =$		log. coeff.
	$0,933838$	$9,970272$
$+$	$0,0195144 \cos 2\eta$	$8,290355$
$-$	$0,0000322 \cos 4\eta$	$5,507856$
$-$	$0,152101 k \cos r$	$9,182132$
$-$	$0,366103 k \cos(2\eta - r)$	$9,563604$
$+$	$0,012829 k \cos(2\eta + r)$	$8,108292$
$+$	$0,002829 k \cos(4\eta - r)$	$7,451633$
$-$	$0,000171 k \cos(4\eta + r)$	$6,232305$
$-$	$0,10133 kk \cos 2r$	$9,005738$
$-$	$0,02057 kk \cos(2\eta - 2r)$	$8,313172$
$+$	$0,01063 kk \cos(2\eta + 2r)$	$8,026598$
$-$	$0,09883 kk \cos(4\eta - 2r)$	$8,994889$
$-$	$0,00004 kk \cos(4\eta + 2r)$	$5,592770$

quae formulae ad motum Lunae horarium tam absolutum quam a sole adhiberi possunt, quemadmodum illa distantiam definiens diametro apparenti et parallaxi horizontali inuestigandae inservit.

§. 23. Quaeramus nunc valorem integralem pro longitudine Lunae Φ, quatenus a sola excentricitate orbitae lunaris pendet, ac ponamus,

$$\Phi = Or + \mathfrak{A}'\sin 2\eta + \mathfrak{a}'kk\sin 2\eta + \mathfrak{B}'\sin 4\eta + \mathfrak{b}'kk\sin 4\eta$$
$$+ \mathfrak{C}'k\sin r + \mathfrak{D}'k\sin(2\eta - r) + \mathfrak{F}'k\sin(4\eta - r)$$
$$+ \mathfrak{E}'k\sin(2\eta + r) + \mathfrak{G}'k\sin(4\eta + r)$$
$$+ \mathfrak{H}'kk\sin 2r + \mathfrak{J}'kk\sin(2\eta - 2r) + \mathfrak{L}'kk\sin(4\eta - 2r)$$
$$+ \mathfrak{K}'kk\sin(2\eta + 2r) + \mathfrak{M}'kk\sin(4\eta + 2r)$$

atque sequentes obtinebimus formulas:

$$+0{,}0188387 = 2\alpha\mathfrak{A}' - \frac{\mathfrak{A}'(2\varkappa B + \mathfrak{B})}{nn} - \frac{2\mathfrak{B}'(2\varkappa A + \mathfrak{A})}{nn}$$

$$-0{,}0000370 = 4\alpha\mathfrak{B}' - \frac{\mathfrak{A}'(2\varkappa A + \mathfrak{A})}{nn}$$

$$-0{,}001231 = \mathfrak{C}' - \frac{\mathfrak{A}'(2\varkappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{A}'(2\varkappa E + \mathfrak{E})}{nn} - \frac{(\mathfrak{D}' + \mathfrak{E}')(2\varkappa A + \mathfrak{A})}{nn}$$

$$-0{,}366103 = (2\alpha - 1)\mathfrak{D}' - \frac{\mathfrak{A}'(2n + \mathfrak{C})}{nn}$$

$$+0{,}012832 = (2\alpha + 1)\mathfrak{E}' - \frac{\mathfrak{A}'(2n + \mathfrak{C})}{nn}$$

$$+0{,}002829 = (4\alpha - 1)\mathfrak{F}' - \frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2\varkappa D + \mathfrak{D})}{nn} - \frac{\mathfrak{D}'(2\varkappa A + \mathfrak{A})}{nn}$$

$$-0{,}000171 = (4\alpha + 1)\mathfrak{G}' - \frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2\varkappa E + \mathfrak{E})}{nn} - \frac{\mathfrak{E}'(2\varkappa A + \mathfrak{A})}{nn}$$

+

$$+0{,}22790 = 2\alpha\mathfrak{a}' - \frac{(\mathfrak{D}' + \mathfrak{E}')(2n + \mathfrak{C})}{nn} - \frac{2\mathfrak{F}'(2\varkappa D + \mathfrak{D})}{nn} - \frac{2\mathfrak{b}'(2\varkappa A + \mathfrak{A})}{nn}$$

$$+0{,}00163 = 4\alpha\mathfrak{b}' - \frac{\mathfrak{a}'(2\varkappa A + \mathfrak{A})}{nn} - \frac{\mathfrak{E}'(2\varkappa D + \mathfrak{D})}{nn} - \frac{2(\mathfrak{F}' + \mathfrak{G}')(2n + \mathfrak{C})}{nn}$$

$$+0{,}01182 = 2\mathfrak{H}' - \frac{\mathfrak{E}'(2\varkappa D + \mathfrak{D})}{nn} - \frac{\mathfrak{A}'(2\varkappa J + \mathfrak{J})}{nn} - \frac{(\mathfrak{J}' + \mathfrak{K}')(2\varkappa A + \mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2\varkappa E + \mathfrak{E})}{nn}$$

$$-0{,}02057 = 2(\alpha - 1)\mathfrak{J}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{D}'(2n + \mathfrak{C})}{nn} - \frac{\mathfrak{A}'(2\varkappa H + \mathfrak{H})}{nn} - \frac{2\mathfrak{L}'(2\varkappa A + \mathfrak{A})}{nn}$$

$$+0{,}01063 = 2(\alpha + 1)\mathfrak{K}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{E}'(2n + \mathfrak{C})}{nn} - \frac{\mathfrak{A}'(2\varkappa H + \mathfrak{H})}{nn} - \frac{2\mathfrak{G}'(2\varkappa D + \mathfrak{D})}{nn}$$

$$-0{,}09883 = 2(2\alpha - 1)\mathfrak{L}' - \frac{3\mathfrak{B}'}{n} + \frac{2\mathfrak{F}'(2n + \mathfrak{C})}{nn} + \frac{\mathfrak{A}'(2\varkappa J + \mathfrak{J})}{nn} - \frac{\mathfrak{J}'(2\varkappa A + \mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2\varkappa D + \mathfrak{D})}{nn}$$

$$-0{,}00004 = 2(2\alpha + 1)\mathfrak{M}' - \frac{3\mathfrak{B}'}{n} - \frac{2\mathfrak{G}'(2n + \mathfrak{C})}{nn}$$

§. 124.

§. 124. Ex his eliciuntur valores sequentes:

$\mathfrak{A}' = +0,0100887$	$l\,\mathfrak{A}' = 8,003837$	$a' = 0,09140$
$\mathfrak{B}' = -0,0000409$	$l{-}\mathfrak{B}' = 5,611723$	$l\,a' = 8,960934$
$\mathfrak{C}' = +0,010146$	$l\,\mathfrak{C}' = 8,006295$	$b' = 0,00089$
$\mathfrak{D}' = -0,420226$	$l{-}\mathfrak{D}' = 9,623483$	$l\,b' = 6,949340$
$\mathfrak{E}' = +0,004992$	$l\,\mathfrak{E}' = 7,698261$	$\mathfrak{A}' + a'kk = 0,0103597$
$\mathfrak{F}' = +0,005286$	$l\,\mathfrak{F}' = 7,723163$	$\mathfrak{B}' + b'kk = -0,0000382$
$\mathfrak{G}' = -0,000086$	$l{-}\mathfrak{G}' = 5,935307$	$l(\mathfrak{A}' + a'kk) = 8,015347$
$\mathfrak{H}' = +0,00420$	$l\,\mathfrak{H}' = 7,623250$	$l{-}(\mathfrak{B}' + b'kk) = 5,582063$
$\mathfrak{J}' = +0,57328$	$l\,\mathfrak{J}' = 9,758367$	
$\mathfrak{K}' = +0,00318$	$l\,\mathfrak{K}' = 7,502427$	
$\mathfrak{L}' = -0,15083$	$l{-}\mathfrak{L}' = 9,178488$	
$\mathfrak{M}' = -0,00002$	$l{-}\mathfrak{M}' = 5,301030$	

§. 125. Pro longitudine ergo Lunae habemus hactenus hanc formulam

		log. coeff.
$\Phi = \text{Const.}$	$+\ 1,0085272\ r$	0,003687
	$+\ 0,0103597 \sin 2\eta$	8,015347
	$-\ 0,0000382 \sin 4\eta$	5,582063
	$+\ 0,010146k \sin r$	8,006295
	$-\ 0,420226k \sin(2\eta - r)$	9,623483
	$+\ 0,004992k \sin(2\eta + r)$	7,698261
	$+\ 0,005286k \sin(4\eta - r)$	7,723163
	$-\ 0,000086k \sin(4\eta + r)$	5,935307
	$+\ 0,00420kk \sin 2r$	7,623250
	$+\ 0,57328kk \sin(2\eta - 2r)$	9,758367
	$+\ 0,00318kk \sin(2\eta + 2r)$	7,402427
	$-\ 0,15083kk \sin(4\eta - 2r)$	9,178488
	$-\ 0,00002kk \sin(4\eta + 2r)$	5,301030

§. 126. Quodsi iam ponamus $k = 0,05445$, et hos coefficientes ad minuta secunda cum partibus decimalibus reducamus, longitudo Φ ita exprimetur vt sit:

		log. coeff.
$\Phi = \text{Const.}$	$+\ 1,0085272\ r$	
	$+\ 2136'',8 \sin 2\eta$	3,329772
	$-\ 7,8 \sin 4\eta$	0,895488
	$+\ 113,9 \sin r$	2,056718
	$-\ 4719,6 \sin(2\eta - r)$	3,673906
	$+\ 56,1 \sin(2\eta + r)$	1,748684
	$+\ 59,4 \sin(4\eta - r)$	1,773586
	$-\ 1,0 \sin(4\eta + r)$	9,985730
	$+\ 2,5 \sin 2r$	0,409671
	$+\ 350,6 \sin(2\eta - 2r)$	2,544788
	$+\ 1,5 \sin(2\eta + 2r)$	0,188848
	$-\ 92,2 \sin(4\eta - 2r)$	1,964909
	$-\ 0,0 \sin(4\eta + 2r)$	8,087451

Hisque formulis praecipuae inaequalitates, quibus motus Lunae perturbatur, continentur.

CAPUT VIII.
DE MOTU APOGEI LUNAE.

§. 127.

His inuentis iam arduam illam de motu apogei Lunae quaestionem examinare, atque adeo decidere licebit. Quanquam enim in praecedentibus calculis vbique verum apogei motum, quem obseruationes ostendunt, introduxi, ita vt id ipsum, quod in controuersia est, assumsisse videar; tamen quoniam in hunc ipsum finem terrae vim, qua luna vrgetur, indefinitam sum contemplatus, dum rationi distantiarum reciprocae duplicatae terminum indefinitum adiunxi, vnde littera μ in calculum est ingressa, iudicium de eo apogei motu, qui Theoriae Neutonianae esset consentaneus, non erit difficile. Quodsi enim valor litterae μ nihilo aequalis reperiatur, hinc concludendum erit Theoriam Neutoni cum phaenomenis perfecte consentire; sin autem pro littera μ notabilis prodeat valor, Theoria ista insufficiens erit censenda.

§. 128. Motus autem apogei, quoniam huius rei in calculo nusquam mentio est facta, in ea continetur proportione, quam motus lunae medius ad motum anomaliae tenere est positus. Cum enim remotis lunae inaequalitatibus, quae regulae Keplerianae aduersantur, longitudo lunae vera obtineatur, si eius anomalia vera r ad longitudinem apogei addatur: denotet ϖ longitudinem apogei, eritque longitudo vera $\phi = \varpi + r$, vnde

ei Lu-
cidere
calcu-
tiones
ntro-
hunc
nitam
rocae
era μ
notu,
erit
jualis
utoni
o lit-
ciens
s rei
ietur
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atur,
vera
gitu-
- r,
nde

vnde fit $\varpi = \phi - r$. Ex quo intelligitur, si $\phi - r$ quantitatem designet constantem, apogeum in quiete relinqui, sin autem $\phi - r$ valorem variabilem obtineat, tum apogeum quoque lunae motum esse habiturum.

