ſupra (206) per ζ indicauimus, ita exprimetur, vt ſit

$$\zeta = C + 1{,}0085272\,p - 1{,}0085272\left(\begin{array}{l} 2k\sin r + \frac{3}{4}kk(1+\frac{3}{8}kk+\frac{3}{32}k^4)\sin 2r + \frac{5}{8}k^3(1+\frac{3}{8}kk)\sin 3r \\ +\frac{3}{32}k^4(1+\frac{3}{8}kk)\sin 4r + \frac{3}{40}k^5\sin 5r + \frac{7}{192}k\sin 6r \end{array}\right)$$

vbi $C+1{,}0085272\,p$ exhibet longitudinem Lunae mediam; quae ſi vocetur $= \xi$, atque in coefficientium partibus minimis pro k ſcribatur valor proximus 0,0545, erit

		log. coeff.	val. in min. ſec.
$\xi = \zeta$	$-2{,}0170544\,k \sin r$	0,304718	$22675'' = 6°,17',55''$
	$-0{,}756770\,kk \sin 2r$	9,878964	$464 = 7{,}44$
	$-0{,}336551\,k^3 \sin 3r$	9,527051	$11\frac{1}{2}$
	$-0{,}15786\,k^4 \sin 4r$	9,198282	$\frac{5}{8}$

vnde patet ſuperfluum futurum fuiſſe, ſi ſuperiores expreſſiones vltra quartam poteſtatem ipſius k extendere voluiſſemus.

CAPUT

CAPUT XVII.

INUESTIGATIO ELEMENTORUM MOTUS LUNAE

§. 272.

Inuentis iam per Theoriam hisce inaequalitatibus, quibus motus Lunae perturbatur, antequam eas ad computum aſtronomicum accommodare liceat, elementa, quae in eas ingrediuntur, per obſeruationes determinari oportet. Primo ſcilicet ad datam epocham cum longitudo Lunae media, tum eius anomalia media, ac locus nodi medius conſtitui debebit, vt eaedem res inde ad quoduis aliud tempus aſſignari queant. Deinde quoque ex obſeruationibus verus valor excentricitatis lunaris colligi debet, a quo potiſſimum quantitas praecipuarum inaequalitatum pendet. Excentricitas autem orbitae ſolaris pro ſatis certa haberi poterit, cum ſit $e = 0{,}0168$. Lunae vero excentricitas tam prope iam conſtat, vt inde ſine errore ad quamlibet anomaliam mediam vera ſatis exacte aſſignari poſſit. Etſi enim in anomalia vera error aliquot minutorum primorum committitur, inaequalitates Lunae inde non vltra aliquot minuta ſecunda afficiuntur.

§. 273. Quodſi autem ſtatim quasuis Lunae obſeruationes ad hunc finem adhibere velimus, ob tam ingentem inaequalitatum numerum, inueſtigatio elementorum maxime moleſta redderetur. Quocirca ex obſer-

obferuationibus eas eligi conueniet, pro quibus numerus inaequalitatum multo fiat minor; dum fcilicet diftantia Lunae a fole feu angulus η datum obtinet valorem. Commodiffimae ergo erunt eae obferuationes, quae in ipfis momentis coniunctionis vel oppofitionis funt inftitutae. Accuratas itaque obferuationes eclipfium lunarium ad hoc negotium adhibebo, quoniam praeter haec tempora, vera vel coniunctionis vel oppofitionis momenta non fatis certo ex obferuationibus colligi licet.

§. 274. Momento autem oppofitionis verae Lunae et Solis, longitudo Lunae fex fignis diftat a longitudine folis, ita vt fit $\theta = \Phi \pm 180°$, ideoque angulus $\eta = 180°$. Pofito autem pro η hoc valore longitudo Lunae vera Φ ex media ξ per fequentes formulas definietur, in quas formulae hactenus inuentae abeunt:

$$\Phi = \xi - 2{,}0170544\,k\sin r - 0{,}756770\,kk\sin 2r - 0{,}33655\,k^3\sin 3r$$

$+0{,}0101460$	$+0{,}004200$	
$+0{,}4202260$	$-0{,}573280$	
$+0{,}0049920$	$+0{,}003180$	
$-0{,}0052860$	$+0{,}150830$	
$-0{,}000860$	$-0{,}000002$	
$+1{,}1959\nu$		
$-0{,}0757\nu$		
$+0{,}00932f$		

$$+(0{,}201385+0{,}021889-0{,}016368-0{,}3959\nu+4{,}1738\nu)e\sin s$$
$$+(0{,}06645 - 0{,}02332 + 0{,}00840)ee\sin 2s$$
$$+(0{,}74760 - 0{,}81430 - 0{,}01420)ek\sin(r-s)$$
$$+(-0{,}61850 - 0{,}23960 - 0{,}00610)ek\sin(r+s)$$

$$-(0{,}002823 + 0{,}000910)f\sin(2\Phi-2\pi)$$
$$-0{,}000028\,f\sin(4\Phi-4\pi)$$
$$+(0{,}01521 - 0{,}00121)fk\sin(2\Phi-2\pi-r)$$
$$+0{,}79079\,fkk\sin(2\Phi-2\pi-2r)$$

§. 275. Cum igitur fit $f = 1{,}09375$, et $\nu = \frac{1}{330}$ erit has formulas colligendo:

$$\Phi = \xi - 1{,}572993\,k\sin r - 1{,}17186\,kk\sin 2r - 0{,}3365\,k^3\sin 3r$$
$$+0{,}21998\,e\sin s + 0{,}05123\,ee\sin 2s - 0{,}0809\,ek\sin(r-s)$$
$$-0{,}8642\,ek\sin(r+s)$$
$$-0{,}002989\sin(2\Phi-2\pi) + 0{,}01531\,k\sin(2\Phi-2\pi-r)$$
$$-0{,}000031\sin(4\Phi-4\pi)$$
$$+0{,}86493\,kk\sin(2\Phi-2\pi-2r)$$

et cum fit $e = 0{,}0168$, erit hoc valore fubftituto:

$$\Phi = \xi - 1{,}572993\,k\sin r - 1{,}17186\,kk\sin 2r - 0{,}3365\,k^3\sin 3r$$
$$+0{,}003697\sin s + 0{,}000014\sin 2s - 0{,}001359\,k\sin(r-s)$$
$$-0{,}014523\,k\sin(r+s)$$
$$-0{,}002989\sin(2\Phi-2\pi) + 0{,}01531\,k\sin(2\Phi-2\pi-r)$$
$$-0{,}000031\sin(2\Phi-4\pi)$$
$$+0{,}86493\,kk\sin(2\Phi-2\pi-2r)$$

§. 276. Affumta iam hypothefi quapiam non nimis a vero aberrante, vnde ad datum quoduis tem-

 pus

pus definiri possit tam longitudo lunae, quam eius anomalia media, ex qua praeterea ope excentricitatis proxime cognitae anomalia vera assignari queat: haec elementa correctione indigebunt, quam ex obseruationibus elici oporteat. Ponamus ergo longitudinem mediam ex tabulis desumtam augeri debere m minutis secundis. Tum vero excentricitas supposita, quae sit $= 0{,}0545$, augeri debeat $\frac{n}{10000}$, vt sit $k = 0{,}0545 + \frac{n}{10000}$: ipsa vero anomalia vera tabularis, quae sit $= v$, augmentum requirat μ minutorum secundorum, vt sit $r = v + \mu''$: eritque $\sin r = \sin v + \mu'' \cos v$; $\sin 2r = \sin 2v + 2\mu'' \cos 2v$; et $\sin 3r = \sin 3v$ in terminis enim minimis haec correctio praetermitti poterit.

