§. 172. Hae autem singulae inaequalitates ad numerum minutorum secundorum reductae dabunt:

			log. coeff.
$\phi =$ Praec.	$+701''$,	$1\ \sin s$	2,845780
	-75,	$8\ \sin(2\eta - s)$	1,879971
	-56,	$7\ \sin(2\eta + s)$	1,753736
	$+3$,	$8\ \sin 2s$	0,585551
	$+1$,	$4\ \sin(2\eta - 2s)$	0,132862
	$+0$,	$5\ \sin(2\eta + 2s)$	9,689472
	$+140$,	$9\ \sin(r - s)$	2,148800
	-116,	$7\ \sin(r + s)$	2,067000
	$+153$,	$7\ \sin(2\eta - r + s)$	2,186530
	-2,	$7\ \sin(2\eta + r - s)$	0,426300
	$+45$,	$2\ \sin(2\eta - r - s)$	1,655200
	$+1$,	$2\ \sin(2\eta + r + s)$	0,064600

Hic scilicet et inaequalitates, quas in capite praecedente inuenimus, et istas in hoc capite eruras simul sum complexus, vt coniunctim conspectui exponerentur.

CAPUT

CAPUT XI.

INUESTIGATIO INAEQUALITATUM LUNAE A PARALLAXI SOLIS PENDENTIUM.

§. 173.

Iam in formulis nostris primariis ad eos quoque terminos progrediamur, qui littera v sunt affecti, et quoniam est $1:v$ vt distantia Solis media ad distantiam Lunae mediam a Terra, erit $1:v$ vt parallaxis Lunae media ad parallaxin solis: ex quo inaequalitates Lunae, quae hinc oriuntur, a parallaxi solis pendere dicuntur. Quoniam vero valor ipsius v est valde paruus, quippe $\frac{1}{280}$ propemodum, alios terminos non contemplabimur, nisi qui per v ac per vk et ve sunt multiplicati, propterea quod magis compositi fiant minimi.

§. 174. Ex terminis ergo iam inuentis hic retineamus eos, qui sunt alicuius momenti, et cum iis nouos determinandos coniungamus; sit ergo:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{C}k\cos r + \mathfrak{D}k\cos(2\eta - r) + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta - s)$$
$$+ \mathfrak{E}k\cos(2\eta + r) \qquad + \mathfrak{R}e\cos(2\eta + s)$$
$$+ \mathfrak{F}v\cos v + \mathfrak{H}vk\cos(\eta - r) + \mathfrak{K}ve\cos(\eta - s)$$
$$+ \mathfrak{G}v\cos 3\eta + \mathfrak{I}vk\cos(\eta + r) + \mathfrak{L}ve\cos(\eta + s)$$

$$v = A\cos 2\eta \;.\;.\; + Dk\cos(2\eta - r) + Pe\cos s + Qe\cos(2\eta - s)$$
$$+ Ek\cos(2\eta + r) \qquad + Re\cos(2\eta + s)$$
$$+ Fv\cos\eta + Hvk\cos(\eta - r) + Kve\cos(\eta - s)$$
$$+ Gv\cos 3\eta + Jvk\cos(\eta + r) + Lve\cos(\eta + s)$$

Non

Non difficulter enim praeuidere licet, terminos, qui angulos r et s cum angulo 3η habeant coniunctos, fore tam exiguos, vt ſine errore praetermitti queant.

§. 175. Quodſi iam retentis litterarum §. 153. inductarum valoribus, praeterea ponamus:

$$f' = \frac{2\varkappa F + \mathfrak{F}}{nn}\ ;\ g' = \frac{2\varkappa G + \mathfrak{G}}{nn}\ ;\ h' = \frac{2\varkappa H + \mathfrak{H}}{nn}$$

$$i' = \frac{2\varkappa J + \mathfrak{J}}{nn}\ ;\ k' = \frac{2\varkappa K + \mathfrak{K}}{nn}\ ;\ l' = \frac{2\varkappa L + \mathfrak{L}}{nn}$$

habebimus:

$$\frac{d\Phi}{dr} = \text{Praec.} - a'\cos 2\eta - \frac{\mathfrak{C}}{nn}k\cos r - d'k\cos(2\eta - r) - p'e\cos s - q'e\cos(2\eta - s)$$
$$- e'k\cos(2\eta + r) \qquad - r'e\cos(2\eta + s)$$
$$- f'v\cos\eta - h'vk\cos(\eta - r) - k've\cos(\eta - s)$$
$$- g'v\cos 3\eta - i'vk\cos(\eta + r) - l've\cos(\eta + s)$$

atque

$$\frac{d\eta}{dr} = a - a'\cos 2\eta - c'k\cos r - d'k\cos(2\eta - r) + \left(\frac{2}{n} - p'\right)e\cos s - q'e\cos(2\eta - s)$$
$$- e'k\cos(2\eta + r) \qquad - r'e\cos(2\eta + s)$$
$$- f'v\cos\eta - h'vk\cos(\eta - r) - k've\cos(\eta - s)$$
$$- g'v\cos\eta - i'vk\cos(\eta + s) - l've\cos(\eta + s)$$

§. 176. Jam vero pro his terminis ab v pendentibus ſequentes colligemus aequationes.

$$R = v\sin\eta\left(\tfrac{3}{8} + \frac{3F - 3G}{2nn}\right)$$
$$v\sin 3\eta\left(\tfrac{15}{8} + \frac{3F}{2nn}\right)$$

$v\,k$

$$vk\sin(\eta - r)\left(\tfrac{15}{16} + \frac{3J}{2nn} + \frac{3F}{nn} - \frac{3G}{nn}\right)$$
$$vk\sin(\eta + r)\left(\tfrac{15}{16} + \frac{3H}{2nn} - \frac{3G}{nn} + \frac{3F}{nn}\right)$$
$$ve\sin(\eta - s)\left(-\tfrac{3}{4} + \frac{3L}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn}\right)$$
$$ve\sin(\eta + s)\left(-\tfrac{3}{4} + \frac{3K}{2nn} + \frac{9G}{4nn} - \frac{9F}{4nn}\right)$$

$$\frac{ddv}{dr^2} =$$

$$v\cos\eta \quad \left\{\tfrac{9}{8} - 6F + \frac{3F}{4nn} + \frac{3G}{4nn} - 2\varkappa\mathfrak{F} + \frac{\mathfrak{A}\mathfrak{F}}{nn} + \frac{\mathfrak{A}\mathfrak{G}}{nn}\right.$$

$$v\cos 3\eta \quad \left\{\tfrac{15}{8} - 6G + \frac{3F}{4nn} - 2\varkappa\mathfrak{G} + \frac{\mathfrak{A}\mathfrak{F}}{nn}\right.$$

$$vk\cos(\eta - r)\left\{\begin{aligned}&\tfrac{45}{16} - 6H + \tfrac{3}{2}bF + \frac{3J}{4nn} + \frac{3F}{2nn} + \frac{3G}{2nn} - 2\varkappa\mathfrak{H}\\ &+ \frac{\mathfrak{A}\mathfrak{J}}{nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} + \frac{\mathfrak{D}\mathfrak{F}}{nn} + \frac{\mathfrak{C}\mathfrak{G}}{nn}\end{aligned}\right.$$

$$vk\cos(\eta + r)\left\{\begin{aligned}&\tfrac{45}{16} - 6J + \tfrac{3}{2}bF + \frac{3H}{4nn} + \frac{3G}{2nn} + \frac{3F}{2nn} - 2\varkappa\mathfrak{J}\\ &+ \frac{\mathfrak{A}\mathfrak{H}}{nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} + \frac{\mathfrak{D}\mathfrak{G}}{nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn}\end{aligned}\right.$$

$v e\cos$

$$v\,e\cos(\eta-s)\begin{cases}-\frac{9}{4}-\mathfrak{E}K+\frac{3L}{4nn}-\frac{3F}{4nn}-\frac{9F}{8nn}-\frac{9G}{8nn}-2\,\kappa\,\mathfrak{K}\\ +\frac{\mathfrak{A}\mathfrak{L}}{nn}+\frac{\mathfrak{F}\mathfrak{P}}{nn}+\frac{\mathfrak{F}\mathfrak{Q}}{nn}+\frac{\mathfrak{G}\mathfrak{R}}{nn}\end{cases}$$

$$v\,e\cos(\eta+s)\begin{cases}-\frac{9}{4}-\mathfrak{E}L+\frac{3K}{4nn}-\frac{3F}{4nn}-\frac{9F}{8nn}-\frac{9G}{8nn}-2\,\kappa\,\mathfrak{L}\\ +\frac{\mathfrak{A}\mathfrak{K}}{nn}+\frac{\mathfrak{F}\mathfrak{P}}{nn}+\frac{\mathfrak{F}\mathfrak{R}}{nn}+\frac{\mathfrak{G}\mathfrak{Q}}{nn}\end{cases}$$

sequentes partes seorsim exponamus:

$$v\cos\eta\begin{cases}+\frac{3\mathfrak{A}F}{nn}+\frac{3\mathfrak{A}G}{nn}+\frac{3A\mathfrak{F}}{nn}+\frac{3A\mathfrak{G}}{nn}+\frac{3AF}{nn}+\frac{3AG}{nn}\\ +\frac{9A}{8nn}+\frac{15A}{8nn}\end{cases}$$

$$v\cos 3\eta\begin{cases}+\frac{3\mathfrak{A}F}{nn}+\frac{3\mathfrak{A}\mathfrak{F}}{nn}+\frac{3AF}{nn}+\frac{9A}{8nn}\end{cases}$$

$$v\,k\cos(\eta-r)\begin{cases}\frac{3\mathfrak{A}\mathfrak{J}}{nn}+\frac{3\mathfrak{C}F}{nn}+\frac{3\mathfrak{D}F}{nn}+\frac{3\mathfrak{C}G}{nn}+\frac{3A\mathfrak{J}}{nn}+\frac{3D\mathfrak{F}}{nn}\\ +\frac{3E\mathfrak{G}}{nn}+\frac{3AJ}{nn}+\frac{3DF}{nn}+\frac{3EG}{nn}-\frac{3AF}{nn}-\frac{3AG}{nn}\\ +\frac{9D}{8nn}+\frac{15E}{8nn}\end{cases}$$

$$v\,k\cos(\eta+r)\begin{cases}\frac{3\mathfrak{A}\mathfrak{H}}{nn}+\frac{3\mathfrak{C}F}{nn}+\frac{3\mathfrak{D}\mathfrak{G}}{nn}+\frac{3\mathfrak{E}F}{nn}+\frac{3A\mathfrak{H}}{nn}\\ +\frac{3D\mathfrak{G}}{nn}+\frac{3E\mathfrak{F}}{nn}+\frac{3AH}{nn}+\frac{3DG}{nn}+\frac{3EF}{nn}\\ -\frac{3AF}{2nn}-\frac{3AG}{2nn}+\frac{9E}{8nn}+\frac{15D}{8nn}\end{cases}$$

$v\,e\cos$

$$v\,e\cos(\eta-s)\begin{cases}\frac{3\mathfrak{A}L}{nn}+\frac{3\mathfrak{P}F}{nn}+\frac{3\mathfrak{Q}F}{nn}+\frac{3\mathfrak{R}G}{nn}+\frac{3A\mathfrak{L}}{nn}\\ +\frac{3P\mathfrak{F}}{nn}+\frac{3Q\mathfrak{F}}{nn}+\frac{3R\mathfrak{G}}{nn}+\frac{3AL}{nn}+\frac{3PF}{nn}+\frac{3QF}{nn}\\ +\frac{3RG}{nn}+\frac{9P}{8nn}+\frac{9Q}{8nn}+\frac{15R}{8nn}\end{cases}$$

$$v\,e\cos(\eta+s)\begin{cases}\frac{3\mathfrak{A}K}{nn}+\frac{3\mathfrak{P}F}{nn}+\frac{3\mathfrak{Q}G}{nn}+\frac{3\mathfrak{R}F}{nn}+\frac{3A\mathfrak{K}}{nn}+\frac{3PF}{nn}\\ +\frac{3Q\mathfrak{G}}{nn}+\frac{3R\mathfrak{F}}{nn}+\frac{3AK}{nn}+\frac{3PF}{nn}+\frac{3QF}{nn}+\frac{3QG}{nn}\\ +\frac{9P}{8nn}+\frac{9R}{8nn}+\frac{15Q}{8nn}\end{cases}$$

§. 177. Verum differentiando nanciscemur

$$R =$$

$$v\sin\eta\ \{\mathfrak{A}f' - \mathfrak{A}g' - \alpha\mathfrak{F} - \tfrac{1}{2}\mathfrak{F}a' + \tfrac{3}{2}\mathfrak{G}a'$$

$$v\sin 3\eta\ \{\mathfrak{A}f' + \tfrac{1}{2}\mathfrak{F}a' - 3\alpha\mathfrak{G}$$

$$v\,k\sin(\eta-r)\{\mathfrak{A}i' + \mathfrak{D}f' - \mathfrak{E}g' + \tfrac{1}{2}\mathfrak{F}c' - \tfrac{1}{2}\mathfrak{F}d' + \tfrac{3}{2}\mathfrak{G}c' - (\alpha-1)\mathfrak{H} - \tfrac{1}{2}\mathfrak{J}a'$$

$$v\,k\sin(\eta+r)\begin{cases}\mathfrak{A}h' - \mathfrak{D}g' + \mathfrak{E}f' + \tfrac{1}{2}\mathfrak{F}c' - \tfrac{1}{2}\mathfrak{F}e' + \tfrac{3}{2}\mathfrak{G}d' - \tfrac{1}{2}\mathfrak{H}a' \\ \qquad - (\alpha+1)\mathfrak{J}\end{cases}$$

$$v\,e\sin(\eta-s)\begin{cases}+\mathfrak{A}l' + \mathfrak{Q}f' - \mathfrak{R}g' - \tfrac{1}{2}\left(\frac{2}{n} - p'\right)\mathfrak{F} - \tfrac{1}{2}\mathfrak{F}q' + \tfrac{3}{2}\mathfrak{G}r' \\ \qquad - \left(\alpha - \frac{1}{n}\right)\mathfrak{K} - \tfrac{1}{2}\mathfrak{L}a'\end{cases}$$

$$v\,e\sin(\eta+s)\begin{cases}+\mathfrak{A}k' - \mathfrak{Q}g' + \mathfrak{R}f' - \tfrac{1}{2}\left(\frac{2}{n} - p'\right)\mathfrak{F} - \tfrac{1}{2}\mathfrak{F}r' + \tfrac{3}{2}\mathfrak{G}q' \\ \qquad - \left(\alpha + \frac{1}{n}\right)\mathfrak{L} - \tfrac{1}{2}\mathfrak{K}a'\end{cases}$$

 Deinde

Deinde posito

$$F' = \alpha F - A(f' - g') + \tfrac{1}{2}F\alpha' - \tfrac{3}{2}G\alpha'$$
$$G' = 3\alpha G - Af' - \tfrac{1}{2}F\alpha'$$
$$H' = (\alpha-1)H - Ai' - Df' + Eg' - \tfrac{1}{2}Fc' + \tfrac{1}{2}Fd' - \tfrac{3}{2}Gc' + \tfrac{1}{2}J\alpha'$$
$$J' = (\alpha+1)J - Ab' + Dg' - Ef' - \tfrac{1}{2}Fc' + \tfrac{1}{2}Fd' - \tfrac{3}{2}Gd' + \tfrac{1}{2}H\alpha'$$
$$K' = (\alpha-1)K - Al' - Qf' + Rg' + \tfrac{1}{2}F(\tfrac{2}{n} - p') + \tfrac{1}{2}Fq' - \tfrac{3}{2}Gr' + \tfrac{1}{2}L\alpha'$$
$$L' = (\alpha+1)L - Ak' + Qg' - Rf' + \tfrac{1}{2}F(\tfrac{2}{n} - p') + \tfrac{1}{2}Fr' - \tfrac{3}{2}Gq' + \tfrac{1}{2}K\alpha'$$

erit

$$\frac{dv}{dr} = -A'\sin 2\eta - D'k\sin(2\eta - r) - P'e\sin s - Q'e\sin(2\eta - s)$$
$$- E'k\sin(2\eta + r) \qquad - R'e\sin(2\eta + s)$$
$$- F'v\sin\eta - H'vk\sin(\eta - r) - K've\sin(\eta - s)$$
$$- G'v\sin 3\eta - J'vk\sin(\eta + r) - L've\sin(\eta + s)$$

§. 178. Hinc iam denuo differentiando consequemur:

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ v\cos\eta \quad [+A'f' + A'g' - \alpha F' + \tfrac{1}{2}F'\alpha' + \tfrac{3}{2}G'\alpha'$$
$$+ v\cos 3\eta \quad [+A'f' + \tfrac{1}{2}F'\alpha' - 3\alpha G'$$
$$+ vk\cos(\eta - r)\begin{cases} +A'i' + D'f' + E'g' + \tfrac{1}{2}F'c' + \tfrac{1}{2}F'd' + \tfrac{3}{2}G'c' \\ \qquad - (\alpha-1)H' + \tfrac{1}{2}J'\alpha' \end{cases}$$
$$+ vk\cos(\eta + r)\begin{cases} +A'b' + E'f' + D'g' + \tfrac{1}{2}F'c' + \tfrac{1}{2}F'c' + \tfrac{3}{2}G'd' \\ \qquad + \tfrac{1}{2}H'\alpha' - (\alpha+1)J' \end{cases}$$
$$+ ve\cos(\eta - s)\begin{cases} +A'l' + Q'f' + R'g' - \tfrac{1}{2}(\tfrac{2}{n} - p')F' + \tfrac{1}{2}F'q' + \tfrac{3}{2}G'r' \\ \qquad - (\alpha - \tfrac{1}{n})K' + \tfrac{1}{2}L'\alpha' \end{cases}$$
$$+$$

+½Ja′ +½Ha′ +½La′ +½Ka′ (2η−s) 2η+s) (η−s) (η+s) eque- a′ G′c′ J′a′ G′d′ 1) J′ G′r′ L′a′ +

$$+ ve\cos(\eta + s)\begin{cases} +A'k' + R'f' + Q'g' - \tfrac{1}{2}(\tfrac{2}{n} - p')F' + \tfrac{1}{2}F'r' + \tfrac{3}{2}G'q' \\ \qquad + \tfrac{1}{2}K'\alpha' - (\alpha + \tfrac{1}{n})L' \end{cases}$$

qui valores cum antecedentibus comparari debent, vt inde valores coefficientium eliciantur.

§. 179. Sumamus primo duos valores ab initio positos, quoniam hi a sequentibus non pendent, atque habebimus,

$$0 = \alpha\mathfrak{F} - \mathfrak{A}f' + \mathfrak{A}g' + \tfrac{1}{2}\mathfrak{F}\alpha' - \tfrac{3}{2}\mathfrak{G}\alpha' + \tfrac{3}{8} + \frac{3F - 3G}{2nn}$$
$$0 = 3\alpha\mathfrak{G} - \mathfrak{A}f' - \tfrac{1}{2}\mathfrak{F}\alpha' + \tfrac{1}{8} + \frac{3F}{2nn}$$
$$0 = \alpha F' - A'f' - A'g' - \tfrac{1}{2}F'\alpha' - \tfrac{3}{2}G'\alpha' + \tfrac{9}{8} - 6F - 2\varkappa\mathfrak{F}$$
$$+ \frac{3(F+G)}{4nn} + \frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{F}+\mathfrak{G}) + \frac{(3A+3A)}{nn}(F+G) + \frac{3A}{nn}$$
$$0 = 3\alpha G' - A'f' - \tfrac{1}{2}F'\alpha' + \tfrac{1}{8} - 6G - 2\varkappa G$$
$$+ \frac{3F}{4nn} + \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{F} + \frac{(3\mathfrak{A}+3A)}{nn}F + \frac{9A}{8nn}$$

et $F' = \alpha F - A(f' - g') + \tfrac{1}{2}F\alpha' - \tfrac{3}{2}G\alpha'$

$G' = 3\alpha G - Af' - \tfrac{1}{2}F\alpha'$

at est $f' = \dfrac{2\varkappa F + \mathfrak{F}}{nn}$ et $g' = \dfrac{2\varkappa G + \mathfrak{G}}{nn}$.

Hic igitur primum litterarum, quae sunt cognitae, valores in numeris substituantur, eritque

 $F' =$

$F' = 0,93905\,F + 0,00753\,\mathfrak{F} + 0,01443\,G - 0,00753\,\mathfrak{G}$

$G' = 2,80121\,G + 0,02505\,F + 0,00753\,\mathfrak{F}$

$0 = 0,92847\,\mathfrak{F} + 0,01776\,F - 0,01776\,G - 0,02501\,\mathfrak{G} + 0,37500$

$0 = 2,80121\,\mathfrak{G} + 0,01447\,\mathfrak{F} + 0,01776\,F + 1,87500$

hincque

$\mathfrak{F} = -0,01930\,F + 0,01913\,G - 0,42145$

$\mathfrak{G} = -0,00623\,F - 0,00010\,G - 0,66717$

§. 180. Inde porro colligemus

$F' = 0,93895\,F + 0,01457\,G + 0,00185$

$G' = 0,02491\,F + 2,80135\,G - 0,00317$

$f' = 0,01138\,F + 0,00011\,G - 0,00240$

$g' = -0,00004\,F + 0,01148\,G - 0,00380$

quibus valoribus substitutis peruenimus ad has aequationes:

$0,09337\,F = +0,05421\,G + 1,96867$

$6,83135\,G = -0,08830\,F - 3,20944$

vnde fit:

$F = 20,65700$	. . .	$l\,F = 1,315067$
$G = -0,73681$	. . .	$l{-}G = 9,867356$
$\mathfrak{F} = -0,83418$	. . .	$l{-}\mathfrak{F} = 9,921260$
$\mathfrak{G} = -0,79580$	. . .	$l{-}\mathfrak{G} = 9,900804$
$F' = +19,38712$	. . .	$l\,F' = 1,287512$
$G' = -1,55266$	. . .	$l{-}G' = 0,191075$
$f' = +0,23259$	. . .	$l\,f' = 9,366501$
$g' = -0,01309$	. . .	$l{-}g' = 8,116940$

0753 𝔊
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aequa-
181.

