

$\rho = r$	Log coeff.
— $2/\text{cof } r$	0,30
— 48 cof 2 r	1,681
+ 11 cof (2 r - r)	1,041
+ 3 cof (2 r + r)	0,48
+ 36 cof (2 ρ -2 r)	1,556
+ 9 cof (2 ρ -2 r - r)	0,95
+ 3 cof (2 ρ -2 r + r)	0,48
+ 23 cof (2 ρ -2 r -2 r)	1,362
+ 484 cof (2 ρ -2 r)	2,6848
+ 9 cof (4 ρ -4 r)	0,95
— 5 cof (2 ρ -2 r - r)	0,70
+ 5 cof (2 ρ -2 r -+ r)	0,70
— 7 cof (2 ρ -2 r - r)	0,84
— 3 cof (2 ρ -2 r + r)	0,48

Tabula autem pro distantia lunae a terra, unde eius parallaxis et diameter apparentes definiatur, ex formulis supra exhibitis facile confuetur.

ADDI-

ADDITAMENTUM

CONTINENS ALIAS METHODOS

INVESTIGANDI MOTUS LUNAE

INAEQUALITATES.

Quia methodum ante descriptam accuratius evolveri, eam quidem in se spectatam satis bonam aequaeque lunae inaequalitatibus definiendis aptam deprehendet; interim tamen fieri cogor, eam non solum maxime esse operosam, sed etiam ita comparatam, ut plures inaequalitates, quae tamen motum lunae imprimis afficere videntur, non satis exacte exhibeat, et quasi in dubio relinquat. Causa huius incertitudinis manifeste in hoc est facta, quod omnes inaequalitates ita inter se sunt connexae, ut nullius valor verus accurate definiti possit, quin simul reliquae inaequalitates omnes fuerint cognitae. Cum igitur eiusmodi methodo approximandi sum usus, ut primo quasdam inaequalitates tanquam cognitae assumserim, ex quibus deinceps reliquas definiuerim, probe notandum est ab his inventis iterum priores, quae erant assumptae, levem quandam mutationem pati; quae si factim ab initio nota fuisset, etiam reliquarum valores aliquantillum mutari prodissent: ac quaedam inaequalitates adeo sunt lubricae, ut facta vel minima mutatione in his, a quibus pendunt, inde non exigam alterationem trahant. Huc imprimis pertinet motus apogei, cuius investigatio omnes omnino inaequalitates

ide eius
formulis

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litates

lites implicat, ita ut sine harum cognitione nevitquam accurate definiri queat.

Cum igitur hæc methodus istis tantis incommodis sit obnoxia, aliam maxime diversam tentavi viam, quæ ab iis esse libera, etiam si negare nequeam, etiam hanc suis non carere incommodis, quæ tamen proflus aliis sunt generis. Ex quo confido his duabus diversis methodis combinandis hæud exiguum fructum in veram motuum lunarium cognitionem esse redundaturum. Præcipuum autem discrimen versatur in electione anomalæ, quæ in superiore metodo non ita est assumta, ut distantia lunæ a terra ferret vel maxima vel minima, si anomalia vel $= 0$ vel $= 180^\circ$ statuitur: neque enim differentiale distantiae dx evanescit, quando sinus anomalæ in nihilum abit, sed præterea etiam nunc ab elongatione solis a luna seu angulo γ pender. Ita secundum hanc methodum neque apogæum lunæ neque perigæum ibi statuitur, ubi angulus, quem motus lunæ direxit cum radio vectore facit, est rectus; sed plerumque in alia puncta incidunt, quæ ab iis locis, ubi luna terræ vel est proxima, vel ab ea maxime remota, notabiliter sint diversa. Erit autem in hoc calculo non veræ lineæ absidum positio consideratur, hinc tamen methodus minime vitiosa est repugnans; propterea quod non est quæsitio, quo nomine quæpiam orbitæ lunaris puncta appellentur, dummodo cunctæ inæqualitates recte exprimentur. Sed quoniam circa has ipsas inæqualitates nonnulla graviora dubia sunt orta, hæud abs re fore arbitror, et alteram methodum hic proponere.

L. Sit

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Sit

I.