§. 277. Quod si haec omnia in minuta secunda conuertantur, prodibit longitudo lunae vera

$$\varphi = \text{Long. med.} + m''$$
$$-17682'' \sin v - 32{,}445\, n'' \sin v - 0{,}085728\, \mu'' \cos v$$
$$- 718'' \sin 2v - 2{,}635\, n'' \sin 2v - 0{,}006962\, \mu'' \cos 2v$$
$$- 11'' \sin 3v + 762'' \sin s + 3'' \sin 2s$$
$$- 15'' \sin (r-s) - 163'' \sin (r+s)$$
$$- 616'' \sin (2\varphi - 2\pi) + 172'' \sin (2\varphi - 2\pi - r)$$
$$- 6'' \sin (4\varphi - 4\pi) + 530 \sin (2\varphi - 2\pi - 2r)$$

Cum

Cum autem postremus terminus sit suspectus, loco eius coefficientis 530 malumus ponere coefficientem indefinitum $100y$, atque ex obseruationibus valorem ipsius y indagare. Deinde sit error anomaliae verae i minutorum primorum, vt calculus commodior reddatur, atque ob $\mu = 60i$, neglectis terminis minimis erit:

$$\varphi = \text{Long. med.} + m''$$
$$-17682'' \sin v - 32{,}445\, n'' \sin v - 5{,}143\, i'' \cos v$$
$$- 718'' \sin 2v - 2{,}635\, n \sin 2v - 0{,}417\, i \cos 2v$$
$$+ 762 \sin s - 15 \sin (r-s) - 163 \sin (r+s)$$
$$- 616 \sin (2\varphi - 2\pi) + 172'' \sin (2\varphi - 2\pi - r)$$
$$+ 100y \sin (2\varphi - 2\pi - 2r)$$

§. 278. Oblata autem obseruatione eclipsis lunae, quaeratur primum momentum medium huius eclipsis, pro quo colligatur longitudo solis, itemque longitudo nodi ascendentis. Punctum autem soli oppositum nondum erit longitudo lunae vera in ecliptica; verumtamen longitudo lunae pro hoc momento eclipsis medio inueniri poterit ope sequentis tabellae.

Subtrahatur longitudo nodi a longitudine ſolis, et aequatio tabulae ſecundum titulos adſcriptos applicetur puncto ſoli oppoſito in ecliptica.

gr.	{ O Sign. VI. Sign. } ſubtrahe	
0	0′, 0″	30
1	0, 32	29
2	1, 6	28
3	1, 39	27
4	2, 12	26
5	2, 45	25
6	3, 17	24
7	3, 49	23
8	4, 21	22
9	4, 53	21
10	5, 24	20
11	5, 56	19
12	6, 26	18
	adde	gr.
	{ V Sign. XI. Sign. }	

§. 279. Quanquam autem hoc momento, ad quod lunae longitudinem hinc colligimus, non vera lunae oppoſitio exiſtit, ſed luna ſecundum longitudinem a puncto ſoli oppoſito diſtat particula, quam haec tabula monſtrat; tamen tuto pro hoc momento ex formula noſtra longitudinem lunae inueſtigare poterimus, viſuri, quam

quam exacte ea conueniat cum longitudine eius ad hoc tempus ex obſeruatione concluſa. Cum enim luna hoc tempore nunquam vltra 5′ a vero oppoſitionis loco diſtet, ſi formula noſtra generali vti vellemus, foret angulus η minor 5 minutis primis; vnde facile perſpicitur, diſcrimen in loco lunae inde oriundum vix vnquam 12″ eſſe ſuperaturum. Quoniam itaque medium cuiusque eclipſis momentum ipſum tam accurate definiri nequit, vt non error dimidii minuti primi ſit pertimeſcendus, ſuperfluum ſane foret in calculo ad iſtiusmodi minutias attendere.

§. 280. Hanc ob cauſam quoque ex calculo, quem inibo, non ſummam praecisionem expectari conuenit; quia ipſae obſeruationes, quibus vtar, non plenae accurationis ſunt capaces. Plus igitur me non effecturum confido, quam vt ſatis prope tam excentricitatem orbitae lunaris, quam longitudinem et anomaliam lunae mediam ad datam epocham definiam. Quod cum fuerit factum maiori confidentia theoriam ad quasuis alias obſeruationes transferre licebit; quae ſi nullis erroribus fuerint inquinatae, non admodum erit difficile reliquas elementorum correctiones, quibus formulae noſtrae ſunt innixae, inde concludere. Imprimis autem hic calculus veram excentricitatem orbitae lunaris ſatis exacte manifeſtabit, vt deinceps accuratius pro quauis anomalia media conuenientem anomalam veram definire valeamus. Hunc igitur in finem nonnullas eclipſes lunares Pariſiis inſtitutas calculo ſubiiciam.

§. 281. Primae igitur eclipſis medium contigiſſe reperio Pariſiis A. 1712. Jan. 23^d, 7^h, $55'$, $16''$ temp. medio. Pro quo momento colligitur:

Longitudo ſolis θ . . .	10^s, 3°, $0'$, $54''$
Anomalia vera ſolis s . . .	6, 24, 25, 13
Deinde ex tabulis meis:	
Longitudo lunae media .	4, 7, 18, 55
Anomalia lunae media .	2, 0, 18, 20
Anomalia lunae vera $v =$	1, 25, 6, 27
Longitudo nodi vera $\pi =$	9, 24, 34, 32
Diſt. nodi a ſole $\theta - \pi =$	0, 8, 26, 22
Hinc aequatio loci lunae .	— — 4, 33
Ergo longitudo lunae vera $\phi =$	4, 2, 56, 21

§. 282. Hinc calculus ſequenti modo inſtituetur:

$v =$ 1, 25, 6, 27	;	$\sin v = +\sin 55^\circ, 6', 27''$ $\cos v = +$
$2v =$ 3, 20, 12, 54	;	$\sin 2v = +\sin 69, 47, 6$ $\cos 2v = -$
$s =$ 6, 24, 25, 13	;	$\sin s = -\sin 24, 25, 13$
$v - s =$ 7, 0, 41, 14	;	$\sin = -\sin 30, 41, 14$
$v + s =$ 8, 19, 31, 40	;	$\sin = -\sin 79, 31, 40$
$\phi - \pi =$ 6, 8, 21, 49		
$2\phi - 2\pi =$ 0, 16, 43, 38	;	$\sin = +\sin 16, 43, 38$
$r =$ 1, 25, 6, 27		
$2\phi - 2\pi - r =$ 10, 21, 37, 11	;	$\sin = -\sin 38, 22, 49$
$2\phi - 2\pi - 2r =$ 8, 26, 30, 44	;	$\sin = -\sin 86, 30, 44$

+

+ 9,91393	+ 9,9139	+ 9,7575
— 4,24753	— 1,5111	— 0,7104
— 4,16146	— 1,4250n	— 0,4679i
+ 9,9724	+ 9,9724	— 9,5385
— 2,8561	— 0,4208	— 9,6201
— 2,8285	— 0,3932n	+ 9,1586i
— 9,6163	— 9,7078	— 9,9927
+ 2,8819	— 1,1761	— 2,2122
— 2,4982	+ 0,8839	+ 2,2049
+ 9,4588	— 9,7930	
— 2,7896	+ 2,2355	— 99,8y
— 2,2484	— 2,0285	

aeq. +	aeq. —		
+ 8	— 14493		— 26,6n
+ 160	— 674		— 2,5n
+ 168	— 315		— 2,9i
— 15766	— 177		+ 0,1i
— 15598	— 107		— 99,8y
— $259'$, $58''$	— 15766		
— 4°, 19, 58 . . aequatio			

Long. med. 4,7,18,55 + m
aeq. −4,19,58
4,2,58,57 − 29,1n − 2,8i − 99,8y = 4,2,56,21
4,2,56,21
Ergo o = 2,36 − 29,1n − 2,8i − 99,8y + m

§. 283.

§. 283. Secundae eclipſi medium contigit:
Pariſiis A. 1713. Dec. 1^d, 15^h, $26'$, $34''$ temp. med.
Pro quo momento colligitur

Longitudo Solis θ	$=$	.	8,	9, 53, 40
Anomalia vera Solis s	$=$	.	5,	1, 46, 43
Longitudo Lunae media		.	2,	5, 2, 26
Anomalia Lunae media		.	9,	12, 27, 42
Anomalia Lunae vera v	$=$	.	9,	18, 24, 49
Longitudo nodi π	$=$	.	8,	17, 46, 10
Diſtantia Solis a nodo		.	11,	22, 7, 30
Aequatio loci Lunae		.	$+$	4, 17
Longitudo Lunae vera Φ	$=$		2,	9, 57, 57