§. 181. His valoribus, qui ad inaequalitates absolutas pertinent, expeditis, progrediamur ad eos, qui ab excentricitate orbitae lunaris pendent, ac his aequationibus continentur:

$$(\alpha-1)\,\mathfrak{H} - \mathfrak{A}i' + \tfrac{1}{2}\mathfrak{J}a' - \mathfrak{D}f' + \mathfrak{E}g' - \tfrac{1}{2}\mathfrak{F}(c'-d') - \tfrac{3}{2}\mathfrak{G}e' + \tfrac{15}{16} + \frac{3J}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$(\alpha+1)\,\mathfrak{J} - \mathfrak{A}b' + \tfrac{1}{2}\mathfrak{H}a' + \mathfrak{D}g' - \mathfrak{E}f' - \tfrac{1}{2}\mathfrak{F}(c'-e') - \tfrac{3}{2}\mathfrak{G}d' + \tfrac{15}{16} + \frac{3H}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$H' = (\alpha-1)\,H - Ai' + \tfrac{1}{2}Ja' - Df' + Eg' - \tfrac{1}{2}F(c'-d') - \tfrac{3}{2}Gc'$$

$$J' = (\alpha+1)\,J - Ab' + \tfrac{1}{2}Ha' + Dg' - Ef' - \tfrac{1}{2}F(c'-e') - \tfrac{3}{2}Gd'$$

$$(\alpha-1)H' - A'i' - \tfrac{1}{2}J'a' - D'f' - E'g' - \tfrac{1}{2}F'(c'+d') - \tfrac{3}{2}G'e'$$
$$+ \tfrac{45}{16} - \mathfrak{E}H - 2u\mathfrak{H} + \tfrac{1}{2}bF + \frac{3J}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{E}\mathfrak{F}}{nn} + \frac{3\mathfrak{E}F}{nn}$$
$$+ \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{J} + \frac{(3\mathfrak{A}+3A)}{nn}J + \frac{(\mathfrak{D}+3D)}{nn}\mathfrak{F} + \frac{(3\mathfrak{D}+3D)}{nn}F$$
$$+ \frac{(\mathfrak{E}+3E)}{nn}\mathfrak{G} + \frac{(3\mathfrak{E}+3E)}{nn}G - \frac{3A(F+G)}{2nn} + \frac{9D+15E}{8nn} = 0$$

$$(\alpha+1)\,J' - A'b' - \tfrac{1}{2}H'a' - E'f' - D'g' - \tfrac{1}{2}F'(c'+e') - \tfrac{3}{2}G'd'$$
$$+ \tfrac{45}{16} - \mathfrak{E}J - 2uJ + \tfrac{1}{2}bF + \frac{3H}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{E}\mathfrak{F}}{nn} + \frac{3\mathfrak{E}F}{nn}$$
$$+ \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{H} + \frac{(3\mathfrak{A}+3A)}{nn}H + \frac{(\mathfrak{D}+3D)}{nn}\mathfrak{G} + \frac{(3\mathfrak{D}+3D)}{nn}G$$
$$+ \frac{(\mathfrak{E}+3E)}{nn}\mathfrak{F} + \frac{(3\mathfrak{E}+3E)}{nn}F - \frac{3A(F+G)}{2nn} + \frac{9E+15D}{8nn} = 0$$

§. 182. In his aequationibus ſubſtituantur valores iam cogniti, atque obtinebimus,

$$-0{,}06626\,\mathfrak{H} + 0{,}81033\,i' - 0{,}00987\,\mathfrak{J} + 0{,}00854\,J \pm 2{,}04620 = 0$$
$$1{,}83374\,\mathfrak{J} + 0{,}81033\,b' - 0{,}00987\,\mathfrak{H} + 0{,}00854\,H + 2{,}10646 = 0$$
$$H' = -0{,}06626\,H + 1{,}31773\,i' - 0{,}00937\,J - 5{,}57458$$
$$J' = 1{,}83374\,J + 1{,}31773\,b' - 0{,}00987\,H - 1{,}55427$$

$$\left.\begin{array}{l} -0{,}06626\,H' + 2{,}47576\,i' + 0{,}00987\,J' - 11{,}86106 \\ -1{,}01591\,H + 0{,}00427\,J - 0{,}02711\,\mathfrak{J} + 2{,}81250 \\ -2{,}01798\,\mathfrak{H} - 0{,}03634\,J + 31{,}31044 \\ \qquad + 10{,}60834 \end{array}\right\} = 0$$

$$\left.\begin{array}{l} 1{,}83374\,J' + 2{,}47576\,b' + 0{,}00987\,H' + 0{,}27762 \\ -1{,}01591\,J + 0{,}00427\,H - 0{,}02711\,\mathfrak{H} + 2{,}81250 \\ -2{,}01798\,\mathfrak{J} - 0{,}03634\,H + 31{,}81380 \\ \qquad - 0{,}81380 \end{array}\right\} = 0$$

§. 183. Subſtituamus primo loco b' et i' valores, atque noſtrae aequationes reducentur ad formas ſequentes,

$$-0{,}06626\,\mathfrak{H} + 0{,}01785\,J - 0{,}00526\,\mathfrak{J} + 2{,}04620 = 0$$
$$+1{,}83374\,\mathfrak{J} + 0{,}01785\,H - 0{,}00526\,\mathfrak{H} + 2{,}10646 = 0$$
$$H' = -0{,}06626\,H + 0{,}00526\,J + 0{,}00750\,\mathfrak{J} - 5{,}57458$$
$$J' = +1{,}83374\,J + 0{,}00526\,H + 0{,}00750\,\mathfrak{H} - 1{,}55427$$

qui valores in ſequentibus ſubſtituti dant

$$-1{,}01147\,H - 2{,}01791\,\mathfrak{H} + 0{,}01417\,J - 0{,}01352\,\mathfrak{J} + 33{,}22429 = 0$$
$$+2{,}34674\,J - 2{,}01791\,\mathfrak{J} + 0{,}00535\,H + 0{,}00073\,\mathfrak{H} + 30{,}68146 = 0$$

vnde elicimus:

$$\mathfrak{H} = \pm 0{,}00077\,H + 0{,}26935\,J + 30{,}96780$$
$$\mathfrak{J} = -0{,}00974\,H + 0{,}00077\,J - 1{,}05989$$

§. 184.

r valores
)4620 = 0
.0646 = 0
57458
55427
106
250
044 = 0
334
762
250
180 = 0
180
i' valo-
formas
20 = 0
46 = 0
57458
55427
429 = 0
146 = 0
§. 184.

§. 184. Hi autem valores in poſterioribus aequationibus ſubſtituti producent

$$1{,}01290\,H + 0{,}52937\,J + 29{,}25137 = 0$$
$$2{,}34539\,J + 0{,}02500\,H + 32{,}84282 = 0$$

hincque tandem concluditur:

$$\begin{array}{lll} H = -21{,}68110 & \ldots & l\text{-}H = 1{,}336081 \\ J = -13{,}77206 & \ldots & l\text{-}J = 1{,}138999 \\ \mathfrak{H} = +27{,}24155 & \ldots & l\,\mathfrak{H} = 1{,}435232 \\ \mathfrak{J} = -1{,}25933 & \ldots & l\text{-}\mathfrak{J} = 0{,}100140 \\ b' = -0{,}09396 & \ldots & l\text{-}b' = 8{,}972943 \\ i' = -0{,}16533 & \ldots & l\text{-}i' = 9{,}218352 \end{array}$$

§. 185. Nunc pro excentricitate orbitae ſolaris hae reſtant aequationes,

$$(\alpha - \tfrac{1}{n})\mathfrak{K} - \mathfrak{A}l' + \tfrac{1}{2}\mathfrak{L}a' - \mathfrak{Q}f' + \mathfrak{R}g' + \tfrac{1}{2}(\tfrac{2}{n} - p')\mathfrak{F} + \tfrac{1}{2}\mathfrak{F}q' - \tfrac{1}{2}\mathfrak{G}r'$$
$$- \tfrac{3}{4} + \frac{3\mathfrak{L}}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn} = 0$$

$$(\alpha + \tfrac{1}{n})\mathfrak{L} - \mathfrak{A}k' + \tfrac{1}{2}\mathfrak{K}a' + \mathfrak{Q}g' - \mathfrak{R}f' + \tfrac{1}{2}(\tfrac{2}{n} - p')\mathfrak{F} + \tfrac{1}{2}\mathfrak{F}r' - \tfrac{1}{2}\mathfrak{G}q'$$
$$- \tfrac{3}{4} + \frac{3K}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn} = 0$$

$$K' = (\alpha - \tfrac{1}{n})K - Al' + \tfrac{1}{2}La' - Qf' + Rg' + \tfrac{1}{2}(\tfrac{2}{n} - p')F + \tfrac{1}{2}Fq' - \tfrac{1}{2}Gr'$$

$$L' = (\alpha + \tfrac{1}{n})L - Ak' + \tfrac{1}{2}Ka' + Qg' - Rf' + \tfrac{1}{2}(\tfrac{2}{n} - p')F + \tfrac{1}{2}Fr' - \tfrac{1}{2}Gq'$$

$$(\alpha - \tfrac{1}{n})K' - A'l' - \tfrac{1}{2}L'a' - Q'f' - R'g' + \tfrac{1}{2}(\tfrac{2}{n} - p')F' - \tfrac{1}{2}F'q' - \tfrac{1}{2}G'r'$$
$$- \tfrac{3}{4} - 6K - 2n\mathfrak{K} - \frac{15F}{8nn} - \frac{9G}{8nn} + \frac{3L}{4nn} + \frac{(\mathfrak{A} + 3A)}{nn}\mathfrak{L} + \frac{(3\mathfrak{A} + 3A)L}{nn}$$

X +

$$+\frac{(\mathfrak{F}+3F)}{nn}(\mathfrak{P}+\mathfrak{Q})+\frac{(3\mathfrak{F}+3F)}{nn}(P+Q)+\frac{(\mathfrak{G}+3G)}{nn}\mathfrak{R}+\frac{(3\mathfrak{G}+3G)}{nn}R$$

$$+\frac{9P}{8nn}+\frac{9Q}{8nn}+\frac{15R}{8nn}=0$$

$$(\alpha+\frac{1}{n})L'-A'k'-\tfrac{1}{2}K'a'-R'f'-Q'g'+\tfrac{1}{2}(\frac{2}{n}-p')F'-\tfrac{1}{2}F'r-\tfrac{1}{2}G'q'$$

$$-\tfrac{2}{4}-\mathfrak{E}L-2\kappa\mathfrak{L}+\frac{3K}{4nn}-\frac{15F}{8nn}-\frac{9G}{8nn}+\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{K}+\frac{(3\mathfrak{A}+3A)}{nn}K$$

$$+\frac{(\mathfrak{F}+3F)}{nn}(\mathfrak{P}+\mathfrak{R})+\frac{(3\mathfrak{F}+3F)}{nn}(P+R)+\frac{(\mathfrak{G}+3G)}{nn}\mathfrak{Q}+\frac{(3\mathfrak{G}+3G)}{nn}Q$$

$$+\frac{9P}{8nn}+\frac{9R}{8nn}+\frac{15Q}{8nn}=0$$

§. 186. Hic autem obſeruo, hanc determinationem maxime eſſe lubricam, cum coefficiens litterae L, quem poſtremó eſt habitura, admodum fiat paruus; vnde is à terminis, quos omiſimus, non mediocrem mutationem perpeti poſſet. Hanc ob cauſam conſultum iudico, in calculum quoque terminos $3\eta-s$ et $3\eta+s$ introducere, quia praeuideo ab iis coefficientes terminorum, quos quaerimus, non leuiter affici. Sequenti ergo modo calculum redintegro.

§. 187. In hunc finem quoque rationem habeamus angulorum $3\eta-s$ et $3\eta+s$, ſitque

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta-s)$$
$$+ \mathfrak{R}e\cos(2\eta+s)$$
$$+\mathfrak{F}v\cos\eta + \mathfrak{K}ve\cos(\eta-s) + \mathfrak{M}ve\cos(3\eta-s)$$
$$+\mathfrak{G}v\cos 3\eta + \mathfrak{L}ve\cos(\eta+s) + \mathfrak{N}ve\cos(3\eta+s)$$

$v =$

$$\varphi = A\cos 2\eta + Pe\cos s + Qe\cos(2\eta-s)$$
$$+ Re\cos(2\eta+s)$$
$$+Fv\cos\eta + Kve\cos(\eta-s) + Mve\cos(3\eta-s)$$
$$+Gv\cos 3\eta + Lve\cos(\eta+s) + Nve\cos(3\eta+s)$$

§. 188. Quodſi iam ponamus:

$$\frac{2\kappa K+\mathfrak{K}}{nn}=k' \;;\quad \frac{2\kappa L+\mathfrak{L}}{nn}=l'$$
$$\frac{2\kappa M+\mathfrak{M}}{nn}=m' \quad\; \frac{2\kappa N+\mathfrak{N}}{nn}=n'$$

erit

$$\frac{d\Phi}{dr} = \text{Praec.} - a'\cos 2\eta - p'e\cos s - q'e\cos(2\eta-s)$$
$$- r'e\cos(2\eta+s)$$
$$- f'v\cos\eta - k've\cos(\eta-s) - m've\cos(3\eta-s)$$
$$- g'v\cos 3\eta - l've\cos(\eta+s) - n've\cos(3\eta+s)$$

atque ob $\frac{ds}{dr}=\frac{1}{n}-\frac{2}{n}e\cos s$ erit

$$\frac{d\eta}{dr} = \alpha - a'\cos 2\eta + (\frac{2}{n}-p')e\cos s - q'e\cos(2\eta-s)$$
$$- r'e\cos(2\eta+s)$$
$$- f'v\cos\eta - k've\cos(\eta-s) - m've\cos(3\eta-s)$$
$$- g'v\cos 3\eta - l've\cos(\eta+s) - n've\cos(3\eta+s)$$

§. 189. Formulas nunc aſſumtas differentiemus, ſolosque terminos, quibus opus habemus, in calculo exprimamus ac reperiemus:

$$R = \text{Praec.}$$

$$+ve\sin(\eta-s)(+\mathfrak{A}l'-\mathfrak{A}m'+\mathfrak{Q}f'-\mathfrak{M}g'-\tfrac{1}{2}\mathfrak{F}(\tfrac{2}{n}-p')-\tfrac{1}{2}\mathfrak{F}q'+\tfrac{1}{2}\mathfrak{G}r'-\mathfrak{K}(\alpha-\tfrac{1}{n})$$
$$-\tfrac{1}{2}\mathfrak{L}a'+\tfrac{1}{2}\mathfrak{M}a'$$
$$+$$

$$+ve\sin(\eta+s)\left(+\mathfrak{A}k' - \mathfrak{A}n' + \mathfrak{R}f' - \mathfrak{Q}g' - \tfrac{1}{2}\mathfrak{F}\left(\tfrac{2}{n}-p'\right) - \tfrac{1}{2}\mathfrak{F}r' + \tfrac{3}{2}\mathfrak{G}q' - \mathfrak{L}\left(\alpha+\tfrac{1}{n}\right)\right.$$
$$\left. - \tfrac{1}{2}\mathfrak{K}a' + \tfrac{3}{2}\mathfrak{N}a'\right)$$

$$+ve\sin(3\eta-s)\left(+\mathfrak{A}k' + \mathfrak{Q}f' + \tfrac{1}{2}\mathfrak{F}q' - \tfrac{3}{2}\mathfrak{G}\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}\mathfrak{K}a' - \mathfrak{M}\left(3\alpha-\tfrac{1}{n}\right)\right)$$

$$+ve\sin(3\eta-s)\left(+\mathfrak{A}l' + \mathfrak{R}f' + \tfrac{1}{2}\mathfrak{F}r' - \tfrac{3}{2}\mathfrak{G}\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}\mathfrak{L}a' - \mathfrak{N}\left(3\alpha+\tfrac{1}{n}\right)\right)$$

Ac si breuitatis gratia ponamus:

$$K' = \left(\alpha-\tfrac{1}{n}\right)K - A(l'-m') - Qf' + Rg' + \tfrac{1}{2}F\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}Fq' - \tfrac{3}{2}Gr' + \tfrac{1}{2}La' - \tfrac{3}{2}Ma'$$

$$L' = \left(\alpha+\tfrac{1}{n}\right)L - A(k'-n') - Rf' + Qg' + \tfrac{1}{2}F\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}Fr' - \tfrac{3}{2}Gq' + \tfrac{1}{2}Ka' - \tfrac{3}{2}Na'$$

$$M' = \left(3\alpha-\tfrac{1}{n}\right)M - Ak' - Qf' - \tfrac{1}{2}Fq' + \tfrac{3}{2}G\left(\tfrac{2}{n}-p'\right) - \tfrac{1}{2}Ka'$$

$$N' = \left(3\alpha+\tfrac{1}{n}\right)N - Al' - Rf' - \tfrac{1}{2}Fr' + \tfrac{3}{2}G\left(\tfrac{2}{n}-p'\right) - \tfrac{1}{2}La'$$

erit

$$\frac{dv}{dr} = -A'\sin 2\eta - P'e\sin s - Q'e\sin(2\eta-s) - R'e\sin(2\eta+s)$$
$$-F'v\sin\eta - K've\sin(\eta-s) - M've\sin(3\eta-s)$$
$$-G'v\sin 3\eta - L've\sin(\eta+s) - N've\sin(3\eta+s)$$

§. 190. Hinc iam denuo differentiando nanciscemur

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ve\cos(\eta-s)\left\{\begin{array}{l} A'l' + A'm' + Q'f' + R'g' - \tfrac{1}{2}F'\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}F'q' \\ + \tfrac{3}{2}G'r' + \tfrac{1}{2}L'a' + \tfrac{3}{2}M'a' - \left(\alpha-\tfrac{1}{n}\right)K' \end{array}\right.$$

+

$$+ve\cos(\eta+s)\left\{\begin{array}{l} A'k' + A'n' + R'f' + Q'g' - \tfrac{1}{2}F'\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}F'r' \\ + \tfrac{3}{2}G'q' + \tfrac{1}{2}K'a' + \tfrac{3}{2}N'a' - \left(\alpha+\tfrac{1}{n}\right)L' \end{array}\right.$$

$$+ve\cos(3\eta-s)\left\{\begin{array}{l} A'k' + Q'f' + \tfrac{1}{2}F'q' - \tfrac{3}{2}G'\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}K'a' \\ - \left(3\alpha-\tfrac{1}{n}\right)M' \end{array}\right.$$

$$+ve\cos(3\eta+s)\left\{\begin{array}{l} A'l' + R'f' + \tfrac{1}{2}F'r' - \tfrac{3}{2}G'\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}L'a' \\ - \left(3\alpha+\tfrac{1}{n}\right)N' \end{array}\right.$$

§. 191. Quodsi autem valores iam inuenti substituantur, habebitur

$$R = \text{Praec.}$$

$$+ve\sin(\eta-s)\left\{\begin{array}{l} -0{,}85830\mathfrak{K} + 0{,}00526\mathfrak{L} - 0{,}02500\mathfrak{M} + 0{,}37592 \\ -0{,}00931L + 0{,}00931M \end{array}\right]$$

$$+ve\sin(\eta+s)\left\{\begin{array}{l} -1{,}00918\mathfrak{L} + 0{,}00526\mathfrak{K} - 0{,}02500\mathfrak{N} + 0{,}34115 \\ -0{,}00931K + 0{,}00931N \end{array}\right]$$

$$+ve\sin(3\eta-s)\left\{\begin{array}{l} -2{,}72578\mathfrak{M} - 0{,}01448\mathfrak{K} + 0{,}49223 \\ -0{,}00931K \end{array}\right]$$

$$+ve\sin(3\eta+s)\left\{\begin{array}{l} -2{,}87666\mathfrak{N} - 0{,}01448\mathfrak{L} + 0{,}47116 \\ -0{,}00931L \end{array}\right]$$

 $\mathfrak{K}' =$

$$K' = 0{,}85830K + 0{,}00527L + 0{,}01447M + 1{,}48970 + 0{,}00750\mathfrak{L} - 0{,}00750\mathfrak{M}$$

$$L' = 1{,}00916L + 0{,}00527K + 0{,}01447N + 1{,}55183 + 0{,}00750\mathfrak{K} - 0{,}00750\mathfrak{N}$$

$$M' = 2{,}72528M + 0{,}02501K - 1{,}17408 + 0{,}00750\mathfrak{K}$$

$$N' = 2{,}87666N + 0{,}02501L - 0{,}95901 + 0{,}00750\mathfrak{L}$$

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ vecf(\eta - s)\begin{Bmatrix} -0{,}85830K' - 0{,}02843L - 0{,}02843M - 0{,}32674 \\ -0{,}00987L' - 0{,}01409\mathfrak{L} - 0{,}01409\mathfrak{M} \\ -0{,}02961M' \end{Bmatrix}$$

$$+ vecf(\eta + s)\begin{Bmatrix} -1{,}00918L' - 0{,}02843K - 0{,}02843N - 0{,}56620 \\ -0{,}00987K' - 0{,}01409\mathfrak{K} - 0{,}01409\mathfrak{N} \\ -0{,}02981N' \end{Bmatrix}$$

$$+ vecf(3\eta - s)\begin{Bmatrix} -2{,}72578M' - 0{,}02843K + 0{,}74683 \\ -0{,}00987K' - 0{,}01409\mathfrak{K} \end{Bmatrix}$$

$$+ vecf(3\eta + s)\begin{Bmatrix} -2{,}87661N' - 0{,}02853L + 0{,}67486 \\ -0{,}00987L' - 0{,}01409\mathfrak{L} \end{Bmatrix}$$

§. 192. Valores autem litterarum commate notatarum hic substitutae dabunt

$$\frac{ddv}{dr^2} =$$

$$vecf(\eta - s)\begin{Bmatrix} -0{,}73747K - 0{,}04264L - 0{,}12156M - 0{,}00014N \\ -0{,}00029\mathfrak{K} - 0{,}02053\mathfrak{L} - 0{,}00765\mathfrak{M} + 0{,}00008\mathfrak{N} \\ -1{,}58590 \end{Bmatrix}$$

ve

$$vecf(\eta + s)\begin{Bmatrix} -1{,}01923L - 0{,}04222K - 0{,}12821N - 0{,}00014M \\ -0{,}00029\mathfrak{L} - 0{,}02166\mathfrak{K} - 0{,}00642\mathfrak{N} + 0{,}00008\mathfrak{M} \\ -2{,}11858 \end{Bmatrix}$$

$$vecf(3\eta - s)\begin{Bmatrix} -7{,}43058M - 0{,}09507K - 0{,}00005L + 3{,}93257 \\ +0{,}00008\mathfrak{M} - 0{,}03453\mathfrak{K} - 0{,}00007\mathfrak{L} \end{Bmatrix}$$

$$vecf(3\eta + s)\begin{Bmatrix} -8{,}27529N - 0{,}11338L - 0{,}00005K + 3{,}41830 \\ +0{,}00008\mathfrak{N} - 0{,}03566\mathfrak{L} - 0{,}00007\mathfrak{K} \end{Bmatrix}$$

§. 193. His expressionibus ita euolutis atque ad calculum numericum praeparatis, quaeramus easdem expressiones ex formulis supra traditis pro R et $\frac{ddv}{dr^2}$, quae continentur in §. 52 et 54. Inde autem omittendis terminis, quos iam tractauimus, consequemur.

$$R = \text{Pr.} + vefin(\eta - s)\left(-\tfrac{3}{4} + \frac{3L}{2nn} - \frac{3M}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn}\right)$$

$$+ vefin(\eta + s)\left(-\tfrac{3}{4} + \frac{3K}{2nn} - \frac{3N}{2nn} + \frac{9G}{4nn} - \frac{9F}{4nn}\right)$$

$$+ vefin(3\eta - s)\left(-\tfrac{15}{4} + \frac{3K}{2nn} - \frac{9F}{4nn}\right)$$

$$+ vefin(3\eta + s)\left(-\tfrac{15}{4} + \frac{3L}{2nn} - \frac{9F}{4nn}\right)$$

ddv

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ve\cos(\eta-s)\begin{cases} -\frac{9}{4} + \frac{9P}{8nn} + \frac{9Q}{8nn} + \frac{15R}{8nn} - 6K + \frac{3L}{4nn} + \frac{3M}{4nn} \\ -\frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2n\mathfrak{K} + \frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{L}+\mathfrak{M}) \\ + \frac{(3\mathfrak{A}+3A)}{nn}(L+M) + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{P} + \frac{(3\mathfrak{P}+3P)}{nn}F \\ + \frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{P} + \frac{(3\mathfrak{Q}+3Q)}{nn}F \quad \frac{(\mathfrak{R}+3R)}{nn}\mathfrak{G} \\ + \frac{(3\mathfrak{R}+3R)}{nn}G \end{cases}$$

$$+ve\cos(\eta+s)\begin{cases} -\frac{9}{4} + \frac{9P}{8nn} + \frac{9R}{8nn} + \frac{15Q}{8nn} - 6L + \frac{3K}{4nn} + \frac{3N}{4nn} \\ -\frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2n\mathfrak{L} + \frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{K}+\mathfrak{N}) \\ + \frac{(3\mathfrak{A}+3A)}{nn}(K+N) + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{F} + \frac{(3\mathfrak{P}+3P)}{nn}F \\ + \frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{G} + \frac{(3\mathfrak{Q}+3Q)}{nn}G + \frac{(\mathfrak{R}+3R)}{nn}\mathfrak{F} \\ + \frac{(3\mathfrak{R}+3R)}{nn}F \end{cases}$$

$$+ve\cos(3\eta-s)\begin{cases} -\frac{15}{4} + \frac{15P}{8nn} + \frac{9Q}{8nn} - 6M + \frac{3K}{4nn} - \frac{3G}{4nn} \\ -\frac{9F}{8nn} - 2n\mathfrak{M} + \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{K} + \frac{(3\mathfrak{A}+3A)}{nn}K + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{G} \\ + \frac{(3\mathfrak{P}+3P)}{nn}G + \frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{F} + \frac{(3\mathfrak{Q}+3Q)}{nn}F \end{cases}$$