Sit igitur ut ante: Massa solis $= \odot$; terræ $= \otimes$ et lunæ $= \mathcal{D}$; atque vis attractiva terræ in distantia d ut $\frac{1}{d^2} - \frac{1}{bb}$; manente vi solis quadrato distantie exacte proportionali. Tum vero sit

Longitudo lunæ $= \phi$; latitudo $= \psi$; et distantia curvata $= x$

Longitudo solis $= \theta$; eiusque a terra distantia $= y$

Longitudo nodi ascendentis lunæ $= \pi$ et inclinatio ad eclipticam $= \varrho$

ac ponatur brevitatis ergo elongatio lunæ a sole $\phi - \theta = \eta$ et distantia $\sqrt{(xx \sec. \psi^2 - 2xy \cos \eta + yy)} = z$.

Quibus positis supra §. 20. vidimus motum lunæ his quatuor æquationibus contineri:

$$I. \quad 2xzd\phi + xdd\phi = -\frac{1}{2}dz^2 \odot \left(\frac{y}{x^2} - \frac{1}{yy} \right) \sin \eta$$

$$II. \quad ddx - 2d\phi^2 = -\frac{1}{2}dz^2 (\zeta + \mathcal{D}) \cos \psi^2 \left(\frac{1}{xx} - \frac{1}{bb} \right) \\ - \frac{1}{2}dz^2 \odot \left(\frac{x-y \cos \eta}{z^2} + \frac{\cos \eta}{yy} \right)$$

$$III. \quad d\pi = -\frac{1}{2}dz^2 \odot \left(\frac{y}{x^2} - \frac{1}{yy} \right) \frac{\sin(\phi - \pi) \sin(\theta - \pi)}{x d\phi}$$

IV. $d \text{ tang} \varrho = \frac{d\pi}{\text{tang}(\phi - \pi)}$, et tang $\psi = \text{tg} \varrho \cos(\phi - \pi)$ ubi elementum temporis dt sumtum est pro constante.

Mm 2

II. Qua-

Quatenus hic motus solis ingreditur, is pro regu-
lari aequae regulis Kepleri conformi haberi poterit: ha-
bebimus ergo

$$2dyd\theta + yd\theta^2 = 0. \text{ et } d^2y - yd\theta^2 = -\frac{1}{2}d^2r. \frac{\odot + \ominus}{yy}$$

vnde si ponamus orbitae solaris:

$$\text{semiparametrum} = r; \text{ eccentricitatem} = e \text{ et ano-} \\ \text{maliam veram} = s$$

$$\text{erit } y = \frac{r}{1-e \cos \eta}; \quad d\eta = \frac{dt}{yy} \sqrt{\frac{1}{2}e(\odot + \ominus)}$$

Sic a semixis transierit orbis solaris, ac tempore = s
sol motu medio absoluti angulum = \omega, quo pro men-
sura temporis s vramur: erit ergo $d\omega = \frac{dt}{2s} \sqrt{\frac{1}{2}e(\odot + \ominus)}$

ideoque $\frac{1}{2}d^2r = \frac{a^3d\omega^2}{\odot + \ominus}$. At est $a = \frac{r}{1-e^2}$. Hinc ergo fit

$$d\omega = d\theta = \frac{a'd\omega}{yy} \sqrt{a'e} = \frac{a'dd\theta}{yy} \sqrt{(1-e^2)} = \frac{d\omega(1-e \cos \eta)^2}{(1-e^2)\sqrt{(1-e^2)}}$$

fiatque tam $d\omega$ quam $d\theta$ per elementum $d\omega$ loco tempo-
ris introductum expressimus. Quia autem massa solis \odot
massam terrae \ominus tam enormiter excedit, sine errore pro
 $\frac{1}{2}d^2r$ scribi poterit $\frac{a^3d\omega^2}{\odot}$, eruntque nostrae aequationes
pro luna:

$$I. \quad 2dx d\phi + x d\phi^2 = -a^3 d\omega^2 \left(\frac{y}{z^3} - \frac{1}{yy} \right) \sin \eta$$

$$II. \quad ddx - x d\phi^2 = - \frac{a^3(\odot + \ominus) d\omega}{z^3} \cos \psi^2 \left(\frac{1}{xx} - \frac{1}{bz} \right) \\ - \frac{a^3 d\omega^2}{z^3} \left(\frac{x-y \cos \eta}{z^3} + \frac{\cos \eta}{yy} \right) \quad III. \quad dx$$

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$$\frac{\odot + \ominus}{yy}$$