§. 284. Hinc calculus ſequens inſtituatur:

v	$=$ 9, 18, 24, 49	; ſin v	$=$ $-$ ſin 71, 35, 11
		cof v	$=$ $+$
$2v$	$=$ 7, 6, 49	; ſin $2v$	$=$ $-$ ſin 36, 49
		cof $2v$	$=$ $-$
s	$=$ 5, 1, 46	; ſin s	$=$ $+$ ſin 28, 14
$v-s$	$=$ 4, 16, 39	; ſin	$=$ $+$ ſin 43, 21
$v+s$	$=$ 2, 20, 11	; ſin	$=$ $+$ ſin 80, 11
$\Phi-\pi$	$=$ 11, 22, 12		
$2\Phi-2\pi$	$=$ 11, 14, 24	; ſin	$=$ $-$ ſin 15, 36
r	$=$ 9, 18, 25		
$2\Phi-2\pi-r$	$=$ 1, 25, 59	; ſin	$=$ $+$ ſin 55, 59
$2\Phi-2\pi-2r$	$=$ 4, 7, 34	; ſin	$=$ $+$ ſin 52, 26

$-$ 9,97717	$-$ 9,9772	$+$ 9,4996
$-$ 4,24753	$-$ 1,5111	$-$ 0,7104
$+$ 4,22470	$+$ 1,4883n	$-$ 0,2100i
$-$ 9,7776	$-$ 9,7776	$-$ 9,9034
$-$ 2,8561	$-$ 0,4208	$-$ 9,6201
$+$ 2,6337	$+$ 0,1984n	$+$ 9,5235i
$+$ 9,6749	$+$ 9,8366	$+$ 9,9936
$+$ 2,8819	$-$ 1,1761	$-$ 2,2122
$+$ 2,5568	$-$ 1,0127	$-$ 2,2058
$-$ 9,4296	$+$ 9,9185	$+$ 79, 2y
$-$ 2,7866	$+$ 2,2355	
$+$ 2,2192	$+$ 2,1530	

aeq. aff.	aequat.	
$+$ 16776	$-$ 10	$+$ 30, 8n
$+$ 430	$-$ 161	$+$ 1, 6n
$+$ 360	$-$ 171	$-$ 1, 6i
$+$ 166		$+$ 0, 3i
$+$ 143		
$+$ 17875		
$-$171		
$+$ 17704		
$+$ 2961′,4″	aequatio	

Long. media	$=$	2, 5, 2, 26 $+$ m
aeq.		$+$4, 55, 4
Long. vera		2, 9, 57, 30, $+$ m
obſ.		2, 9, 57, 57
Ergo o	$=$	$-$ 0′,27″ $+$ m $+$ 32, 4n $-$1, 3i $+$ 79, 2y

§. 285. Tertiae eclipsis medium contigit Parisiis A. 1717 Mart. 26^d, 15^b, $21'$, $20''$ temp. med. Pro quo tempore colligitur

Longitudo solis vera $\odot$ =	0°,	6°,	19′,	56″,
Anomalia solis vera s =	8,	28,	0,	17
Longitudo media lunae . .	6,	1,	37,	2
Anomalia media lunae . .	8,	24,	7,	21
Anomalia lunae vera v =	9,	0,	19,	10
Longitudo nodi vera π =	6,	13,	30,	22
Distantia nodi a sole . .	5,	22,	49,	29
aeq. pro loco lunae	+	3,	57	
Ergo longitudo lunae vera $\mathfrak{C}$ =	6,	6,	23,	53

§. 286. Calculus igitur ita se habebit

v = 9, 0, 19, 10	; sin v =	− sin 89°, 40′, 50″
	cos. =	+
$2v$ = 6, 0, 38,	; sin $2v$ =	− sin 0°, 38′
	cos =	−
s = 8, 28, 0	; sin s =	− sin 88°, 0′
$v - s$ = 0, 2, 19	; sin =	+ sin 2°, 19
$v + s$ = 5, 28, 19	; sin =	+ sin 1, 41
$\mathfrak{C} - \pi$ = 5, 22, 53		
$2\mathfrak{C} - 2\pi$ = 11, 15, 46	; sin =	− sin 14, 14
r = 9, 0, 19		
$2\mathfrak{C} - 2\pi - r$ = 2, 15, 27	; sin =	+ sin 75, 27
$2\mathfrak{C} - 2\pi - 2r$ = 5, 15, 8	; sin =	+ sin 14, 52

— 9,99999	−	10,0000	+	7,7425	
— 4,24753	−	1,5111	−	0,7104	
+ 4,24752	+	1,5111n	−	8,4526i	
— 8,0435	−	8,0435	−	9,9999	
— 2,8561	−	0,4208	−	9,6201	
+ 0,8996	+	8,4643n	+	9,6200i	
— 9,9997	+	8,6066	+	8,4680	
+ 2,8819	−	1,1761	−	2,2122	
— 2,8816	−	9,7827	−	0,0802	
— 9,3907	+	9,9858	+	25,65y	
— 2,7893	+	2,2355			
+ 2,1893	+	2,2213			

aeq. aff.	aeq. neq.		
+ 17682	− 762	+ 32, 4n	
+ 8	− 1	+ 0, 0n	
+ 152	− 4	− 0, 0i	
+ 166	− 767	+ 0, 2i	
+ 18008	+ 18008		
	+ 17241		
	+ 287,21		
	+ 4, 47, 21	aequatio	

Long. media $\mathfrak{C}$ =	6, 1, 37, 2	
aeq.	+ 4, 47, 21	
Long. $\mathfrak{D}$ vera =	6, 6, 24, 23	
obs.	6, 6, 23, 53	

Ergo s = + 30 + m + 32,4n + 0,2i + 25,65y

§. 287. Quartae eclipſis medium erat Pariſiis A. 1718 Sept. 9^d, 8^h, $1'$, $1''$ temp. medio
Pro quo tempore colligitur

Longitudo ſolis vera $\odot$ =	5, 16, 40, 58
Anomalia ſolis vera s =	2, 8, 19, 59
Longitudo lunae media	11, 17, 25, 16
Anomalia lunae media	0, 10, 41, 28
Anomalia lunae vera v =	0, 9, 36, 52
Longitudo nodi vera π =	5, 15, 59, 35
Diſtantia nodi a ſole	0, 0, 41, 23
aeq. pro loco lunae	— 22
Longitudo lunae obſ. $\mathfrak{C}$ =	11, 16, 40, 36

§. 288. Calculus ergo ſequens habebitur.

v = 0, 9, 36, 52	; ſin v =	+ ſin 9, 36, 52
	cof =	+
$2v$ = 0, 19, 14	; ſin $2v$ =	+ ſin 19, 14
	cof =	+
s = 2, 8, 20	; ſin s =	+ ſin 68, 20
$v-s$ = 10, 1, 17	; ſin =	− ſin 58, 43
$v+s$ = 2, 17, 57	; ſin =	+ ſin 77, 57
$\mathfrak{C}-\pi$ = 0, 0, 41		
$2\mathfrak{C}-2\pi$ = 0, 1, 22	; ſin =	+ ſin 1, 22
r = 0, 9, 37		
$2\mathfrak{C}-2\pi-r$ = 11, 21, 45	; ſin =	− ſin 8, 15
$2\mathfrak{C}-2\pi-2r$ = 11, 12, 8	; ſin =	− ſin 17, 52

+ 9,22274	+ 9,2227	+ 9,9938
— 4,24753	− 1,5111	− 0,7104
— 3,47027	− 0,7338 n	− 0,7042 i
+ 9,5177	+ 9,5177	+ 9,9750
— 2,8561	− 0,4208	− 9,6201
— 2,3738	− 9,9385 n	− 9,5951 i
+ 9,9682	− 9,9318	+ 9,9903
+ 2,8819	− 1,1761	− 2,2122
+ 2,8501	+ 1,1079	− 2,2025
+ 8,3775	− 9,1568	− 30,68 y
— 2,7896	+ 2,2355	
— 1,1671	− 1,3923	

aeq. aff.	aeq. neg.	
+ 708	− 2953	− 5, 4 n
+ 13	− 237	− 0, 8 n
+ 721	− 159	− 5, 1 i
— 3389	− 15	− 0, 4 i
— 2668	− 25	
aeq. = — 44′, 28″	− 3389	

Long. $\mathfrak{C}$ med.	11, 17, 25, 16
aeq.	— 44, 28
Long. vera	11, 16, 40, 48
obſ.	11, 16, 40, 36

Ergo o = + 12 + m − 6, 2 n − 5, 5 i − 30, 68 y

§. 289. Quintae eclipſis medium erat:
Pariſiis A. 1719 Aug. $29^d, 8^h, 33', 19''$ temp. med.
Pro quo tempore colligitur:

Longitudo ſolis vera $\theta =$	5, 5, 47, 14
Anomalia ſolis vera $s =$	1, 27, 25, 24
Longitudo lunae media .	11, 2, 9, 40
Anomalia lunae media . .	10, 15, 59, 25
Anomalia lunae vera $v =$	10, 20, 5, 19
Longitudo nodi vera $\pi =$	4, 27, 44, 39
Diſtantia nodi a ſole $==$	0, 8, 2, 35
Aequ. pro loco lunae . .	— — 4, 22
Long. lunae obſeruata . .	11, 5, 42, 52

§. 290. Calculus ergo ita ſe habebit:

$v = 10, 20, 5, 19$	; $\sin = -\sin 39, 54, 41$ $\cos = +$
$2v = 9, 10, 11$	; $\sin 2v = -\sin 79, 49$ $\cos = +$
$v = 10, 20, 5$	
$s = 1, 27, 25$	; $\sin s = +\sin 57, 25$
$v-s = 8, 22, 40$	; $\sin = -\sin 82, 40$
$v+s = 0, 17, 30$	; $\sin = +\sin 17, 30$
$\Phi-\pi = 0, 7, 58$	
$2\Phi-2\pi = 0, 15, 16$	; $\sin = +\sin 15, 56$
$r = 10, 20, 5$	
$2\Phi-2\pi-r = 1, 25, 15$	; $\sin = +\sin 55, 51$
$2\Phi-2\pi-2r = 3, 5, 46$	; $\sin = +\sin 84, 14$

—	9, 80726	−	9, 8073	+	9, 8849
—	4, 24753	−	1, 5111	−	0, 7104
+	4, 05479	+	$1, 3184n$	−	$0, 5953i$
—	9, 9931	−	9, 9931	+	9, 2475
—	2, 8561	−	0, 4208	−	9, 6201
+	2, 8492	+	$0, 4139n$	−	$8, 8676i$
+	9, 9256	−	9, 9964	+	9, 4781
+	2, 8819	−	1, 1761	−	2, 2122
+	2, 8075	+	1, 1725	−	1, 6903
+	9, 4386	+	9, 9178	+	99, 5J
—	2, 7896	+	2, 2355		
—	2, 2282	+	2, 1533		

aeq. aff.	aeq. neg.	
+ 11345	− 49	$+ 20, 8n$
+ 707	− 170	$+ 2, 6n$
+ 642	− 219	$- 3, 9i$
+ 15	+ 12851	$- 0, 1i$
+ 142	+ 12632	
+ 12851	210, 32	

+ 3, 30, 32 aequatio

Long. lunae med.	11, 2, 9, 40
aeq.	+ 3, 30, 32
Long. lunae vera	11, 5, 40, 12
obſ.	11, 5, 42, 52

$$\epsilon = -2,40 + m + 23,4n - 4,0i + 99,5J$$

§. 291.

§. 291. Sextae eclipſis medium erat Pariſiis A. 1722. Jun. $28^d, 13^h, 58', 41''$ temp. med. Pro quo tempore habetur:

Longitudo ſolis vera $\odot$ =	3, 6, 51, 7
Anomalia ſolis vera s =	11, 28, 26, 56
Longitudo lunae media .	9, 9, 31, 50
Anomalia lunae media .	4, 28, 8, 18
Anomalia lunae vera v =	4, 24, 39, 53
Longitudo nodi vera π =	3, 2, 36, 2
Diſtantia nodi a ſole . .	0, 4, 15, 5
Aequatio loci lunae . .	— 2, 20
Longitudo lunae obſeruata	9, 6, 48, 47

§. 292. Calculus ergo ita ſe habebit

v = 4, 24, 39, 53	;	$\sin v = +\sin 35^b, 20', 7''$
		$\cos v = -$
$2v$ = 9, 19, 20	;	$\sin 2v = -\sin 70, 40$
		$\cos 2v = +$
r = 4, 24, 40		
s = 11, 28, 27	;	$\sin s = -\sin 1, 33$
$r-s$ = 4, 26, 13	;	$\sin = +\sin 33, 47$
$r+s$ = 4, 23, 7	;	$\sin = +\sin 36, 53$
$\Phi-\pi$ = 0, 4, 13		
$2\Phi-2\pi$ = 0, 8, 26	;	$\sin = +\sin 8, 26$
r = 4, 24, 40		
$2\Phi-2\pi-r$ = 7, 13, 46	;	$\sin = -\sin 43, 46$
$2\Phi-2\pi-2r$ = 2, 19, 6	;	$\sin = +\sin 79, 6$

+ 9, 76220	+ 9,7622	− 9, 9116	
— 4, 24753	− 1, 5111	− 0, 7104	
— 4, 00973	− 1, 2733n	+ 0, 6220i	
— 9, 9748	− 9, 9748	+ 9, 5199	
— 2, 8561	− 0, 4208	− 9, 6201	
+ 2, 8309	+ 0, 3956n	− 9, 1400i	
— 8, 4321	+ 9, 7451	+ 9, 7783	
+ 2, 8819	− 1, 1761	− 2, 2122	
— 1, 3140	− 0, 9212	− 1, 9905	
+ 9, 1663	− 9, 8399	+ 98, 2y	
— 2, 7896	+ 2, 2355		
— 1, 9559	− 2, 0754		

aeq. aff.	aeq. neg.	
+ 678	− 10227	− 18, 8n
— 10563	− 21	+ 2, 5n
— 9885	− 8	
164,45	− 98	+ 4, 2i
aeq. = — 2,44,45	− 90	− 0, 1i
	− 119	
	−101563	

Long. lun. med.	9, 9, 31, 50
	− 2, 44, 45
Long. lun.	9, 6, 47, 5
obſ.	9, 6, 48, 47

Ergo $o = -1, 42 + m - 16, 3n + 4, 1i + 98, 9y$

§. 293. Septimae eclipſis medium obſeruatum eſt Pariſiis A. 1724 Oct. $31^d, 15^h, 34', 17''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera $\vartheta =$	7,	8,	56,	1
Anomalia ſolis vera $s =$	4,	0,	29,	44
Longitudo lunae media	1,	9,	23,	59
Anomalia lunae media	5,	22,	38,	2
Anomalia lunae vera $v =$	5,	21,	46,	51
Longitudo nodi vera	1,	16,	36,	22
Diſtantia nodi a ſole	5,	22,	19,	39
aequatio loci lunae	+		4,	10
Long. lunae obſeruata	1,	9,	0,	11

§. 294. Calculus ergo ita ineatur:

$v =$	5,	21,	46,	51	;	$\sin v =$	+ $\sin$	$8°, 13', 9''$
						$\cos =$	−	
$2v =$	11,	13,	34,		;	$\sin 2v =$	− $\sin$	$16°, 26'$
$r =$	5,	21,	47			$\cos =$	+	
$s =$	4,	0,	30		;	$\sin s =$	+ $\sin$	$59°, 30'$
$r - s =$	1,	21,	17		;	$\sin$ =	+ $\sin$	51, 17
$r + s =$	9,	22,	17		;	$\sin$ =	− $\sin$	67, 43
$\varphi - \pi =$	5,	22,	24					
$2\varphi - 2\pi =$	11,	14,	48		;	$\sin$ =	− $\sin$	15, 12
$r =$	5,	21,	47					
$2\varphi - 2\pi - r =$	5,	23,	1		;	$\sin$ =	+ $\sin$	6, 59
$2\varphi - 2\pi - 2r =$	0,	1,	14		;	$\sin$ =	+ $\sin$	1, 14

+ 9, 15520	+ 9, 1552	− 9, 9955
− 4, 24753	− 1, 5111	− 0, 7104
− 3, 40273	− 0, 6663m	+ 0, 7059i
− 9, 4516	− 9, 4516	+ 9, 9819
− 2, 8561	− 0, 4208	− 9, 6201
+ 2, 3077	+ 9, 8724n	− 9, 6020i
+ 9, 9353	+ 9, 8922	− 9, 9663
+ 2, 8819	− 1, 1761	− 2, 2122
− 2, 8172	− 1, 0683	+ 2, 1785
− 9, 4186	+ 9, 0849	+ 2, 15y
− 2, 7896	+ 2, 2355	
+ 2, 2082	+ 1, 3024	

aeq. aff.	aeq. neg.	
+ 203	− 2528	− 4, 6m
+ 657	− 11	− 7, 5m
+ 151	− 2539	
+ 161	+ 1193	+ 5, 1i
+ 21	− 1346	− 0, 4i
+ 1193	− 22, 26	aequatio