§. 129. Cum autem terminos illos omnes, qui sinus angulorum implicant, ideoque inaequalitates periodicas continent, quibus apogei motus non afficitur, omittimus, per integrationem deducimur ad huiusmodi formulam $\phi = \text{Const.} + Or$, vnde propterea habetur longitudo apogei $\varpi = \text{Const.} + (O-1)\,r$. Hinc consequimur sequentes proportiones:

I. Vt 1 ad $O-1$, ita motus anomaliae lunae ad motum apogei.

II. Vt O ad 1, ita motus lunae medius ad motum anomaliae.

III. Vt O ad $O-1$, ita motus lunae medius ad motum apogei.

§. 130. Si obseruationes consulamus, valor litterae O reperitur $= 1,0085272$, quem etiam in calculo vbique adhibui; propterea quod propositum erat non tam in istum valorem a priori inquirere, quam ipsam potius Theoriam ita instituere, atque si opus fuerit, emendare, vt motus inde apogei experientiae consentaneus resultaret. Vicissim autem Theoria stabilita, siue Neutoniana siue alia, quae ex determinato pro μ substituto valore oriatur, facile erit valorem ipsius O a priori eruere, quem deinceps cum valore vero 1,0085272 conferre licebit. Vel inuento valore ipsius O, apogeum

 lunae

lunae interuallo menfis apogiftici progredietur per fpatium $(O-1)\ 360°$, interuallo autem menfis periodici per fpatium $(1-\frac{1}{O})\ 360°$. Secundum obferuationes autem apogeum promouetur
vno menfe apogiftico per fpatium 3°, 4′, 11″
vno menfe periodico per fpatium 3, 2, 38

§. 131. Ex calculo autem §. 107. expofito valor litterae O ex elementis ante affumtis ita definitur, vt fit $O+0{,}000649 = x+0{,}000285$ fiue $O = x-0{,}000364$. Etfi enim haec exigua particula 0,000364 iam ex valore ipfius x veritati confentanee affumto eft orta, tamen perfpicuum eft, leuem differentiam nullius hic momenti futuram fuiffe. Verum littera x per Theoriam ita erat affumta, vt effet

$$x = \sqrt{(1+\frac{3+4\mu+\delta}{2nn})}$$

vbi quidem valor ipfius nn ex motu medio lunae ad motum folis relato habetur, ita vt fit fine refpectu ad motum apogei habito $nn = 175{,}71795$. Ergo pro Theoria Neutoniana eft

$$x = \sqrt{(1+\frac{3+\delta}{2nn})} \quad \text{et} \quad O = \sqrt{(1+\frac{3+\delta}{2nn})} - 0{,}000364.$$

§. 132. Hic igitur patet totam hanc inueftigationem ad inuentionem litterae δ reduci, cuius valor, vti ex fuperiori calculo manifeftum eft, a pluribus litteris et coefficientibus terminorum, quos ante eruere oportebat, pendet, ita vt neglecta hac littera δ motus apogei nullo modo recte definiri queat. Initio quidem vbi hanc litteram

ıer fpa- ʼlici per ; autem ɔ [valor r, vt fit ɔoo364 ex va- rta, ta- ıic mo- ıeoriam nae ad ɛctu ad Theo- ɔo364. ligatio- or, vti litteris rtebat, nullo ınc lit- teram

teram in calculum induximus, quod factum eft §. 44. haec res leuis momenti eft vifa; cum enim pro CC, quae erat conftans per integrationem in calculum ingreffa, valorem vero proximum inueniffemus $1+\frac{3+4\mu}{2nn}$, quoniam facile erat praeuidere, reliquis adhibitis elementis ad motum lunae pertinentibus, hunc valorem aliquantum immutari poffe, pro vero valore ipfius $\frac{CC}{1-kk}$ pofuimus $1+\frac{3+4\mu+\delta}{2nn}$. Deinde autem valor ipfius δ potiffimum pendet a valore litterae γ, qua vfi fumus ad verum valorem conftantis $\frac{m}{nn} = 1+\frac{2+3\mu+\gamma}{nn}$ obtinendum, cum proxime verus effet inuentus $= 1+\frac{2+3\mu}{nn}$.

§. 133. Ab his ergo litteris γ et δ, quae initio nullius fere vfus effe videbantur, determinatio motus apogei potiffimum pendet, quae cum ex pluribus atque adeo omnibus inaequalitatibus lunae ab excentricitate orbitis determinari debeant, mirum fane non eft, quod legitima motus apogei defignatio, cum tantis implicata fit difficultatibus, tam dudum fuerit abfcondita. Plerique enim, qui motum apogei ex fola Theoria concludere funt annifi, ad omnes has inaequalitates non refpexerunt, atque calculum perinde adminiftrauerunt, ac fi hic litteras γ et δ neglexiffemus. Ac fi non defuere, qui fibi perfuaferunt, motum apogei cum Theoria Neutoniana confentire, ii plerumque per errorem calculi feducti ad veritatem perueniffe fibi funt vifi. Quin etiam Ipfe Neutonus

tonus Theoriae suae in motu apogei determinando parum tribuisse videtur.

§. 134. Hinc ex neglectu harum litterarum γ et δ, seu ex alia omissione eodem recidente, factum est, vt Theoria Neutoni obseruationibus circa motum apogei lunae institutis plane non satisfacere sit putata; quae opinio etiam ita inualuit, vt perspicacissimus quisque hanc Theoriam insufficientem pronunciaret. Atque sagacissimus Clairaltius huic opinioni vehementissime erat addictus, antequam publice in contrarias partes discesserat. Eadem scilicet ratione ob neglectum minutarum illarum particularum erat deceptus, qua et ego fateri cogor, me per complures annos constanter esse opinatum, ex Theoria Neutoni pro motu apogei Lunae non vltra semissem prodire, ita vt error vltra semissem exsurgens committeretur.

§. 135. Fons itaque huius erroris, qui nisi summa circumspectio adhibeatur, vix euitatur, in eo latet, quod in calculo debita illa constantium determinatio, pro qua equidem hic litteras γ et δ adhibui, negligatur. Quemadmodum per hanc omissionem dimidius tantum apogei motus eliciatur, ostendisse iuuabit. Sit igitur $\delta = 0$, atque littera illius O secundum Theoriam Neutonianam, qua est $\mu = 0$, valor erit $O = \sqrt{(1 + \frac{3}{2nn})} - 0{,}000364$; qui euolutus fit: $O = 1{,}0042592 - 0{,}000364$. Quare etiamsi particula 0,000364 vtpote ex profundiori indagine nata praetermittatur, tamen iste valor pro $O = 1{,}0042592$, si cum vero per obseruationes cognito $O = 1{,}0085272$ comparetur, exacte fere dimidium motum apogei praebet;

nando pa-
um γ et δ,
m est, vt
n apogei
quae opi-
que hanc
sagacissi-
erat addi-
iscesserat.
n illarum
gor, me
Theoria
em prodi-
itteretur.
i summa
tet, quod
pro qua
Quemad-
ogei mo-
0, atque
am, qua
364; qui
e etiamsi
gine na-
042592,
085272
ei prae-
bet;

bet; atque adeo haec tam accurata medietas non parum digna videtur.

§. 136. Jam videamus, quam prope valorem litterae δ adhibendo ad veritatem perducamur. Inuenimus autem (115) $\delta = 3{,}20892$, vnde prodit

$$\sqrt{(1 + \frac{3+\delta}{2nn})} = 1{,}0087947$$

qui valor iam maior est quam verus 1,0085272, sed recordandum est inde subtrahi debere 0,000364, sicque relinquetur $O = 1{,}0084307$, ex quo motus progressiuus apogei pro interuallo mensis apogistici prodibit $= 3° 2' 9''$ et pro interuallo mensis periodici $= 3° 0' 37''$, qui numeri duobus tantum minutis a vero deficiunt. Ad hunc defectum supplendum litterae μ tribui poterit valor conueniens ex formula $\mu = \frac{1}{2}(\varkappa\varkappa - 1)nn - \frac{3}{4} - \frac{1}{4}\delta$, vnde reperitur $\mu = 0{,}03782$, qui valor tantillus est, ut nisi de motu apogei sit quaestio, semper pro nihilo haberi possit.

§. 137. Verum nullo modo affirmare possumus, valores illos pro γ et δ inuentos ita esse absolutos, vt nulla amplius correctione indigeant. Quin potius, si formulas supra exhibitas attentius perpendamus, tantum abest vt eas pro completis habere possimus, vt potius manifestum sit, omnes reliquas inaequalitates motus lunae perinde ac eas quas iam definiuimus, terminos quoque in eas suppeditare. Qui etsi admodum erunt parui, tamen omnino sufficere poterunt ad exiguum istud supplementum, quo adhuc a vero distamus, conficiendum.

Cum

Cum enim sola fere inaequalitas ab angulo $2\eta - r$ pendens motum apogei a dimidio tantopere auxisset, vt valor ipsius O ab 1,0042592 vsque ad 1,0084307 increvisset, nullum fere est dubium, quin leuis defectus huius numeri a vero valore 1,0085272 a reliquis inaequalitatibus proficiscatur.

§. 138. Hinc igitur concludere debemus, Theoriam Neutonianam cum motu apogei obseruato tam exacte conuenire, vt aberratio, si quidem vlla locum habeat, tam sit exigua, vt merito pro nihilo reputari possit: neque etiam calculi ope ob summam paruitatem eam certo definire licebit. Cum itaque hoc pacto Theoria Neutoniana a fortissima obiectione sit vindicata, gloria huius insignis inuenti cum industriae tum candori excellentissimi Clairalti debetur, qui primus egregium hunc Theoriae consensum cum veritate detexit et publice est professus: cui ea re eo maiores debemus gratias, quod sine eius studio summo, quod in hac inuestigatione consumsit, Theoria Neutoniana fortasse vix vnquam ab hac suspicione insufficientiae esset liberata. Atque nunc demum pleno lumine veritas istius Theoriae, cui vni Astronomiae Theoria vniuersa innititur, fulgere est censenda, cum antea non mediocribus tenebris fuisset inuoluta.

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CAPUT IX.

INUESTIGATIO INAEQUALITATUM LUNAE A SOLA EXCENTRICITATE ORBITAE SOLIS PENDENTIUM.

§. 139.

Quoniam in hac inuestigatione excentricitas orbitae lunaris non in censum venit, inaequalitates quas scrutamur partim ab anomalia vera solis s partim ab angulo 2η pendebunt. Cum igitur sit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n}k\cos r - \frac{2}{n}e\cos s - \frac{2}{n}ek\cos(r-s)$$
$$+ \frac{3}{2n}kk\cos 2r + \frac{1}{2n}ee\cos 2s - \frac{2}{n}ek\cos(r+s)$$

hinc differentiale ds ad differentiale dr reducitur. Atque hoc quidem capite, quia ad excentricitatem Lunae non attendimus, erit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} - \frac{2}{n}e\cos s + \frac{1}{2n}ee\cos 2s.$$

§. 140. Incipiamus ergo a formulis $\int R\,dr$ et v, quas omissis terminis ab angulo r pendentibus ponamus

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta - s) + \mathfrak{R}e\cos(2\eta + s)$$
$$+ \mathfrak{S}ee\cos 2s + \mathfrak{T}ee\cos(2\eta - 2s) + \mathfrak{V}ee\cos(2\eta + 2s)$$
$$v = A\cos 2\eta + Pe\cos s + Qe\cos(2\eta - s) + Re\cos(2\eta + s)$$
$$+ See\cos 2s + Tee\cos(2\eta - 2s) + Vee\cos(2\eta + 2s)$$

vbi quidem pro $\mathfrak{A}$ et A valores ſupra inuentos completos accipi oportet, ita vt in iis termini akk et αkk ſint comprehenſi; erit ergo

$$\mathfrak{A} = -0,81033 \qquad l{-}\mathfrak{A} = 9,908662$$
$$A = -1,31773 \qquad l{-}A = 0,119826$$

Valores autem hinc deriuati erunt:

$$\frac{2\varkappa A + \mathfrak{A}}{nn} = -0,019744, \quad l\frac{-(2\varkappa A + \mathfrak{A})}{nn} = 8,295442$$
$$A' = -2,47576 \qquad l{-}A' = 0,393708$$
$$\mathfrak{A}' = +0,01036 \qquad l\,\mathfrak{A}' = 8,015347$$

Terminos autem angulum quadruplum 4η inuoluentes hic ob ſummam paruitatem omiſi, quoniam in combinatione cum angulo s plane fierent imperceptibiles.