+ve

$$+ve\cos(3\eta+s)\begin{cases} -\frac{15}{4} + \frac{15P}{8nn} + \frac{9R}{8nn} - 6N + \frac{3L}{4nn} - \frac{3G}{4nn} - \frac{9F}{8nn} - 2n\mathfrak{N} \\ + \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{L} + \frac{(3\mathfrak{A}+3A)}{nn}L + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{G} + \frac{(3\mathfrak{P}+3P)}{nn}G \\ + \frac{(\mathfrak{R}+3R)}{nn}\mathfrak{F} + \frac{(3\mathfrak{R}+3R)}{nn}F \end{cases}$$

§. 194. Introducantur hic quoque valores iam cogniti, ac prodibit

$$\begin{aligned} R = \text{Pr.} &+ ve\sin(\eta-s)[-1{,}02394 + 0{,}00854L - 0{,}00854M] \\ &+ ve\sin(\eta+s)[-1{,}02394 + 0{,}00854K - 0{,}00854N] \\ &+ ve\sin(3\eta-s)[-4{,}01450 + 0{,}00854K] \\ &+ ve\sin(3\eta+s)[-4{,}01450 + 0{,}00854L] \end{aligned}$$

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ve\cos(\eta-s)\begin{bmatrix} -1{,}01591K - 0{,}03207L - 0{,}03207M - 1{,}16867 \\ -2{,}01798\mathfrak{K} - 0{,}02711\mathfrak{L} - 0{,}02711\mathfrak{M} \end{bmatrix}$$

$$+ve\cos(\eta+s)\begin{bmatrix} -1{,}01591L - 0{,}03207K - 0{,}03207N - 1{,}93904 \\ -2{,}01798\mathfrak{L} - 0{,}02711\mathfrak{K} - 0{,}02711\mathfrak{N} \end{bmatrix}$$

$$+ve\cos(3\eta-s)\begin{bmatrix} -1{,}01591M - 0{,}03207K - 2{,}49545 \\ -2{,}01798\mathfrak{M} - 0{,}02711\mathfrak{K} \end{bmatrix}$$

$$+ve\cos(3\eta+s)\begin{bmatrix} -1{,}01591N - 0{,}03207L - 2{,}78450 \\ -2{,}01798\mathfrak{N} - 0{,}02711\mathfrak{L} \end{bmatrix}$$

§. 195. Hinc ergo octo sequentes aequationes resultabunt

$$\text{I.}\quad 0{,}85830\,\mathfrak{K} = +\,0{,}00526\,\mathfrak{L} - 0{,}02500\,\mathfrak{M} + 1{,}39986 - 0{,}01785\,L + 0{,}01785\,M$$

$$\text{II.}\quad 1{,}00918\,\mathfrak{L} = +\,0{,}00526\,\mathfrak{K} - 0{,}02500\,\mathfrak{N} + 1{,}36509 - 0{,}01785\,K + 0{,}01785\,N$$

$$\text{III.}\quad 2{,}72578\,\mathfrak{M} = -\,0{,}01448\,\mathfrak{K} + 4{,}50673 - 0{,}01785\,K$$

$$\text{IV.}\quad 2{,}87666\,\mathfrak{N} = -\,0{,}01448\,\mathfrak{L} + 4{,}48566 - 0{,}01785\,L$$

$$\text{V.}\quad +0{,}27844K - 0{,}01057L - 0{,}08949M - 0{,}00014N + 0{,}10081 = 0$$
$$+2{,}01769\,\mathfrak{K} + 0{,}00658\,\mathfrak{L} - 0{,}01946\,\mathfrak{M} + 0{,}00008\,\mathfrak{N}$$

$$\text{VI.}\quad -0{,}00332L - 0{,}01015K - 0{,}09614N - 0{,}00014M - 0{,}17954 = 0$$
$$+2{,}01769\,\mathfrak{L} + 0{,}00545\,\mathfrak{K} + 0{,}02069\,\mathfrak{N} + 0{,}00008\,\mathfrak{M}$$

$$\text{VII.}\quad -6{,}41487\,M - 0{,}06300\,K - 0{,}00005\,L + 6{,}42802 = 0$$
$$+2{,}01806\,\mathfrak{M} - 0{,}00742\,\mathfrak{K} - 0{,}00007\,\mathfrak{L}$$

$$\text{VIII.}\quad -7{,}25938\,N - 0{,}08131\,L - 0{,}00005\,K + 6{,}20280 = 0$$
$$+2{,}01806\,\mathfrak{N} - 0{,}00855\,\mathfrak{L} - 0{,}00007\,\mathfrak{K}$$

§. 196. Ex aequationibus III et IV statim eliciuntur hi valores

$$\mathfrak{M} = -\,0{,}00531\,\mathfrak{K} - 0{,}00655\,K + 1{,}65336$$
$$\mathfrak{N} = -\,0{,}00503\,\mathfrak{L} - 0{,}00621\,L + 1{,}55933$$

qui

equationes
1,39986
1,36509
0081=0
7954=0
102=0
80=0
eliciun-
qui

qui in I et II substituti praebent:

$$0{,}85817\,\mathfrak{K} = 1{,}35853 + 0{,}00526\,\mathfrak{L} - 0{,}01785(L-M) + 0{,}00016K$$
$$1{,}00905\,\mathfrak{L} = 1{,}32611 + 0{,}00526\,\mathfrak{K} - 0{,}01785(K-N) + 0{,}00015L$$

vnde obtinetur:

$$\mathfrak{K} = +\,1{,}59116 - 0{,}02080(L-M) + 0{,}00008\,K \pm 0{,}00010\,N$$
$$\mathfrak{L} = +\,1{,}32251 - 0{,}01769(K-N) + 0{,}00005\,L + 0{,}00011\,M$$
$$\mathfrak{M} = +\,1{,}64491 - 0{,}00655\,K + 0{,}00010(L-M)$$
$$\mathfrak{N} = +\,1{,}55268 - 0{,}00621\,L + 0{,}00009\,(K-N)$$

§. 197. His valoribus substitutis caeterae aequationes abibunt in formas sequentes:

$$0{,}27862K - 0{,}05252L - 0{,}04754M + 0{,}00017N + 3{,}35313 = 0$$
$$-0{,}00346L - 0{,}04584K - 0{,}06045N + 0{,}00020M + 2{,}53002 = 0$$
$$-6{,}41502M - 0{,}07622K + 0{,}00030L + 9{,}73587 = 0$$
$$-7{,}25971N - 0{,}09384L + 0{,}00028K + 9{,}32499 = 0$$

ex quarum binis postremis statim obtinetur:

$$M = -\,0{,}01188\;K + 0{,}00005\;L + 1{,}51765$$
$$N = -\,0{,}01293\;L + 0{,}00004\;K + 1{,}28448$$

vnde colligitur:

$$+0{,}27918\;K - 0{,}05252\;L + 3{,}28120 = 0$$
$$-0{,}00268\;L - 0{,}04584\;K + 2{,}45267 = 0$$

ac denique

$$K = +\;49{,}44710 \;\ldots\; lK = 1{,}606887$$
$$L = +\,368{,}40200 \;\ldots\; lL = 2{,}566322$$
$$M = +\;1{,}05555 \;\ldots\; lM = 0{,}023478$$
$$N = -\;3{,}47730 \;\ldots\; l\text{-}N = 0{,}541242$$

 §. 197.

§. 198. Litterarum germanicarum valores hinc erunt:

$\mathfrak{K} = -5,04677$	. . .	$l\text{-}\mathfrak{K} = 0,703013$
$\mathfrak{L} = +0,56302$	. . .	$l\,\mathfrak{L} = 9,750524$
$\mathfrak{M} = +1,41672$	. . .	$l\,\mathfrak{M} = 0,151283$
$\mathfrak{N} = -0,73119$	. . .	$l\,\mathfrak{N} = 9,864030$

ac litterarum hinc deriuatarum:

$k' = +0,43578$	. . .	$l\,k' = 9,639267$
$l' = +4,23401$	. . .	$l\,l' = 0,626752$
$m' = +0,02018$	. . .	$l\,m' = 8,304921$
$n' = -0,04409$	. . .	$l\text{-}n' = 8,644340$

§. 199. Nunc igitur intelligimus inaequalitates ab angulis $3\eta - s$ et $3\eta + s$ pendentes tam esse paruas, vt sine vllo errore reiici queant, etiamsi valores K et L aliquantum immutauerint. Distantia ergo lunae curtata a terra $x = \frac{(1-kk)\,au}{1-k\cos r}$ ita ab his inaequalitatibus parallacticis pendebit, vt sit

		Log. coeff.
$x = \text{Praec.}$	$+\,0,11756\ v\ \cos\eta$	9,070249
	$-\,0,00419\ v\ \cos 3\eta$	7,622540
	$-\,0,1234\ vk\ \cos(\eta - r)$	9,091265
	$-\,0,0784\ vk\ \cos(\eta + r)$	8,894183
	$+\,0,2302\ ve\ \cos(\eta - s)$	9,362071
	$+\,2,0965\ ve\ \cos(r + s)$	0,321506

Motus

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Motus autem momentaneus ita hinc afficietur, vt sit

$\frac{d\Phi}{dr} = \text{Praec.}$	$-\,0,23259\ v\ \cos\eta$	9,366591
	$+\,0,01309\ v\ \cos 3\eta$	8,116940
	$+\,0,0939\ vk\cos(\eta - r)$	8,972943
	$+\,0,1653\ vk\cos(\eta + r)$	9,218352
	$-\,0,4358\ ve\cos(\eta - s)$	9,639267
	$-\,4,2340\ ve\cos(\eta + s)$	0,626752

§. 200. Quodsi iam ipsam longitudinem lunae, quatenus ab his inaequalitatibus parallacticis pendet, ponamus:

$$\Phi = \text{Praec.} + \mathfrak{F}'v\sin\eta + \mathfrak{H}'vk\sin(\eta - r) + \mathfrak{K}'ve\sin(\eta - s) + \mathfrak{G}'v\sin 3\eta + \mathfrak{J}'vk\sin(\eta + r) + \mathfrak{L}'ve\sin(\eta + s)$$

sequentes obtinebimus aequationes pro horum coefficientium determinatione:

$$-0,23259 = \alpha\mathfrak{F}' - \mathfrak{A}'f' - \mathfrak{A}'g' - \tfrac{1}{2}\mathfrak{J}'a' - \tfrac{1}{2}\mathfrak{G}'a'$$

$$+0,01303 = 3\alpha\mathfrak{G}' - \mathfrak{A}'f' - \tfrac{1}{2}\mathfrak{F}'a'$$

$$+0,0939 = (\alpha - 1)\mathfrak{H}' - \mathfrak{A}'i' - \tfrac{1}{2}\mathfrak{J}'a' - \mathfrak{D}'f' - \mathfrak{E}'g' - \tfrac{1}{2}\mathfrak{F}'c' - \tfrac{1}{2}\mathfrak{F}'d' - \tfrac{1}{2}\mathfrak{G}'e'$$

$$+0,1653 = (\alpha + 1)\mathfrak{J}' - \mathfrak{A}'b' - \tfrac{1}{2}\mathfrak{H}'a' - \mathfrak{E}'f' - \mathfrak{D}'g' - \tfrac{1}{2}\mathfrak{F}'c' - \tfrac{1}{2}\mathfrak{F}'e' - \tfrac{1}{2}\mathfrak{G}'d'$$

$$-0{,}4358 = \left(a-\frac{1}{n}\right)\mathfrak{K}' - \mathfrak{N}'l - \tfrac{1}{2}\mathfrak{L}'d - \mathfrak{Q}'f - \mathfrak{M}'g$$
$$+\tfrac{1}{2}\mathfrak{F}'\left(\frac{2}{n}-p'\right) - \tfrac{1}{2}\mathfrak{F}'q' - \tfrac{1}{2}\mathfrak{G}'r'$$

$$-4{,}2340 = \left(a+\frac{1}{n}\right)\mathfrak{L}' - \mathfrak{N}'k' - \tfrac{1}{2}\mathfrak{K}'d' - \mathfrak{M}'f - \tfrac{1}{2}\mathfrak{Q}'g'$$
$$+\tfrac{1}{2}\mathfrak{F}'\left(\frac{2}{n}-p'\right) - \tfrac{1}{2}\mathfrak{F}'r' - \tfrac{1}{2}\mathfrak{G}'q'$$

§. 201. Valoribus autem iam cognitis hic ſubſtitutis, aequationes iſtae in ſequentes abibunt formas:

$$-0{,}23031 = +0{,}94361\,\mathfrak{F}' + 0{,}02961\,\mathfrak{G}'$$
$$+0{,}01550 = +2{,}80122\,\mathfrak{G}' + 0{,}00987\,\mathfrak{F}'$$
$$-0{,}0056 = -0{,}06626\,\mathfrak{H}' + 0{,}00987\,\mathfrak{J}' - 0{,}25665\,\mathfrak{F}' + 0{,}01926\,\mathfrak{G}'$$
$$+0{,}1710 = +1{,}93374\,\mathfrak{J}' + 0{,}00987\,\mathfrak{H}' - 0{,}06717\,\mathfrak{F}' - 0{,}54918\,\mathfrak{G}'$$
$$-0{,}3968 = +0{,}85830\,\mathfrak{K}' + 0{,}00987\,\mathfrak{L}' + 0{,}06357\,\mathfrak{F}' - 0{,}04509\,\mathfrak{G}'$$
$$-4{,}2330 = +1{,}00918\,\mathfrak{L}' + 0{,}00987\,\mathfrak{K}' + 0{,}06729\,\mathfrak{F}' - 0{,}05625\,\mathfrak{G}'$$

vnde colligitur fore

$\mathfrak{F}' = -0{,}24427$ . . . $l{-}\mathfrak{F}' = 9{,}387868$

$\mathfrak{G}' = +0{,}00639$ . . . $l\,\mathfrak{G}' = 7{,}805991$

$\mathfrak{H}' = +1{,}1959$ . . . $l\,\mathfrak{H}' = 0{,}077694$

$\mathfrak{J}' = +0{,}0757$ . . . $l\,\mathfrak{J}' = 8{,}879096$

$\mathfrak{K}' = -0{,}3959$ . . . $l{-}\mathfrak{K}' = 9{,}597508$

$\mathfrak{L}' = -4{,}1738$ . . . $l{-}\mathfrak{L}' = 0{,}620530$

§. 202. Hinc ergo habebimus ſequentes partes pro longitudine Lunae, quas ſimul ope valorum proxime cognitorum pro v, k, e ad minuta ſecunda reducamus:

			Log. coeff.	Val. coeff. in min. ſec.
Φ = Praec.	$-0{,}24427$	$v\ \sin\eta$	9,387868	$-175''$
	$+0{,}00639$	$v\ \sin 3\eta$	7,805991	$+4''$
	$+1{,}1959$	$vk\ \sin(\eta-r)$	0,077694	$+59''$
	$+0{,}0757$	$vk\ \sin(\eta+r)$	8,879096	$+4''$
	$-0{,}3959$	$ve\ \sin(\eta-s)$	9,597508	$-5''$
	$-4{,}1738$	$ve\ \sin(\eta+s)$	0,620530	$-49''$

Sicque omnes iam adepti ſumus motus lunae inaequalitates, quae quidem ab inclinatione eius orbitae ad eclipticam non pendent. Interim tamen non diffiteor, dari aliquas inſuper inaequalitates, quae alicuius forte ſint momenti, quas in hac inueſtigatione praeterimus, cuiusmodi ſunt eae, quae ab angulis $2\eta-3r$ et $2\eta-2r+s$ pendent, quae ad plura minuta ſecunda aſſurgere poſſe videntur. Verum earum determinatio tam eſt taedioſa, vt malim eam obſeruationibus relinquere.

§. 203. Quae ergo hactenus inuenimus, in vnum colligamus ac primo pro diſtantia lunae a terra curtata $x = \frac{(1-kk)\,au}{1-k\cos r}$ erit

	Log. coeff.	coeff. integri
$u = 1 - 0,0074991 \cos 2\eta$	7,875009	−0,007499
$+ 0,0000532 \cos 4\eta$	5,725912	+0,000053
$+ 0,191557\, k \cos(2\eta - r)$	9,282297	+0,010430
$- 0,003293\, k \cos(2\eta + r)$	7,517525	−0,000179
$- 0,003321\, k \cos(4\eta - r)$	7,521296	−0,000181
$+ 0,000049\, k \cos(4\eta + r)$	5,692584	+0,000005
$- 0,00511\, kk \cos 2r$	7,708601	−0,000015
$- 0,08022\, kk \cos(2\eta - 2r)$	8,904280	−0,000238
$- 0,00237\, kk \cos(2\eta + 2r)$	7,375072	−0,000007
$+ 0,07892\, kk \cos(4\eta - 2r)$	8,897172	+0,000234
$+ 0,00001\, kk \cos(4\eta + 2r)$	4,872349	+0,000000
$- 0,006400\, e \cos s$	7,806180	−0,000206
$+ 0,014801\, e \cos(2\eta - s)$	8,170303	+0,000249
$+ 0,011415\, e \cos(2\eta + s)$	8,057492	+0,000192
$+ 0,00364\, ee \cos 2s$	7,539670	+0,000001
$- 0,01482\, ee \cos(2\eta - 2s)$	8,170799	−0,000004
$- 0,00584\, ee \cos(2\eta + 2s)$	7,766856	−0,000001
$- 0,37957\, ek \cos(r - s)$	9,579297	−0,000347
$+ 0,30693\, ek \cos(r + s)$	9,487039	+0,000281
$- 0,36337\, ek \cos(2\eta - r + s)$	9,560344	−0,000332
$+ 0,00993\, ek \cos(2\eta + r - s)$	7,997004	+0,000009
$- 0,07752\, ek \cos(2\eta - r - s)$	8,889425	−0,000071
$+ 0,00305\, ek \cos(2\eta + r + s)$	7,484024	+0,000003
$+ 0,11756\, \nu \cos \eta$	9,070249	+0,000408
$- 0,00419\, \nu \cos 3\eta$	7,622540	−0,000015
$- 0,1234\, \nu k \cos(\eta - r)$	9,091265	−0,000024
$- 0,0784\, \nu k \cos(\eta + r)$	8,894183	−0,000015
$+ 0,2302\, \nu e \cos(\eta - s)$	9,362071	+0,000013
$+ 2,0965\, \nu e \cos(\eta + s)$	0,321506	+0,000122

Hic ad latus adiunxi valores coefficientium integrorum in numeris absolutis expressos, ponendo $k = 0,05445$, $e = 0,01680$ et $\nu = \frac{1}{11}$; quos proinde, si hi valores aliter per observationes determinentur, facile erit emendare.

§. 204.

§. 204. Pro motu autem lunae momentaneo, ex quo eius motus horarius definiri poterit, habebimus:

	Log. coeff.	Coeff. integri
$\frac{d\Phi}{dr} = 1,009176$	0,003967	+1,009176
$+ 0,0195144 \cos 2\eta$	8,290355	+0,019514
$- 0,0000322 \cos 4\eta$	5,507856	−0,000032
$- 0,001231\, k \cos r$	7,090258	−0,000067
$- 0,366103\, k \cos(2\eta - r)$	9,563604	−0,019934
$+ 0,012832\, k \cos(2\eta + r)$	8,108292	+0,000699
$+ 0,002829\, k \cos(4\eta - r)$	7,451633	+0,000154
$- 0,000171\, k \cos(4\eta + r)$	6,232305	−0,000009
$+ 0,01182\, kk \cos 2r$	8,072618	+0,000035
$- 0,02057\, kk \cos(2\eta - 2r)$	8,313172	−0,000061
$+ 0,01063\, kk \cos(2\eta + 2r)$	8,026598	+0,000032
$- 0,09883\, kk \cos(4\eta - 2r)$	8,994889	−0,000293
$- 0,00004\, kk \cos(4\eta + 2r)$	5,592770	−0,000000
$+ 0,013760\, e \cos s$	8,138618	+0,000231
$- 0,037487\, e \cos(2\eta - s)$	8,573878	−0,000630
$- 0,030062\, e \cos(2\eta + s)$	8,478023	−0,000505
$- 0,00722\, ee \cos 2s$	7,858166	−0,000002
$+ 0,03470\, ee \cos(2\eta - 2s)$	8,540319	+0,000010
$+ 0,01533\, ee \cos(2\eta + 2s)$	8,185614	+0,000005
$+ 0,75204\, ek \cos(r - s)$	9,876241	+0,000688
$- 0,62626\, ek \cos(r + s)$	9,796755	−0,000573
$+ 0,69420\, ek \cos(2\eta - r + s)$	9,841484	+0,000635
$- 0,02440\, ek \cos(2\eta + r - s)$	8,387390	−0,000022
$+ 0,12160\, ek \cos(2\eta - r - s)$	9,084933	+0,000111
$- 0,02000\, ek \cos(2\eta + r + s)$	9,301030	−0,000018
$- 0,23259\, \nu \cos \eta$	9,366591	−0,000808
$+ 0,01309\, \nu \cos 3\eta$	8,116940	+0,000045
$+ 0,0939\, \nu k \cos(\eta - r)$	8,972943	+0,000018
$+ 0,1653\, \nu k \cos(\eta + r)$	9,218352	+0,000031
$- 0,4358\, \nu e \cos(\eta - s)$	9,639267	−0,000025
$- 4,2340\, \nu e \cos(\eta + s)$	0,626753	−0,000247

§. 105.

§. 205. Si iam longitudo Lunae per ſolam excentricitatem ſecundum regulas Keplerianas determinata ponatur $=\zeta$, ita vt poſita eius anomalia vera $=r$, futurum ſit $\zeta = C + 1{,}0085272\, r$, erit longitudo vera per hactenus inuentas inaequalitates.

$\Phi =$	Log. coeff.	Val. coeff. in min. sec.
$\zeta + 0{,}0103597 \sin 2\eta$	8,015347	$+2137''$
$- 0{,}0000382 \sin 4\eta$	5,582063	$-$ 8
$+ 0{,}010146k \sin r$	8,006295	$+$ 114
$- 0{,}420226k \sin(2\eta - r)$	9,623483	$-$ 4720
$+ 0{,}004992k \sin(2\eta + r)$	7,698261	$+$ 56
$+ 0{,}005286k \sin(4\eta - r)$	7,723163	$+$ 59
$- 0{,}000086k \sin(4\eta + r)$	5,935307	$-$ 1
$+ 0{,}00420kk \sin 2r$	7,623250	$+ 2\frac{1}{2}$
$+ 0{,}57328kk \sin(2\eta - 2r)$	9,758367	$+$ 351
$+ 0{,}00318kk \sin(2\eta + 2r)$	7,402427	$+ 1\frac{1}{2}$
$- 0{,}15083kk \sin(4\eta - 2r)$	9,178488	$-$ 92
$- 0{,}00002kk \sin(4\eta + 2r)$	5,301030	$-$ 0
$+ 0{,}201385e \sin s$	9,304026	$+$ 701
$- 0{,}021889e \sin(2\eta - s)$	8,340237	$-$ 76
$- 0{,}016368e \sin(2\eta + s)$	8,214002	$-$ 57
$+ 0{,}06615ee \sin 2s$	8,820508	$+$ 4
$+ 0{,}02332ee \sin(2\eta - 2s)$	8,367825	$+$ 1
$+ 0{,}00840ee \sin(2\eta + 2s)$	7,924429	$+ \frac{1}{2}$
$+ 0{,}74760ek \sin(r - s)$	9,873165	$+$ 141
$- 0{,}61850ek \sin(r + s)$	9,791317	$-$ 118
$+ 0{,}81430ek \sin(2\eta - r + s)$	9,910800	$+$ 154
$- 0{,}01420ek \sin(2\eta + r - s)$	8,150690	$-$ 3
$+ 0{,}23960ek \sin(2\eta - r - s)$	9,379550	$+$ 45
$- 0{,}00610ek \sin(2\eta + r + s)$	7,788910	$-$ 1
$- 0{,}24427\nu \sin\eta$	9,387868	$-$ 175
$+ 0{,}00639\nu \sin 3\eta$	7,805991	$+$ 4
$+ 1{,}1959\nu k \sin(\eta - r)$	0,077694	$+$ 59
$+ 0{,}0757\nu k \sin(\eta + r)$	8,879096	$+$ 4
$- 0{,}3939\nu e \sin(\eta - s)$	9,597508	$-$ 5
$- 4{,}1738\nu e \sin(\eta + s)$	0,620530	$-$ 49

CAPUT XII.