Long. ☽ med. 1, 9, 23, 59
− 22, 26
Long. calc. 1, 9, 1, 33
Long. obſ. 1, 9, 0, 11

$o = + 1,22 + m - 3,9n + 4,7i + 2,15y.$

§. 295. Octauae eclipſis medium obſeruatum eſt Pariſiis A. 1729. Febr. 13^d, 9^h, $6'$, $56''$ temp. med. Pro quo tempore colligitur:

Longitudo ſolis vera	$\odot$ =	10^s, 25°, $13'$, $23''$
Anomalia ſolis vera	s =	7, 16, 43, 34
Longitudo lunae media	.	5, 0, 5, 27
Anomalia lunae media	.	3, 18, 53, 24
Anomalia lunae vera	v =	3, 12, 54, 9
Longitudo nodi vera	π =	10, 24, 4, 30
Diſtantia nodi a ſole	=	0, 1, 8, 53
aequatio pro long. lunae	.	— 0, 37
Longitudo lunae obſeruata		4, 25, 12, 46

§. 296. Calculus ergo ita ſe habebit:

v = 3, 12, 54, 9	;	$\sin v$ = + $\sin 77^\circ, 5', 51''$
		$\cos v$ = −
$2v$ = 6, 25, 48	;	$\sin 2v$ = − $\sin 25, 48$
		$\cos 2v$ = −
r = 3, 12, 54		
s = 7, 16, 44	;	$\sin s$ = − $\sin 46, 44$
$r-s$ = 7, 26, 10	;	$\sin$ = − $\sin 56, 10$
$r+s$ = 10, 29, 38	;	$\sin$ = − $\sin 30, 22$
$\odot-\pi$ = 0, 1, 8,		
$2\odot-2\pi$ = 0, 2, 16,	;	$\sin$ = + $\sin 2, 16$
r = 3, 12, 54		
$2\odot-2\pi-r$ = 8, 19, 22	;	$\sin$ = − $\sin 79, 22$
$2\odot-2\pi-2v$ = 5, 6, 28	;	$\sin$ = + $\sin 23, 32$

+ 9,98889	+	9,9889	−	9,3488
— 4,24757	−	1,5111	−	0,7104
— 4,23642	−	$1,5000n$	+	$0,0502i$
— 9,6387	−	9,6387	−	9,9544
— 2,8561	−	0,4208	−	9,6201
+ 2,4948	+	$0,0595n$	+	$9,5745i$
— 9,8622	−	9,9194	−	9,7037
+ 2,8819	−	1,1761	−	2,2122
— 2,7441	+	1,0955	+	1,9159
+ 8,5971	−	9,9925	+	$39,9y$
— 2,7896	+	2,2355		
— 1,3867	−	2,2280		

aeq. aff.	aeq. neg.		
+ 312	− 17235	−	$31,7n$
+ 12	− 555	+	$1,1n$
+ 82	− 24	+	$1,1i$
+ 406	− 169	+	$0,3i$
	− 17983		
	+ 406		
	− 17577		
	− 292,57		
	− 4,52,57	aequatio	

Long. lunae media	=	5, 0, 5, 27
aeq.		− 4, 52, 57
Long. lunae calc.		4, 25, 12, 30
Long. lunae obſ.		4, 25, 12, 46

$\epsilon = -16 + m - 30,6n + 1,4i + 39,9y$

§. 297. Nonae eclipſis medium obſeruatum eſt Pariſiis A. 1729. Aug. 8^d, 13^h, $14'$, $14''$ temp. med. Pro quo tempore reperitur.

Longitudo ſolis vera	$\odot$ =	4,	16,	17, 29
Anomalia ſolis vera	s =	1,	7,	47, 12
Longitudo lunae media		10,	11,	23, 57
Anomalia lunae media		8,	10,	36, 19
Anomalia lunae vera	v =	8,	16,	34, 40
Longitudo nodi vera	π =	10,	14,	58, 21
Diſtantia nodi a ſole		6,	1,	19, 8
Aequatio pro loco lunae			—	43
Long. lunae obſeruata		10,	16,	16, 46

§. 298. Calculus ergo ita inſtituetur:

v = 8, 16, 34, 40 ; $\sin v = -\sin$ 76, 34, 40
$\cos v = -$

$2v$ = 5, 3, 9 ; $\sin 2v = +\sin$ 26, 51
$\cos 2v = -$

r = 8, 16, 35
s = 1, 7, 47 ; $\sin s = +\sin$ 37, 47
$r - s$ = 7, 8, 48 ; $\sin = -\sin$ 38, 48
$r + s$ = 9, 24, 22 ; $\sin = -\sin$ 65, 38
$\odot - \pi$ = 6, 1, 19
$2\odot - 2\pi$ = 0, 2, 38 ; $\sin = +\sin$ 2, 38
r = 8, 16, 35
$2\odot - 2\pi - r$ = 3, 16, 3 ; $\sin = +\sin$ 73, 57
$2\odot - 2\pi - 2v$ = 6, 29, 28 ; $\sin = -\sin$ 29, 28

— 9,93797	— 9,9880	— 9,3655
— 4,24755	— 1,5111	— 0,7104
+ 4,23550	+ 1,4991n	+ 0,0759i
+ 9,6548	+ 9,6548	— 9,9505
— 2,8561	— 0,4208	— 9,6201
— 2,5109	— 0,0756n	+ 9,5706i
+ 9,7872	— 9,7970	— 9,9595
+ 2,8819	— 1,1761	— 2,2122
+ 2,6691	+ 0,9731	+ 2,1717
+ 8,6622	+ 9,9827	— 49, 2y.
— 2,7896	+ 2,2355	
— 1,4518	+ 2,2182	

aeq. aff.	aeq. neg.	
+ 17199	— 324	+ 31, 6n
+ 467	— 28	— 1, 2n
+ 9	— 352	+ 1, 2i
+ 148	+ 17988	+ 0, 4i
+ 165	+ 17636	
+ 17988	+ 293, 56	
	+ 4, 53, 56	aequatio

Long. $\odot$ med. = 10, 11, 23, 57
aeq. + 4, 53, 56
Long. $\odot$ calc. = 10, 16, 17, 53
Long. $\odot$ obſ. 10, 16, 16, 46
$o = +1, 7 + n + 30, 4n + 1, 6i - 49, 2y$

§. 299.

§. 299. Decimae eclipſis medium obſeruatum eſt Pariſiis A. 1731. Jun. $19^d, 13^h, 55', 13''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera $\theta =$	2,	28°,	5',	41''
Anomalia ſolis vera $s =$	11,	19,	48,	47
Longitudo lunae media .	9,	1,	45,	1
Anomalia lunae media .	4,	15,	9,	43
Anomalia lunae vera $v =$	4,	10,	34,	21
Longitudo nodi vera $\pi =$	9,	8,	6,	38
Diſtantia nodi a ſole .	5,	19,	59,	3
Aequatio pro loco lunae .		+	5,	24
Longitudo lunae obſeruata	8,	28,	11,	5

§. 300. Calculus ergo ita inſtituetur

$v = 4, 10, 34, 21$; $\sin v = +\sin 49, 25, 39$
$\cos v = -$.

$2v = 8, 21, 9$; $\sin 2v = -\sin 81, 9$
$\cos 2v = -$

$r = 4, 10, 34$
$s = 11, 19, 49$; $\sin s = -\sin 10, 11$
$r - s = 4, 20, 45$; $\sin = +\sin 39, 15$
$r + s = 4, 0, 23$; $\sin = +\sin 59, 37$
$\Phi - \pi = 5, 20, 4$
$2\Phi - 2\pi = 11, 10, 8$; $\sin = -\sin 29, 52$
$s = 4, 10, 34$
$2\Phi - 2\pi - r = 6, 29, 34$; $\sin = -\sin 29, 34$
$2\Phi - 2\pi - 2r = 2, 19, 0$; $\sin = +\sin 79, 0$

+

eſt

+ 9,88057	+ 9,8806	− 9,8131
− 4,24753	− 1,5111	− 0,7104
− 4,12810	− 1,3917π	+ 0,2100 i
− 9,9948	− 9,9948	− 9,1871
− 2,8561	− 0,4208	− 9,6201
+ 2,8509	+ 0,4156π	+ 8,8072 i
− 9,2475	+ 9,8012	+ 9,9358
+ 2,8819	− 1,1761	− 2,2122
− 2,1294	− 0,9773	− 2,1480
− 9,5313	− 9,6932	+ 98, 1 y
− 2,7896	+ 2,2355	
+ 2,3209	− 1,9287	

aeq. aff.	aeq. neg.	
+ 709	− 13431	− 24, 8π
+ 209	− 135	+ 2, 6π
+ 918	− 9	+ 3, 3 i
− 13801	− 141	+ 0, 1 i
− 12883	− 85	
− 214, 43	− 13801	
aeq. − 3, 34, 43		