§. 141. Hinc iam primo colligitur:

$$\frac{d\Phi}{dr} = \varkappa - \frac{(2\varkappa A + \mathfrak{A})}{nn}\cos 2\eta - \frac{(2\varkappa P + \mathfrak{P})}{nn}\, e\cos s$$
$$- \frac{(2\varkappa Q + \mathfrak{Q})}{nn}\, e\cos(2\eta - s) - \frac{(2\varkappa R + \mathfrak{R})}{nn}\, e\cos(2\eta + s)$$
$$- \frac{(2\varkappa S + \mathfrak{S})}{nn}\, ee\cos 2s$$
$$- \frac{(2\varkappa T + \mathfrak{T})}{nn}\, ee\cos(2\eta - 2s) - \frac{(2\varkappa V + \mathfrak{V})}{nn}\, ee\cos(2\eta + 2s)$$

atque porro

$$\frac{d\eta}{dr} = \alpha - \frac{(2\varkappa A + \mathfrak{A})}{nn}\cos 2\eta - \left(\frac{2\varkappa P + \mathfrak{P}}{nn} - \frac{2}{n}\right) e\cos s$$
$$- \frac{(2\varkappa Q + \mathfrak{Q})}{nn}\, e\cos(2\eta - s) + \frac{(2\varkappa R + \mathfrak{R})}{nn}\, e\cos(2\eta + s)$$
$$- \left(\frac{(2\varkappa S + \mathfrak{S})}{nn} - \frac{1}{2n}\right) ee\cos 2s$$

Deinde

Deinde quia eſt proxime $kk = 9ee$, erit

$$R = \tfrac{3}{2}(1 + \tfrac{27}{2}ee)\sin 2\eta + \left(\frac{3Q}{2nn} - \frac{3R}{2nn}\right) e\sin s$$
$$- \left(\tfrac{9}{4} - \frac{3P}{2nn}\right) e\sin(2\eta - s) - \left(\tfrac{9}{4} - \frac{3P}{2nn}\right) e\sin(2\eta + s)$$
$$+ \left(\frac{3T}{2nn} - \frac{3V}{2nn}\right) ee\sin 2s$$
$$+ \left(\tfrac{9}{8} + \frac{3S}{2nn}\right) ee\sin(2\eta - 2s) + \left(\tfrac{9}{8} + \frac{3S}{2nn}\right) ee\sin(2\eta + 2s)$$

atque omiſſis terminis, quibus non eſt opus

$$\frac{ddv}{dr^2} = e\cos s \left\{ \begin{array}{l} -\tfrac{3}{2} - 2\varkappa\mathfrak{P} - 6P - \frac{9A}{4nn} + \frac{\mathfrak{A}\mathfrak{Q}}{nn} + \frac{\mathfrak{A}\mathfrak{R}}{nn} \\ + \frac{3AQ}{nn} + \frac{3AR}{nn} + \frac{3\mathfrak{A}(Q+R)}{nn} + \frac{3A(\mathfrak{Q}+\mathfrak{R})}{nn} \end{array} \right.$$

$$+ e\cos(2\eta - s) \left\{ \begin{array}{l} -\tfrac{9}{4} - 2\varkappa\mathfrak{Q} - 6Q - \frac{3A}{4nn} + \frac{3P}{4nn} \\ + \frac{\mathfrak{A}\mathfrak{P}}{nn} + \frac{3AP}{nn} + \frac{3\mathfrak{A}P}{nn} + \frac{3A\mathfrak{P}}{nn} \end{array} \right.$$

$$+ e\cos(2\eta + s) \left\{ \begin{array}{l} -\tfrac{9}{4} - 2\varkappa\mathfrak{R} - 6R - \frac{3A}{4nn} + \frac{3P}{4nn} \\ + \frac{\mathfrak{A}\mathfrak{P}}{nn} + \frac{3AP}{nn} + \frac{3\mathfrak{A}P}{nn} + \frac{3A\mathfrak{P}}{nn} \end{array} \right.$$

$$+ ee\cos 2s \left\{ \begin{array}{l} +\tfrac{3}{4} - 2\varkappa\mathfrak{S} - 6S - \frac{3P}{4nn} + \frac{\mathfrak{P}\mathfrak{P}}{2nn} \\ + \frac{3PP}{2nn} + \frac{3P\mathfrak{P}}{nn} \end{array} \right.$$

Q 2 +

$$+ee\cos(2\eta-2s)\begin{cases}+\frac{9}{8}-2\kappa\mathfrak{T}-6T-\frac{9P}{8nn}+\frac{\mathfrak{A}\mathfrak{S}}{nn}\\ +\frac{3AS}{nn}+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}\end{cases}$$

$$+ee\cos(2\eta+2s)\begin{cases}+\frac{9}{8}-2\kappa\mathfrak{V}-6V-\frac{9P}{8nn}+\frac{\mathfrak{A}\mathfrak{S}}{nn}\\ +\frac{3AS}{nn}+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}\end{cases}$$

§. 142. Quodsi iam forma pro $\int R\,dr$ assumta differentietur, orietur:

$$R = -2\alpha\mathfrak{A}\sin 2\eta$$
$$+e\sin s\left(-\frac{1}{n}\mathfrak{P}-\frac{(\mathfrak{Q}-\mathfrak{R})(2\kappa A+\mathfrak{A})}{nn}\right)$$
$$+e\sin(2\eta-s)\left(-\frac{2\mathfrak{A}}{n}\pm\frac{\mathfrak{A}(2\kappa P+\mathfrak{P})}{nn}-(2\alpha-\frac{1}{n})\mathfrak{Q}\right)$$
$$+e\sin(2\eta+s)\left(-\frac{2\mathfrak{A}}{n}+\frac{\mathfrak{A}(2\kappa P+\mathfrak{P})}{nn}-(2\alpha+\frac{1}{n})\mathfrak{R}\right)$$
$$+ee\sin 2s\left(\frac{1}{n}\mathfrak{P}-\frac{2}{n}\mathfrak{S}-\frac{(\mathfrak{T}-\mathfrak{V})(2\kappa A+\mathfrak{A})}{nn}\right)$$
$$+ee\sin(2\eta-2s)\left(\frac{1}{2n}\mathfrak{A}-\frac{3}{n}\mathfrak{Q}-(2\alpha-\frac{2}{n})\mathfrak{T}\right)$$
$$+ee\sin(2\eta+2s)\left(\frac{1}{2n}\mathfrak{A}-\frac{1}{n}\mathfrak{R}-(2\alpha+\frac{2}{n})\mathfrak{V}\right)$$

§. 143. Comparatione ergo instituta habebitur

$$-\frac{1}{n}\mathfrak{P}-\frac{(\mathfrak{Q}-\mathfrak{R})(2\kappa A+\mathfrak{A})}{nn}=\frac{3(Q-R)}{2nn}$$

—

$\frac{P}{nn}+\frac{\mathfrak{A}\mathfrak{S}}{nn}$ $\frac{A\mathfrak{S}}{nn}$ $\frac{P}{n}+\frac{\mathfrak{A}\mathfrak{S}}{nn}$ $\frac{\mathfrak{S}}{n}$ mta dif- $\mathfrak{A})$ $-\frac{1}{n})\mathfrak{Q})$ $\frac{1}{n})\mathfrak{R})$ $A+\mathfrak{A})$ $\frac{2}{n})\mathfrak{T})$ $)\mathfrak{V})$ tur —

$$-\frac{2}{n}\mathfrak{A}+\frac{\mathfrak{A}(2\kappa P+\mathfrak{P})}{nn}-(2\alpha-\frac{1}{n})\mathfrak{Q}=-\frac{9}{4}+\frac{3P}{2nn}$$
$$-\frac{2}{n}\mathfrak{A}+\frac{\mathfrak{A}(2\kappa P+\mathfrak{P})}{nn}-(2\alpha+\frac{1}{n})\mathfrak{R}=-\frac{9}{4}+\frac{3P}{2nn}$$
$$\frac{1}{n}\mathfrak{P}-\frac{2}{n}\mathfrak{S}-\frac{(\mathfrak{T}-\mathfrak{V})(2\kappa A+\mathfrak{A})}{nn}=\frac{3(T-V)}{2nn}$$
$$\frac{1}{2n}\mathfrak{A}-\frac{3}{n}\mathfrak{Q}-(2\alpha-\frac{2}{n})\mathfrak{T}=\frac{9}{8}+\frac{3S}{2nn}$$
$$\frac{1}{2n}\mathfrak{A}-\frac{1}{n}R-(2\alpha+\frac{2}{n})\mathfrak{V}=\frac{9}{8}+\frac{3S}{2nn}$$

vnde deinceps valores litterarum germanicarum $\mathfrak{P}$, $\mathfrak{Q}$, $\mathfrak{R}$, $\mathfrak{S}$, $\mathfrak{T}$, $\mathfrak{V}$ sumus inuestigaturi.

§. 144. Differentietur simili modo quantitas v, ac ponatur:

$$\frac{dv}{dr}=-A'\sin 2\eta-P'e\sin s-Q'e\sin(2\eta-s)-S'ee\sin 2s-T'ee\sin(2\eta-2s)$$
$$-R'e\sin(2\eta+s)\qquad -V'ee\sin(2\eta+2s)$$

eritque

$A'=2\alpha\mathfrak{A}$, cuius quidem valor iam supra habetur

$$P'=\frac{1}{n}P+\frac{(Q-R)(2\kappa A+\mathfrak{A})}{nn}$$
$$Q'=(2\alpha-\frac{1}{n})Q+\frac{2}{n}A-\frac{A(2\kappa P+\mathfrak{P})}{nn}$$
$$R'=(2\alpha+\frac{1}{n})R+\frac{2}{n}A-\frac{A(2\kappa P+\mathfrak{P})}{nn}$$
$$S'=\frac{2}{n}S-\frac{1}{n}P+\frac{(T-V)(2\kappa A+\mathfrak{A})}{nn}$$

$$T' = (2\alpha - \frac{2}{n})T + \frac{3}{n}Q - \frac{1}{2n}A$$

$$V' = (2\alpha + \frac{2}{n})V + \frac{1}{n}R - \frac{1}{2n}A$$

vnde denuo differentiando eruitur.

$$\frac{ddv}{dr^2} = e\cos s\,(-\frac{1}{n}P' + \frac{(Q'+R')(2\kappa A+\mathfrak{A})}{nn}$$

$$e\cos(2\eta - s)\,(-(2\alpha - \frac{1}{n})Q' - \frac{2}{n}A' + \frac{A'(2\kappa P+\mathfrak{P})}{nn}$$

$$e\cos(2\eta + s)\,(-(2\alpha + \frac{1}{n})R' - \frac{2}{n}A' + \frac{A'(2\kappa P+\mathfrak{P})}{nn}$$

$$ee\cos 2s\,(-\frac{2}{n}S' + \frac{1}{n}P' + \frac{(T'+V')(2\kappa A+\mathfrak{A})}{nn}$$

$$ee\cos(2\eta - 2s)\,(-(2\alpha - \frac{2}{n})T' + \frac{1}{2n}A' - \frac{3}{n}Q'$$

$$ee\cos(2\eta + 2s)\,(-(2\alpha + \frac{2}{n})V' + \frac{1}{2n}A' - \frac{1}{n}R'$$