INUESTIGATIO INAEQUALITATUM MOTUM LINEAE NODORUM AFFICIENTIUM.

§. 206.

Antequam reliquas motus Lunae inaequalitates, quae ab inclinatione eius orbitae ad eclipticam pendent, definire licet, cum variationes, quae in motu lineae nodorum Lunae, tum eas, quae in ipſa inclinatione eius orbitae ad eclipticam deprehenduntur, inueſtigari oportet. Reſidua enim pars aequationis noſtrae principalis, qua omnes motus Lunae inaequalitates continentur, litteras π et ϱ implicat, quarum illa longitudinem nodi aſcendentis, haec vero ϱ inclinationem ad eclipticam deſignat. Niſi igitur vtriusque huius quantitatis incrementa vel decrementa ad differentiale dr reduxerimus, reſiduas motus Lunae inaequalitates determinare non poterimus.

207. Aequatio autem ſupra (55) pro motu lineae nodorum tradita, cum ſit $\frac{2\varkappa v + \int R\, dr}{nn} = u + \frac{1+2ee}{nn} \cdot \frac{d\Phi}{dr}$

ideoque $\frac{2\varkappa v + \int R\, dr}{nn} = a' \cos 2\eta - \left(\frac{2}{n} - c'\right) k \cos r + d' k \cos(2\eta - r)$

$$+ e' k \cos(2\eta + r)$$
$$+ p' e \cos s + q' e \cos(2\eta - s)$$
$$+ r' e \cos(2\eta + s)$$

ſi ponamus breuitatis gratia $\frac{3(1+2kk+\frac{3}{2}ee)}{\varkappa\varkappa\, nn} = i$, vt ſit $i = 0,0168918$, induet formam ſequentem:

$$\frac{d\pi}{dr} = -i\left(\varkappa + \alpha' \cos 2\eta - \left(\frac{2}{n} - \iota'\right) k \cos r \begin{matrix} + \delta' k \cos(2\eta - r) + \text{etc.} \\ + \epsilon' k \cos(2\eta + r) \end{matrix}\right)$$

$$\left(\begin{matrix} 1 + 4k\cos r + 5kk\cos 2r - 6ek\cos(r-s) \\ -3e\cos s + \frac{3}{2}ee\cos 2s - 6ek\cos(r+s) \end{matrix}\right)\left(\tfrac{3}{4} + \tfrac{3}{4}\cos 2\eta \begin{matrix} -\frac{3}{4}\cos(2\Phi - 2\pi) \\ -\frac{3}{4}\cos(2\theta - 2\pi) \end{matrix}\right)$$

$$-\frac{\varkappa i \nu}{4}\left(\tfrac{15}{4}\cos\eta + \tfrac{5}{4}\cos 3\eta - \tfrac{5}{2}\cos(\Phi+\theta-2\pi) \begin{matrix} -\frac{5}{4}\cos(3\Phi - \theta - 2\pi) \\ -\frac{5}{4}\cos(3\theta - \Phi - 2\pi) \end{matrix}\right)$$

vbi quidem plurimi termini tam fiunt parui, vt facile negligi queant.

§. 208. Productum autem ex duobus prioribus factoribus, quoniam id in formula pro inclinatione recurrit, ſeorſim exhibeamus: fiet id autem reiectis terminis, qui prae reliquis admodum ſunt parui vt ſequitur:

	$-\varkappa i + 2i\left(\frac{2}{n} - \iota'\right)kk - \frac{3}{2}ip'ee$	$-0,017043$
$\cos 2\eta$	$(-i\alpha' - 2i\alpha' kk$	$+0,000161 \cos 2\eta$
$k\cos r$	$\left(i\left(\frac{2}{n} - \iota'\right) - 4\varkappa i\right.$	$-0,068110\, k\cos r$
$k\cos(2\eta - r)$	$(-i\delta' - 2i\alpha'$	$-0,005791\, k\cos(2\eta - r)$
$k\cos(2\eta + r)$	$(-i\epsilon' - 2i\alpha'$	$+0,000828\, k\cos(2\eta + r)$
$kk\cos 2r$	$\left(-5\varkappa i + 2i\left(\frac{2}{n} - \iota'\right)\right.$	$-0,085091\, kk\cos 2r$
$e\cos s$	$(-ip' + 3\varkappa i$	$+0,051362\, e\cos s$
$e\cos(2\eta - s)$	$(iq' + \frac{3}{2}i\alpha'$	$-0,000929\, e\cos(2\eta - s)$
$e\cos(2\eta + s)$	$(-ir' + \frac{3}{2}i\alpha'$	$-0,000804\, e\cos(2\eta + s)$
$ee\cos 2s$	$(-\frac{3}{2}\varkappa i + \frac{3}{2}ip'$	$-0,025914\, ee\cos 2s$

§. 209.

§. 209. His valoribus ſubſtitutis prodibit

$\frac{d\pi}{dr} = -$	$0,004261 + 0,000020$
$\cos 2\eta$	$(+0,000040 - 0,004261$
$k\cos r$	$(-0,017043 - 0,000620$
$k\cos(2\eta - r)$	$(-0,001448 - 0,008514) - 0,010636\, kk\cos(2\eta - 2r)$
$k\cos(2\eta + r)$	$(+0,000207 - 0,008514) - 0,010636\, kk\cos(2\eta + 2r)$
$kk\cos 2r$	$(-0,021273$
$e\cos s$	$(+0,012840 - 0,000217$
$e\cos(2\eta - s)$	$(-0,000232 + 0,006420) - 0,003239\, ee\cos(2\eta - 2s)$
$e\cos(2\eta + s)$	$(-0,000201 + 0,006420) - 0,003239\, ee\cos(2\eta + 2s)$
$ee\cos 2s$	$(-0,006479)$
$\cos 2(\Phi - \pi)$	$(+0,003261 - 0,000020) - 0,000041 \cos\eta$ $+ 0,000019 \cos(3\Phi - \theta - 2\pi)$
$\cos 2(\theta - \pi)$	$(+0,004261 - 0,000020) - 0,000019 \cos 3\eta$ $+ 0,000019 \cos(3\theta - \Phi - 2\pi)$
$k\cos(2\Phi - 2\pi - r)$	$(+0,008514 + 0,000724)$ $+ 0,000022 \cos(\Phi + \theta - 2\pi)$
$k\cos(2\Phi - 2\pi + r)$	$(+0,008514 - 0,000103)$ $+ 0,010636\, kk\cos 2(\Phi - v - \pi)$
$k\cos(2\theta - 2\pi - r)$	$(+0,008514 - 0,000103)$
$k\cos(2\theta - 2\pi + r)$	$(+0,008514 + 0,000724)$
$e\cos(2\Phi - 2\pi - s)$	$(-0,006420 + 0,000116)$
$e\cos(2\Phi - 2\pi + s)$	$(-0,006420 + 0,000100)$
$e\cos(2\theta - 2\pi - s)$	$(-0,006420 + 0,000100)$ $+ 0,003239\, ee\cos 2(\theta - s - \pi)$
$e\cos(2 - \theta 2\pi + s)$	$(-0,006420 + 0,000116)$

§. 110.

§. 210. Habebimus ergo

$$\begin{aligned}
\frac{d\pi}{dr} = &-0{,}004241 & &-0{,}000041 \cos\eta \\
&-0{,}004221 \cos 2\eta & &-0{,}000019 \cos 3\eta \\
&-0{,}017663\, k \cos r & &+0{,}000020 \cos 4\eta \\
&-0{,}009962\, k \cos(2\eta-r) & &+0{,}004241 \cos(2\Phi-2\pi) \\
&-0{,}008307\, k \cos(2\eta+r) & &+0{,}004241 \cos(2\theta-2\pi) \\
&-0{,}010636\, kk \cos(2\eta-2r) & &+0{,}000022 \cos(\Phi+\theta-2\pi) \\
&-0{,}010636\, kk \cos(2\eta+2r) & &+0{,}000019 \cos(3\Phi-\theta-2\pi) \\
&-0{,}021273\, kk \cos r & &+0{,}000019 \cos(3\theta-\Phi-2\pi) \\
&+0{,}012623\, e \cos s \\
&+0{,}006188\, e \cos(2\eta-s) \\
&+0{,}006219\, e \cos(2\eta+s) \\
&-0{,}006479\, ee \cos 2s \\
&-0{,}003239\, ee \cos(2\eta-2s) \\
&-0{,}003239\, ee \cos(2\eta+2s) \\
&+0{,}009238\, k \cos(2\Phi-2\pi-r) & &-0{,}006304\, e \cos(2\Phi-2\pi-s) \\
&+0{,}008411\, k \cos(2\Phi-2\pi+r) & &-0{,}006320\, e \cos(2\Phi-2\pi+s) \\
&+0{,}008411\, k \cos(2\theta-2\pi-r) & &-0{,}006320\, e \cos(2\theta-2\pi-s) \\
&+0{,}009238\, k \cos(2\theta-2\pi+r) & &-0{,}006304\, e \cos(2\theta-2\pi+s) \\
&+0{,}010636\, kk \cos(2\Phi-2\pi-2r) & &+0{,}003239\, ee \cos(2\theta-2\pi-2s)
\end{aligned}$$

§. 211. Quanquam plurimi horum terminorum tam sunt parui, vt in se spectati tuto reiici possent, tamen quidam per integrationem ad magnitudinem satis notabilem excrescere possunt. Huius autem indolis sunt illi termini, qui eiusmodi complectuntur angulos, quorum

$-2\pi)$ $-2\pi)$ $\theta-2\pi)$ $-\theta-2\pi)$ $\Phi-2\pi)$ $-2\pi-s)$ $-2\pi+s)$ $-2\pi-s)$ $2\pi+s)$ $2\pi-2s)$ torum nt, ta- a satis is sunt , quo- rum

rum differentialia ad dr admodum paruam tenent rationem, cuiusmodi sunt anguli s, $2s$, $2\theta-2\pi$, $2\theta-2\pi-s$, $2\theta-2\pi+s$, $2\Phi-2\pi-2r$ et $2\theta-2\pi-2s$; quorum natura differentialium ex sequentibus formulis colligi potest:

$$\frac{d\eta}{dr} = \alpha - a'\cos 2\eta - c'k\cos r - d'k\cos(2\eta-r) - e'k\cos(2\eta+r)$$
$$+\left(\frac{2}{n}-p'\right) e\cos s - q'e\cos(2\eta-s) - r'e\cos(2\eta+s)$$

$$\frac{d\Phi}{dr} = \alpha + \frac{1+2ee}{n} - a'\cos 2\eta + \left(\frac{2}{n}-c'\right) k\cos r$$
$$- d'k\cos(2\eta-r) - p'e\cos s - q'e\cos(2\eta-s)$$
$$- e'k\cos(2\eta+r) \qquad - r'e\cos(2\eta+s)$$

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n}k\cos r - \frac{2}{n}e\cos s$$

§. 212. Quaeramus primo inaequalitates motus nodorum, quae neque ab excentricitate orbitae lunaris neque solaris pendent, sitque:

$$\pi = \text{Const.} - \mathfrak{O}r + \mathfrak{A}\sin 2\eta + \mathfrak{B}\sin(2\Phi-2\pi) + \mathfrak{C}\sin(2\theta-2\pi)$$

reiectis reliquis terminis, quos praeuidemus fore minimos, ac differentiando obtinebimus:

$$\frac{d\pi}{dr} = -\mathfrak{O} - \mathfrak{A}a' - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C}$$
$$+\cos 2\eta\,(2\alpha\mathfrak{A} - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C}$$
$$+\cos(2\Phi-2\pi)\,\left(2\left(\alpha+\frac{1}{n}\right)\mathfrak{B} + 0{,}008482\,\mathfrak{B} + 0{,}004221\,\mathfrak{C}\right.$$
$$+\cos(2\theta-2\pi)\,\left(-\mathfrak{B}a' + 0{,}004221\,\mathfrak{B} + \frac{2\mathfrak{C}}{n} + 0{,}008482\,\mathfrak{C}\right.$$

vnde

vnde oritur:

$$O - 0{,}019744\,\mathfrak{A} + 0{,}004241\,\mathfrak{B} + 0{,}004241\,\mathfrak{C} = 0{,}004241$$
$$1{,}867476\,\mathfrak{A} - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C} = -0{,}004221$$
$$2{,}026834\,\mathfrak{B} + 0{,}004221\,\mathfrak{C} = 0{,}004241$$
$$0{,}023965\,\mathfrak{B} + 0{,}159358\,\mathfrak{C} = 0{,}004241$$

§. 213. Valores hinc igitur prodibunt sequentes:

$$O = +0{,}004078 \quad \ldots \quad l\,O = 7{,}610447$$
$$\mathfrak{A} = -0{,}002196 \quad \ldots \quad l{-}\mathfrak{A} = 7{,}341634$$
$$\mathfrak{B} = +0{,}002037 \quad \ldots \quad l\,\mathfrak{B} = 7{,}308991$$
$$\mathfrak{C} = +0{,}026307 \quad \ldots \quad l\,\mathfrak{C} = 8{,}420081$$

Vbi primum obseruo valorem ipsius O iam proxime accedere ad motum medium lunae nodorum, vti per obseruationes constat; inde enim esse deberet $O = 0{,}004053$ facile autem intelligitur, hunc exiguum defectum per reliquas inaequalitates suppleri posse. Quocirca hinc erit

	Valores in min. sec.
$\pi = \text{Const.} - 0{,}004078\,r$	
$-0{,}002196 \sin 2\eta$	$-\ 453''$
$+0{,}002037 \sin(2\phi - 2\pi)$	$+\ 420$
$+0{,}026307 \sin(2\theta - 2\pi)$	$+\ 5426$

quae inaequalitates mirifice conueniunt cum obseruationibus. His addi potest terminus:

$$+0{,}000336 \sin(4\theta - 4\pi)$$

cuius in minutis secundis valor est $+69''$, qui terminus cum postremo illo facile coniungi potest.

§. 214. Quaeramus iam seorsim inaequalitates, quae ab excentricitate orbitae lunaris pendent, sitque

$\pi =$

$$\pi = \text{Const.} - Or + \mathfrak{A}\sin 2\eta + \mathfrak{B}\sin(2\phi - 2\pi) + \mathfrak{C}\sin(2\theta - 2\pi)$$
$$+ \mathfrak{D}k\sin(2\eta - r) + \mathfrak{E}k\sin(2\eta + r) + \mathfrak{F}k\sin r$$
$$+ \mathfrak{G}kk\sin(2\eta - 2r) + \mathfrak{H}kk\sin 2r + \mathfrak{J}k\sin(2\phi - 2\pi - r)$$
$$+ \mathfrak{K}k\sin(2\phi - 2\pi + r) + \mathfrak{L}k\sin(2\theta - 2\pi - r) + \mathfrak{M}k\sin(2\theta - 2\pi + r)$$
$$+ \mathfrak{N}kk\sin(2\phi - 2\pi - 2r)$$

eritque differentiando

$$\frac{d\pi}{dr} = \text{Pr.} + k\cos(2\eta - r)(-\mathfrak{A}c' - 0{,}009238\,\mathfrak{B} - 0{,}009238\,\mathfrak{C} + (2\alpha - 1)\mathfrak{D}$$
$$+ k\cos(2\eta + r)(-\mathfrak{A}c' - 0{,}008411\,\mathfrak{B} - 0{,}008411\,\mathfrak{C} + (2\alpha + 1)\mathfrak{E}$$
$$+ k\cos r(-\mathfrak{A}d' - \mathfrak{A}e' - 0{,}017649\,\mathfrak{B} - 0{,}017649\,\mathfrak{C} + \mathfrak{F} - \mathfrak{D}a' - \mathfrak{E}a'$$
$$+ kk\cos 2r \quad (-0{,}010636\,\mathfrak{B} + 2\mathfrak{H} - \mathfrak{D}e' - \mathfrak{E}d' - \mathfrak{G}a'$$
$$+ kk\cos(2\eta - 2r)(-0{,}010636\,\mathfrak{C} + 2(\alpha - 1)\mathfrak{G} - \mathfrak{D}c'$$

$$+ k\cos(2\theta - 2\pi - r)\left\{\begin{array}{l} -\mathfrak{B}c' + 0{,}008307\,\mathfrak{B} + \frac{2}{n}\mathfrak{C} + 0{,}017663\,\mathfrak{C} \\ \qquad + \left(\frac{2}{n} - 1\right)\mathfrak{L} + 0{,}008482\,\mathfrak{L} \end{array}\right.$$

$$+ k\cos(2\theta - 2\pi + r)\left\{\begin{array}{l} -\mathfrak{B}d' + 0{,}009962\,\mathfrak{B} + \frac{2}{n}\mathfrak{C} + 0{,}017663\,\mathfrak{C} \\ \qquad + \left(\frac{2}{n} + 1\right)\mathfrak{M} + 0{,}008482\,\mathfrak{M} \end{array}\right.$$

$$+ k\cos(2\phi - 2\pi - r)\left\{\begin{array}{l} +0{,}017663\,\mathfrak{B} + 0{,}009962\,\mathfrak{C} + 2\left(\alpha + \frac{1}{n}\right)\mathfrak{J} \\ \qquad - \mathfrak{J} + 0{,}008482\,\mathfrak{J} \end{array}\right.$$

$$+ k\cos(2\phi - 2\pi + r)\left\{\begin{array}{l} +0{,}017663\,\mathfrak{B} + 0{,}008307\,\mathfrak{C} + 2\left(\alpha + \frac{1}{n}\right)\mathfrak{K} \\ \qquad + \mathfrak{K} + 0{,}008482\,\mathfrak{K} \end{array}\right.$$

$$+ kk\cos(2\phi - 2\pi - 2r)\left\{\begin{array}{l} +0{,}021273\,\mathfrak{B} + 0{,}010636\,\mathfrak{C} + 2\left(\alpha + \frac{1}{n}\right)\mathfrak{N} \\ \qquad - 2\mathfrak{N} + 0{,}008482\,\mathfrak{N} \end{array}\right.$$

 §. 215.

§. 215. Superfluum foret maiorem curam in his differentialibus adhibere, quia vero proxime tantum rem determinare sufficit; erit ergo:

$$0,867476\,\mathfrak{D} = -0,009962$$
$$2,867476\,\mathfrak{E} = -0,008307$$

Hincque et in min sec.

$\mathfrak{D} = -0,011480$	$\mathfrak{D}k = -129''$
$\mathfrak{E} = -0,002900$	$\mathfrak{E}k = -33''$

quae inaequalitates in loco nodi vix alicuius sunt momenti, vnde eas exactius determinare non est opus.

§. 216. Calculo autem euoluto erit

$\pi = \text{Pr.}$	$-\ 0,011480k\ \sin(2\eta - r)$	8,059940	$-129''$
	$-\ 0,002900k\ \sin(2\eta + r)$	7,462400	$-\ 33$
	$-\ 0,017663k\ \sin r$	8,247064	-198
	$+\ 0,090497kk\ \sin(2\eta - 2r)$	8,956634	$+\ 55$
	$-\ 0,011978kk\ \sin 2r$	8,078384	$-\ 7$
	$+\ 0,008707k\ \sin(2\Phi - 2\pi - r)$	7,939851	$+\ 98$
	$+\ 0,002701k\ \sin(2\Phi - 2\pi + r)$	7,431516	$+\ 30$
	$-\ 0,004680k\ \sin(2\theta - 2\pi - r)$	7,670224	$-\ 53$
	$+\ 0,004685k\ \sin(2\theta - 2\pi + r)$	7,670680	$+\ 53$
	$+\ 0,384848kk\ \sin(2\Phi - 2\pi - 2r)$	9,585289	$+235$

§. 217. Simili modo inuestigemus inaequalitates motus nodorum, quae pendent ab excentricitate orbitae solaris sitque:

$\pi =$

$$\pi = \text{Const.} - \mathfrak{O}r + \mathfrak{A}\sin 2\eta + \mathfrak{B}\sin(2\Phi - 2\pi) + \mathfrak{C}\sin(2\theta - 2\pi)$$
$$+ \mathfrak{D}e\sin s + \mathfrak{E}e\sin(2\eta - s) + \mathfrak{F}e\sin(2\eta + s)$$
$$+ \mathfrak{G}ee\sin 2s + \mathfrak{H}e\sin(2\Phi - 2\pi - s) + \mathfrak{K}e\sin(2\theta - 2\pi - s)$$
$$+ \mathfrak{J}e\sin(2\Phi - 2\pi + s) + \mathfrak{L}e\sin(2\theta - 2\pi + s)$$
$$+ \mathfrak{M}ee\sin(2\theta - 2\pi - 2s)$$

vnde differentiando pro terminis quaesitis erit:

$$\frac{d\pi}{dr} = \text{Praec.}$$

$$+ e\cos s \begin{cases} -\mathfrak{A}q' - \mathfrak{A}r' + 0,006304\mathfrak{B} + 0,006320\mathfrak{C} + \frac{1}{n}\mathfrak{D} - \mathfrak{E}s' - \mathfrak{F}s' \\ + 0,006320\mathfrak{B} + 0,006304\mathfrak{C} \end{cases}$$

$$+ e\cos(2\eta - s)\left\{\mathfrak{A}\left(\frac{2}{n} - p'\right) + 0,006304\mathfrak{B} + 0,006304\mathfrak{C} + \left(2\alpha - \frac{1}{n}\right)\mathfrak{E}\right.$$

$$+ e\cos(2\eta + s)\left\{\mathfrak{A}\left(\frac{2}{n} - p'\right) + 0,006320\mathfrak{B} + 0,006320\mathfrak{C} + \left(2\alpha + \frac{1}{n}\right)\mathfrak{F}\right.$$

$$+ ee\cos 2s\left\{-0,003239\,\mathfrak{C} - \frac{1}{n}\mathfrak{D} - \mathfrak{E}r' - \mathfrak{F}q' + \frac{2}{n}\mathfrak{G}\right.$$

$$+ e\cos(2\Phi - 2\pi - s)\begin{cases} -\mathfrak{B}p' - 0,012623\,\mathfrak{B} - 0,006188\,\mathfrak{C} \\ + \left(2\alpha \pm \frac{1}{n} + 0,008482\right)\mathfrak{H} \end{cases}$$

$$+ e\cos(2\Phi - 2\pi + s)\begin{cases} -\mathfrak{B}p' - 0,012623\,\mathfrak{B} - 0,006219\,\mathfrak{C} \\ + \left(2\alpha + \frac{3}{n} + 0,008482\right)\mathfrak{J} \end{cases}$$

+

$$+ s \cos(2\theta - 2\pi - s)\left\{\begin{array}{l} -\mathfrak{B}r' - 0{,}006219\mathfrak{B} - \frac{2}{n}\mathfrak{C} - 0{,}012623\mathfrak{C} \\ +\left(\frac{1}{n} + 0{,}008482\right)\mathfrak{K} \end{array}\right.$$

$$+ e \cos(2\theta - 2\pi + s)\left\{\begin{array}{l} -\mathfrak{B}q' - 0{,}006188\mathfrak{B} - \frac{2}{n}\mathfrak{C} - 0{,}012623\mathfrak{C} \\ +\left(\frac{3}{n} + 0{,}008482\right)\mathfrak{L} \end{array}\right.$$

$$+ ee \cos(2\theta - 2\pi - 2s)\left\{\begin{array}{l} +0{,}003239\mathfrak{B} + \frac{1}{2n}\mathfrak{C} + 0{,}006479\mathfrak{C} \\ +0{,}008482\mathfrak{M} - \frac{1}{n}\mathfrak{K} - 0{,}012623\mathfrak{K} \end{array}\right.$$

§. 218. Hinc reperiuntur sequentes valores

$\mathfrak{D} = 0{,}159070$.	$l\,\mathfrak{D} = 9{,}201585$;	$\mathfrak{D}e = 551''$
$\mathfrak{E} = 0{,}003562$.	$l\,\mathfrak{E} = 7{,}551680$;	$\mathfrak{E}e = 12\frac{1}{2}$
$\mathfrak{F} = 0{,}003301$.	$l\,\mathfrak{F} = 7{,}518677$;	$\mathfrak{F}e = 11\frac{1}{4}$
$\mathfrak{G} = 0{,}031650$.	$l\,\mathfrak{G} = 8{,}587191$;	$\mathfrak{G}ee = 2''$
$\mathfrak{H} = -0{,}003153$.	$l - \mathfrak{H} = 7{,}498692$;	$\mathfrak{H}e = -11''$
$\mathfrak{I} = -0{,}002932$.	$l - \mathfrak{I} = 7{,}467118$;	$\mathfrak{I}e = -10''$
$\mathfrak{K} = -0{,}025750$.	$l - \mathfrak{K} = 8{,}410784$;	$\mathfrak{K}e = -90$
$\mathfrak{L} = -0{,}009076$.	$l - \mathfrak{L} = 7{,}957885$;	$\mathfrak{L}e = -32$

At valor ipsius $\mathfrak{M}$ tam sit paruus, vt merito pro nihilo haberi possit.