9

Long. lunae media	9,	1,	45,	1
aeq.			− 3, 34,	43
Long. lunae calc.	8,	28,	10,	18,
Long. lunae obſ.	8,	28,	11,	5

Ergo $\epsilon = -47'' + \pi - 22, 2\pi + 3, 4i + 98, 1y$

§. 301. Eclipsis undecimae medium obseruatum est Parisiis A. 1732 Dec. $1^d, 9^h, 48', 23''$ temp. med. Pro quo tempore colligitur:

Longitudo solis vera θ =	8,	10,	3,	6
Anomalia solis vera s =	5,	1,	29,	50
Longitudo lunae media .	2,	6,	8,	19
Anomalia lunae media . .	7,	19,	24,	12
Anomalia lunae vera v =	7,	24,	19,	39
Longitudo nodi vera π =	8,	10,	41,	14
Distantia nodi a sole =	11,	29,	21,	52
Aequ. pro loco lunae . .		+		21
Long. lunae obseruata . .	2,	10,	3,	27

§. 302. Calculus ergo ita se habebit:

v = 7, 24, 19, 39	; $\sin v$ =	$-\sin$ 54, 19, 39
	$\cos v$ =	$-$
$2v$ = 3, 18, 39	; $\sin 2v$ =	$+\sin$ 71, 21
	cos =	$-$
r = 7, 24, 20		
s = 5, 1, 30	; $\sin s$ =	$+\sin$ 28, 30
$r-s$ = 2, 22, 50	; sin =	$+\sin$ 82, 50
$r+s$ = 0, 25, 50	; sin =	$+\sin$ 25, 50
$\Phi-\pi$ = 11, 29, 22		
$2\Phi-2\pi$ = 11, 28, 44	; sin =	$+\sin$ 1, 16
r = 7, 24, 40		
$2\Phi-2\pi-r$ = 4, 4, 4	; sin =	$+\sin$ 55, 56
$2\Phi-2\pi-2r$ = 8, 9, 24	; sin =	$+\sin$ 69, 24

$-$ 9,90975	$-$ 9,9097	$-$ 9,7657
$-$ 4,24753	$-$ 1,5111	$-$ 0,7104
$+$ 4,15728	$+$ 1,4208n	$+$ 0,4761i
$+$ 9,9766	$+$ 9,9766	$-$ 9,5048
$-$ 2,8561	$-$ 0,4208	$-$ 9,6201
$-$ 2,8327	$-$ 0,3974n	$+$ 9,1249i
$+$ 9,6787	$+$ 9,9969	$+$ 9,6444
$+$ 2,8819	$-$ 1,1761	$-$ 2,2122
$+$ 2,5606	$-$ 1,1730	$-$ 1,8566
$-$ 8,3445	$+$ 9,9182	$-$ 93, 6y
$-$ 2,7896	$+$ 2,2355	
$+$ 1,1341	$+$ 2,1537	

aeq. aff.	aeq. neq.	$+$ 26, 4n
$+$ 14364	$-$ 680	$-$ 2, 5n
$+$ 364	$-$ 15	$+$ 3, 0i
$+$ 14	$-$ 72	$+$ 0, 1i
$+$ 142	$-$ 767	
$+$ 14884	$+$ 14884	
	$+$ 14117	
	$+$ 235, 17	
	$+$ 3, 55, 17	aequatio

Long. lunae media	2, 6, 8, 19	
aeq.	$+$ 3, 55, 17	
Long. lunae calc.	2, 10, 3, 36	
obs.	2, 10, 3, 27	

s = $+9 + m + 23, 9n + 3, 1i - 93, 6y$.

§. 303. Eclipſis duodecimae medium obſeruatum eſt Pariſiis A. 1736 Mart. 26^d, 12^b, 14^l, $36''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera $\theta =$	0^s,	6°,	35^l,	$42''$
Anomalia ſolis vera $s =$	8,	27,	58,	24
Longitudo lunae media	6,	4,	5,	0
Anomalia lunae media	7,	3,	25,	43
Anomalia lunae vera $v =$	7,	7,	2,	56
Longitudo nodi vera $\pi =$	6,	6,	24,	31
Diſtantia nodi a ſole	6,	0,	11,	11
aeq. pro long. lunae	—			6
Longitudo lunae obſ.	6,	6,	35,	36

§. 304. Calculus ergo ita inſtituatur.

$v =$ 7, 7, 2, 56 ; $\sin v = -\sin$ 57, 2, 56
$\cos = -$

$2v =$ 2, 14, 6 ; $\sin 2v = +\sin$ 74, 6
$\cos = +$

$r =$ 7, 7, 3
$s =$ 8, 27, 58 ; $\sin s = -\sin$ 87, 58
$r - s =$ 10, 9, 5 ; $\sin = -\sin$ 50, 55
$r + s =$ 4, 5, 1 ; $\sin = +\sin$ 54, 59
$\varphi - \pi =$ 6, 0, 11
$2\varphi - 2\pi =$ 0, 0, 22 ; $\sin = +\sin$ 0, 22
$r =$ 7, 7, 3
$2\varphi - 2\pi - r =$ 4, 23, 19 ; $\sin = +\sin$ 36, 41
$2\varphi - 2\pi - 2r =$ 9, 16, 16 ; $\sin = -\sin$ 73, 44

— 9,77995	— 9,7799	— 9,9021
— 4,24753	— 1,5111	— 0,7104
+ 4,02748	+ 1,2910n	+ 0,6125i
+ 9,9831	+ 9,9831	+ 9,4377
— 2,8561	— 0,4208	— 9,6201
— 2,8392	— 0,4039n	— 9,0578i
— 9,9997	— 9,8900	+ 9,9133
+ 2,8819	— 1,1761	— 2,2122
— 2,8816	+ 1,0661	— 2,1255
+ 7,8061	+ 9,7763	— 96, 0y
— 2,7896	+ 2,2355	
— 0,5957	+ 2,0118	

aeq. aff.	aeq. neg.	+ 19, 6n
+ 10653	— 691	— 2, 5n
+ 12	— 761	
+ 103	— 133	+ 4, 1i
+ 10768	— 4	— 0, 1i
— 1589	— 1589	
+ 9179		
+ 152, 59		
aeq. = + 2,32,59		

Long. ☾ med.	6,	4,	5,	0
aeq.	+ 2,	32,	59	
Long. ☽ calc.	6,	6,	37,	59
obſ.	6,	6,	35,	36

$\epsilon = +2', 23'' + m + 17, 1n + 4, 0i - 96, 0y$

§. 305. Eliplis decima tertiae medium obſeruatum eſt Pariſiis A. 1736 Sept. 19^d, 14^b, $59'$, $36''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera $\theta =$	5^s, 27°,21′, 39″,
Anomalia ſolis vera $s =$	2^l, 18, 43, 51
Longitudo lunae media . .	11, 27, 48, 53
Anomalia lunae media . .	0, 7, 25, 42
Anomalia lunae vera $v =$	0, 6, 40, 44
Longitudo nodi vera	5, 27, 15, 4
Diſtantia nodi a ſole . .	0, 0, 6, 35
aeq. pro long. lunae	— 4
Longitudo lunae obſeruata	11, 27, 21, 35

§. 306. Calculus ergo ita inſtituetur

$v = 0, 6, 40, 44$; $\sin v = + \sin 6, 40, 50$
$\cos v = +$
$2v = 0, 13, 21$; $\sin 2v = + \sin 13, 21$
$\cos 2v = +$
$r = 0, 6, 41$
$s = 2, 18, 44$; $\sin s = + \sin 88, 44$
$r - s = 9, 17, 57$; $\sin = - \sin 72, 3$
$r + s = 2, 25, 25$; $\sin = + \sin 85, 25$
$\Phi - \pi = 0, 0, 6$
$2\Phi - 2\pi = 0, 0, 12$; $\sin = + \sin 0, 12$
$r = 0, 6, 41$
$2\Phi - 2\pi - r = 11, 23, 31$; $\sin = - \sin 6, 29$
$2\Phi - 2\pi - 2r = 11, 16, 50$; $\sin = - \sin 13, 10$