§. 145. Sequentes ergo aequationes resoluendae occurrent

$$-\frac{3}{4} - 2\kappa\mathfrak{P} - 6P - \frac{9A}{4nn} + \frac{(\mathfrak{A}+3A)(Q+R)}{nn} + \frac{(3\mathfrak{A}+3A)(Q+R)}{nn} = -\frac{1}{n}P' + \frac{(Q'+R')(2\kappa A+\mathfrak{A})}{nn}$$

$$-\frac{2}{4} - 2\kappa\mathfrak{Q} - 6Q - \frac{3A}{4nn} + \frac{3P}{4nn} + \frac{(\mathfrak{A}+3A)\mathfrak{P}}{nn} + \frac{(3\mathfrak{A}+3A)P}{nn} = -(2\alpha - \frac{1}{n})Q' - \frac{2}{n}A' + \frac{A'(2\kappa P+\mathfrak{P})}{nn}$$

$$-\frac{2}{4} - 2\kappa\mathfrak{R} - 6R - \frac{3A}{4nn} + \frac{3P}{4nn} + \frac{(\mathfrak{A}+3A)\mathfrak{P}}{nn} + \frac{(3\mathfrak{A}+3A)P}{nn} = -(2\alpha + \frac{1}{n})R' - \frac{2}{n}A' + \frac{A'(2\kappa P+\mathfrak{P})}{nn}$$

$$+\frac{3}{4} - 2\kappa\mathfrak{S} - 6S - \frac{3P}{4nn} + \frac{\mathfrak{P}\mathfrak{P} + 6P\mathfrak{P} + 3PP}{2nn} = -\frac{2}{n}S' + \frac{1}{n}P' + \frac{(T'+V')(2\kappa A+\mathfrak{A})}{nn}$$

$$+\frac{9}{8} - 2\kappa\mathfrak{T} - 6T - \frac{9P}{8nn} + \frac{(\mathfrak{A}+3A)\mathfrak{S}}{nn} + \frac{(3\mathfrak{A}+3A)S}{nn} = -(2\alpha - \frac{2}{n})T' + \frac{1}{2n}A' - \frac{3}{n}Q'$$

$$+\frac{9}{8} - 2\kappa\mathfrak{V} - 6V - \frac{9P}{8nn} + \frac{(\mathfrak{A}+3A)\mathfrak{S}}{nn} + \frac{(3\mathfrak{A}+3A)S}{nn} = -(2\alpha + \frac{2}{n})V' + \frac{1}{2n}A' - \frac{1}{n}R'$$

Neglectis primo terminis minimis, qui adhuc sunt incogniti, reperitur:

$\mathfrak{Q} = +1,3238$	$Q = +2,5714$	$Q' = 4,40924$
$\mathfrak{R} = +1,2210$	$R = +2,0571$	$R' = 3,79793$
$\mathfrak{P} = -0,0313$	$P = -1,4807$	$P' = -0,12185$

§. 146. Ex his autem accuratius ita definientur vt sit:

$\mathfrak{Q} = +1,33859$ . . . $l\mathfrak{Q} = 0,126649$
$\mathfrak{R} = +1,23468$ . . . $l\mathfrak{R} = 0,091545$
$Q = +2,60087$ . . . $lQ = 0,415119$
$R = +2,00590$ . . . $lR = 0,302308$
$Q' = 4,44801$; $R' = 3,67581$

et

et $\mathfrak{P} = -0{,}04010$ . . . $l\text{-}\mathfrak{P} = 8{,}603133$
$P = -1{,}46488$ . . . $l\text{-}P = 0{,}165801$
$P' = -0{,}12222$

Deinde reperitur

$\mathfrak{S} = +0{,}03911$ . . . $l\mathfrak{S} = 8{,}592230$
$S = +0{,}60882$ . . . $lS = 9{,}784486$
$\mathfrak{T} = -0{,}85267$ . . . $l\text{-}\mathfrak{T} = 9{,}930827$
$\mathfrak{V} = -0{,}62125$ . . . $l\text{-}\mathfrak{V} = 9{,}793266$
$T = -2{,}60380$ . . . $l\text{-}T = 0{,}465615$
$V = -1{,}02720$ . . . $l\text{-}V = 0{,}011672$

Pro sequentibus vero calculis est

$\frac{2\varkappa P + \mathfrak{P}}{nn} = -0{,}017051$ . . . $l\text{-}\frac{(2\varkappa P + \mathfrak{P})}{nn} = 8{,}231755$

$\frac{2\varkappa Q + \mathfrak{Q}}{nn} = +0{,}037487$ . . . $l+\frac{2\varkappa Q + \mathfrak{Q}}{nn} = 8{,}573878$

$\frac{2\varkappa R + \mathfrak{R}}{nn} = +0{,}030062$ . . . $l+\frac{2\varkappa R + \mathfrak{R}}{nn} = 8{,}478023$

§. 147. Hinc igitur pro distantia lunae a terra curtata $x = \frac{(1-kk)au}{1-k\cos r}$, pars quantitatis x ab excentricitate orbitae solaris tantum pendens erit

		log. coeff.
$x =$ Praeced.	$-\ 0{,}008336e\ \cos s$	7,920985
	$+\ 0{,}014801e\ \cos(2\eta - s)$	8,170303
	$+\ 0{,}011415e\ \cos(2\eta + s)$	8,057492
	$+\ 0{,}00364ee\ \cos 2s$	7,539670
	$-\ 0{,}01482ee\ \cos(2\eta - 2s)$	8,170799
	$-\ 0{,}00584ee\ \cos(2\eta + 2s)$	7,766856

Deinde

31755
73878
78023
a cur-
icitate
coeff.
:0985
0303
7492
9670
:799
5856
:inde

Deinde vero erit

		log. coeff.
$\frac{d\Phi}{ds} =$ Praec.	$+\ 0{,}017041e\cos s$	8,231755
	$-\ 0{,}037487e\cos(2\eta - s)$	8,573878
	$-\ 0{,}030062e\cos(2\eta + s)$	8,478023
	$-\ 0{,}00722ee\cos 2s$	7,858166
	$+\ 0{,}03470ee\cos(2\eta - 2s)$	8,540319
	$+\ 0{,}01533ee\cos(2\eta + 2s)$	8,185614

§. 148. Ponatur nunc integrale

$$\Phi = \text{Praec.} + \mathfrak{A}'\sin 2\eta + \mathfrak{P}'e\sin s + \mathfrak{Q}'e\sin(2\eta - s) + \mathfrak{R}'e\sin(2\eta + r) + \mathfrak{S}'ee\sin 2s + \mathfrak{T}'ee\sin(2\eta - 2s) + \mathfrak{V}'ee\sin(2\eta + 2s)$$

erit differentiatione peracta:

$$+0{,}017051 = \frac{1}{n}\mathfrak{P}' - \frac{(\mathfrak{Q}' + \mathfrak{R}')(2\varkappa A + \mathfrak{A})}{nn}$$

$$-0{,}037489 = \left(2\alpha - \frac{1}{n}\right)\mathfrak{Q}' + \frac{2}{n}\mathfrak{A}' - \frac{\mathfrak{A}'(2\varkappa P + \mathfrak{P})}{nn}$$

$$-0{,}030062 = \left(2\alpha + \frac{1}{n}\right)\mathfrak{R}' + \frac{2}{n}\mathfrak{A}' - \frac{\mathfrak{A}'(2\varkappa P + \mathfrak{P})}{nn}$$

$$-0{,}00722 = \frac{2}{n}\mathfrak{S}' - \frac{1}{n}\mathfrak{P}' - \frac{(\mathfrak{T}' + \mathfrak{V}')(2\varkappa A + \mathfrak{A})}{nn}$$

$$+0{,}03470 = \left(2\alpha - \frac{2}{n}\right)\mathfrak{T}' - \frac{1}{2n}\mathfrak{A}' + \frac{3}{n}\mathfrak{Q}'$$

$$+0{,}01533 = \left(2\alpha + \frac{2}{n}\right)\mathfrak{V}' - \frac{1}{2n}\mathfrak{A}' + \frac{1}{n}\mathfrak{R}'$$

 fietque

fietque his valoribus determinatis

		log. coeff.
$\Phi =$ Praec.	$+\ 0{,}236034\, e\ \sin s$	9,372974
	$-\ 0{,}021889\, e\ \sin(2\eta - s)$	8,340237
	$-\ 0{,}016368\, e\ \sin(2\eta + s)$	8,214002
	$+\ 0{,}06615\, ee\ \sin 2s$	8,820508
	$+\ 0{,}02332\, ee\ \sin(2\eta - 2s)$	8,367825
	$+\ 0{,}00840\, ee\ \sin(2\eta + 2s)$	7,924429

§. 149. Reducamus has inaequalitates etiam ad minuta secunda, ponendo excentricitatem orbitae solaris $e = 0{,}01680$, atque habebimus

		log. coeff.
$\Phi =$ Praec.	$+\ 817'',9\ \sin s$	2,912708
	$-\ 75,\ 8\ \sin(2\eta - s)$	1,879971
	$-\ 56,\ 7\ \sin(2\eta + s)$	1,753736
	$+\ 3,\ 8\ \sin 2s$	0,585551
	$+\ 1,\ 4\ \sin(2\eta - 2s)$	0,132868
	$+\ 0,\ 5\ \sin(2\eta + 2s)$	9,689472

Denotat hic s anomaliam veram solis; vnde patet eam Lunae inaequalitatem, quae sinui huius anomaliae est proportionalis, admodum esse notabilem, dum ad 13′, 38″ exsurgit. Tabulae autem Astronomicae, vbi haec inaequalitas aequatio solaris nominatur, eam multo minorem faciunt, cuius rei causam inuestigari adhuc conveniet.

§. 150.

. coeff.
72974
40237
14002
20508
67825
24429
i ad mi-
e solaris
coeff.
12708
79971
53736
35551
32868
39472
tet eam
liae est
13′, 38″
aec in-
mino-
c con-
§. 150.

§. 150. Quodsi enim litteram $\mathfrak{P}'$ accuratius definire velimus, habebimus has formulas resoluendas:

$$-\frac{1}{n}\mathfrak{P} + \frac{\mathfrak{A}(2\varkappa Q+\mathfrak{Q})}{nn} - \frac{\mathfrak{A}(2\varkappa R+\mathfrak{R})}{nn} - \frac{(\mathfrak{Q}-\mathfrak{R})(2\varkappa A+\mathfrak{A})}{nn} - \frac{3(Q-R)}{nn}$$

$$P' = \frac{1}{n}P - \frac{A(2\varkappa Q+\mathfrak{Q})}{nn} + \frac{A(2\varkappa R+\mathfrak{R})}{nn} + \frac{(Q-R)(2\varkappa A+\mathfrak{A})}{nn}$$

$$-\tfrac{3}{2} - 2\varkappa\mathfrak{P} - \mathfrak{C}P - \frac{9A}{4nn} + \frac{(\mathfrak{A}+3A)(\mathfrak{Q}+\mathfrak{R})}{nn} + \frac{(3\mathfrak{A}+3A)(Q+R)}{nn} =$$

$$-\frac{1}{n}P' + \frac{A'(2\varkappa Q+\mathfrak{Q})}{nn} + \frac{A'(2\varkappa R+\mathfrak{R})}{nn} + \frac{(Q'+R')(2\varkappa A+\mathfrak{A})}{nn}$$

vnde elicimus:

$$\mathfrak{P} = -0{,}1183;\quad P = -1{,}1356;\quad \frac{P}{nn} = -0{,}0064$$

atque $\frac{2\varkappa P + \mathfrak{P}}{nn} = -0{,}01376$, fierique iam oportet

$$+0{,}01376 = \frac{1}{n}\mathfrak{P}' - \frac{\mathfrak{A}'(2\varkappa Q+\mathfrak{Q})}{nn} - \frac{\mathfrak{A}'(2\varkappa R+\mathfrak{R})}{nn} - \frac{(\mathfrak{Q}'+\mathfrak{R}')(2\varkappa A+\mathfrak{A})}{nn}$$

vnde oritur $\mathfrak{P}' = +0{,}201385$. Quare accuratius habemus

$s =$ Praec.	$-\ 0{,}006400\, e\ \cos s$	7,806180
$\frac{d\Phi}{dr} =$ Praec.	$+\ 0{,}013760\, e\ \cos s$	8,138618
$\Phi =$ Praec.	$+\ 0{,}201385\, e\ \sin s$	9,304026
	seu	
$\Phi =$ Praec.	$+\ 701'',1\ \sin s$	2,845780

Ergo aequatio sinui Anguli s proportionalis tantum est 11′, 41″.

CAPUT

CAPUT X.

INUESTIGATIO INAEQUALITATVM LUNAE AB VTRIUSQUE ORBITAE EXCENTRICITATE SIMUL PENDENTIUM.

§. 151.