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$$\left(\frac{1}{z} - \frac{1}{bz} \right) \\ III. \quad dx$$

$$III. \quad dx = - \frac{a^3 d\omega^2}{xxd\phi} \left(\frac{y}{z^3} - \frac{1}{yy} \right) \sin(\phi - \pi) \sin(\theta - \pi)$$

$$IV. \quad d l \text{ tang } \rho = \frac{dx}{\text{tg}(\phi - \pi)}; \text{ aequae ob } \text{tg } \psi = \text{tg } \rho \text{ c}(\phi - \pi), \\ \text{ habebitur proxime } \cos \psi^2 = 1 - \frac{1}{2} \text{tg } \rho^2 - \frac{1}{2} \text{tg } \rho^2 \text{ c}^2(\phi - \pi).$$

III.

Incipiamus a duabus aequationibus prioribus, ac
ponamus brevitate gratia

$$a^2 \left(\frac{y}{z^3} - \frac{1}{yy} \right) \sin \eta = M. \text{ et}$$

$$\frac{a^2(\odot + \ominus)}{\odot} \cos \psi^2 \left(\frac{1}{xx} - \frac{1}{bz} \right) + a^2 \left(\frac{x-y \cos \eta}{z^3} + \frac{\cos \eta}{yy} \right) = \frac{A}{xx} + N$$

quandocumque haec posterior expressio terminum in-
voluit formae $\frac{A}{xx}$ prae ceteris incomparabiliter maiorem;
aeque habebimus has duas aequationes:

$$2dx d\phi + x d\phi^2 = -M d\omega^2 \text{ et } ddx - x d\phi^2 = - \frac{A d\omega^2}{xx} - N d\omega^2$$

quarum prior per $2x^3 d\phi$ multiplicata ob $d\omega$ constans
habebit integrale:

$$x^4 d\phi^2 = -2d\omega^2 \int Mx^3 d\phi$$

Tum prior multiplicata per $2x d\phi$ addatur ad postero-
riorem per $2dx$ multiplicatam, eritque, aggregatum:

$$2x dx d\phi^2 + 2xx d\phi d\phi + 2dx ddx = -2Mx d\omega^2 d\phi \\ - \frac{2A d\omega^2 dx}{xx} - 2N d\omega^2 dx$$

Cuius integrale erit:

$$dx^2 + xx d\phi^2 = + \frac{2A d\omega^2}{x} - 2d\omega^2 \int (Mx^2 d\phi + N dx)$$

Mm 3

IV.

IV.

Ponantur formulae integrales, quae in his expressionibus insunt:

$$\int Mx^2 d\phi = P \text{ et } \int (Mx d\phi + Ndx) = Q$$

ut habeamus has duas aequationes:

$$x^2 d\phi^2 = 2P d\omega^2 \text{ et } dx^2 + x^2 d\phi^2 = \frac{2Ad\omega^2}{x} + 2Qd\omega^2$$

unde cum sit $x^2 d\phi^2 = \frac{2P d\omega^2}{x^2}$ erit

$$dx^2 = 2d\omega^2 \left(Q + \frac{A}{x} - \frac{P}{xx} \right) \text{ et } dx = \pm d\omega \sqrt{2 \left(Q + \frac{A}{x} - \frac{P}{xx} \right)}$$

haecque differentiale dx per $d\omega$ exprimitur. Deinde vero habetur

$$d\phi = \frac{d\omega}{x} \sqrt{2P}$$

etque per hypothesin:

$$dP = -Mx\omega^2 \sqrt{2P} \text{ et } dQ = -\frac{Md\omega}{x} \sqrt{2P} + N\omega^2 \sqrt{2 \left(Q + \frac{A}{x} - \frac{P}{xx} \right)}$$

vbi quidem signorum ambiguum inferius locum habere statuimus, quia motum ab apogeo numerare in animo est, ita ut hinc exundo distantia x minuat.

V.