+ 9, 06561	+ 9, 0656	+ 9, 9970
— 4, 24753	— 1, 5111	— 0, 7104
— 3, 31314	— 0, 5767n	— 0, 7074i
+ 9, 3634	+ 9, 3634	+ 9, 9881
— 2, 8561	— 0, 4208	— 9, 6201
— 2, 2195	— 9, 7842n	— 9, 6082i
+ 9, 9915	— 9, 9783	+ 9, 9986
+ 2, 8819	— 1, 1751	— 2, 2122
+ 2, 8734	+ 1, 1534	— 2, 2108
+ 7, 5429	— 9, 0527	— 22, 6y
— 2, 7896	+ 2, 2355	
— 0, 3325	— 1, 2882	

aeq. aff.	aeq. neg.	
+ 747	— 2057	— 3, 8n
+ 14	— 166	— 0, 6n
+ 761	— 2	— 5, 1i
— 2406	— 19	— 0, 4i
— 1645	— 2406	
— 27′,25″	aequatio	

Long. ☽ med. 11, 27, 48, 53
aeq. — 27, 25
Long. ☽ calc. 11, 27, 21, 28
Long. ☽ obſ. 11, 27, 21, 35

$\epsilon = - 0,7'' + m - 4,4n - 5,5i - 22,6y$

§. 307.

§. 307. Ex his ergo tredecim eclipſibus nacti ſumus aequationes, ex quibus cum tabularum, quibus ſum vſus, correctiones, tum verus valor aequationis ab angulo $2\Phi - 2\pi - 2r$ pendentis definiri debebit:

Aequationes autem inde ortae ſunt ſequentes.

I. $0 = +156'' + m - 29,1n - 2,8i - 99,8y$
II. $0 = -27 + m + 32,4n - 1,3i + 79,2y$
III. $0 = +30 + m + 32,4n + 0,2i + 25,6y$
IV. $0 = +12 + m - 6,2n - 5,5i - 30,7y$
V. $0 = -160 + m + 23,4n - 4,0i + 99,5y$
VI. $0 = -102 + m - 16,3n + 4,1i + 98,2y$
VII. $0 = +82 + m - 3,9n + 4,7i + 2,1y$
VIII. $0 = -16 + m - 30,6n + 1,4i + 39,9y$
IX. $0 = +67 + m + 30,4n + 1,6i - 49,2y$
X. $0 = -47 + m - 22,2n + 3,4i - 98,1y$
XI. $0 = +9 + m + 23,9n + 3,1i - 93,6y$
XII. $0 = +143 + m + 17,1n + 4,0i - 96,0y$
XIII. $0 = -7 + m - 4,4n - 5,5i - 22,6y$

§. 308. Hic ſtatim commode euenit, vt errores calculi ab obſeruationibus infra tria minuta prima ſubſiſtant, qui autem infra ſesquiminutum primum deprimuntur, ſimul ac litterae y valor tribuitur vnitati fere aequalis. Hinc que ergo cognoſcimus valorem ipſius y, quem quinario maiorem inueneramus, merito nobis fuiſſe ſuſpectum, cum iam perſpiciamus, eum vnitatem ſuperare non poſſe. Quamobrem ponamus $y = 1$, ſeu in formula noſtra pro longitudine lunae ſcribamus terminum $100''$ ſin $(2\Phi - 2\pi - 2r)$. Quod autem ad litteras m, n et i attinet, tentanti mox patebit, quoscunque ipſis valores tribuamus, errores inde non admodum poſſe diminui; interim tamen decem circiter minutis ſecundis diminuentur, ſi ponatur $y = \frac{4}{5}$; $n = \frac{1}{2}$; $i = -3$ et $m = -4$; quo facto errores vix vnum minutum primum ſuperabunt.

CAPUT XVIII.

CONSTITUTIO ELEMENTORUM PRO TABULIS LUNARIBUS.

§. 309.

Tabulae autem, quibus in praec[illegible] calculo sum vsus, praebent pro meridiano Parisino ad epocham 1701 seu ad meridiem diei v[illegible]imi anni 1700 tempore medio

Longitudinem Lunae mediam $5^s, 20^\circ, 19', 47''$
et Anomaliam Lunae mediam $6, 13, 26, 51$

Hinc accuratius habebimus haec elementa pro eodem tempore eodemque loco scilicet

Longitudinem Lunae mediam $5^s, 20^\circ, 19', 43''$
Anomaliam Lunae mediam $6, 13, 24, 0$
unde Longitudo Apogei $11, 6, 55, 43$

§. 310. Si haec elementa comparemus cum Tabulis astronomicis Cel. Cassini et Monnierii, reperiemus pro eodem tempore et loco

	Cassini	Monnier
Long. mediam Lunae	5, 20, 18, 19	5, 20, 19, 28
Anom. mediam Lunae	6, 13, 10, 48	6, 13, 13, 2
Long. Apogei	11, 7, 7, 27	11, 7, 6, 26

Hic quidem longitudo media satis conuenit cum ea, quam ex obseruationibus concluſimus; verum anomalia media inuenta superat Cassinianam $13'$, $12''$, Monnierianam autem $11'$, quod discrimen satis est notabile.

Verum

Verum si perpendamus motum lunae a tam multis variisque inaequalitatibus perturbari, mirum sane non est, anomaliam mediam per solas obseruationes accuratius definiri non potuisse; praesertim cum error $15'$ in anomalia media commissus in loco lunae ad summum errorem $1', 45''$ gignere valeat.

§. 311. Excentricitatem autem orbitae lunaris, quam statueram $= 0,0545$ iam $\frac{1}{2000}$ vel $0,00005$ augeri oportet, ita vt nunc sit excentricitatis valor $k = 0,05455$; qui a supra assumto tam parum discrepat, vt anomalia vera inde ex media collecta pro satis exacta haberi possit: aequationes autem ab excentricitate pendentes aliquod augmentum capient, quod nunc quidem diligentius definiri oportet. Primum ergo formulam pro longitudine lunae inuentam hinc corrigamus; deinde vero etiam formulas pro distantia lunae a terra, pro eius motu momentaneo, et pro loco nodi veraque inclinatione orbitae lunaris ad eclipticam hinc euoluamus.

§. 312. Ante omnia autem oportebit formulam exhibere, cuius ope ex data quauis anomalia lunae media p elicere liceat, conuenientem anomaliam veram r. Ac substituto quidem pro k vero eius valore nunc inuento, coefficientibusque in minuta secunda conuersis, formula supra (§. 306) exhibita sequentem induet formam:

$$r = p - \underset{4,352086}{22495''} \sin p + \underset{2,884229}{766''} \sin 2p - \underset{1,55630}{36''} \sin 3p$$

Huius ergo formulae ope haud difficulter tabula computabitur, quae ad singulos anomaliae mediae gradus exhibeat valores anomaliae verae.

§. 313. Inuenta autem anomalia vera r, si habeatur quoque anomalia vera solis s, vna cum angulo η et longitudinibus Φ, θ, π saltem proxime, formula longitudinem veram Φ datae mediae ξ respondentem exhibens, sequenti modo habebitur expressa:

$\Phi = \xi$

	log. coeff.	
$-$ 22466″ sin r	4,351535	I
$-$ 462 sin $2r$	2,66456	
$-$ 11 sin $3r$	1,0518	
$+$ 701 sin s	2,84572	II
$+$ 4 sin $2s$	0,602	
$+$ 141 sin $(r-s)$	2,1492	III
$-$ 118 sin $(r+s)$	2,0719	IV
$-$ 175 sin η	2,2430	V
$+$ 2115 sin 2η	3,32531	
$+$ 4 sin 3η	0,602	
$-$ 8 sin 4η	0,903	
$+$ 59 sin $(\eta-r)$	1,7708	VI
$+$ 352 sin $(2\eta-2r)$	2,5465	
$-$ 2729 sin $(2\eta-r)$	3,67477	VII
$-$ 93 sin $(4\eta-2r)$	1,9685	
$+$ 56 sin $(2\eta+r)$	1,7482	VIII
$+$ 59 sin $(4\eta-r)$	1,7708	IX
$-$ 49 sin $(\eta+s)$	1,6902	X
$-$ 76 sin $(2\eta-s)$	1,8808	XI
$-$ 57 sin $(2\eta+s)$	1,7559	XII
$+$ 154 sin $(2\eta-r+s)$	2,1875	XIII

$+$ 45 sin $(2\eta-r-s)$	1,6532	XIV
$-$ 411 sin $(2\Phi-2\pi)$	2,6138	XV
$-$ 205 sin $(2\theta-2\pi)$	2,3117	XVI
$-$ 6 sin $(4\theta-4\pi)$	0,778	
$+$ 187 sin $(2\Phi-2\pi-r)$	2,2718	XVII
$+$ 80 sin $(2\Phi-2\pi-2r)$	1,9031	XVIII
$-$ 15 sin $(2\theta-2\pi-r)$	1,176	XIX
$-$ 10 sin $(2\theta-2\pi+r)$	1,000	XX

§. 314. Inaequalitates has ita disposui, vt eas, quae vna tabula comprehendi possunt, coniunctim exposuerim, quo facilius calculus expediri queat. Hinc igitur patet omissis iis inaequalitatibus, quae 10″ non superant, locum lunae per viginti inaequalitates corrigi debere, antequam vera eius longitudo obtineatur.