Quoniam praeuidemus inaequalitates huius generis, quae altiores litterarum k et e potestates simul complectuntur, minimas esse futuras, alios terminos non scrutabimur, nisi qui producto simplici ek sint affecti. Habebimus ergo

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1}{n} + \frac{2}{n} k \operatorname{cos} r - \frac{2}{n} e \operatorname{cos} s - \frac{2}{n} ek \operatorname{cos}(r-s) - \frac{2}{n} ek \operatorname{cos}(r+s)$$

Cum igitur ad hanc inuestigationem opus non sit illis terminis ex praecedentibus, qui vel per k^2 vel per e^2 erant affecti, quia litterae alphabethi deficere incipiunt, litteris S, T et sequentibus denuo utemur; quare cavendum, ne istae litterae cum ante adhibitis confundantur.

§. 152. Assumtis ergo ex terminis iam ante definitis, iis qui in eos, quos iam inuestigamus, vim exserunt, ponamus

$$\int R\,dr = \mathfrak{A} \operatorname{cs} 2\eta + \mathfrak{C} k \operatorname{cs} r + \mathfrak{D} k \operatorname{cs}(2\eta - r) + \mathfrak{P} e \operatorname{cs} s + \mathfrak{Q} e \operatorname{cs}(2\eta - s) + \mathfrak{E} k \operatorname{cs}(2\eta + r) + \mathfrak{R} e \operatorname{cs}(2\eta + s)$$
$$+ \mathfrak{S} ek \operatorname{cos}(r-s) + \mathfrak{V} ek \operatorname{cos}(2\eta - r + s) + \mathfrak{Y} ek \operatorname{cos}(2\eta - r - s) + \mathfrak{T} ek \operatorname{cos}(r+s) + \mathfrak{X} ek \operatorname{cos}(2\eta + r + s) + \mathfrak{Z} ek \operatorname{cos}(2\eta + r + s)$$

$v =$

INAE
TE
eneris,
simul
s ter-
ici ek
$s)$
$s)$
t illis
er e^2
piunt,
e ca-
nfun-
defi-
exse-
$2\eta - s)$
$\eta + s)$
$-r-s)$
$r+s)$
$v =$

$$v = A \operatorname{cos} 2\eta \ldots + D k \operatorname{cos}(2\eta - r) + P e \operatorname{cos} s + Q e \operatorname{cos}(2\eta - s) + E k \operatorname{cos}(2\eta + r) + R e \operatorname{cos}(2\eta + s)$$
$$+ S ek \operatorname{cos}(r-s) + V ek \operatorname{cos}(2\eta - r + s) + Y ek \operatorname{cos}(2\eta - r - s)$$
$$+ T ek \operatorname{cos}(r+s) + X ek \operatorname{cos}(2\eta + r - s) + Z ek \operatorname{cos}(2\eta + r + s)$$

eritque ex praecedentibus

$\mathfrak{A} = -0{,}81033$;	$l\text{-}\mathfrak{A} = 9{,}908662$
$\mathfrak{C} = -0{,}64383$;	$l\text{-}\mathfrak{C} = 9{,}808771$
$\mathfrak{D} = -3{,}59362$;	$l\text{-}\mathfrak{D} = 0{,}555531$
$\mathfrak{E} = -1{,}08732$;	$l\text{-}\mathfrak{E} = 0{,}036358$
$\mathfrak{P} = -0{,}11830$;	$l\text{-}\mathfrak{P} = 9{,}072985$
$\mathfrak{Q} = +1{,}33859$;	$l\,\mathfrak{Q} = 0{,}126649$
$\mathfrak{R} = +1{,}23468$;	$l\,\mathfrak{R} = 0{,}091545$
$A = -1{,}31773$;	$l\text{-}A = 0{,}119826$
$D = +33{,}6600$;	$l\,D = 1{,}527113$
$E = -0{,}5785$;	$l\text{-}E = 9{,}762341$
$P = -1{,}1356$;	$l\text{-}P = 0{,}055225$
$Q = +2{,}60087$;	$l\text{-}Q = 0{,}415119$
$R = +2{,}00590$;	$l\,R = 0{,}302308$

§. 153. Reliqui vero valores hinc deriuati, quibus opus habemus, sunt:

$A' = -2{,}47576$;	$l\text{-}A' = 0{,}393708$
$C' = -0{,}13847$;	$l\text{-}C' = 9{,}141356$
$D' = 29{,}39153$;	$l\,D' = 1{,}468222$
$E' = -1{,}47347$;	$l\text{-}E' = 0{,}168341$
$P' = -0{,}0260$;	$l\text{-}P' = 8{,}414973$
$Q' = 4{,}40924$;	$l\,Q' = 0{,}644363$
$R' = 3{,}79793$;	$l\,R' = 0{,}579548$

$\mathfrak{A}' = +0{,}01036$; $l\,\mathfrak{A}' = 8{,}015347$
$\mathfrak{C}' = +0{,}01015$; $l\,\mathfrak{C}' = 8{,}006295$
$\mathfrak{D}' = -0{,}42023$; $l-\mathfrak{D}' = 9{,}623483$
$\mathfrak{E}' = +0{,}00499$; $l-\mathfrak{E}' = 7{,}698261$
$\mathfrak{P}' = +0{,}20138$; $l\,\mathfrak{P}' = 9{,}304016$
$\mathfrak{Q}' = -0{,}02189$; $l-\mathfrak{Q}' = 8{,}340237$
$\mathfrak{R}' = -0{,}01637$; $l-\mathfrak{R}' = 8{,}214003$

Sit breuitatis gratia

$a' = \frac{2\varkappa A + \mathfrak{A}}{nn} = -0{,}019744$; $l - \frac{(2\varkappa A + \mathfrak{A})}{nn} = 8{,}295442$

$d' = \frac{2\varkappa D + \mathfrak{D}'}{nn} = +0{,}36611$; $l\,\frac{2\varkappa D + \mathfrak{D}}{nn} = 9{,}563604$

$e' = \frac{2\varkappa E + \mathfrak{E}}{nn} = -0{,}01283$; $l - \frac{(2\varkappa E + \mathfrak{E})}{nn} = 8{,}108292$

$p' = \frac{2\varkappa P + \mathfrak{P}}{nn} = -0{,}01376$; $l - \frac{(2\varkappa P + \mathfrak{P})}{nn} = 8{,}138618$

$q' = \frac{2\varkappa Q + \mathfrak{Q}}{nn} = +0{,}03749$; $l\,\frac{2\varkappa Q + \mathfrak{Q}}{nn} = 8{,}573878$

$r' = \frac{2\varkappa R + \mathfrak{R}}{nn} = +0{,}03006$; $l\,\frac{2\varkappa R + \mathfrak{R}}{nn} = 8{,}478023$

§. 154. Si simili modo vlterius ponatur:

$$s' = \frac{2\varkappa S + \mathfrak{S}}{nn};\quad t' = \frac{2\varkappa T + \mathfrak{T}}{nn};\quad v' = \frac{2\varkappa V + \mathfrak{V}}{nn};$$
$$x' = \frac{2\varkappa X + \mathfrak{X}}{nn};\quad y' = \frac{2\varkappa Y + \mathfrak{Y}}{nn};\quad z' = \frac{2\varkappa Z + \mathfrak{Z}}{nn}$$

habe-

5442
3604
3292
3618
3878
1023
;
ibe-

habebimus:

$$\frac{d\Phi}{dr} = \text{Praec.} - a'\,\mathrm{cs}\,2\eta - \frac{\mathfrak{C}}{nn}k\,\mathrm{cs}\,r - d'k\,\mathrm{cs}(2\eta - r) - p'e\,\mathrm{cs}\,s - q'e\,\mathrm{cs}(2\eta - s)$$
$$- e'k\,\mathrm{cs}(2\eta + r) \qquad - r'e\,\mathrm{cs}(2\eta + s)$$
$$- s'ek\cos(r - s) - v'ek\cos(2\eta - r + s) - y'ek\cos(2\eta - r - s)$$
$$- t'ek\cos(r + s) - x'ek\cos(2\eta + r - s) - z'ek\cos(2\eta + r + s)$$

atque posito $\frac{2}{n} + \frac{\mathfrak{C}}{nn} = c' = 0{,}147197$; $l\,c' = 9{,}167900$

$$\frac{d\eta}{dr} = \alpha - a'\,\mathrm{cs}\,2\eta - c'k\,\mathrm{cs}\,r - d'k\,\mathrm{cs}(2\eta - r) + \left(\frac{2}{n} - p'\right)e\,\mathrm{cs}\,s - q'e\,\mathrm{cs}(2\eta - s)$$
$$- e'k\,\mathrm{cs}(2\eta + r) \qquad - r'e\,\mathrm{cs}(2\eta + s)$$
$$+ \left(\frac{2}{n} - s'\right)ek\,\mathrm{cs}(r - s) - v'ek\,\mathrm{cs}(2\eta - r + s) - y'ek\,\mathrm{cs}(2\eta - r - s)$$
$$+ \left(\frac{2}{n} - t'\right)ek\,\mathrm{cs}(r + s) - x'ek\,\mathrm{cs}(2\eta + r - s) - z'ek\,\mathrm{cs}(2\eta + r + s)$$

vbi cum sit $\frac{2}{n} = 0{,}150876$, erit $\frac{2}{n} - p' = 0{,}16464$

§. 155. Nunc termini coefficiente ek affecti, qui in formis R = et $\frac{ddv}{dr^2}$ insunt, colligantur: eritque

$$R = ek\sin(r - s)\left(+\frac{3}{2nn}V - \frac{3}{2nn}X - \frac{3Q}{nn} + \frac{3R}{nn} - \frac{9D}{4nn} + \frac{9E}{4nn}\right)$$
$$ek\sin(r + s)\left(+\frac{3}{2nn}Y - \frac{3}{2nn}Z - \frac{3R}{nn} + \frac{3Q}{nn} + \frac{9E}{4nn} - \frac{9D}{4nn}\right)$$
$$ek\sin(2\eta - r + s)\left(-\tfrac{3}{2} + \frac{3}{2nn}S + \frac{3P}{nn}\right)$$
$$ek\sin(2\eta + r - s)\left(-\tfrac{3}{2} + \frac{3}{2nn}S + \frac{3P}{nn}\right)$$

eh

$$ek\sin(2\eta-r-s)\left(-\tfrac{9}{2}+\frac{3}{2nn}T+\frac{3P}{nn}\right)$$

$$ek\sin(2\eta+r+s)\left(-\tfrac{9}{2}+\frac{3}{2nn}T+\frac{3P}{nn}\right)$$

et $\frac{ddv}{dr^2}$ =

$$ek\cos(r-s)\begin{cases}-3-2\kappa\mathfrak{S}-\epsilon S+\tfrac{1}{2}bP+\frac{3V}{4nn}+\frac{3X}{4nn}+\frac{3Q}{2nn}+\frac{3R}{2nn}\\ -\frac{9D}{8nn}-\frac{9E}{8nn}+\frac{\mathfrak{A}\mathfrak{V}}{nn}+\frac{\mathfrak{A}\mathfrak{Y}}{nn}+\frac{\mathfrak{C}\mathfrak{P}}{nn}+\frac{\mathfrak{D}\mathfrak{Q}}{nn}+\frac{\mathfrak{C}\mathfrak{R}}{nn}\\ +\frac{3AV}{nn}+\frac{3AX}{nn}+\frac{3DQ}{nn}+\frac{3ER}{nn}-\frac{3AQ}{2nn}-\frac{3AR}{2nn}\\ +\frac{3\mathfrak{A}V}{nn}+\frac{3A\mathfrak{V}}{nn}+\frac{3\mathfrak{A}X}{nn}+\frac{3A\mathfrak{Y}}{nn}+\frac{3\mathfrak{C}P}{nn}\\ +\frac{3\mathfrak{D}Q}{nn}+\frac{3D\mathfrak{Q}}{nn}+\frac{3\mathfrak{C}R}{nn}+\frac{3E\mathfrak{R}}{nn}\end{cases}$$