Cum igitur differentiale dx in apogeo et perigeo evanescat, necesse est ut his locis formula irrationalis $\sqrt{2 \left(Q + \frac{A}{x} - \frac{P}{xx} \right)}$ in nihilum abeat, in reliquis autem locis valorem sortitur realem. Commodissime ergo haec formula per sinum cuiuspiam anguli ψ exhibebitur, qui cum in apogeo evanescat, in perigeo autem duobus re-

Etis

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$$) = Q$$

$$Qd\omega^2$$

$$\left(\frac{P}{xx} \right)$$

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Etis

Etis aequalis fiat, anomaliam lunae referet: idque sensu vero, ita ut distantia x in apogeo prodeat maxima, in perigeo vero minima. Sic igitur ut formam motus regularis sequamur:

$$\text{semilunus rectum orbitae lunaris} = p$$

$$\text{excentricitas orbitae} = q$$

$$\text{et anomalia vera lunae} = \psi$$

$$\text{etique hinc per eandem legem distantia } x = \frac{p}{1 - q \cos \psi}$$

Verum hic quantitates p et q , quae in motu regulari essent constantes, nunc pro variabilibus sunt habendae, earumque variabilitas per variabilitatem quantitarum P et Q , quae in motu regulari iidem sunt constantes, determinari debet.

VI.

$$\text{Substituamus ergo valorem assumptum } x = \frac{p}{1 - q \cos \psi}$$

in formula irrationali $\sqrt{2 \left(Q + \frac{A}{x} - \frac{P}{xx} \right)}$, quae abibit in

$$\frac{1}{p} \sqrt{ (Qp^2 + Ap^2 (1 - q \cos \psi)^2 - P (1 - q \cos \psi)^2) }$$

$$\frac{1}{p} \sqrt{ (Qpp + Ap^2 - Apg \cos \psi + 2Pq \cos \psi - Pqq \cos^2 \psi) }$$

quae ut reducatur ad formam $\sqrt{\sin \psi}$, fiat

$$\text{primo } 2P - Ap^2 = 0$$

$$\text{tum vero } Qpp + Ap^2 - P = Pqq$$

$$\text{ac nostra formula fiet } = \frac{1}{p} \sqrt{Pqq \sin \psi^2} = \frac{q \sin \psi}{p} \sqrt{P},$$

Etis

$$dx = -\frac{qd\omega \sin v}{p} \sqrt{2P} \text{ et}$$

$$dQ = -\frac{M d\omega}{x} \sqrt{2P} + \frac{N q d\omega \sin v}{p} \sqrt{2P}$$

VII.

Cum iam sit $2P - Ap = e$; erit $P = \frac{1}{2} Ap$: quo valore in altera formula substituto oritur:

$$Qpp + \frac{1}{2} Ap = \frac{1}{2} Apqq \text{ seu } Q = -\frac{A}{2p} (1 - qq)$$

Sumantur nunc differentia; eritque

$dP = -Mx d\omega \sqrt{2P} = \frac{1}{2} A dp$, quae ob $2P = Ap$ abir in hanc

$$-Mx d\omega \sqrt{Ap} = \frac{1}{2} A dp, \text{ siue } dp = -\frac{2Mx d\omega}{A} \sqrt{Ap}$$

vel etiam $\sqrt{Ap} = -\sqrt{Mx d\omega}$

Simili modo erit

$$dQ = +\frac{A dp (1 - qq)}{2pp} + \frac{A q dq}{p} = -\frac{Mx d\omega (1 - qq)}{pp} \sqrt{Ap} + \frac{A q dq}{p}$$

ideoque

$$\frac{A q dq}{p} = M d\omega \left(\frac{x(1 - qq)}{pp} - \frac{1}{x} \right) \sqrt{Ap} + \frac{N q d\omega \sin v}{p} \sqrt{Ap}$$

$$\text{At est } \frac{x(1 - qq)}{pp} \cdot 1 = \frac{x}{pp} (1 - qq - \frac{pp}{xx}) = \frac{x}{pp} (1 - qq + 2q \cos v - q^2 \cos^2 v)$$

$$\text{siue } \frac{x(1 - qq)}{pp} - \frac{1}{x} = \frac{q^2}{pp} (2 \cos v - q - q \cos^2 v)$$

Hinc ergo colligitur:

$$dq = \frac{Mx d\omega}{Ap} (2 \cos v - q - q \cos^2 v) \sqrt{Ap} + \frac{N d\omega \sin v}{A} \sqrt{Ap} \text{ siue}$$

$$dq = d\omega \left(\frac{M}{A} (2 \cos v - \frac{q \sin v^2}{1 - q \cos^2 v}) + \frac{N}{A} \sin v \right) \sqrt{Ap}$$

VIII.

VIII.