§. 219.

§. 219. Colligamus ergo has inaequalitates in vnam summam, atque obtinebimus longitudinem veram nodi ascendentis

		Valor. in minut. sec.
$\pi = $ Const.	$-0{,}004053\ r$	
	$-0{,}002196 \sin 2\eta$	$-\ 453''$
	$+0{,}002037 \sin(2\Phi - 2\pi)$	$+\ 420$
	$+0{,}026307 \sin(2\theta - 2\pi)$	$+\ 5426$
	$+0{,}000370 \sin(4\theta - 4\pi)$	$+\ 75$
	$-0{,}01766k \sin r$	$-\ 198$
	$-0{,}01148k \sin(2\eta - r)$	$-\ 129$
	$-0{,}00290k \sin(2\eta + r)$	$-\ 33$
	$+0{,}0905kk \sin(2\eta - 2r)$	$+\ 55$
	$-0{,}0120kk \sin 2r$	$-\ 7$
	$+0{,}00871k \sin(2\Phi - 2\pi - r)$	$+\ 98$
	$+0{,}00270k \sin(2\Phi - 2\pi + r)$	$+\ 30$
	$-0{,}00468k \sin(2\theta - 2\pi - r)$	$-\ 53$
	$+0{,}00468k \sin(2\theta - 2\pi + r)$	$+\ 53$
	$+0{,}3848kk \sin(2\Phi - 2\pi - 2r)$	$+\ 235$
	$+0{,}15907e \sin s$	$+\ 551$
	$-0{,}02575e \sin(2\theta - 2\pi - s)$	$-\ 90$
	$-0{,}00907e \sin(2\theta - 2\pi + s)$	$-\ 32$

omissis scilicet iis inaequalitatibus, quae non supra 30'' exsurgunt.

CAPUT XIII.

INUESTIGATIO INCLINATIONIS ORBITAE LUNARIS AD ECLIPTICAM.

§ 220.

Pro inclinatione orbitae lunaris ad eclipticam inuenienda, forma §. 208. euoluta multiplicari debet per $-\frac{1}{4}\sin 2\eta + \frac{1}{4}\sin 2(\Phi-\pi) + \frac{1}{4}\sin 2(\theta-\pi)$, ac productum erit $= \frac{d.\,l\,\text{tang}\,\varrho}{dr}$: Hinc ergo habebitur:

$$\frac{d.\,l\,\text{tang}\,\varrho}{dr} = \begin{aligned}[t] &+0{,}004261 \sin 2\eta \qquad -0{,}000020\sin 4\eta\\ &+0{,}008514k \sin(2\eta-r)\\ &+0{,}008514k \sin(2\eta+r)\\ &+0{,}000827k \sin r\\ &-0{,}004261 \sin(2\Phi-2\pi) \quad \text{adiice}\\ &-0{,}004261 \sin(2\theta-2\pi) \quad \text{ad coeff.}\\ &-0{,}008514k \sin(2\Phi-2\pi-r)-0{,}000724\\ &-0{,}008514k \sin(2\Phi-2\pi+r)+0{,}000103\\ &-0{,}008514k \sin(2\theta-2\pi-r)+0{,}000103\\ &-0{,}008514k \sin(2\theta-2\pi+r)-0{,}000724\\ &-0{,}010636kk\sin(2\Phi-2\pi-2r)\\ &+0{,}006420\,e \sin(2\theta-2\pi-s)-0{,}000100\\ &+0{,}006420\,e \sin(2\theta-2\pi+s)-0{,}000116 \end{aligned}$$

§. 221.

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§. 221.

§. 221. Quaeramus primo terminos, qui a neutra excentricitate pendent, sitque

$$l\frac{\text{tang}\,\varrho}{\text{tang}\,\epsilon} =$$
$$\mathfrak{A}\cos 2\eta + \mathfrak{a}\cos 4\eta + \mathfrak{B}\cos(2\Phi-2\pi) + \mathfrak{C}\cos(2\theta-2\pi) + \mathfrak{c}\cos(4\theta-4\pi)$$

eritque differentiando:

$$\frac{d\,l\,\text{tang}\,\varrho}{dr} = \begin{aligned}[t] &\sin 2\eta\,[-2\alpha\mathfrak{A} + 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C}\\ &\sin 4\eta\,[-4\alpha\mathfrak{a} + \mathfrak{A}\alpha'\\ &\sin(2\Phi-2\pi)\left\{-2(\alpha+\tfrac{1}{n})\mathfrak{B} - 0{,}008482\mathfrak{B} - 0{,}004227\mathfrak{C}\right.\\ &\sin(2\theta-2\pi)\left\{-\tfrac{2}{n}\mathfrak{C} + \mathfrak{B}\alpha' - 0{,}004221\mathfrak{B} - 0{,}008482\mathfrak{C}\right.\\ &\sin(4\theta-4\pi)\left\{+0{,}004241\,\mathfrak{C} - \tfrac{4}{n}\,\mathfrak{c}\right. \end{aligned}$$

§. 222. Ex his iam reperitur:

$\mathfrak{A} = -0{,}002630$ . . . $l-\mathfrak{A} = 7{,}419914$
$\mathfrak{B} = +0{,}002037$ . . . $l\,\mathfrak{B} = 7{,}308991$
$\mathfrak{C} = +0{,}026307$ . . . $l\,\mathfrak{C} = 8{,}420081$
$\mathfrak{a} = +0{,}000019$ . . . $l\,\mathfrak{a} = 5{,}278753$
$\mathfrak{c} = +0{,}000370$ . . . $l\,\mathfrak{c} = 6{,}567931$

ita vt hinc sit:

$$l\frac{\text{tang}\,\varrho}{\text{tang}\,\epsilon} = \begin{aligned}[t] &-0{,}002630 \cos 2\eta\\ &+0{,}000019 \cos 4\eta\\ &+0{,}002037 \cos(2\Phi-2\pi)\\ &+0{,}026307 \cos(2\theta-2\pi)\\ &+0{,}000370 \cos(4\theta-4\pi) \end{aligned}$$

§. 223.

§. 223. Quaeramus iam seorsim terminos ab excentricitate Lunae pendentes: sitque

$$l\frac{\tang\varrho}{\tang\varepsilon} = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos(2\Phi-2\pi) + \mathfrak{C}\cos(2\theta-2\pi)$$
$$+\mathfrak{D}k\cos(2\eta-r) + \mathfrak{E}k\cos(2\eta+r) + \mathfrak{F}k\cos r$$
$$+\mathfrak{G}k\cos(2\Phi-2\pi-r) + \mathfrak{J}k\cos(2\theta-2\pi-r)$$
$$+\mathfrak{H}k\cos(2\Phi-2\pi+r) + \mathfrak{K}k\cos(2\theta-2\pi+r)$$
$$+\mathfrak{L}k^2\cos(2\Phi-2\pi-2r)$$

vnde differentialibus sumendis habebitur: $\frac{d.\, l\tang\varrho}{dr} =$

$$k\sin(2\eta-r)\begin{cases}+\mathfrak{A}c'+0{,}09238\,\mathfrak{B}-0{,}009238\,\mathfrak{C}+0{,}004241\,\mathfrak{G}\\ +(2\alpha-1)\mathfrak{D}-0{,}004241\,\mathfrak{K}\end{cases}$$

$$k\sin(2\eta+r)\begin{cases}+\mathfrak{A}c'+0{,}008411\,\mathfrak{B}-0{,}008411\,\mathfrak{C}+(2\alpha+1)\mathfrak{E}\\ \mp 0{,}004241\,\mathfrak{H}-0{,}004241\,\mathfrak{J}\end{cases}$$

$$k\sin r\begin{cases}+\mathfrak{A}d'-\mathfrak{A}c'+0{,}000827\,\mathfrak{B}-0{,}000827\,\mathfrak{C}-\mathfrak{F}-0{,}004241\,\mathfrak{G}\\ +0{,}004241\,\mathfrak{H}-\mathfrak{D}a'+\mathfrak{E}a'-0{,}004241\,\mathfrak{J}+0{,}004241\,\mathfrak{K}\end{cases}$$

$$k\sin(2\Phi-2\pi-r)\begin{cases}-0{,}017663\,\mathfrak{B}-0{,}009962\,\mathfrak{C}-2(\alpha+\frac{1}{n})\mathfrak{G}+\mathfrak{G}\\ -0{,}008482\,\mathfrak{G}-0{,}004221\,\mathfrak{J}\end{cases}$$

$$k\sin(2\Phi-2\pi+r)\begin{cases}-0{,}017663\,\mathfrak{B}-0{,}008307\,\mathfrak{C}-2(\alpha+\frac{1}{n})\mathfrak{H}-\mathfrak{H}\\ -0{,}008482\,\mathfrak{H}-0{,}004221\,\mathfrak{K}\end{cases}$$

$$k\sin(2\theta-2\pi-r)\begin{cases}+\mathfrak{B}e'-0{,}008307\,\mathfrak{B}-\frac{2}{n}\mathfrak{C}-0{,}017663\,\mathfrak{C}\\ +\mathfrak{G}a'-0{,}004221\,\mathfrak{G}-\frac{2}{n}\mathfrak{J}+\mathfrak{J}-0{,}008482\,\mathfrak{J}\end{cases}$$

k sin

$$k\sin(2\theta-2\pi+r)\begin{cases}+\mathfrak{B}d'-0{,}009962\,\mathfrak{B}-\frac{2}{n}\mathfrak{C}-0{,}017663\,\mathfrak{C}\\ +\mathfrak{H}a'-0{,}004221\,\mathfrak{H}-\frac{2}{n}\mathfrak{K}-\mathfrak{K}-0{,}008482\,\mathfrak{K}\end{cases}$$

$$kk\sin(2\Phi-2\pi-2r)\begin{cases}-0{,}021273\,\mathfrak{B}-0{,}010636\,\mathfrak{C}-2(\alpha+\frac{1}{n})\mathfrak{L}+2\mathfrak{L}\\ -0{,}008482\,\mathfrak{L}\end{cases}$$

hincque reperitur:

$\mathfrak{D} = 0{,}010487$	. . .	$l\,\mathfrak{D} = 8{,}020638$
$\mathfrak{E} = 0{,}003166$	. . .	$l\,\mathfrak{E} = 7{,}500439$
$\mathfrak{F} = -0{,}001600$	. . .	$l{-}\mathfrak{F} = 7{,}204120$
$\mathfrak{G} = +0{,}008719$	. . .	$l\,\mathfrak{G} = 7{,}940484$
$\mathfrak{H} = +0{,}002699$	. . .	$l\,\mathfrak{H} = 7{,}431136$
$\mathfrak{J} = -0{,}004460$	. . .	$l{-}\mathfrak{J} = 7{,}649305$
$\mathfrak{K} = +0{,}004717$	. . .	$l\,\mathfrak{K} = 7{,}623628$
$\mathfrak{L} = +0{,}384890$	. . .	$l\,\mathfrak{L} = 9{,}585335$

§. 224. Nunc denique pro inaequalitatibus ab excentricitate orbitae solaris pendentibus ponatur.

$$l\frac{\tang\varrho}{\tang\varepsilon} = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos(2\Phi-2\pi) + \mathfrak{M}e\cos(2\theta-2\pi-s)$$
$$+\mathfrak{C}\cos(2\theta-2\pi) + \mathfrak{N}e\cos(2\theta-2\pi+s)$$

ac differentiando prodibit: $\frac{d.\, l\tang\varrho}{dr} =$

$$e\sin(2\theta-2\pi-s)\begin{cases}+\mathfrak{B}r'+0{,}006219\,\mathfrak{B}+\frac{2}{n}\mathfrak{C}+0{,}012623\,\mathfrak{C}\\ -\frac{1}{n}\mathfrak{M}-0{,}008482\,\mathfrak{M}\end{cases}$$

e sin

$$e\sin(2\theta-2\pi+s)\left\{\begin{array}{l}+\mathfrak{B}q'+0{,}06188\,\mathfrak{B}+\frac{2}{n}\mathfrak{C}+0{,}012623\,\mathfrak{C}\\ -\frac{3}{n}\mathfrak{N}-0{,}008482\,\mathfrak{N}\end{array}\right.$$

vnde reperitur

$$\mathfrak{M}=-0{,}024034 \quad . \quad . \quad l\text{-}\mathfrak{M}=8{,}380835$$

$$\mathfrak{N}=-0{,}008519 \quad . \quad . \quad l\text{-}\mathfrak{N}=7{,}930332$$

§. 225. Si ergo ε denotet inclinationem mediam orbitae lunaris ad eclipticam, et ϱ inclinationem veram, erit

			log. coeff.
$l\,\frac{\tang\varrho}{\tang\varepsilon}=$	$-0{,}002630$	$\cos 2\eta$	7,419915
	$+0{,}000019$	$\cos 4\eta$	5,278753
	$+0{,}002037$	$\cos(2\Phi-2\pi)$	7,308991
	$+0{,}026307$	$\cos(2\theta-2\pi)$	8,420081
	$+0{,}000370$	$\cos(4\theta-4\pi)$	6,567931
	$+0{,}01049k$	$\cos(2\eta-r)$	8,020638
	$+0{,}00317k$	$\cos(2\eta+r)$	7,500439
	$-0{,}00160k$	$\cos r$	7,204120
	$+0{,}00872k$	$\cos(2\Phi-2\pi-r)$	7,940484
	$+0{,}00270k$	$\cos(2\Phi-2\pi+r)$	7,431136
	$-0{,}00446k$	$\cos(2\theta-2\pi-r)$	7,649305
	$+0{,}00472k$	$\cos(2\theta-2\pi+r)$	7,673628
	$+0{,}3849kk$	$\cos(2\Phi-2\pi-2r)$	9,585335
	$-0{,}02403e$	$\cos(2\theta-2\pi-s)$	8,380835
	$-0{,}00852e$	$\cos(2\theta-2\pi+s)$	7,930332

§. 226.

§. 226. Quodsi iam ponatur $l\,\frac{\tang\varrho}{\tang\varepsilon}=S$, erit ad numeros ipsos procedendo $\frac{\tang\varrho}{\tang\varepsilon}=1+S+\frac{1}{2}SS$ Hinc igitur negligendo terminos minimos, consequemur:

$$\frac{\tang\varrho}{\tang\varepsilon}=1-0{,}002604\cos 2\eta$$
$$+0{,}000020\cos 4\eta$$
$$+0{,}002003\cos(2\Phi-2\pi)$$
$$+0{,}026307\cos(2\theta-2\pi)$$
$$+0{,}000490\cos(4\theta-4\pi)$$
$$-0{,}00160k\cos r$$
$$+0{,}01049k\cos(2\eta-r)$$
$$+0{,}00317k\cos(2\eta+r)$$
$$+0{,}00885k\cos(2\Phi-2\pi-r)$$
$$+0{,}00472k\cos(2\Phi-2\pi+r)$$
$$-0{,}00448k\cos(2\theta-2\pi-r)$$
$$+0{,}00470k\cos(2\theta-2\pi+r)$$
$$+0{,}3849kk\cos(2\Phi-2\pi-2r)$$
$$-0{,}02403e\cos(2\theta-2\pi-s)$$
$$-0{,}00852e\cos(2\theta-2\pi+s)$$

§. 227.

§. 227. Cum in aequatione noſtra principali, quae motum Lunae continet, inſit terminus $\frac{\text{tang } \varrho^2}{\text{tang } \epsilon^2}$, huius quoque valorem euolui conueniet: erit ergo

$$\frac{\text{tang } \varrho^2}{\text{tang } \epsilon^2} = 1$$
$$- 0,005156 \text{ coſ } 2\eta \quad + 0,000040 \text{ coſ } 4\eta$$
$$+ 0,003938 \text{ coſ}(2\Phi - 2\pi)$$
$$+ 0,052614 \text{ coſ}(2\theta - 2\pi) + 0,001320 \text{ coſ}(4\theta - 4\pi)$$
$$- 0,00290k \text{ coſ } r$$
$$+ 0,02098k \text{ coſ}(2\eta - r)$$
$$+ 0,00634k \text{ coſ}(2\eta + r)$$
$$+ 0,01796k \text{ coſ}(2\Phi - 2\pi - r)$$
$$+ 0,00556k \text{ coſ}(2\Phi - 2\pi + r)$$
$$- 0,00896k \text{ coſ}(2\theta - 2\pi - r)$$
$$+ 0,00940k \text{ coſ}(2\theta - 2\pi + r)$$
$$+ 0,7698kk \text{ coſ}(2\Phi - 2\pi - 2r)$$
$$+ 0,04806e \text{ coſ}(2\theta - 2\pi - s)$$
$$+ 0,01704e \text{ coſ}(2\theta - 2\pi + s)$$

Hicque ergo valor in ſuperiori illa aequatione ſubſtitui poterit.

quae
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η
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tui
8.

§. 228. Celeb. autem Clairaut concluſit inclinationem mediam ϵ ex obſeruationibus exquiſitiſſimis 5° 8′ 9″, ex qua igitur ad quoduis tempus inclinationem veram elicere licebit. Sit enim $\varrho = \epsilon + \omega$, erit tang $\varrho = \frac{\text{tg } \epsilon + \omega}{1 - \omega \text{ tg } \epsilon}$ $= \text{tang } \epsilon + \frac{\omega}{\text{coſ } \epsilon^2} = V \text{ tang } \epsilon$, ponendo V pro expreſſione ipſius $\frac{\text{tang } \varrho}{\text{tang } \epsilon}$. Hinc erit $\omega = (V-1) \text{ ſin } \epsilon \text{ coſ } \epsilon = \frac{1}{2}(V-1) \text{ ſin } 2\epsilon = 0,08915(V-1)$: vnde reperitur in minutis ſecundis

$$\varrho = \epsilon - 48'' \text{ coſ } 2\eta$$
$$+ 36 \text{ coſ } (2\Phi - 2\pi)$$
$$+ 484 \text{ coſ } (2\theta - 2\pi)$$
$$+ 9 \text{ coſ } (4\theta - 4\pi)$$
$$- 2 \text{ coſ } r$$
$$+ 11 \text{ coſ } (2\eta - r)$$
$$+ 3 \text{ coſ } (2\eta + r)$$
$$+ 9 \text{ coſ } (2\Phi - 2\pi - r)$$
$$+ 3 \text{ coſ } (2\Phi - 2\pi + r)$$
$$- 5 \text{ coſ } (2\theta - 2\pi - r)$$
$$+ 5 \text{ coſ } (2\theta - 2\pi + r)$$
$$+ 23 \text{ coſ } (2\Phi - 2\pi - 2r)$$
$$- 7 \text{ coſ } (2\theta - 2\pi - s)$$
$$- 3 \text{ coſ } (2\theta - 2\pi + s)$$

§. 229. Hic notandum eſt, etiamſi valor inclinationis mediae ϵ aliquantillum immutetur, aequationes has tamen inde vix alterari, ita vt eae ſemper eaedem ſint manſurae. Perſpicuum quoque eſt in calculo aſtronomico ſufficere tres inaequalitates primores, et reliquas omnes ſine errore ſenſibili praetermitti poſſe; niſi forte aequatio 23 coſ $(2\phi-2\pi-2r)$ retinenda cenſeatur, quae inter reliquas eſt maxima. Exprimit autem angulus $2\phi-2\pi-2r$ duplam diſtantiam apogei Lunae ab eius nodo, a quo angulo quoque locum nodi non mediocriter affici vidimus, cum correctio hinc oriunda pro loco nodi vsque ad $235''$ aſſurgere poſſit.

inclina- ationes eaedem aſtro- eliquas niſi for- ſeatur, angu- ab eius medio- la pro

PUT

CAPUT XIV.

INUESTIGATIO INAEQUALITATUM MOTUS LUNAE AB EIUS INCLINATIONE AD ECLIPTICAM ORIUNDARUM.

§. 230.