$$ek\cos(r+s)\begin{cases}-3-2\kappa\mathfrak{T}-\epsilon T+\tfrac{1}{2}bP+\frac{3Y}{4nn}-\frac{3Z}{4nn}+\frac{3Q}{2nn}+\frac{3R}{2nn}\\ -\frac{9D}{8nn}-\frac{9E}{8nn}+\frac{\mathfrak{A}\mathfrak{Y}}{nn}+\frac{\mathfrak{A}\mathfrak{Z}}{nn}+\frac{\mathfrak{C}\mathfrak{P}}{nn}+\frac{\mathfrak{D}\mathfrak{R}}{nn}+\frac{\mathfrak{C}\mathfrak{Q}}{nn}\\ +\frac{3AY}{nn}+\frac{3AZ}{nn}+\frac{3DR}{nn}+\frac{3EQ}{nn}-\frac{3AQ}{2nn}-\frac{3AR}{2nn}\\ +\frac{3\mathfrak{A}Y}{nn}+\frac{3A\mathfrak{Y}}{nn}+\frac{3\mathfrak{A}Z}{nn}+\frac{3A\mathfrak{Z}}{nn}+\frac{3\mathfrak{C}P}{nn}\\ +\frac{3\mathfrak{C}Q}{nn}+\frac{3E\mathfrak{Q}}{nn}+\frac{3\mathfrak{D}R}{nn}+\frac{3D\mathfrak{R}}{nn}\end{cases}$$

$$ek\cos(2\eta-r+s)\begin{cases}-\tfrac{9}{2}-2\kappa\mathfrak{V}-\epsilon V+\tfrac{1}{2}bR+\frac{3S}{4nn}-\frac{3D}{4nn}+\frac{3P}{2nn}\\ +\frac{\mathfrak{A}\mathfrak{S}}{nn}+\frac{\mathfrak{C}\mathfrak{R}}{nn}+\frac{\mathfrak{D}\mathfrak{P}}{nn}+\frac{3AS}{nn}+\frac{3DP}{nn}-\frac{3AP}{2nn}\\ +\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}+\frac{3\mathfrak{C}R}{nn}+\frac{3\mathfrak{D}P}{nn}+\frac{3D\mathfrak{P}}{nn}\end{cases}$$

$$ek\cos(2\eta+r-s)\begin{cases}-\tfrac{9}{2}-2\kappa\mathfrak{X}-\epsilon X+\tfrac{1}{2}bQ+\frac{3S}{4nn}-\frac{3E}{4nn}+\frac{3P}{2nn}\\ +\frac{\mathfrak{A}\mathfrak{S}}{nn}+\frac{\mathfrak{C}\mathfrak{Q}}{nn}+\frac{\mathfrak{C}\mathfrak{P}}{nn}+\frac{3AS}{nn}+\frac{3EP}{nn}-\frac{3AP}{2nn}\\ +\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}+\frac{3\mathfrak{C}Q}{nn}+\frac{3\mathfrak{C}P}{nn}+\frac{3E\mathfrak{P}}{nn}\end{cases}$$

$$ek\cos(2\eta-r-s)\begin{cases}-\tfrac{9}{2}-2\kappa\mathfrak{Y}-\epsilon Y+\tfrac{1}{2}bQ+\frac{3T}{4nn}-\frac{3D}{4nn}+\frac{3P}{2nn}\\ +\frac{\mathfrak{A}\mathfrak{T}}{nn}+\frac{\mathfrak{C}\mathfrak{Q}}{nn}+\frac{\mathfrak{D}\mathfrak{P}}{nn}+\frac{3AT}{nn}+\frac{3DP}{nn}-\frac{3AP}{2nn}\\ +\frac{3\mathfrak{A}T}{nn}+\frac{3A\mathfrak{T}}{nn}+\frac{3\mathfrak{C}Q}{nn}+\frac{3\mathfrak{D}P}{nn}+\frac{3D\mathfrak{P}}{nn}\end{cases}$$

$$ek\cos(2\eta+r+s)\begin{cases}-\tfrac{9}{2}-2\kappa\mathfrak{Z}-\epsilon Z+\tfrac{1}{2}bR+\frac{3T}{4nn}-\frac{3E}{4nn}+\frac{3P}{2nn}\\ +\frac{\mathfrak{A}\mathfrak{T}}{nn}+\frac{\mathfrak{C}\mathfrak{R}}{nn}+\frac{\mathfrak{C}\mathfrak{P}}{nn}+\frac{3AT}{nn}+\frac{3EP}{nn}-\frac{3AP}{2nn}\\ +\frac{3\mathfrak{A}T}{nn}+\frac{3A\mathfrak{T}}{nn}+\frac{3\mathfrak{C}R}{nn}+\frac{3\mathfrak{C}P}{nn}+\frac{3E\mathfrak{P}}{nn}\end{cases}$$

§. 156. Quaeramus ergo quoque ex formula assumta $\int R\,dr$ differentiale, quod erit

$$R =\!=\!=\!=$$

$$ek\sin(r-s)(+\mathfrak{A}v'-\mathfrak{A}x'-\mathfrak{D}q'+\mathfrak{E}r'+\frac{1}{n}\mathfrak{P}+\mathfrak{Q}d'-\mathfrak{R}e'-(1-\frac{1}{n})\mathfrak{S}-\mathfrak{V}a'+\mathfrak{X}a')$$

$$ek\sin(r+s)(+\mathfrak{A}y'-\mathfrak{A}z'-\mathfrak{D}r'+\mathfrak{E}q'-\frac{1}{n}\mathfrak{P}-\mathfrak{Q}e'+\mathfrak{R}d'-(1+\frac{1}{n})\mathfrak{T}-\mathfrak{Y}a'+\mathfrak{Z}a')$$

$$ek\sin(2\eta-r+s)(-\mathfrak{A}(\frac{2}{n}-s')-\mathfrak{D}(\frac{2}{n}-p')+\mathfrak{R}c'-\frac{1}{n}\mathfrak{R}-(2\alpha-1+\frac{1}{n})\mathfrak{V})$$

$$ek\sin(2\eta+r-s)(-\mathfrak{A}(\frac{2}{n}-s')-\mathfrak{E}(\frac{2}{n}-p')+\mathfrak{Q}c'+\frac{1}{n}\mathfrak{Q}-(2\alpha+1-\frac{1}{n})\mathfrak{X})$$

$$ek\sin(2\eta-r-s)(-\mathfrak{A}(\frac{2}{n}-t')-\mathfrak{D}(\frac{2}{n}-p')+\mathfrak{Q}c'+\frac{1}{n}\mathfrak{Q}-(2\alpha-1-\frac{1}{n})\mathfrak{Y})$$

$$ek\sin(2\eta+r+s)(-\mathfrak{A}(\frac{2}{n}-t')-\mathfrak{E}(\frac{2}{n}-p')+\mathfrak{R}c'-\frac{1}{n}\mathfrak{R}-(2\alpha+1+\frac{1}{n})\mathfrak{Z})$$

§. 157. Ponatur ex differentiatione formae v;

$$S' = (1-\frac{1}{n})S-A(v'-x')+Dq'-Er'-\frac{1}{n}P-Qd'+Re'+(V-X)a'$$

$$T' = (1+\frac{1}{n})T-A(y'-z')+Dr'-Eq'+\frac{1}{n}P+Qe'-Rd'+(Y-Z)a'$$

$$V' = (2\alpha-1+\frac{1}{n})V+A(\frac{2}{n}-s')+D(\frac{2}{n}-p')-Rc'+\frac{1}{n}R$$

$$X' = (2\alpha+1-\frac{1}{n})X+A(\frac{2}{n}-s')+E(\frac{2}{n}-p')-Qc'-\frac{1}{n}Q$$

$$Y' = (2\alpha-1-\frac{1}{n})Y+A(\frac{2}{n}-t')+D(\frac{2}{n}-p')-Qc'-\frac{1}{n}Q$$

$$Z' = (2\alpha+1+\frac{1}{n})Z+A(\frac{2}{n}-t')+E(\frac{2}{n}-p')-Rc'+\frac{1}{n}R$$

vt

la as- '+X a') '+Z a') ; -X)a' -Z)a' R Q Q R vt

vt habeatur

$$\frac{dv}{dr} = -A'\sin 2\eta - C'k\sin r - D'k\sin(2\eta-r) - E'k\sin(2\eta+r)$$
$$-P'e\sin s - Q'e\sin(2\eta-s) - R'e\sin(2\eta+s)$$
$$-S'ek\sin(r-s) - V'ek\sin(2\eta-r+s) - Y'ek\sin(2\eta-r-s)$$
$$-T'ek\sin(r+s) - X'ek\sin(2\eta+r-s) - Z'ek\sin(2\eta+r+s)$$

§. 158. Haec iam forma denuo differentiata dabit

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ek\cos(r-s)(+A'(v'+x')+D'q'+E'r'-\frac{1}{n}P'+Q'd'+R'e'-(1-\frac{1}{n})S'+(V'+X')a'$$

$$+ek\cos(r+s)(+A'(y'+z')+D'r'+E'q'-\frac{1}{n}P'+Q'e'+R'd'-(1+\frac{1}{n})T'+(Y'+Z')a'$$

$$+ek\cos(2\eta-r+s)(-A'(\frac{2}{n}-s')-D'(\frac{2}{n}-p')+R'c'-\frac{1}{n}R'-(2\alpha-1+\frac{1}{n})V')$$

$$+ek\cos(2\eta+r-s)(-A'(\frac{2}{n}-s')-E'(\frac{2}{n}-p')+Q'c'+\frac{1}{n}Q'-(2\alpha+1-\frac{1}{n})X')$$

$$+ek\cos(2\eta-r-s)(-A'(\frac{2}{n}-t')-D'(\frac{2}{n}-p')+Q'c'+\frac{1}{n}Q'-(2\alpha-1-\frac{1}{n})Y')$$

$$+ek\cos(2\eta+r+s)(-A'(\frac{2}{n}-t')-E'(\frac{2}{n}-p')+R'c'-\frac{1}{n}R'-(2\alpha+1+\frac{1}{n})Z')$$

§. 159. Priores autem expressiones, si litterarum cognitarum valores substituantur, sequenti modo prodibunt.

$$R = ek\sin(r-s)\,(-0{,}44856 + 0{,}00854\,(V-X))$$
$$ek\sin(r+s)\,(-0{,}42826 + 0{,}00854\,(Y-Z))$$
$$ek\sin(2\eta-r+s)\,(-4{,}51938 + 9{,}00854\,S)$$
$$ek\sin(2\eta+r-s)\,(-4{,}51938 + 0{,}00854\,S)$$
$$ek\sin(2\eta-r-s)\,(-4{,}51939 + 0{,}00854\,T)$$
$$ek\sin(2\eta+r+s)\,(-4{,}51938 + 0{,}00854\,T)$$

 §. 160.