§. 315. Haec autem expressio adhuc isto defectu laborat, quod pleraeque inaequalitates ipsam lunae longitudinem veram Φ, quae tamen demum quaeritur, involuant, ideoque calculus, cum longitudo lunae etiamnunc est incognita, commode expediri non possit. Quoniam tamen sufficit longitudinem lunae proxime tantum nosse, cum longitudo media per quatuor priores inaequalitates fuerit correcta, ea pro sequentibus inaequalitatibus loco longitudinis verae vsurpari poterit, sicque tandem longitudo lunae multo exactior reperietur. Quo facto si accuratior desideretur, omnes inaequalitates post 4 priores denuo ad calculum reuocari conueniet, iisque euolutis longitudo lunae vera prodibit, quae nulla amplius correctione indigebit. Interim

tamen ne calculum per ſe ſatis taedioſum bis repetere opus ſit, non difficulter hanc expreſſionem ita transformare licet, vt locus lunae per quatuor tantum priores inaequalitates correctus ſine errore in ſequentibus loco Φ adhiberi poſſit.

§. 316. Cum autem longitudo lunae iam per obſeruationes fuerit cognita, haec expreſſio ſine vlla immutatione ad calculum accommodabitur, vt hoc modo conſenſus theoriae cum veritate exploretur. In inaequalitatibus enim determinandis pro littera Φ vbique longitudo lunae obſeruata introducetur, calculoque peracto patebit, quantum locus lunae per calculum definitus etiamnunc diſcrepet ab eius loco vero obſeruato. Atque ſi hoc modo plurimae obſeruationes calculo ſubiiciantur, ex aberrationibus a veritate non ſolum elementa, quibus haec formula innititur, accuratius definire licebit, ſed etiam inaequalitates, quae nondum ſatis certae videntur, inde emendari poterunt. Quin etiam nouae inaequalitates, quas per Theoriam determinare non licuerat, hoc modo forte certius colligi poterunt.

§. 317. Antequam autem huiusmodi calculi ſpecimen exhiberi queat, neceſſe eſt vt aequationem pro loco nodi vero inueniendo ad calculum accommodemus. Formulae autem ſupra (219) exhibitae, ſi pro r ſubſtituamus valorem inuentum $r = p - 2k \sin r - \frac{3}{4} kk \sin 2r$, pars Conſt. $- 0,004053\,p$ indicabit longitudinem nodi mediam. Hincque longitudo nodi vera erit

$\pi =$

repetere
transfor-
priores
ius loco

per ob-
vlla im-
c modo
in inae-
vbique
ue per-
im defi-
ſeruato.
ilo ſub-
im ele-
us defi-
ium ſa-
Quin
deter-
colligi

uli ſpe-
tionem
ommo-
, ſi pro
· ſin r
dicabit
o nodi
$\pi =$

			Log.coeff.
$\pi =$ Long.med.	$-$	$107''\sin r$	2, 0294
	$-$	$6 \sin 2r$	0, 778
	$+$	$551 \sin s$	2, 7411
	$-$	$453 \sin 2\eta$	2, 6561
	$-$	$129 \sin (2\eta - r)$	2, 1106
	$-$	$33 \sin (2\eta + r)$	1, 518
	$+$	$55 \sin (2\eta - 2r)$	1, 740
	$+$	$420 \sin (2\Phi - 2\pi)$	2, 6232
	$+$	$98 \sin (2\Phi - 2\pi - r)$	1, 991
	$+$	$30 \sin (2\Phi - 2\pi + r)$	1, 477
	$+$	$235 \sin (2\Phi - 2\pi - 2r)$	2, 3711
	$+$	$5426 \sin (2\theta - 2\pi)$	3, 73448
	$+$	$75 \sin (4\theta - 4\pi)$	1, 875
	$-$	$53 \sin (2\theta - 2\pi - r)$	1, 724
	$+$	$53 \sin (2\theta - 2\pi + r)$	1, 724
	$-$	$90 \sin (2\theta - 2\pi - s)$	1, 954
	$-$	$32 \sin (2\theta - 2\pi + s)$	1, 505

§. 318. In hoc calculo plerasque inaequalitates omittere licet, ſiquidem tantum longitudinem lunae inueſtigare ſit propoſitum: manifeſtum enim eſt, etiamſi in loco nodi error plurium minutorum primorum committatur, inde vix errorem aliquot minutorum ſecundorum in longitudinem lunae redundare. Quodſi vero eclipſis cuiuspiam omnia phaenomena diligenter definire velimus, tum locum nodi exactiſſime cognitum eſſe oportet. Praeterea vero pro latitudine aſſignanda vera inclinatio orbitae lunaris ad eclipticam ex media accuratiſſime erit definienda ope huius formulae:

$\varrho =$

		Log coeff.
$\ell = \epsilon$	$- 2''\cos r$	0,30
	$- 48 \cos 2\eta$	1,681
	$+ 11 \cos(2\eta - r)$	1,041
	$+ 3 \cos(2\eta + r)$	0,48
	$+ 36 \cos(2\Phi - 2\pi)$	1,556
	$+ 9 \cos(2\Phi - 2\pi - r)$	0,95
	$+ 3 \cos(2\Phi - 2\pi + r)$	0,48
	$+ 23 \cos(2\Phi - 2\pi - 2r)$	1,362
	$+ 484 \cos(2\theta - 2\pi)$	2,6848
	$+ 9 \cos(4\theta - 4\pi)$	0,95
	$- 5 \cos(2\theta - 2\pi - r)$	0,70
	$+ 5 \cos(2\theta - 2\pi - + r)$	0,70
	$- 7 \cos(2\theta - 2\pi - s)$	0,84
	$- 3 \cos(2\theta - 2\pi + s)$	0,48

Tabula autem pro diſtantia lunae a terra, vnde eius parallaxis et diameter apparens definiatur, ex formulis ſupra exhibitis facile conſtruetur.

ADDI-

ADDITAMENTUM

CONTINENS ALIAS METHODOS INUESTIGANDI MOTUS LUNAE INAEQUALITATES.

Qui methodum ante deſcriptam accuratius euoluerit, eam quidem in ſe ſpectatam ſatis bonam atque plerisque lunae inaequalitatibus definiendis aptam deprehendet; interim tamen fateri cogor, eam non ſolum maxime eſſe operoſam, ſed etiam ita comparatam, vt plures inaequalitates, quae tamen motum lunae imprimis afficere videntur, non ſatis exacte exhibeat, et quaſi in dubio relinquat. Cauſa huius incertitudinis manifeſto in hoc eſt ſita, quod omnes inaequalitates ita inter ſe ſunt connexae, vt nullius valor verus accurate definiri poſſit, quin ſimul reliquae inaequalitates omnes fuerint cognitae. Cum igitur eiusmodi methodo approximandi fim vſus, vt primo quasdam inaequalitates tanquam cognitas aſſumſerim, ex quibus deinceps reliquas definiuerim, probe notandum eſt ab his inuentis iterum priores, quae erant aſſumtae, leuem quandam mutationem pati; quae ſi ſtatim ab initio nota fuiſſet, etiam reliquarum valores aliquantillum mutati prodiiſſent: at quaedam inaequalitates adeo ſunt lubricae, vt facta vel minima mutatione in iis, a quibus pendent, inde non exiguam alterationem trahant. Huc imprimis pertinet motus apogei, cuius inueſtigatio omnes omnino inaequa-