Ponamus more adhuc vſitato:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{C}k\cos r + \mathfrak{D}k\cos(2\eta-r) + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta-s)$$
$$+ \mathfrak{E}k\cos(2\eta+r) \qquad + \mathfrak{R}e\cos(2\eta+s)$$
$$+ \mathfrak{F}f\cos 2\eta + \mathfrak{G}f\cos(2\phi-2\pi) + \mathfrak{J}fk\cos r + \mathfrak{K}fk\cos(2\eta-r)$$
$$+ \mathfrak{H}f\cos(2\theta-2\pi) \qquad + \mathfrak{L}fk\cos(2\eta+r)$$
$$+ \mathfrak{M}fk\cos(2\phi-2\pi-r) + \mathfrak{S}fk\cos(2\theta-2\pi-r)$$
$$+ \mathfrak{N}fk\cos(2\phi-2\pi+r) + \mathfrak{T}fk\cos(2\theta-2\pi+r)$$
$$+ \mathfrak{O}fkk\cos(2\phi-2\pi-2r) + \mathfrak{U}fe\cos(2\theta-2\pi-s)$$
$$+ \mathfrak{V}fe\cos(2\theta-2\pi+s)$$

$$\text{et } v = A\cos 2\eta \;..\; + Dk\cos(2\eta-r) + Pe\cos s + Qe\cos(2\eta-s)$$
$$+ Ek\cos(2\eta+r) \qquad + Re\cos(2\eta+s)$$
$$+ Ff\cos 2\eta + Gf\cos(2\phi-2\pi) + Jfk\cos r + Kfk\cos(2\eta-r)$$
$$+ Hf\cos(2\theta-2\pi) \qquad + Lfk\cos(2\eta+r)$$
$$+ Mfk\cos(2\phi-2\pi-r) + Sfk\cos(2\theta-2\pi-r)$$
$$+ Nfk\cos(2\phi-2\pi+r) + Tfk\cos(2\theta-2\pi+r)$$
$$+ Ofkk\cos(2\phi-2\pi-2r) + Ufe\cos(2\theta-2\pi-s)$$
$$+ Vfe\cos(2\theta-2\pi+s)$$

§. 231. His valoribus fubftitutis in formula §. 52. orietur

$$R = f\sin 2\eta \Big(\;\ldots\; \quad fk\sin(2\theta-2\pi-r)\Big(-\frac{3M}{2nn}-\frac{3G}{nn}$$

$$f\sin(2\Phi-2\pi)\Big(+\frac{3H}{2nn} \;\ldots\; fk\sin(2\theta-2\pi+r)\Big(-\frac{3N}{2nn}-\frac{3G}{nn}$$

$$f\sin(2\theta-2\pi)\Big(-\frac{3G}{2nn} \;\ldots\; fe\sin(2\theta-2\pi-s)\Big(+\frac{9G}{4nn}$$

$$fk\sin r\Big(+\frac{3K}{2nn}-\frac{3L}{2nn} \;\ldots\; fe\sin(2\theta-2\pi+s)\Big(+\frac{9G}{4nn}$$

$$fk\sin(2\eta-r)\Big(+\frac{3I}{2nn}$$

$$fk\sin(2\eta-r)\Big(+\frac{3I}{2nn}$$

$$fk\sin(2\Phi-2\pi-r)\Big(+\frac{3S}{2nn}+\frac{3H}{nn}$$

$$fk\sin(2\Phi-2\pi+r)\Big(+\frac{3T}{2nn}+\frac{3H}{nn}$$

$$fk\sin(2\Phi-2\pi-2r)\Big(+\frac{3S}{nn}$$

§. 232. Altera vero aequatio fundamentalis induet formam fequentem:

$$\frac{ddv}{dr^2} = \text{Praec.} + f\cos 2\eta \;[-6F - 2\kappa\mathfrak{F} - 0{,}005156 - 0{,}026307$$
$$+ f\cos(2\Phi-2\pi) \;[-6G - 2\kappa\mathfrak{G} + 0{,}003938 - 1$$
$$+ f\cos(2\theta-2\pi) \;[-6H - 2\kappa\mathfrak{H} + 0{,}052614 + 0{,}002578$$

la §. 52.
3M 3G / 2nn nn
3N 3G / 2nn nn
9G / 4nn
9G / 4nn
induet
026307
578

$$+ fk\cos r \;[-6J - 2\kappa\mathfrak{J} - 0{,}00290 - 0{,}00895$$
$$-0{,}00098 - 0{,}00278 - 0{,}00098$$
$$+ fk\cos(2\eta-r) \;[-6K + \tfrac{1}{2}bF - 2\kappa\mathfrak{K} + 0{,}02098 - 0{,}00470$$
$$- 0{,}00258 - 0{,}01315$$
$$+ fk\cos(2\eta+r) \;[-6L + \tfrac{1}{2}bF - 2\kappa\mathfrak{L} + 0{,}00634 + 0{,}00448$$
$$- 0{,}00258 - 0{,}01315$$
$$+ fk\cos(2\Phi-2\pi-r) \;[-6M + \tfrac{1}{2}bG - 2\kappa\mathfrak{M} + 0{,}01796 + 0{,}00145$$
$$+ 0{,}00197 - \tfrac{x}{2}$$
$$+ fk\cos(2\Phi-2\pi+r) \;[-6N + \tfrac{1}{2}bG - 2\kappa\mathfrak{N} + 0{,}00556 + 0{,}00145$$
$$+ 0{,}00197 - \tfrac{x}{2}$$
$$+ fkk\cos(2\Phi-2\pi-2r) \;[-6O + \tfrac{1}{2}bM + \tfrac{3}{2}G - 2\kappa\mathfrak{O} + 0{,}7698 + 0{,}0089$$
$$+ 0{,}0010 + 0{,}0007 - \tfrac{3}{4}$$
$$+ fk\cos(2\theta-2\pi-r) \;[-6S + \tfrac{1}{2}bH - 2\kappa\mathfrak{S} - 0{,}00896 - 0{,}00317$$
$$+ 0{,}02631 + 0{,}00129$$
$$+ fk\cos(2\theta-2\pi+r) \;[-6T + \tfrac{1}{2}bH - 2\kappa\mathfrak{T} + 0{,}00940 - 0{,}01049$$
$$+ 0{,}02631 + 0{,}00129$$
$$+ fe\cos(2\theta-2\pi-s) \;[-6U - 2\kappa\mathfrak{U} - 0{,}04806$$
$$+ fe\cos(2\theta-2\pi+s) \;[-6V - 2\kappa\mathfrak{V} - 0{,}01704$$

§. 233. Quoniam manifeftum eft, coefficientes F, G, H etc. admodum fore paruos; cum maxima huius generis inaequalitas aliquot minuta prima non excedat, hi iidem coefficientes per nn diuifi tam euadent paruí, vt fine errore reiici queant. Hoc autem facto quoque litterae germanicae $\mathfrak{F}$, $\mathfrak{G}$, $\mathfrak{H}$ etc. pro nihilo effent habendae, ex quo fola pofterior aequatio differentio-differentialis refoluenda fupererit; in qua ob eandem rationem terminos ex diuifione coefficientium per nn oriundos omifimus, cum in tam operofo calculo fuffi-

 ciat

ciat correctiones inde resultantes proxime saltem determinasse; praesertim cum haec praetermissio vix ad aliquot minuta secunda sit ascensura.

§. 234. Ob eandem rationem licebit in valoribus differentialium $\frac{d\Phi}{dr}$ et $\frac{d\eta}{dr}$ particulas ab inclinatione pendentes negligere, vnde erit: $\frac{dv}{dr} =$

$f \sin 2\eta\ [-2\alpha F = -F']$

$f \sin(2\phi - 2\pi)\ [-2(\alpha + \frac{1}{n})G - 0,008482\,G = -G']$

$f \sin(2\theta - 2\pi)\ [+G\epsilon' - \frac{2}{n})H - 0,008482\,H = -H']$

$fk \sin r\ [Fd' - Fe' - J = -J']$

$fk \sin(2\eta - r)\ [+Fe' - (2\alpha - 1)K = -K']$

$fk \sin(2\eta + r)\ [+Fe' - (2\alpha + 1)L = -L']$

$fk \sin(2\phi - 2\pi - r)\ [-2(\alpha + \frac{1}{n})M + M - 0,008482\,M = -M']$

$fk \sin(2\phi - 2\pi + r)\ [-2(\alpha + \frac{1}{n})N - N - 0,008482\,N = -N']$

$fkk \sin(2\phi - 2\pi - 2r)\ [-2(\alpha + \frac{1}{n})O + 2O - 0,008482\,O = -O']$

$fk \sin(2\theta - 2\pi - r)\ [+G\epsilon' - \frac{2}{n}H - \frac{2}{n}S + S - 0,008482\,S = -S']$

$fk \sin(2\theta - 2\pi + r)\ [+Gd' - \frac{2}{n}H - \frac{2}{n}T - T - 0,008482\,T = -T']$

$fe \sin(2\theta - 2\pi - s)\ [+G\gamma' + \frac{2}{n}H - \frac{1}{n}U - 0,008482\,U = -U']$

$fe \sin(2\theta - 2\pi + s)\ [+Gg' + \frac{2}{n}H - \frac{3}{n}V - 0,008482\,V = -V']$

§. 235.

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§.

§. 235. Si nunc simili modo denuo differentiemus, prodibit: $\frac{ddv}{dr^2} =$

$f \cos 2\eta\ [-2\alpha F']$

$f \cos(2\phi - 2\pi)\ [-2(\alpha + \frac{1}{n})G' - 0,008482\,G']$

$f \cos(2\theta - 2\pi)\ [+G'\epsilon' - \frac{2}{n}H' - 0,008482\,H']$

$fk \cos r\ [F'd' + Fe' - J']$

$fk \cos(2\eta - r)\ [F'e' - (2\alpha - 1)K']$

$fk \cos(2\eta + r)\ [F'e' - (2\alpha + 1)L']$

$fk \cos(2\phi - 2\pi - r)\ [-2(\alpha + \frac{1}{n})M' + M' - 0,008482\,M']$

$fk \cos(2\phi - 2\pi + r)\ [-2(\alpha + \frac{1}{n})N' - N' - 0,008482\,N']$

$fkk \cos(2\phi - 2\pi - 2r)\ [-2(\alpha + \frac{1}{n})O' + 2O' - 0,008482\,O']$

$fk \cos(2\theta - 2\pi - r)\ [G'\epsilon' - \frac{2}{n}H' - \frac{2}{n}S' + S' - 0,008482\,S']$

$fk \cos(2\theta - 2\pi + r)\ [G'd' - \frac{2}{n}H' - \frac{2}{n}T' - T' - 0,008482\,T']$

$fe \cos(2\theta - 2\pi - s)\ [G'\gamma' + \frac{2}{n}H' - \frac{1}{n}U' - 0,008482\,U']$

$fe \cos(2\theta - 2\pi + s)\ [G'g' + \frac{2}{n}H' - \frac{3}{n}V' - 0,008482\,V']$

§. 236. Hinc autem sequentes eliciuntur valores

F =	0,01273	...	l F = 8,104833
G =	0,32213	...	l G = 9,508032
H =	0,06976	...	l H = 8,843590
J =	−1,87800	...	l−J = 0,273710

K =

$K = +0,05615$ . . . $l\text{-}K = 8,749352$
$L = -0,00077$ . . . $l\text{-}L = 6,888904$
$M = -0,29638$ . . . $l\text{-}M = 9,471854$
$N = +0,00012$ . . . $l\,N = 6,089109$
$O = +0,32287$ . . . $l\,O = 9,509034$
$S = +0,39091$ . . . $l\,S = 9,592073$
$T = +0,69579$ . . . $l\,T = 9,842475$
$U = -0,07922$ . . . $l\text{-}U = 8,898830$
$V = -0,05141$ . . . $l\text{-}V = 8,711093$

§. 237. Pro diſtantia ergo lunae a ſole curtata $x = \frac{(1-kk)au}{1-k\cos r}$ erit

$$u = \text{Praec.}$$

		Log. coeff.	Valores coeff. integri
$+ 0,000072f$	$\cos 2\eta$	5,860017	$+ 0,000079$
$+ 0,001833f$	$\cos(2\Phi-2\pi)$	7,263216	$+ 0,002005$
$+ 0,000397f$	$\cos(2\theta-2\pi)$	6,598774	$+ 0,000434$
$- 0,01069fk$	$\cos r$	8,028894	$- 0,000634$
$+ 0,00032fk$	$\cos(2\eta-r)$	6,504536	$+ 0,000019$
$- 0,00000fk$	$\cos(2\eta+r)$	4,644088	$- 0,000000$
$- 0,00169fk$	$\cos(2\Phi-2\pi-r)$	7,227038	$- 0,000100$
$+ 0,00000fk$	$\cos(2\Phi-2\pi+r)$	3,834293	$+ 0,000000$
$+ 0,00184fk^2$	$\cos(2\Phi-2\pi-2r)$	7,264218	$+ 0,000006$
$+ 0,00223fk$	$\cos(2\theta-2\pi-r)$	7,347257	$+ 0,000132$
$+ 0,00396fk$	$\cos(2\theta-2\pi+r)$	7,597659	$+ 0,000235$
$- 0,00045fe$	$\cos(2\theta-2\pi-s)$	6,654014	$- 0,000000$
$- 0,00029fe$	$\cos(2\theta-2\pi+s)$	6,466277	$- 0,000000$

vbi notandum eſt eſſe $f = 1,093756$, et $l\,f = 0,038921$.

§. 238.

§. 238. Deinde pro motu momentaneo habebitur

$$\frac{d\Phi}{dr} = \text{Praec.}$$

		Log. coeff.	Valores coeff. in numeris.
$- 0,000146f$	$\cos 2\eta$	6,164934	$- 0,000160$
$- 0,003700f$	$\cos(2\Phi-2\pi)$	7,568133	$- 0,004046$
$- 0,000801f$	$\cos(2\theta-2\pi)$	6,903691	$- 0,000876$
$+ 0,02157fk$	$\cos r$	8,333811	$+ 0,001286$
$- 0,00065fk$	$\cos(2\eta-r)$	6,809453	$- 0,000038$
$+ 0,00000fk$	$\cos(2\eta+r)$	4,949005	$+ 0,000001$
$+ 0,00340fk$	$\cos(2\Phi-2\pi-r)$	7,531955	$+ 0,000203$
$- 0,00000fk$	$\cos(2\Phi-2\pi+r)$	4,139210	$- 0,000000$
$- 0,00371fk^2$	$\cos(2\Phi-2\pi-2r)$	7,569135	$- 0,000012$
$- 0,00449fk$	$\cos(2\theta-2\pi-r)$	7,652174	$- 0,000267$
$- 0,00799fk$	$\cos 2\theta-2\pi+r)$	7,902576	$- 0,000476$
$+ 0,00091fe$	$\cos 2\theta-2\pi-s)$	6,958931	$+ 0,000000$
$+ 0,00059fe$	$\cos(2\theta-2\pi+s)$	6,771194	$+ 0,000000$

§. 239. Pro correctione longitudinis verae hinc oriunda ponatur,

$$\Phi = \text{Praec.}$$

$$+ \mathfrak{A}'f\sin 2\eta \quad + \mathfrak{J}'fk\sin r \quad + \mathfrak{M}'fk\sin(2\Phi-2\pi-r)$$
$$+ \mathfrak{G}'f\sin(2\Phi-2\pi) + \mathfrak{K}'fk\sin(2\eta-r) + \mathfrak{N}'fk\sin(2\Phi-2\pi+r)$$
$$+ \mathfrak{H}'f\sin(2\theta-2\pi) + \mathfrak{L}'fk\sin(2\eta+r) + \mathfrak{O}'fkk\sin(2\Phi-2\pi-2r)$$
$$+ \mathfrak{S}'fk\sin(2\theta-2\pi-r) \; + \; \mathfrak{U}'fe\sin(2\theta-2\pi-s)$$
$$+ \mathfrak{T}'fk\sin(2\theta-2\pi+r) \; + \; \mathfrak{V}'fe\sin(2\theta-2\pi+s)$$

eris.

eritque:

$$2\alpha\,\mathfrak{F}' = -0{,}000146$$
$$0{,}026834\,\mathfrak{G}' = -0{,}003700$$
$$0{,}159358\,\mathfrak{H}' - \mathfrak{G}'a' = -0{,}000801$$
$$\mathfrak{J}' - \mathfrak{F}'d' - \mathfrak{F}'e' = +0{,}02157$$
$$0{,}867476\,\mathfrak{K}' - \mathfrak{F}'c' = -0{,}00065$$
$$2{,}867476\,\mathfrak{L}' - \mathfrak{F}'c' = -0{,}00000$$
$$1{,}026834\,\mathfrak{M}' = +0{,}00340$$
$$3{,}026834\,\mathfrak{N}' = -0{,}00000$$
$$0{,}026834\,\mathfrak{O}' = -0{,}00371$$
$$-0{,}840642\,\mathfrak{S}' - \mathfrak{G}'e' + \frac{2}{n}\,\mathfrak{H}' = -0{,}00449$$
$$+1{,}159358\,\mathfrak{T}' - \mathfrak{G}'d' + \frac{2}{n}\,\mathfrak{H}' = -0{,}00799$$
$$+0{,}083920\,\mathfrak{U}' - \mathfrak{G}'r' - \frac{2}{n}\,\mathfrak{H}' = +0{,}00091$$
$$+0{,}234796\,\mathfrak{V}' - \mathfrak{G}'q' - \frac{2}{n}\,\mathfrak{H}' = +0{,}00059$$

§. 240. Expeditis igitur his formulis orietur:

	Log. coeff.	coeff. tot. in sec.
$\Phi = \text{Pr.} - 0{,}000078 f \sin 2\eta$	5,893680	$-$ 18″
$-0{,}001825 f \sin(2\Phi - 2\pi)$	7,261316	$-$ 422
$-0{,}004800 f \sin(2\theta - 2\pi)$	7,681286	$-$ 1083
$+0{,}02154 fk \sin r$	8,333246	$+$ 264
$-0{,}00074 fk \sin(2\eta - r)$	6,867925	$-$ 9
$+0{,}00332 fk \sin(2\Phi - 2\pi - r)$	7,520465	$+$ 41
$-0{,}13818 fk^2 \sin(2\Phi - 2\pi - 2r)$	9,140450	$-$ 92
$+0{,}00446 fk \sin(2\theta - 2\pi - r)$	7,649421	$+$ 55
$-0{,}00685 fk \sin(2\theta - 2\pi + r)$	7,835624	$-$ 84
$+0{,}00310 fe \sin(2\theta - 2\pi - s)$	7,491107	$+$ 11
$-0{,}00030 fe \sin(2\theta - 2\pi + s)$	6,474418	$-$ 1

§. 242.

r:
coeff. tot. in sec.
- 18″
- 422
-1083
- 264
- 9
- 41
- 92
- 55
- 84
- 11
- 1
§. 242.

§. 241. Haec omnia satis conueniunt cum notis inaequalitatibus motus lunae, nisi quod inaequalitas ab angulo $2\theta - 2\pi$ pendens plane aduersari videatur, cum nullum eius vestigium in tabulis astronomicis occurrat; quod quidem eo magis est mirandum, cum correctio inde oriunda ad 18′, 3″ exsurgat. Lubens equidem agnosco, in hoc calculo non omnem curam esse adhibitam, vt hanc aequationem tanquam omnibus numeris absolutam spectare liceat, quoniam ad plurimos terminos, quos formulae nostrae suppeditant, non respexi. Interim tamen calculum repetenti mox patebit, non admodum enormiter esse aberratum, praesertim cum aequatio ab angulo $2\Phi - 2\pi$ pendens, quae pari passu procedit, veritati perquam consentanea prodierit, cum ea reductio lunae ad eclipticam contineatur. Ac si quidem haec inaequalitas ad semissem vsque diminuatur, tamen tanta remanet, vt merito dubitare debeamus, eius effectum ab Astronomis non esse animaduersum; cum eius omissio vix per aliam aequationem compensari queat. Hancobrem, siue omissio terminorum neglectorum sit in causa, siue etiam in calculo numerico error fuerit admissus, quod facile euenire potuit, istam inuestigationem in capite sequenti accuratius suscipiamus.

CAPUT XV.

ACCURATIOR INUESTIGATIO INAEQUALITATUM LUNAE AB INCLINATIONE EJUS ORBITAE PENDENTIUM.

§. 242.

Quoniam praecipuum dubium circa inaequalitatem ab angulo $2\theta-2\pi$ pendentem verſatur, noſtram inueſtigationem ab iis inaequalitatibus, quae ſimul ab alterutra excentricitate pendent, abſtrahamus. Ponamus ergo:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{F} f\cos 2\eta + \mathfrak{G} f\cos(2\Phi-2\pi) + \mathfrak{H} f\cos(2\theta-2\pi) + \mathfrak{J} f\cos(4\theta-4\pi)$$

et $$v = A\cos 2\eta + F f\cos 2\eta + G f\cos(2\Phi-2\pi) + H f\cos(2\theta-2\pi) + J f\cos(4\theta-4\pi)$$

Poſitoque $\frac{2\varkappa F+\mathfrak{F}}{nn}=f'$; $\frac{2\varkappa G+\mathfrak{G}}{nn}=g'$; $\frac{2\varkappa H+\mathfrak{H}}{nn}=h'$; $\frac{2\varkappa J+\mathfrak{J}}{nn}=i'$ erit:

$$\frac{d\Phi}{dr} = \alpha + \frac{1}{\varkappa} - \alpha'\cos 2\eta - f' f\cos 2\eta - g' f\cos(2\Phi-2\pi) - h' f\cos(2\theta-2\pi) - i' f\cos(4\theta-4\pi)$$

$$\frac{d\eta}{dr} = \alpha - \alpha'\cos 2\eta - f' f\cos 2\eta - g' f\cos(2\Phi-2\pi) - h' f\cos(2\theta-2\pi) - i' f\cos(4\theta-4\pi)$$

$$\frac{d\theta}{dr} = \frac{1}{n} \text{ et } \frac{d\pi}{dr} = -0{,}004241 - 0{,}004221\cos 2\eta + 0{,}004241\cos(2\Phi-2\pi) + 0{,}004241\cos(2\theta-2\pi)$$

§. 243.

UALI- JS litatem oſtram ſimul Po- θ−4π) θ−4π) =h'; −4π) θ−4π) cos 2η 243.

§. 243. His valoribus in formulis principalibus ſubſtitutis habebimus has aequationes:

$$R = -\frac{3G}{2nn} f\sin(2\theta-2\pi) + \frac{3H}{2nn} f\sin(2\Phi-2\pi)$$

$$\frac{ddv}{dr^2} = f\cos 2\eta\left\{-\beta F - 2\varkappa\mathfrak{F}\right.$$

$$f\cos(2\Phi-2\pi)\left\{\begin{array}{l} -\beta G + \frac{3H}{4nn} - 2\varkappa\mathfrak{G} + \frac{\mathfrak{A}\mathfrak{H}}{nn} + \frac{3A\mathfrak{H}}{nn} \\ + \frac{3\mathfrak{A}H}{nn} + \frac{3AH}{nn} \end{array}\right.$$

$$f\cos(2\theta-2\pi)\left\{\begin{array}{l} -\beta H + \frac{3G}{4nn} - 2\varkappa\mathfrak{H} + \frac{\mathfrak{A}\mathfrak{G}}{nn} + \frac{3A\mathfrak{G}}{nn} \\ + \frac{3\mathfrak{A}G}{nn} + \frac{3AG}{nn} \end{array}\right.$$

$$f\cos(4\theta-4\pi)\left[-\beta J - 2\varkappa\mathfrak{J}\right.$$

$$f\cos 2\eta\left(-0{,}005156 - 0{,}026307 + 0{,}001969\frac{A}{nn}\right)$$

$$f\cos(2\Phi-2\pi)\left(+0{,}003938 - 1 - 0{,}052614\frac{A}{nn} - 0{,}002578\frac{A}{nn}\right)$$

$$f\cos(2\theta-2\pi)\left(+0{,}052614 + 0{,}002578 - 0{,}001969\frac{2A}{nn} + \frac{A}{nn}\right)$$

$$f\cos(4\theta-4\pi)\left(+0{,}001320 + 0{,}026307\frac{A}{nn}\right)$$

§. 244. Vel in numeris erit

$$R = 0{,}008540 H f\sin(2\Phi-2\pi) - 0{,}008540 G f\sin(2\theta-2\pi)$$

 $ddv =$

$$\frac{ddv}{dr^2} =\!=\!=$$

$$f \cos 2\eta \;[-1{,}01591\,F - 2\varkappa\,\mathfrak{F} - 0{,}031478$$

$$f\cos(2\Phi-2\pi)\begin{cases}-1{,}01591\,G - 2\varkappa\mathfrak{G} + 0{,}000636\,H - 0{,}845648\\ -0{,}027110\,\mathfrak{H}\end{cases}$$

$$f\cos(2\theta-2\pi)\begin{cases}-1{,}01591\,H - 2\varkappa\mathfrak{H} + 0{,}000636\,G + 0{,}047722\\ -0{,}027110\,\mathfrak{G}\end{cases}$$

$$f\cos(4\theta-4\pi)[-1{,}01591\,J - 2\varkappa\mathfrak{J} + 0{,}001123$$

Nunc autem ex formulis assumtis erit

$$R = f \sin 2\eta \;[-2\,\alpha\,\mathfrak{F} + 0{,}004241\,\mathfrak{G} - 0{,}004241\,\mathfrak{H}]$$
$$f\sin(2\Phi-2\pi)[+\mathfrak{A}\,b' - 2{,}026834\,\mathfrak{G} + 0{,}004221\,\mathfrak{H}]$$
$$f\sin(2\theta-2\pi)[-\mathfrak{A}\,g' - 0{,}023965\,\mathfrak{G} - 0{,}159358\,\mathfrak{H}]$$
$$f\sin(4\theta-4\pi)[+0{,}004241\,\mathfrak{H} - 0{,}318716\,\mathfrak{J}]$$

at est

$\mathfrak{A}b' = -0{,}00931H - 0{,}00461\mathfrak{H}$; et $\mathfrak{A}g' = -0{,}00931G - 0{,}00461\mathfrak{G}$

vnde fit:

$$1{,}867476\,\mathfrak{F} = 0{,}004241\,(\mathfrak{G}-\mathfrak{H})$$
$$2{,}026874\,\mathfrak{G} = -0{,}01785\,H - 0{,}00039\,\mathfrak{H}$$
$$0{,}159358\,\mathfrak{H} = +0{,}01785\,G - 0{,}01935\,\mathfrak{G}$$
$$0{,}318716\,\mathfrak{J} = +0{,}004241\,\mathfrak{H}$$

§. 245. Tum simili modo differentiando valorem ipsius v ponatur

$$F' = 1{,}867476\,F - 0{,}004241\,G + 0{,}004241\,H$$
$$G' = 2{,}026834\,G + 0{,}01091\,H + 0{,}00750\,\mathfrak{H}$$
$$H' = 0{,}159358\,H + 0{,}03909\,G - 0{,}00750\,\mathfrak{G}$$
$$J' = 0{,}318716\,F - 0{,}004241\,H$$

vt

45648 47722 41 $\mathfrak{H}$] 21 $\mathfrak{H}$] 58 $\mathfrak{H}$] 461 $\mathfrak{G}$)) lorem H $\mathfrak{H}$ $\mathfrak{G}$ vt

vt fit $\frac{dv}{dr} =\!=$

$$A' \sin 2\eta - F' f \sin 2\eta - G' f \sin(2\Phi-2\pi) - H' f \sin(2\theta-2\pi) - J' f \sin(4\theta-4\pi)$$

eritque

$$\frac{ddv}{dr^2} = f \cos 2\eta \;[-2\alpha F' + 0{,}004241\,G' + 0{,}004241\,H']$$
$$f\cos(2\Phi-2\pi)[+A'b' - 2{,}026834\,G' - 0{,}004221\,H']$$
$$f\cos(2\Phi-2\pi)[+A'g' - 0{,}159358\,H' - 0{,}023965\,G']$$
$$f\cos(4\theta-4\pi)[+0{,}004241\,H' - 0{,}318716\,J']$$

seu $\frac{ddv}{dr^2} =\!=\!=$

$$f\cos 2\eta \begin{cases}-3{,}48745\,F + 0{,}01668\,G - 0{,}00721\,H - 0{,}00003\,\mathfrak{G}\\ +0{,}00007\,\mathfrak{H}\end{cases}$$
$$f\cos(2\Phi-2\pi)\,[-4{,}10820\,G - 0{,}05121\,H - 0{,}02929\,\mathfrak{H} + 0{,}0000,\mathfrak{G}$$
$$f\cos(2\theta-2\pi)\,[-0{,}02565\,H - 0{,}08323\,G - 0{,}01290\,\mathfrak{G} - 0{,}00018\,\mathfrak{H}$$
$$f\cos(4\theta-4\pi)\,[-0{,}10158\,J + 0{,}00016\,G + 0{,}00202\,H - 0{,}00003\,\mathfrak{G}$$

§. 246. Hinc pro $\mathfrak{F}$ et $\mathfrak{J}$ substitutis valoribus

$$\mathfrak{F} = 0{,}00227\,(\mathfrak{G}-\mathfrak{H}) \quad\text{et}\quad \mathfrak{J} = 0{,}01331\,\mathfrak{H}$$

habebimus has aequationes

$$+2{,}47154\,F - 0{,}01668\,G + 0{,}00721\,H - 0{,}00456\,\mathfrak{G} + 0{,}00456\,\mathfrak{H} = 0{,}031478$$
$$+3{,}09229\,G + 0{,}05185\,H + 0{,}00218\,\mathfrak{H} - 2{,}01799\,\mathfrak{G} = 0{,}995648$$
$$-0{,}99026\,H + 0{,}08387\,G - 0{,}01421\,\mathfrak{G} - 2{,}01780\,\mathfrak{H} = -0{,}047722$$
$$-0{,}91433\,J - 0{,}00016\,G - 0{,}00202\,H + 0{,}00003\,\mathfrak{G} - 0{,}02685\,\mathfrak{H} = -0{,}001123$$

§. 247.