§. 160. Altera vero forma pro $\frac{ddv}{dr^2}$ fit

$$\frac{ddv}{dr^2} =$$

$ek\,c\text{ſ}(r-s)(-2{,}83505-2k\mathfrak{S}-\beta S-0{,}02711(\mathfrak{V}+\mathfrak{X})-0{,}03207(V+X))$

$ek\,c\text{ſ}(r+s)(-3{,}21669-2k\mathfrak{T}-\beta T-2{,}02711(\mathfrak{Y}+\mathfrak{Z})-0{,}03207(Y+Z))$

$ek\,c\text{ſ}(2\eta-r+s)(-2{,}26927-2k\mathfrak{V}-\beta V-0{,}02711\,\mathfrak{S}-0{,}03207\,S)$

$ek\,c\text{ſ}(2\eta+r-s)(-0{,}53441-2k\mathfrak{X}-\beta X-0{,}02711\,\mathfrak{S}-0{,}03207\,S)$

$ek\,c\text{ſ}(2\eta-r-s)(-1{,}36456-2k\mathfrak{Y}-\beta Y-0{,}02711\,\mathfrak{T}-0{,}03207\,T)$

$ek\,c\text{ſ}(2\eta+r+s)(-1{,}43912-2k\mathfrak{Z}-\beta Z-0{,}02711\,\mathfrak{T}-0{,}03207\,T)$

§. 161. Deinde ſimili modo alterae formulae per differentiationem erutae, ſubſtitutis valoribus cognitis ita ſe habebunt.

$$R =$$

$ek\,\text{ſin}(r-s)\,(+0{,}59883+\mathfrak{A}(v'-x')\cdot(1-\frac{1}{n})\mathfrak{S})+0{,}01974(\mathfrak{V}-\mathfrak{X})$

$ek\,\text{ſin}(r+s)(+0{,}54556+\mathfrak{A}(y'-z')-(1+\frac{1}{n})\mathfrak{T})+0{,}01974(\mathfrak{Y}-\mathfrak{Z})$

$ek\,\text{ſin}(2\eta-r+s)\,(+0{,}80251+\mathfrak{A}s'-(2\alpha-1+\frac{1}{n})\,\mathfrak{V})$

$ek\,\text{ſin}(2\eta+r-s)\,(+0{,}59936+\mathfrak{A}s'-(2\alpha+1-\frac{1}{n})\,\mathfrak{X})$

$ek\,\text{ſin}(2\eta-r-s)\,(+1{,}01199+\mathfrak{A}s'-(2\alpha-1-\frac{1}{n})\,\mathfrak{Y})$

$ek\,\text{ſin}(2\eta+r+s)\,(+0{,}38988+\mathfrak{A}s'-(\alpha+1+\frac{1}{n})\,\mathfrak{Z})$

Porro

3207(V+X))
3207(Y+Z))
-0,03207 S)
0,03207 S)
0,03207 T)
0,03207 T)
mulae per
cognitis ita
974(𝔙-𝔛)
974(𝔜-𝔷)
𝔙)
𝔛)
𝔜)
𝔷)
Porro

Porro reperiemus ſequentes valores

$S' = (1-\frac{1}{n})\,S - A\,(v'-x') + 0{,}38693 - 0{,}01974\,(V-X)$

$T' = (1+\frac{1}{n})T - A\,(y'-z') + 0{,}18018 - 0{,}01974\,(Y-Z)$

$V' = (2\alpha-1+\frac{1}{n})\;V - A\,s' + 5{,}19902$

$X' = (2\alpha+1-\frac{1}{n})\;X - A\,s' - 0{,}87311$

$Y' = (2\alpha-1-\frac{1}{n})\;Y - A\,s' + 4{,}76391$

$Z' = (2\alpha+1+\frac{1}{n})\;Z - A\,s' - 0{,}43800$

ac denique $\quad \frac{ddv}{dr^2} = \text{Praec.}$

$+ek\,c\text{ſ}(r-s)(-(1-\frac{1}{n})S'+A'(v'+x')+2{,}62592-0{,}01974\,(V'+X'))$

$+ek\,c\text{ſ}(r+s)(-(1+\frac{1}{n})T'+A'(y'+z')+2{,}16417-0{,}01974\,(Y'+Z'))$

$+ek\,\text{coſ}(2\eta-r+s)(-(2\alpha-1+\frac{1}{n})\;V'+A's'-4{,}19296)$

$+ek\,\text{coſ}(2\eta+r-s)(-(2\alpha+1-\frac{1}{n})\;X'+A's'+1{,}59777)$

$+ek\,\text{coſ}(2\eta-r-s)(-(2\alpha-1-\frac{1}{n})\;Y'+A's'-3{,}48384)$

$+ek\,\text{coſ}(2\eta+r+s)(-(2\alpha+1+\frac{1}{n})\;Z'+A's'+0{,}88865)$

§. 162. Hinc ergo pro determinandis coefficientibus sequentes obtinemus aequationes

$(1-\frac{1}{n})\mathfrak{S}=1,04739-0,00854(V-X)+\mathfrak{A}(v'-x')+0,01974(\mathfrak{V}-\mathfrak{X})$

$(1+\frac{1}{n})\mathfrak{T}=0,97382-0,00858(Y-Z)+\mathfrak{A}(y'-z')+0,01974(\mathfrak{Y}-\mathfrak{Z})$

$(2\alpha-1+\frac{1}{n})\,\mathfrak{V}=5,32189-0,00854\,S+\mathfrak{A}\,s'$

$(2\alpha+1-\frac{1}{n})\,\mathfrak{X}=5,11874-0,00854\,S+\mathfrak{A}\,s'$

$(2\alpha-1-\frac{1}{n})\,\mathfrak{Y}=5,53137-0,00854\,T+\mathfrak{A}\,t'$

$(2\alpha+1+\frac{1}{n})\,\mathfrak{Z}=4,90926-0,00854\,T+\mathfrak{A}\,t'$

Deinde

$+5,46097=(1-\frac{1}{n})S'-2\kappa\mathfrak{S}-\mathfrak{E}S-0,02711(\mathfrak{V}+\mathfrak{X})-0,03207(V+X)$
$-A'(v'+x')+0,01974(V'+X')$

$+5,38086=(1+\frac{1}{n})T'-2\kappa\mathfrak{T}-\mathfrak{E}T-0,02711(\mathfrak{Y}+\mathfrak{Z})-0,03207(Y+Z)$
$-A'(y'+z')+0,01974(Y'+Z')$

$-1,92371=(2\alpha-1+\frac{1}{n})V'-2\kappa\mathfrak{V}-\mathfrak{E}V-0,02711\mathfrak{S}-0,03207\,S-A's'$

$+2,13218=(2\alpha+1-\frac{1}{n})X'-2\kappa\mathfrak{X}-\mathfrak{E}X-0,02711\mathfrak{S}-0,03207\,S-A's'$

$-2,11928=(2\alpha-1-\frac{1}{n})Y'-2\kappa\mathfrak{Y}-\mathfrak{E}Y-0,02711\mathfrak{T}-0,03207\,T-A't'$

$+2,32777=(2\alpha+1+\frac{1}{n})Z'-2\kappa\mathfrak{Z}-\mathfrak{E}Z-0,02711\mathfrak{T}-0,03207\,T-A't'$

§. 163.

§. 163. Pro vlteriori calculo est

$1-\frac{1}{n}=0,924562$ . . . . . $l(1-\frac{1}{n})=9,965935$

$1+\frac{1}{n}=1,075438$ . . . . . $l(1+\frac{1}{n})=0,031570$

$2\alpha-1+\frac{1}{n}=0,942914$ . . . $l(2\alpha-1+\frac{1}{n})=9,965935$

$2\alpha+1-\frac{1}{n}=2,792038$ . . . $l(2\alpha+1-\frac{1}{n})=0,445915$

$2\alpha-1-\frac{1}{n}=0,792038$ . . . $l(2\alpha-1-\frac{1}{n})=9,898747$

$2\alpha+1+\frac{1}{n}=2,942914$ . . . $l(2\alpha+1+\frac{1}{n})=0,468775$

Hinc in aequationibus posterioribus valores litterarum S', T', V' substituantur, et ob $\mathfrak{E}=1,01591$, erit

$0,16110S=-5,10323-2\kappa\mathfrak{S}-A'(v'+x')-0,02711(\mathfrak{V}+\mathfrak{X})$
$+0,01974(V'+X')$
$+1,21835(v'-x')-0,03207(V+X)$
$-0,01825(V-X)$

$0,14059T=+5,18709+2\kappa\mathfrak{T}+A'(y'+z')+0,02711(\mathfrak{Y}+\mathfrak{Z})$
$-0,01974(Y'+Z')$
$-1,41710(y'-z')+0,03207(Y+Z)$
$+0,02124(Y-Z)$

$0,12683\,V=+6,82591-2\kappa\mathfrak{V}+3,718\,s'-0,02711\mathfrak{S}$
$-0,03207\,S$

$6,77934\,X=+4,56988+2\kappa\mathfrak{X}-6,1548\,s'+0,02711\mathfrak{S}$
$+0,03207\,S$

$0,38858\,Y=+5,89253-2\kappa\mathfrak{Y}+3,5194\,t'-0,02711\,\mathfrak{T}$
$-0,03207\,T$

$7,64474\,Z=+3,61676+2\kappa\mathfrak{Z}-6,3536\,t'+0,02711\,\mathfrak{T}$
$+0,03207\,T$

§. 164.

§. 164. Commodissime hi coefficientes inueniri videntur, si primo $\mathfrak{V}$, $\mathfrak{X}$, $\mathfrak{Y}$, $\mathfrak{Z}$ et V, X, Y, Z proxime quaerantur, quod fiet terminos minimos negligendo:

$\mathfrak{V} = +5,7560$ . . . $l\,\mathfrak{V} = 0,760125$
$\mathfrak{X} = +1,8334$ . . . $l\,\mathfrak{X} = 0,263245$
$\mathfrak{Y} = +6,9837$ . . . $l\,\mathfrak{Y} = 0,844088$
$\mathfrak{Z} = +1,6643$ . . . $l\,\mathfrak{Z} = 0,221240$
$V = -37,7650$ . . . $l{-}V = 1,577086$
$X = +1,2198$ . . . $l\,X = 0,086293$
$Y = -21,1040$ . . . $l{-}Y = 1,324360$
$Z = +0,9125$ . . . $l\,Z = 9,960209$
$V' = -29,7170$ . . . $v' = -0,398$
$X' = +2,5326$ . . . $x' = +0,024$
$Y' = -11,9510$ . . . $y' = -0,201$
$Z' = +2,2472$ . . . $z' = +0,020$

§. 165. Hic autem valores pro V′ et Y′ tam fiunt magni, vt vicissim post inuentas litteras S, T nimium valores modo erutos afficiant, vnde necesse erit resolutionem harum aequationum ordinario modo instruere. Reperitur ergo

$\mathfrak{V} = 5,7560 - 0,0193\,S - 0,0050\,\mathfrak{S}$
$\mathfrak{X} = 1,8333 - 0,0064\,S - 0,0016\,\mathfrak{S}$
$\mathfrak{Y} = 6,9837 - 0,0225\,T - 0,0058\,\mathfrak{T}$
$\mathfrak{Z} = 1,6643 - 0,0061\,T - 0,0016\,\mathfrak{T}$
$2\varkappa\,\mathfrak{V} = 11,6155 - 0,0390\,S - 0,0100\,\mathfrak{S}$
$2\varkappa\,\mathfrak{X} = 3,6996 - 0,0129\,S - 0,0033\,\mathfrak{S}$
$2\varkappa\,\mathfrak{Y} = 14,0930 - 0,0455\,T - 0,0118\,\mathfrak{T}$
$2\varkappa\,\mathfrak{Z} = 3,3586 - 0,0122\,T - 0,0032\,\mathfrak{T}$

qui

qui valores substituti dant:

$V = -37,7647 + 0,3912\,S + 0,0031\,\mathfrak{S}$
$X = +1,2198 - 0,0076\,S - 0,0016\,\mathfrak{S}$
$Y = -21,1038 + 0,1385\,T + 0,0121\,\mathfrak{T}$
$Z = +0,9125 - 0,0069\,T - 0,0016\,\mathfrak{T}$

$2\varkappa V = -76,2085 + 0,7893\,S + 0,0064\,\mathfrak{S}$
$2\varkappa X = +2,4616 - 0,0153\,S - 0,0033\,\mathfrak{S}$
$2\varkappa Y = -42,5870 + 0,2794\,T + 0,0244\,\mathfrak{T}$
$2\varkappa Z = +1,8413 - 0,0140\,T - 0,0033\,\mathfrak{T}$

§. 166. Hinc porro valores deriuati erunt

$V' = -29,7167 + 0,3767\,S + 0,0094\,\mathfrak{S}$
$X' = +2,5326 - 0,0061\,S + 0,0029\,\mathfrak{S}$
$Y' = -11,9511 + 0,1248\,T + 0,0161\,\mathfrak{T}$
$Z' = +2,2472 - 0,0054\,T + 0,0028\,\mathfrak{T}$

$v' = -0,4009 + 0,0044\,S + 0,0001\,\mathfrak{S}$
$x' = +0,0244$
$y' = -0,2026 + 0,0015\,T + 0,0001\,\mathfrak{T}$
$z' = +0,0200$

ac porro

$\mathfrak{V} - \mathfrak{X} = 3,9227 - 0,0129\,S - 0,0034\,\mathfrak{S}$
$\mathfrak{Y} - \mathfrak{Z} = 5,3194 - 0,0164\,T - 0,0042\,\mathfrak{T}$
$V - X = -38,9845 + 0,3988\,S + 0,0047\,\mathfrak{S}$
$Y - Z = -22,0163 + 0,1454\,T + 0,0137\,\mathfrak{T}$

$\mathfrak{V} + \mathfrak{X} = 7,5893 - 0,0257\,S - 0,0066\,\mathfrak{S}$
$\mathfrak{Y} + \mathfrak{Z} = 8,6480 - 0,0286\,T - 0,0074\,\mathfrak{T}$
$V + X = -36,5449 + 0,3836\,S + 0,0015\,\mathfrak{S}$
$Y + Z = -20,1913 + 0,1316\,T + 0,0105\,\mathfrak{T}$

T V′+X′

$$V'+X' = -27,1841 + 0,3706\,S + 0,0123\,\mathfrak{S}$$
$$Y'+Z' = -9,7039 + 0,1194\,T + 0,0189\,\mathfrak{T}$$
$$v'-x' = -0,4253 + 0,0044\,S + 0,0001\,\mathfrak{S}$$
$$y'-z' = -0,2226 + 0,0015\,T + 0,0001\,\mathfrak{T}$$
$$v'+x' = -0,3765 + 0,0044\,S + 0,0001\,\mathfrak{S}$$
$$y'+z' = -0,1826 + 0,0015\,T + 0,0001\,\mathfrak{T}$$

§. 167. His valoribus substitutis reperitur

$$(1-\tfrac{1}{n})\,\mathfrak{S} = +1,8024 - 0,0070\,S$$
$$(1+\tfrac{1}{n})\,\mathfrak{T} = +1,4453 - 0,0027\,T$$

vnde concluditur

$\mathfrak{S} = 1,9495 - 0,0075\,S$	$2n\mathfrak{S} = 3,9340 - 0,0160\,S$
$\mathfrak{T} = 1,3440 - 0,0025\,T$	$2n\mathfrak{T} = 2,7121 - 0,0053\,T$
$0,1401\,S = -9,2444$	$0,1479\,T = +7,9767$

§. 168. Nunc igitur habebimus

$S = -66,6980$ . . .	$l{-}S = 1,824113$
$T = +53,9330$ . . .	$l\,T = 1,731855$
$\mathfrak{S} = +2,4497$ . . .	$l\,\mathfrak{S} = 0,389113$
$\mathfrak{T} = +1,2092$ . . .	$l\,\mathfrak{T} = 0,082498$
$s' = -0,75204$ . . .	$l{-}s' = 9,876241$
$t' = +0,62626$ . . .	$l\,t' = 9,796755$
$v' = -0,6942$ . . .	$l{-}v' = 9,841484$
$x' = +0,0244$ . . .	$l\,x' = 8,387390$
$y' = -0,1216$ . . .	$l{-}y' = 9,084933$
$z' = +0,0200$ . . .	$l\,z' = 9,301030$

$\mathfrak{V} =$

$\mathfrak{V} = +7,0311$ . . .	$l\,\mathfrak{V} = 0,847029$
$\mathfrak{X} = +2,2561$ . . .	$l\,\mathfrak{X} = 0,353358$
$\mathfrak{Y} = +5,7659$ . . .	$l\,\mathfrak{Y} = 0,760867$
$\mathfrak{Z} = +1,3339$ . . .	$l\,\mathfrak{Z} = 0,125123$
$V = -63,8498$ . . .	$l{-}V = 1,805160$
$X = +1,7451$ . . .	$l\,X = 0,241820$
$Y = -13,6222$ . . .	$l{-}Y = 1,134241$
$Z = +0,5356$ . . .	$l\,Z = 9,728840$

§. 169. His iam valoribus inuentis pro distantia lunae $x = \frac{(1-kk)\,au}{1-k\cos r}$ erit valoris ipsius u portio ab his terminis pendens;

		Log. coeff.
$u =$ Praec.	$-\ 0,3796\ ek \cos(r-s)$	9,579297
	$+\ 0,3069\ ek \cos(r+s)$	9,487093
	$-\ 0,3634\ ek \cos(2\eta-r+s)$	9,560344
	$+\ 0,0099\ ek \cos(2\eta+r-s)$	7,997004
	$-\ 0,0775\ ek \cos(2\eta-r-s)$	8,889425
	$+\ 0,0030\ ek \cos(2\eta+r+s)$	7,484024

et pro longitudine lunae

$$\frac{d\Phi}{dr} = \text{Praec.} + 0,7520\ ek \cos(r-s)$$
$$-\ 0,6263\ ek \cos(r+s)$$
$$+\ 0,6942\ ek \cos(2\eta-r+s)$$
$$-\ 0,0244\ ek \cos(2\eta+r-s)$$
$$+\ 0,1216\ ek \cos(2\eta-r-s)$$
$$-\ 0,0200\ ek \cos(2\eta+r+s)$$

 cuius

cuius integrale si ponatur:

$$\Phi = \text{Praec.} + \mathfrak{S}'ek\sin(r-s) + \mathfrak{B}'ek\sin(2\eta-r+s) + \mathfrak{Y}'ek\sin(2\eta-r-s)$$
$$+ \mathfrak{T}'ek\sin(r+s) + \mathfrak{X}'ek\sin(2\eta+r-s) + \mathfrak{Z}ek\sin(2\eta+r+s)$$

erit

$$+0{,}7520 = (1-\tfrac{1}{n})\mathfrak{S}' - \mathfrak{A}'(v'+x') - \mathfrak{D}'q' - \mathfrak{E}'r' + \tfrac{1}{n}\mathfrak{P}' - \mathfrak{Q}'d' - \mathfrak{R}'e' - (\mathfrak{B}'+\mathfrak{X}')a'$$

$$-0{,}6263 = (1+\tfrac{1}{n})\mathfrak{T}' - \mathfrak{A}'(y'+z') - \mathfrak{D}'r' - \mathfrak{E}'q' + \tfrac{1}{n}\mathfrak{P}' - \mathfrak{Q}'e' - \mathfrak{R}'d' - (\mathfrak{Y}'+\mathfrak{Z}')a'$$

$$+0{,}6942 = (2\alpha-1+\tfrac{1}{n})\mathfrak{B}' + \mathfrak{A}'(\tfrac{2}{n}-s') + \mathfrak{D}'(\tfrac{2}{n}-p') - \mathfrak{R}'(c'-\tfrac{1}{n})$$

$$-0{,}0244 = (2\alpha+1-\tfrac{1}{n})\mathfrak{X}' + \mathfrak{A}'(\tfrac{2}{n}-s') + \mathfrak{E}'(\tfrac{2}{n}-p') - \mathfrak{Q}'(c'+\tfrac{1}{n})$$

$$+0{,}1216 = (2\alpha-1-\tfrac{1}{n})\mathfrak{Y} + \mathfrak{A}'(\tfrac{2}{n}-s') + \mathfrak{D}'(\tfrac{2}{n}-p') - \mathfrak{Q}'(c'+\tfrac{1}{n})$$

$$-0{,}0200 = (2\alpha+1+\tfrac{1}{n})\mathfrak{Z} + \mathfrak{A}'(\tfrac{2}{n}-s') + \mathfrak{E}'(\tfrac{2}{n}-p') - \mathfrak{R}'(c'-\tfrac{1}{n})$$

§. 170. Hinc autem reperitur

$\mathfrak{S}' = +0,7467$ . . . $l\,\mathfrak{S}' = 9,873165$
$\mathfrak{T}' = -0,6185$ . . . $l{-}\mathfrak{T}' = 9,791317$
$\mathfrak{B}' = +0,8143$ . . . $l\,\mathfrak{B}' = 9,910800$
$\mathfrak{X}' = -0,0142$ . . . $l{-}\mathfrak{X}' = 8,150690$
$\mathfrak{Y}' = +0,2396$ . . . $l\,\mathfrak{Y}' = 9,379550$
$\mathfrak{Z}' = -0,0061$ . . . $l{-}\mathfrak{Z}' = 7,788910$

§. 171. Quatenus ergo longitudo Lunae ab excentricitate orbitae solis pendet, erit

			log. coeff.
$\Phi =$ Praec.	$+ 0,201385\,e$	$\sin s$	9,304026
	$- 0,021889\,e$	$\sin(2\eta - s)$	8,340237
	$- 0,016368\,e$	$\sin(2\eta + s)$	8,214002
	$+ 0,06615\,ee$	$\sin 2s$	8,820508
	$+ 0,02332\,ee$	$\sin(2\eta - 2s)$	8,367825
	$+ 0,00840\,ee$	$\sin(2\eta + 2s)$	7,924429
	$+ 0,7476\,ek$	$\sin(r - s)$	9,873165
	$- 0,6185\,ek$	$\sin(r + s)$	9,791317
	$+ 0,8143\,ek$	$\sin(2\eta - r + s)$	9,910800
	$- 0,0142\,ek$	$\sin(2\eta + r - s)$	8,150690
	$+ 0,2396\,ek$	$\sin(2\eta - r - s)$	9,379550
	$- 0,0061\,ek$	$\sin(2\eta + r + s)$	7,788910

§. 172. Hae autem singulae inaequalitates ad numerum minutorum secundorum reductae dabunt:

			log. coeff.
$\Phi =$ Praec.	$+ 701'', 1$	$\sin s$	2,845780
	$- 75, 8$	$\sin(2\eta - s)$	1,879971
	$- 56, 7$	$\sin(2\eta + s)$	1,753736
	$+ 3, 8$	$\sin 2s$	0,585551
	$+ 1, 4$	$\sin(2\eta - 2s)$	0,132862
	$+ 0, 5$	$\sin(2\eta + 2s)$	9,689472
	$+ 140, 9$	$\sin(r - s)$	2,148800
	$- 116, 7$	$\sin(r + s)$	2,067000
	$+ 153, 7$	$\sin(2\eta - r + s)$	2,186530
	$- 2, 7$	$\sin(2\eta + r - s)$	0,426300
	$+ 45, 2$	$\sin(2\eta - r - s)$	1,655200
	$+ 1, 2$	$\sin(2\eta + r + s)$	0,064600

Hic scilicet et inaequalitates, quas in capite praecedente inuenimus, et istas in hoc capite erutas simul sum complexus, vt coniunctim conspectui exponerentur.

CAPUT

CAPUT XI.

INUESTIGATIO INAEQUALITATUM LUNAE A PARALLAXI SOLIS PENDENTIUM.

§. 173.

Iam in formulis nostris primariis ad eos quoque terminos progrediamur, qui littera ν sunt affecti, et quoniam est $1:\nu$ vt distantia Solis media ad distantiam Lunae mediam a Terra, erit $1:\nu$ vt parallaxis Lunae media ad parallaxin solis: ex quo inaequalitates Lunae, quae hinc oriuntur, a parallaxi solis pendere dicuntur. Quoniam vero valor ipsius ν est valde paruus, quippe $\frac{1}{280}$ propemodum, alios terminos non contemplabimur, nisi qui per ν ac per νk et νe sunt multiplicati, propterea quod magis compositi fiant minimi.

§. 174. Ex terminis ergo iam inuentis hic retineamus eos, qui sunt alicuius momenti, et cum iis nouos determinandos coniungamus; sit ergo:

$$\int R\,dr = \mathfrak{A}\,c\,2\eta + \mathfrak{C}k\cos r + \mathfrak{D}k\,c(2\eta - r) + \mathfrak{P}e\,cs + \mathfrak{Q}e\,c(2\eta - s)$$
$$+ \mathfrak{E}k\,c(2\eta + r) \qquad + \mathfrak{R}e\,c(2\eta + s)$$
$$+ \mathfrak{F}\nu\cos\nu + \mathfrak{H}\nu k\cos(\eta - r) + \mathfrak{K}\nu e\cos(\eta - s)$$
$$+ \mathfrak{G}\nu\cos 3\eta + \mathfrak{J}\nu k\cos(\eta + r) + \mathfrak{L}\nu e\cos(\eta + s)$$

$$v = A\cos 2\eta \ldots + Dk\cos(2\eta - r) + Pe\cos s + Qe\cos(2\eta - s)$$
$$+ Ek\cos(2\eta + r) \qquad + Re\cos(2\eta + s)$$
$$+ F\nu\cos\eta + H\nu k\cos(\eta - r) + K\nu e\cos(\eta - s)$$
$$+ G\nu\cos 3\eta + J\nu k\cos(\eta + r) + L\nu e\cos(\eta + s)$$

Non