§. 247. Deinde reperitur

$$\mathfrak{G} = -0{,}00881\,H - 0{,}00002\,G$$
$$\mathfrak{H} = +0{,}11201\,G + 0{,}00107\,H$$

qui valores substituti praebent:

$$+2{,}47154\,F - 0{,}01618\,G + 0{,}00725\,H = 0{,}031478$$
$$+3{,}09256\,G + 0{,}06964\,H = 0{,}995648$$
$$+0{,}99229\,H + 0{,}14239\,G = 0{,}047722$$
$$-0{,}91430\,J - 0{,}00297\,G - 0{,}00205\,H = -0{,}001123$$

vnde tandem reperitur

$F = +0{,}014830 \quad . \; . \; . \quad l\,F = 8{,}171166$
$G = +0{,}321910 \quad . \; . \; . \quad l\,G = 9{,}507734$
$H = +0{,}001901 \quad . \; . \; . \quad l\,H = 7{,}278927$
$J = +0{,}000180 \quad . \; . \; . \quad l\,J = 6{,}256381$

atque

$\mathfrak{F} = -0{,}000080 \quad . \; . \; . \quad l{-}\mathfrak{F} = 5{,}903090$
$\mathfrak{G} = -0{,}000023 \quad . \; . \; . \quad l{-}\mathfrak{G} = 5{,}361728$
$\mathfrak{H} = +0{,}036056 \quad . \; . \; . \quad l\,\mathfrak{H} = 8{,}556977$
$\mathfrak{J} = +0{,}000480 \quad . \; . \; . \quad l\,\mathfrak{J} = 6{,}681241$

§. 248. Hinc ergo pro distantia $x = \frac{(1-kk)\,au}{1-k\cos r}$ reperitur

			log.coeff.	val.coeff.
$x = \text{Pr.}$	$+\,0{,}000084 f$	$\cos 2\eta$	5,926350	+0,000092
	$+\,0{,}001832 f$	$\cos(2\Phi-2\pi)$	7,262918	+0,002004
	$+\,0{,}000011 f$	$\cos(2\theta-2\pi)$	5,034111	+0,000012
	$+\,0{,}000001 f$	$\cos(4\theta-4\pi)$	4,011565	+0,000001

At

At pro motu momentaneo erit

			log.coeff.	val.coeff.
$\frac{d\Phi}{dr} = \text{Pr.}$	$-\,0{,}000169 f$	$\cos 2\eta$	6,227887	−0,000185
	$-\,0{,}003697 f$	$\cos(2\Phi-2\pi)$	7,567849	−0,004043
	$-\,0{,}000227 f$	$\cos(2\theta-2\pi)$	6,356026	−0,000248
	$-\,0{,}000005 f$	$\cos(4\theta-4\pi)$	4,698970	−0,000005

vnde quidem iam patet inaequalitatem ab angulo $2\theta-2\pi$ pendentm multo fore minorem, quam supra inueneramus, in quo non paruum veritatis criterium cernitur.

§. 249. Pro ipsa iam longitudine lunae ponamus:

$$\Phi = \text{Pr.} + \mathfrak{A}' \sin 2\eta + \mathfrak{F}' f \sin 2\eta + \mathfrak{G}' f \sin(2\Phi-2\pi) + \mathfrak{H}' f \sin(2\theta-2\pi) + \mathfrak{J}' f \sin(4\theta-4\pi)$$

atque obtinebimus has aequationes:

$$2\,a\,\mathfrak{F}' - 0{,}004241\,(\mathfrak{G}' + \mathfrak{H}') = -0{,}000169$$
$$2{,}026834\,\mathfrak{G}' + 0{,}004221\,\mathfrak{H}' - 0{,}000227\,\mathfrak{A}' = -0{,}003697$$
$$0{,}159358\,\mathfrak{H}' + 0{,}023965\,\mathfrak{G}' - 0{,}003697\,\mathfrak{A}' = -0{,}000227$$
$$0{,}318716\,\mathfrak{J}' - 0{,}004241\,\mathfrak{H}' = -0{,}000005$$

vnde erit

			log. coeff.	vol.tot.coeff. in min. sec.
$\Phi = \text{Pr.}$	$-\,0{,}000096 f$	$\sin 2\eta$	5,984018	− 22″
	$-\,0{,}001823 f$	$\sin(2\Phi-2\pi)$	7,260310	−411
	$-\,0{,}000910 f$	$\sin(2\theta-2\pi)$	6,959131	−205
	$-\,0{,}000028 f$	$\sin(4\theta-4\pi)$	5,450835	− 6

Patet ergo reuera aequationem ab angulo $2\theta-2\pi$ ortam multo essa minorem, quam capite praecedente inueneramus; atque nunc quidem non vltra 205″ seu 3′, 25″ ascendere. Nullum igitur est dubium, quin haec aequatio tabulas lunares ad multo maiorem perfectionem sit ehectura.

 §. 250.

§. 250. Cum igitur neglectus terminorum, minimorum tantum errorem pepererit in aequatione ab angulo $2\theta - 2\pi$ pendente, operae erit pretium, etiam aequationes insuper ab excentricitate orbitae lunaris pendentes curatius inuestigare, quae quidem alicuius videntur esse momenti. In hunc finem ponamus.

$$\begin{aligned}\int R\,dr = {} & \mathfrak{A}\operatorname{cs}2\eta + \mathfrak{C}k\operatorname{cs}r + \mathfrak{D}k\operatorname{cs}(2\eta - r) + \mathfrak{E}k\operatorname{cs}(2\eta + r) + 32kk\operatorname{cs}(2\eta - 2r)\\ & + \mathfrak{G}f\cos(2\theta - 2\pi) \quad + \mathfrak{H}f\cos(2\theta - 2\pi)\\ & + \mathfrak{J}fk\cos r + \mathfrak{M}fk\cos(2\Phi - 2\pi - r) + \mathfrak{S}fk\cos(2\theta - 2\pi - r)\\ & + \mathfrak{O}fkk\operatorname{cs}(2\Phi - 2\pi - 2r) + \mathfrak{T}fk\cos(2\theta + 2\pi + r)\end{aligned}$$

$$\begin{aligned}\text{et } v = {} & A\operatorname{cs}2\eta + Dk\operatorname{cs}(2\eta - r) + Ek\cos(2\eta + r) - 14kk\cos(2\eta - 2r)\\ & + Gf\cos(2\Phi - 2\pi) \quad + Hf\cos(2\theta - 2\pi)\\ & + Jfk\cos r + Mfk\cos(2\Phi - 2\pi - r) + Sfk\cos(2\theta - 2\pi - r)\\ & + Ofkk\cos(2\Phi - 2\pi - 2r) + Tfk\cos(2\theta - 2\pi + r)\end{aligned}$$

§. 251. His valoribus in aequationibus nostris principalibus substitutis habebimus:

$R = fk\sin r \quad (\mathrm{o})$

$$\begin{aligned} fk\sin(2\Phi - 2\pi - r) &\ (0{,}00854\,S + 0{,}01708\,H)\\ fkk\sin(2\Phi - 2\pi - 2r) &\ (0{,}01708\,S)\\ fk\sin(2\theta - 2\pi - r) &\ (-0{,}00854\,M - 0{,}01708\,G)\\ fk\sin(2\theta - 2\pi + r) &\ (-0{,}01708\,G)\end{aligned}$$

et: $\dfrac{ddv}{dr^2} =$

$$fk\cos r\begin{cases}-\mathcal{E}J - 2\varkappa\mathfrak{J} - 0{,}00290 - 0{,}00893 - 0{,}00278\\ -0{,}00098 - 0{,}00098 - 0{,}00448\dfrac{A}{nn} + 0{,}00470\dfrac{A}{nn}\\ + 0{,}01315\dfrac{A}{nn} + 0{,}01315\dfrac{A}{nn} + 0{,}0263\dfrac{D}{nn}\\ + 0{,}0263\dfrac{E}{nn}\end{cases}$$

fk

$$fk\cos(2\Phi - 2\pi - r)\begin{cases}-\mathcal{E}M - 2\varkappa\mathfrak{M} + \tfrac{1}{2}bG + 0{,}00427S - 0{,}00854H\\ -0{,}02711\mathfrak{S} - 0{,}03634S + \dfrac{\mathfrak{C}}{nn}\mathfrak{G} + \dfrac{3\mathfrak{C}}{nn}G\\ + 0{,}55422\mathfrak{H} + 0{,}51332H - \dfrac{3AH}{2nn} + 0{,}01796\\ + 0{,}00145 + 0{,}001969 - \tfrac{1}{2} + 0{,}00896\dfrac{A}{nn}\\ + 0{,}01049\dfrac{A}{nn} - 0{,}02631\dfrac{A}{nn} - 0{,}00197\dfrac{A}{nn}\\ - 0{,}00129\dfrac{A}{nn} - 0{,}05261\dfrac{D}{nn} - 0{,}00258\dfrac{E}{nn}\end{cases}$$

$$fkk\operatorname{cs}(2\Phi - 2\pi - 2r)\begin{cases}-\mathcal{E}O - 2\varkappa\mathfrak{O} + \tfrac{1}{2}bM + \tfrac{3}{4}\mathfrak{G} - \dfrac{10}{nn}\mathfrak{H} + \dfrac{54}{nn}H\\ -\dfrac{3DH}{2nn} + 0{,}7698 + 0{,}00898 + 0{,}00098\\ + 0{,}00072 - \tfrac{1}{4} + 0{,}00448\dfrac{A}{nn} + 0{,}0052\dfrac{A}{nn}\\ - 0{,}00064\dfrac{A}{nn} + 0{,}00896\dfrac{D}{nn} - 0{,}01315\dfrac{A}{nn}\\ - 0{,}02630\dfrac{D}{nn} + 0{,}01049\dfrac{E}{nn} - 0{,}00129\dfrac{E}{nn}\\ - 0{,}00129\dfrac{E}{nn} - 0{,}00042\end{cases}$$

$$fk\cos(2\theta - 2\pi - r)\begin{cases}-\mathcal{E}S - 2\varkappa\mathfrak{S} + \tfrac{1}{2}bH + 0{,}00427M - 0{,}00854G\\ -0{,}02711\mathfrak{M} - 0{,}03634M + \dfrac{\mathfrak{C}}{nn}\mathfrak{H} + \dfrac{3\mathfrak{C}}{nn}H\\ - 0{,}01606\mathfrak{G} - 0{,}02844G - \dfrac{3AG}{2nn} - 0{,}00896\\ - 0{,}00327 + 0{,}026307 + 0{,}00129 - 0{,}00179\dfrac{A}{nn}\\ - 0{,}00145\dfrac{A}{nn} + \dfrac{A}{2nn} - 0{,}00394\dfrac{E}{nn} + \dfrac{E}{nn}\end{cases}$$

fk

$$fk\cos(2\theta-2\pi+r)\begin{cases} -\beta T-2\kappa\mathfrak{T}+\tfrac{1}{2}bH+0{,}00854\,G+\frac{\mathfrak{E}}{nn}\mathfrak{H}+\frac{3\mathfrak{E}}{nn}H \\ +0{,}55422\mathfrak{G}+0{,}51332\,G-\frac{3AG}{2nn}+0{,}00940 \\ -0{,}01049+0{,}026307+0{,}00129-0{,}00556\frac{A}{nn} \\ -0{,}00145\frac{A}{nn}-0{,}003938\frac{A}{nn}+\frac{A}{2nn} \\ -0{,}0394\frac{D}{nn}+\frac{D}{nn} \end{cases}$$

§. 252. Hae autem formulae intricatae reducuntur ad sequentes

$$\frac{ddv}{dr}=\text{Pr.}+fk\cos r\,(-\beta J-2\kappa\mathfrak{J}-0{,}01182)$$
$$fk\cos(2\phi-2\pi-r)\begin{pmatrix}-\beta M-2\kappa\mathfrak{M}-0{,}03207S+0{,}53619\\ -0{,}02711\mathfrak{G}\end{pmatrix}$$
$$fkk\cos(2\phi-2\pi-2r)\,(-\beta O-2\kappa\mathfrak{O}+1{,}52115M+0{,}76615)$$
$$fk\cos(2\theta-2\pi-r)\begin{pmatrix}-\beta S-2\kappa\mathfrak{S}-0{,}03207M+0{,}00286\\ -0{,}02711\mathfrak{M}\end{pmatrix}$$
$$fk\cos(2\theta-2\pi+r)\,(-\beta T-2\kappa\mathfrak{T}+0{,}38147)$$

§. 253. Nunc eosdem valores ex formulis assumtis eruamus, ac posito more adhuc recepto $\frac{2\kappa J+\mathfrak{J}}{nn}=i'$, $\frac{2\kappa M+\mathfrak{M}}{nn}=m'$ etc. erit

$$\frac{d\Phi}{dr}=a+\frac{1}{n}-a'\cos 2\eta-d'k\cos(2\eta-r)\quad -g'f\cos(2\Phi-2\pi)$$
$$-e'k\cos(2\eta+r)\quad -b'f\cos(2\theta-2\pi)$$
$$-i'fk\cos r\ -m'fk\cos(2\Phi-2\pi-r)\ -s'fk\cos(2\theta-2\pi-r)$$
$$-n'fkk\cos(2\Phi-2\pi-2r)\ -t'fk\cos(2\theta-2\pi+r)$$

$d\eta$

$$\frac{d\eta}{dr}=a-a'\cos 2\eta-e'k\cos r-d'k\cos(2\eta-r)-g'k\cos(2\Phi-2\pi)$$
$$-e'k\cos(2\eta+r)-b'k\cos(2\theta-2\pi)$$
$$-i'fk\cos r-m'fk\cos(2\Phi-2\pi-r)-s'fk\cos(2\theta-2\pi-r)$$
$$-n'fkk\cos(2\Phi-2\pi-2r)-t'fk\cos(2\theta-2\pi+r)$$

$$\frac{d\theta}{dr}=\frac{1}{n}+\frac{2}{n}k\cos r+\frac{3}{2n}kk\cos 2r \quad \text{atque}$$

$$\frac{d\pi}{dr}=$$

$$-0{,}004241\qquad +0{,}004241\,\cos(2\Phi-2\pi)$$
$$-0{,}004221\cos 2\eta+0{,}004241\,\cos(2\theta-2\pi)$$
$$-0{,}01766k\cos r-0{,}00996k\cos(2\eta-r)\ -0{,}010636kk\cos(2\eta-2r)$$
$$-0{,}00831k\cos(2\eta+r)\ -0{,}021273kk\cos 2r$$
$$+0{,}00924k\cos(2\Phi-2\pi-r)\quad +0{,}00841k\cos(2\theta-2\pi-r)$$
$$+0{,}00841k\cos(2\Phi-2\pi+r)\quad +0{,}00924k\cos(2\theta-2\pi+r)$$
$$+0{,}01064kk\cos(2\Phi-2\pi-2r)$$

§. 254. His iam valoribus introducendis differentiemus formulas nostras assumtas pro $\int \mathfrak{R}dr$ et v; atque obtinebimus primo:

$$R=\text{Praec.}$$
$$+fk\sin r\begin{Bmatrix}+0{,}00924\mathfrak{G}+0{,}00841\mathfrak{H}-0{,}004241\mathfrak{M}\\ -0{,}00841\mathfrak{G}-0{,}00924\mathfrak{H}-0{,}004241\mathfrak{S}\\ +0{,}004241\,\mathfrak{T}\end{Bmatrix}$$
$$+fk\sin(2\Phi-2\pi-r)\begin{Bmatrix}+\mathfrak{A}s'-0{,}01766\mathfrak{G}-0{,}00996\mathfrak{H}\\ -1{,}026834\mathfrak{M}-0{,}004221\mathfrak{S}\end{Bmatrix}$$
$$+fkk\sin(2\Phi-2\pi-2r)\begin{Bmatrix}+\mathfrak{D}s'+32h'-0{,}02127\mathfrak{G}-0{,}01064\mathfrak{H}\\ -0{,}026834\mathfrak{O}-0{,}01766\mathfrak{M}-0{,}00996\mathfrak{S}\end{Bmatrix}$$

Ee +

$$+fk\sin(2\theta-2\pi-r)\left\{\begin{matrix}-\mathfrak{A}m'+\mathfrak{D}b'+\mathfrak{G}e'-\frac{2}{n}\mathfrak{H}+0{,}840642\mathfrak{S}+\mathfrak{M}a'\\ -\mathfrak{E}g'-0{,}00831\mathfrak{G}-0{,}01766\mathfrak{H}-0{,}004221\mathfrak{M}\end{matrix}\right\}$$

$$+fk\sin(2\theta-2\pi+r)\left\{\begin{matrix}-\mathfrak{D}g'+\mathfrak{E}b'+\mathfrak{G}d'-\frac{2}{n}\mathfrak{H}-1{,}159358\mathfrak{T}\\ +0{,}00996\mathfrak{G}-0{,}01766\mathfrak{H}\end{matrix}\right\}$$

§. 255. Hinc igitur consequimur istas aequationes

$$0{,}00083\,(\mathfrak{G}-\mathfrak{H})-\mathfrak{J}-0{,}004241\,(\mathfrak{M}+\mathfrak{S}-\mathfrak{T})=0$$

$$\mathfrak{A}s'-0{,}01766\mathfrak{G}-0{,}00996\mathfrak{H}-0{,}004221\mathfrak{S}-1{,}026834\mathfrak{M}=0{,}01708\,H+0{,}00854\,S$$

$$\mathfrak{D}s'+32b'-0{,}002127\mathfrak{G}-0{,}01064\mathfrak{H}-0{,}026834\mathfrak{O}-0{,}01766\mathfrak{M}-0{,}00996\mathfrak{S}=0{,}01708\,S$$

$$-\mathfrak{A}m'+\mathfrak{D}b'-\mathfrak{E}g'+\mathfrak{G}e'-0{,}00831\mathfrak{G}-\frac{2}{n}\mathfrak{H}-0{,}01766\mathfrak{H}+\mathfrak{M}a'-0{,}004221\mathfrak{M}+0{,}840642\mathfrak{S}=-0{,}01708G-0{,}00854M$$

$$-\mathfrak{D}g'+\mathfrak{E}b'+\mathfrak{G}d'-0{,}00996\mathfrak{G}-\frac{2}{n}\mathfrak{H}-0{,}01766\mathfrak{H}-1{,}159358\mathfrak{T}=-0{,}01708G$$

§. 256. Ponatur nunc vlterius:

$$J'=\mathfrak{J}-0{,}00083\,(G-H)+0{,}004241\,(M+S-T)$$

$$M'=1{,}026834M-As'+0{,}01766G+0{,}00996H+0{,}004221S$$

$$O'=0{,}026834\,O-Ds'+14b'+0{,}02127\,G+0{,}01064\,H+0{,}01766\,M+0{,}00996\,S$$

$$S'=-0{,}840642\,S+Am'-Db'+Eg'-Ge'+0{,}00831G+\frac{2}{n}H+0{,}01766H-Ma'+0{,}004221M$$

$$T'=1{,}159358\,T+Dg'-Eb'-Gd'+0{,}00996\,G+\frac{2}{n}H+0{,}01766H$$

eritque

eritque $\frac{ddv}{dr}=$ Praec.

$$+fk\cos r\,[+0{,}01766\,(G'+H')-J'+0{,}004241\,(M'+S'+T')]$$

$$+fk\cos(2\varphi-2\pi-r)\left\{\begin{matrix}A's'-0{,}01766\,G'-0{,}00996\,H'\\ -0{,}004221S'-1{,}026834\,M'\end{matrix}\right\}$$

$$+fkk\cos(2\varphi-2\pi-2r)\left\{\begin{matrix}D's'-2{,}6b'-0{,}02127G'-0{,}01064\,H'\\ -0{,}01766\,M'-0{,}00996\,S'\\ -0{,}026834\,O'\end{matrix}\right\}$$

$$+fk\cos(2\theta-2\pi-r)\left\{\begin{matrix}A'm'+D'b'+E'g'+G'e'+0{,}840642\,S'\\ -0{,}00831\,G'-\frac{2}{n}H'-0{,}01766H'\\ +M'a'-0{,}004221M'\end{matrix}\right\}$$

$$+fk\cos(2\theta-2\pi+r)\left\{\begin{matrix}D'g'+E'b'+G'd'-0{,}00996\,G-\frac{2}{n}H'\\ -0{,}01766\,H'-1{,}159358T'\end{matrix}\right\}$$

§. 257. Prioris ordinis aequationes huc reducuntur:

$$\mathfrak{J}=-0{,}004241\,(\mathfrak{M}+\mathfrak{S}+\mathfrak{T})$$

$$1{,}026834\,\mathfrak{M}=-0{,}01785\,S-0{,}00883\,\mathfrak{S}-0{,}00039$$

$$0{,}026834\,\mathfrak{O}=-0{,}05835\,S-0{,}03041\,\mathfrak{S}-0{,}01766\,\mathfrak{M}+0{,}00690$$

$$-0{,}840642\,\mathfrak{S}=+0{,}01785\,M-0{,}01935\,\mathfrak{M}+0{,}00263$$

$$1{,}159358\,\mathfrak{T}=+0{,}01247$$

Hinc fit

$$\mathfrak{J}=+0{,}00007\,S+0{,}00008\,M+0{,}00006 \text{ seu } \mathfrak{J}=0$$

$$\mathfrak{M}=-0{,}01738\,S+0{,}00018\,M-0{,}00035$$

$$\mathfrak{O}=-2{,}16266\,S+0{,}02394\,M+0{,}26092$$

$$\mathfrak{S}=-0{,}00040\,S-0{,}02123\,M-0{,}00313$$

$$\mathfrak{T}=-0{,}01076$$

§. 258. Porro reperietur

$J' = J + 0{,}004241\,(M+S-T) - 0{,}00026$

$M' = 1{,}026834\,M + 0{,}01935\,S - 0{,}00016\,M + 0{,}00568$

$O' = 0{,}026834\,O - 0{,}37652\,S + 0{,}01805\,M + 0{,}01063$

$S' = 0{,}840642\,S + 0{,}00013\,S + 0{,}00883\,M - 0{,}00167$

$T' = 1{,}159358\,T + 0{,}01023$

ac succinctius habebitur $\frac{ddv}{dr^2} =$ Praec.

$$+fk \cos r\,[-J' + 0{,}004241(M'+S'+T') + 0{,}01175]$$

$$+fk \cos(2\phi-2\pi-r)\left\{\begin{matrix}-1{,}026834\,M' - 0{,}00422\,S' - 0{,}02842\,S\\ +0{,}00030\,M - 0{,}01160\end{matrix}\right\}$$

$$+fkk \cos(2\phi-2\pi-2r)\left\{\begin{matrix}-0{,}026834\,O' - 0{,}01766\,M' - 0{,}00354\,M\\ -0{,}01511 - 0{,}00996\,S' + 0{,}33746\,S\end{matrix}\right\}$$

$$+fk \cos(2\theta-2\pi-r)\left\{\begin{matrix}+0{,}840642\,S' - 0{,}02396\,M' - 0{,}02843\,M\\ -0{,}01472 + 0{,}00025\,S\end{matrix}\right\}$$

$$+fk \cos(2\theta-2\pi+r)\,[-1{,}159358\,T' + 0{,}33856]$$

§. 259. Hinc tandem valores quaesiti eliciuntur

$J = -0{,}81144$ . . . $l{-}J = 9{,}909256$

$M = -1{,}25325$ . . . $l{-}M = 0{,}098046$

$O = -2{,}12630$ . . . $l{-}O = 0{,}327624$

$S = -0{,}13490$ . . . $l{-}S = 9{,}130012$

$T = -0{,}10080$ . . . $l{-}T = 9{,}003441$

ac

+ 0,00568
+ 0,01063
- 0,00167
0,01175]
,02842S
,01160]
00354M
33746S]
2843M
0025 S]
tur
ac

ac $\mathfrak{J} = -0{,}00005$ . . . $l{-}\mathfrak{J} = 5{,}698970$

$\mathfrak{M} = +0{,}00177$ . . . $l\,\mathfrak{M} = 7{,}247973$

$\mathfrak{O} = +0{,}52266$ . . . $l\,\mathfrak{O} = 9{,}718219$

$\mathfrak{S} = +0{,}02355$ . . . $l\,\mathfrak{S} = 8{,}371991$

$\mathfrak{T} = +0{,}01076$ . . . $l\,\mathfrak{T} = 8{,}031812$

§. 260. Ex his iam pro distantia Lunae a terra erit

$u =$ Praec.

	Log. coeff.	Valor. coeff. totius
$-0{,}00462\,fk \cos 2\eta$	7,664440	$-0{,}000275$
$-0{,}00773\,fk \cos(2\phi-2\pi-r)$	7,853230	$-0{,}000424$
$-0{,}01210\,fkk \cos(2\phi-2\pi-2r)$	8,082808	$-0{,}000039$
$-0{,}00077\,fk \cos(2\theta-2\pi-r)$	6,885196	$-0{,}000046$
$-0{,}00057\,fk \cos(2\theta-2\pi+r)$	6,758625	$-0{,}000034$

et pro motu momentaneo

$$\frac{d\Phi}{dr} = \text{Praec.}$$

	Log. coeff.	Val. coeff.
$+0{,}00932\,fk \cos r$	7,969416	$+0{,}000555$
$+0{,}01558\,fk \cos(2\phi-2\pi-r)$	8,192568	$+0{,}000928$
$+0{,}02142\,fkk \cos(2\phi-2\pi-2r)$	8,330819	$+0{,}000069$
$+0{,}00142\,fk \cos(2\theta-2\pi-r)$	7,152288	$+0{,}000085$
$+0{,}00109\,fk \cos(2\theta-2\pi+r)$	7,037426	$+0{,}000064$

§. 261. Pro longitudine autem lunae sequentes resolui debent aequationes:

$+0{,}00932 = \mathfrak{J}' - 0{,}01766\,(\mathfrak{S}'+\mathfrak{H}') - 0{,}004241\,(\mathfrak{M}'+\mathfrak{S}'+\mathfrak{T}')$

$+0{,}01558 = 1{,}026834\,\mathfrak{M}' - \mathfrak{A}'s' + 0{,}01766\,\mathfrak{S}' + 0{,}00996\,\mathfrak{H}' + 0{,}004221\,\mathfrak{S}'$

$$+0{,}02142 = 0{,}026834\,\mathfrak{O}' - \mathfrak{D}'s' + 2{,}6b' + 0{,}02127\,\mathfrak{G}'$$
$$+0{,}01064\,\mathfrak{H}' + 0{,}01766\,\mathfrak{M}' + 0{,}00996\,\mathfrak{S}'$$
$$+0{,}00142 = -0{,}840642\,\mathfrak{S}' - \mathfrak{A}'m' - \mathfrak{D}'b' - \mathfrak{E}'g' + 0{,}02114\,\mathfrak{G}'$$
$$+0{,}16853\,\mathfrak{H}' + 0{,}02396\,\mathfrak{M}'$$
$$+0{,}00109 = 1{,}159358\,\mathfrak{T}' - \mathfrak{D}'g' - \mathfrak{E}'b' - 0{,}35615\,\mathfrak{G}'$$
$$+0{,}16850\,\mathfrak{H}'$$

hincque prodit pro longitudine vera:

		Log. coeff.	Val. coeff. in min. sec.
$\Phi =$ Pr. $+0{,}00932fk$	fin r	7,969416	+115
$+0{,}01521fk$	fin $(2\Phi-2\pi-r)$	8,182130	+187
$+0{,}79079fkk$	fin $(2\Phi-2\pi-2r)$	9,898060	+529
$-0{,}00121fk$	fin $(2\theta-2\pi-r)$	7,083939	−15
$-0{,}00082fk$	fin $(2\theta-2\pi+r)$	6,913527	−10

§. 262. Ob inclinationem ergo orbitae lunaris ad eclipticam omnes correctiones huc redeunt, vt fit

I. Pro distantia lunae a terra:

$u =$ Praec.

		Log. coeff.	Val. coeff.
$+0{,}000084f$	cof 2η	5,926350	+0,000092
$+0{,}001832f$	cof $(2\Phi-2\pi)$	7,262918	+0,002004
$+0{,}000011f$	cof $(2\theta-2\pi)$	5,034111	+0,000012
$+0{,}000001f$	cof $(4\theta-4\pi)$	4,011565	+0,000001
$-0{,}00462fk$	cof r	7,664440	−0,000275
$-0{,}00773fk$	cof $(2\Phi-2\pi-r)$	7,853230	−0,000424
$-0{,}01210fkk$	cof $(2\Phi-2\pi-2r)$	8,082808	−0,000039
$-0{,}00077fk$	cof $(2\theta-2\pi-r)$	6,885196	−0,000046
$-0{,}00057fk$	cof $(2\theta-2\pi+r)$	1,758625	−0,000034

II. Pro

II. Pro motu momentaneo:

$$\frac{d\Phi}{dr} = \text{Praec.}$$

		Log. coeff.	Val. coeff.
$-0{,}000169f$	cof 2η	6,227887	−0,000185
$-0{,}003697f$	cof $(2\Phi-2\pi)$	7,567849	−0,004043
$-0{,}000227f$	cof $(2\theta-2\pi)$	6,356026	−0,000248
$-0{,}000005f$	cof $(4\theta-4\pi)$	4,698970	−0,000005
$+0{,}00932fk$	cof r	7,969416	+0,000555
$+0{,}01558fk$	cof $(2\Phi-2\pi-r)$	8,192568	+0,000928
$+0{,}02142fkk$	cof $(2\Phi-2\pi-2r)$	8,330819	+0,000069
$+0{,}00142fk$	cof $(2\theta-2\pi-r)$	7,152288	+0,000085
$+0{,}00109fk$	cof $(2\theta-2\pi+r)$	7,037426	+0,000064

III. Pro longitudine lunae vera

		Log. coeff.	Val. coeff. in min. sec.
$\Phi =$ Pr. $-0{,}000096f$	fin 2η	5,984018	−22″
$-0{,}001823f$	fin $(2\Phi-2\pi)$	7,260310	−411
$-0{,}000910f$	fin $(2\theta-2\pi)$	6,959131	−205
$-0{,}000028f$	fin $(4\theta-4\pi-r)$	5,450835	−6
$+0{,}00932fk$	fin r	7,969416	+115
$+0{,}01521fk$	fin $(2\theta-2\pi-r)$	8,182130	+187
$+0{,}79079fkk$	fin $(2\Phi-2\pi-2r)$	9,898060	+529
$-0{,}00121fk$	fin $(2\theta-2\pi-r)$	7,083939	−15
$-0{,}00082fk$	fin $(2\theta-2\pi+r)$	6,913527	−10

CAPUT

CAPUT XVI.

EXPOSITIO INAEQUALITATUM LUNAE HACTENUS INUENTARUM.

§. 263.

Quas igitur inuenimus hactenus lunae inaequalitates eae primum, ſi originem earum ſpectemus, ad ſex claſſes reducuntur. Quatenus enim luna in motu ſuo a regulis Keplerianis, in quibus quidem motum apogei compeſtimur, recedit, eius errores vel primo a ſolo lunae aſpectu, ſeu eius diſtantia a ſole pendent, ſeu quod eodem redit, per angulum η tantum definiuntur, quibus variatio lunae continetur. Ad ſecundam claſſem refero eas lunae inaequalitates, quae inſuper ab excentricitate eius orbitae pendent. Tertia claſſis eas complectitur inaequalitates, quae ab excentricitate orbitae ſolis ortum trahunt. Quarta vero eas, quae per vtramque excentricitatem coniunctim determinantur. Quintae porro claſſi annumeramus eas inaequalitates, quae parallaxin ſolis inuoluunt, atque errores quatuor ante memoratorum generum implicant. Sexta denique claſſis ſuppeditat eas inaequalitates, quae praeterea ab inclinatione orbitae lunaris ad eclipticam pendent.

§. 264. Quodſi vero ad vſum harum inaequalitatum attendamus, prouti eae, ad lunam accommodari debent, tum eae in quinque claſſes commodiſſime diſtribuuntur. Primo enim perpendendae ſunt eae inaequalitates, quarum ope vera diſtantia lunae a terra determinatur, vt inde porro tam lunae diameter apparens, quam eius parallaxis horizontalis aſſignari poſſit. Secundo loco formulae erunt collocandae illae, quae motui momentaneo definiendo inſeruiunt, ex quibus deinceps motus lunae horarius accurate exhiberi poterit. Tertium locum occupabunt eae inaequalitates, quae veram longitudinem lunae ad eclipticam relatam praebent. Quarto vero poſitio lineae nodorum lunae, ſeu longitudo nodi aſcendentis; ac quinto vera inclinatio orbitae lunaris ad eclipticam inueniri debebit; vt deinde vera lunae latitudo concludi poſſit. Manifeſtum autem eſt, has inaequalitates plurimum inter ſe permiſceri, ita vt vix vllum habeatur genus, cuius inaequalitates non a reliquis generibus pendeant; cui tamen incommodo facile medela adhibetur.

§. 265. Quanquam numerus inaequalitatum, quas ſumus conſecuti, tantopere increuit, vt calculus ſine maxima moleſtia expediri nequeat, tamen iam monui, non omnes inaequalitates, quibus motus Lunae perturbatur, eſſe definitas, ſed potius earum numerum omnino eſſe infinitum. Facile quidem intelligitur, pleraſque has praetermiſſas inaequalitates nullius fere eſſe momenti, atque ſine notabili errore iis ſuperſederi poſſe: verum tamen ſunt inter eas nonnullae, quae ad plura minuta ſecunda aſſurgere videntur, quarum argumenta ſupra iam innui; ex quo omnino operae eſſet pretium in eas omni cura inquirere. Sed earum inueſtigatio tam eſt lubrica et incerta, vt leuiſſima omiſſio in calculo facta eas

maxime

maxime afficiat. Cum igitur in calculo plurimos terminos reiicere cogamur, istam inuestigationem frustra plane susciperemus, quamdiu scilicet rem sine appropinquatione exequi non licet. Cuius defectus eximium habemus exemplum in inaequalitatibus postremo loco inuentis, quae statim atque in negligendo minus largi fueramus, mirum quantum prodierunt immutatae; ac nullum plane est dubium, si calculum adhuc accuratius prosequi liceret, quin valores inuenti notabilem insuper mutationem sint subiturae. Imprimis autem aequatio ab angulo $2\Phi-2\pi-2r$ seu a dupla distantia apogei a nodo pendens, est suspecta, ac minime pro certa haberi potest, cum leuissima circumstantia eam magnopere perturbare valeat.

§. 266. Si enim in causam inquiramus, cur analysis posterior tam diuersos valores pro his inaequalitatibus suppeditauerit, primo quidem statim patet, neglectum litterarum germanicarum 𝔍, 𝔐, 𝔒, etc. in calculo priori potissimum hoc discrimen produxisse: ingens enim valor litterae 𝔒 imprimis aequationem ab angulo $2\Phi-2\pi-2r$ pendentem tantopere auxit. Praeterea vero etiam non parum augmenti haec aequatio inde est nacta, quod in calculo posteriori rationem quoque habuimus termini $\operatorname{cos}(2\eta-2r)$, qui tam in valore $\frac{d\Phi}{dr}$ quam $\frac{d\eta}{dr}$ inesse est deprehensus; vnde tuto colligere licet, si alios quoque terminos similes veluti $2\theta-2\pi-2r$, etsi per se sunt minimi, in calculum introduxissemus, coefficien-

efficientes terminorum $2\Phi-2\pi-2r$ non mediocrem inde mutationem subituros fuisse. Quamobrem plus hinc colligere non possumus, nisi inaequalitatem Lunae ab angulo hoc $2\Phi-2\pi-2r$ pendentem minime esse contemnendam, etiamsi fortasse tanta non sit, quam inuenimus. Vera autem eius quantitas certius ex obseruationibus quam ex Theoria colligi posse videtur.

§. 267. Quoniam vero hae inaequalitates omnes ad anomaliam Lunae veram referuntur, antequam eas ad vsum adhibere liceat, modum tradi conueniet ad quodvis tempus propositum anomaliam Lunae veram determinandi. Cognita autem excentricitate orbitae lunaris k et motu anomaliae mediae, inde ad quodvis tempus facile anomalia media p colligitur. Verum ex anomalia media p et excentricitate k anomalia vera r definiri debet ope huius aequationis $dp=\frac{(1-kk)^{\frac{3}{2}}\,dr}{(1-k\operatorname{cos}r)^2}$; vnde quidem non difficulter, si nota esset anomalia vera r, vicissim inueniri posset anomalia media p. Calculo enim peracto, si breuitatis gratia ponatur

$$\delta=\frac{1-\sqrt{(1-kk)}}{k}=\tfrac{1}{2}k+\frac{1.1}{2.4}k^3+\frac{1.1.3}{2.4.6}k^5+\text{etc.}$$

reperitur:

$$p=r+2k\operatorname{sin}r+2\delta(k-\tfrac{1}{2}\delta)\operatorname{sin}2r+2\delta\delta(k-\tfrac{2}{3}\delta)\operatorname{sin}3r$$
$$+2\delta^3(k-\tfrac{3}{4}\delta)\operatorname{sin}4r\ \text{etc.}$$

cuius seriei progressio primo intuitu patet. Cum autem k ac proinde δ sit valde paruum, erit satis exacte:

$p=$

$$p = r + 2k\sin r + (\tfrac{3}{4}kk + \tfrac{1}{8}k^4 + \tfrac{3}{64}k^6)\sin 2r + (\tfrac{1}{3}k^3 + \tfrac{1}{8}k^5)\sin 3r$$
$$+ (\tfrac{5}{32}k^4 + \tfrac{3}{32}k^6)\sin 4r + \tfrac{3}{40}k^5 \sin 5r + \tfrac{7}{192}k^6 \sin 6r$$

§. 268. Data ergo anomalia media Lunae p, eius anomalia vera r elici debebit ex hac aequatione

$$r = p - 2k\sin r - (\tfrac{3}{4}kk + \tfrac{1}{3}k^4 + \tfrac{3}{64}k^6)\sin 2r - (\tfrac{1}{3}k^3 + \tfrac{1}{8}k^5)\sin 3r$$
$$- (\tfrac{5}{32}k^4 + \tfrac{3}{32}k^6)\sin 4r - \tfrac{3}{40}k^5 \sin 5r - \tfrac{7}{192}k^6 \sin 6r$$

cuius quidem ope, ſi cognita fuerit excentricitas k, calculus non difficulter expedietur. Quoniam enim termini ſinus inuoluentes ſunt admodum parui, in iis ſtatim poni poterit $r = p$, vnde valor verior pro r eruetur, qui deinde iterum in his terminis adhibitus, iuſtiorem valorem pro r ſuppeditabit. Atque hoc modo poſt aliquot operationes verus tandem valor pro anomalia vera r obtinebitur. Interim tamen, quo iſte calculus facilius perfici queat, aequatio haec ita poteſt transformari, vt loco ſinuum anomaliae verae r, ſinus anomaliae mediae p introducantur; id quod ſequenti modo praeſtabitur.

§. 269 Ponatur breuitatis gratia:

$$\tfrac{3}{4} + \tfrac{1}{8}kk + \tfrac{3}{64}k^4 = \alpha;\quad \tfrac{1}{3} + \tfrac{1}{8}kk = \beta;\quad \tfrac{5}{32} + \tfrac{3}{32}kk = \gamma$$
$$\tfrac{3}{40} = \delta \text{ et } \tfrac{7}{192} = \epsilon$$

vt ſit:

$$r = p - 2k\sin r - \alpha kk \sin 2r - \beta k^3 \sin 3r - \gamma k^4 \sin 4r - \delta k^5 \sin 5r - \epsilon k^6 \sin 6r$$

ac ponatur denuo

$$2k\sin r + \alpha kk\sin 2r + \beta k^3 \sin 3r + \gamma k^4 \sin 4r + \delta k^5 \sin 5r + \epsilon k^6 \sin 6r = R$$

et

k^5) ſin 3r
n6r
p, eius
) ſin 3r
ſin 6r
k, calm teriis ſta- erueus, iumodo o anote calpoteſt ſinus quenti
k = γ
ſin6r
= R
et

et cum ſit $r = p - R$ habebitur:

$$\sin r = (1 - \tfrac{1}{2}RR + \tfrac{1}{24}R^4)\sin p - (R - \tfrac{1}{6}R^3 + \tfrac{1}{120}R^5)\cos p$$
$$\sin 2r = (1 - 2RR + \tfrac{2}{3}R^4)\sin 2p - (2R - \tfrac{4}{3}R^3)\cos 2p$$
$$\sin 3r = (1 - \tfrac{9}{2}RR)\sin 3p - (3R - \tfrac{9}{2}R^3)\cos 3p$$
$$\sin 4r = (1 - 8RR)\sin 4p - 4R\cos 4p$$
$$\sin 5r = \sin 5p - 5R\cos 5p$$
$$\sin 6r = \sin 6p$$

negligendo ſcilicet terminos, qui ipſius k poteſtates ſexta altiores continent.

§. 270. Euolutio autem huius calculi fit maxime prolixa, ſi quidem ad ſextam poteſtatem ipſius k aſcendere velimus. Facile autem expedietur, ſi ad quartam ſubſiſtamus, tum autem reperietur:

$$r = p - (2k - \tfrac{1}{4}k^3)\sin p + (\tfrac{5}{4}kk - \tfrac{11}{24}k^4)\sin 2p - \tfrac{13}{12}k^3 \sin 3p - \tfrac{103}{96}k^4 \sin 4p$$

quae expreſſio ſatis accurate pro quauis anomalia media p conuenientem anomaliam veram indicabit. Calculus autem, ſi excentricitas k conſtet, facili negotio abſoluetur. Formula haec quoque ita repraeſentari poterit, vt ſit

$$r = p - 2k(1 - \tfrac{1}{8}kk)\sin p + \tfrac{5}{4}kk(1 - \tfrac{11}{30}kk)\sin 2p - \tfrac{13}{12}k^3 \sin 3p - \tfrac{103}{96}k^4 \sin 4p$$

Hinc igitur tabulam conſtrui conueniet, vnde pro quauis anomalia media propoſita ipſi reſpondens anomalia vera excerpi queat.

§. 271. Cum autem inuenta fuerit anomalia vera r, longitudo Lunae regula Kepleriana inuenienda, quam

 ſupra

ſupra (206) per ζ indicauimus, ita exprimetur, vt ſit

$$\zeta = C + 1{,}0085272\,p$$

$$-1{,}0085272\left(\begin{array}{l}2k\sin r + \tfrac{3}{4}kk(1+\tfrac{2}{3}kk+\tfrac{7}{16}k^4)\sin 2r + \tfrac{1}{3}k^3(1+\tfrac{3}{8}kk)\sin 3r \\ \quad + \tfrac{5}{32}k^4(1+\tfrac{3}{5}kk)\sin 4r + \tfrac{3}{40}k^5\sin 5r + \tfrac{7}{192}k\sin 6r\end{array}\right)$$

vbi $C + 1{,}0085272\,p$ exhibet longitudinem Lunae mediam; quae ſi vocetur $= \xi$, atque in coefficientium partibus minimis pro k ſcribatur valor proximus 0,0545, erit

	log. coeff.	val. in min. ſec.
$\zeta = \xi - 2{,}0170544\,k\ \sin r$	0,304718	$22675'' = 6^\circ, 17', 55''$
$- 0{,}756770\,kk\ \sin 2r$	9,878964	$464 = 7, 44$
$- 0{,}336551\,k^3\ \sin 3r$	9,527051	$11\tfrac{1}{2}$
$- 0{,}15786\,k^4\ \sin 4r$	9,198282	$\tfrac{1}{2}$

vnde patet ſuperfluum futurum fuiſſe, ſi ſuperiores expreſſiones vltra quartam poteſtatem ipſius k extendere voluiſſemus.

vt ſit
kk)ſin3r
kſin6r
nae me-
um par-
0,0545,
n. ſec.
,17',55''
7, 44
res ex-
tendere
APUT

CAPUT XVII.

INUESTIGATIO ELEMENTORUM MOTUS LUNAE.

§. 272.

Inuentis iam per Theoriam hiſce inaequalitatibus, quibus motus Lunae perturbatur, antequam eas ad computum aſtronomicum accommodare liceat, elementa, quae in eas ingrediuntur, per obſeruationes determinari oportet. Primo ſcilicet ad datam epocham cum longitudo Lunae media, tum eius anomalia media, ac locus nodi medius conſtitui debebit, vt eaedem res inde ad quoduis aliud tempus aſſignari queant. Deinde quoque ex obſeruationibus verus valor excentricitatis lunaris colligi debet, a quo potiſſimum quantitas praecipuarum inaequalitatum pendet. Excentricitas autem orbitae ſolaris pro ſatis certa haberi poterit, cum ſit $e = 0{,}0168$. Lunae vero excentricitas tam prope iam conſtat, vt inde ſine errore ad quamlibet anomaliam mediam vera ſatis exacte aſſignari poſſit. Etſi enim in anomalia vera error aliquot minutorum primorum committitur, inaequalitates Lunae inde non vltra aliquot minuta ſecunda afficiuntur.

§. 273. Quodſi autem ſtatim quasuis Lunae obſeruationes ad hunc finem adhibere velimus, ob tam ingentem inaequalitatum numerum, inueſtigatio elementorum maxime moleſta redderetur. Quocirca ex obſer-