Inventa iam relatione differentialium dx , dp et dq ad differentiale temporis $d\omega$ scilicet:

$$dx = -\frac{qd\omega \sin v}{q} \sqrt{Ap}; \quad dp = -\frac{2Mx d\omega}{A} \sqrt{Ap}$$

$$\text{et } dq = d\omega \left(\frac{M}{A} (2 \cos v - \frac{q \sin v^2}{1 - q \cos^2 v}) + \frac{N}{A} \sin v \right) \sqrt{Ap}$$

superest, ut quoque relationem elementi anomaliae $d\omega$ definiamus. Cum igitur sit

$$x = \frac{p}{1 - q \cos v}, \text{ erit } 1 - q \cos v = \frac{p}{x}; \text{ hincque differentiendo}$$

$$qd\omega \sin v = dq \cos v + \frac{dp}{x} - \frac{p dx}{xx};$$

substantur valores pro dq , dp et dx inventi; ac diuisione facta per $q \sin v$ prodibit

$$d\omega = \frac{d\omega}{xx} \sqrt{Ap} - \frac{d\omega}{q} \left(\frac{M}{A} (2 \sin v + \frac{q \sin v \cos v}{1 - q \cos^2 v}) - \frac{N}{A} \cos v \right) \sqrt{Ap}$$

Pro elemento autem longitudinis $d\Phi$ ob $2P = Ap$, ex antecedentibus habemus:

$$d\Phi = \frac{d\omega}{xx} \sqrt{Ap} = \frac{d\omega (1 - q \cos v)^2}{pp} \sqrt{Ap}$$

IX.

Ex his formulis statim se offert motus apogei; cum enim longitudo apogei sit $= \Phi - v$, erit eius differentiale pro tempore $d\omega$:

$$d\Phi - dv = \frac{d\omega}{q} \left(\frac{M}{A} (2 \sin v + \frac{q \sin v \cos v}{1 - q \cos^2 v}) - \frac{N}{A} \cos v \right) \sqrt{Ap}$$

cuius ergo integrale praebit verum motum apogei cum omnibus inaequalitatibus; quibus perturbatur. Vnde

N n

qui-

quidem perficitur, quod per se est manifestum, si quantitates M et N evanescerent, motum apogei fore nullum, seu apogeuin perpetuo in loco fixo esse permanurum. Deinde etiam inuabit nouasse has formulas:

$$d q \cos v = -q d \phi \sin v + \frac{2M}{A} d \omega \sqrt{A p}$$

$$d q \sin v = +q d \phi \cos v + d \omega \left(\frac{N}{A} - \frac{M}{A} \cdot \frac{q \sin v}{1 - q \cos v} \right) \sqrt{A p}$$

Tandem quoque habemus ex motu solis $d u = d \theta = \frac{d \omega (1 - e \cos u)^2}{(1 - e^2) \sqrt{1 - e^2}}$ ideoque

$$d \eta = d \phi - d \theta = d \omega \left(\frac{1 - q \cos v}{p p} \sqrt{A p} - \frac{(1 - e \cos u)^2}{(1 - e^2) \sqrt{1 - e^2}} \right)$$

X.

Inuentis nunc omnium differentialium relationibus ad elementum temporis $d \omega$, euoluamus valores litterarum M et N, ac primo quidem cum sit

$z = \sqrt{(y y - 2 x y \cos \eta + x x \sec \psi^2)}$; quoniam quantitas z non nisi in terminis minimis occurrit, pro sec. ψ tuto unitas scribi poterit, et quia y tantopere excedit x , erit proxime

$$\frac{1}{z^2} = \frac{1}{y^2} + \frac{3x^2}{y^4} \cos^2 \eta + \frac{3x^2}{2y^5} (5 \cos^2 \eta - 1) \text{ siue}$$

$$\frac{1}{z^2} = \frac{1}{y^2} + \frac{3x^2}{y^4} \cos^2 \eta + \frac{3x^2}{4y^5} (3 + 5 \cos^2 \eta)$$

Ideoq; hinc habebitur:

$$\frac{y}{z^3} - \frac{1}{y y} = \frac{3x^2}{y^3} \cos^2 \eta + \frac{3x^2}{4y^5} (3 + 5 \cos^2 \eta)$$

Vnde

im, si fore e per- iulas:

$$\sqrt{A p}$$

$$d \theta =$$

)

nibus terna-

itas 2 unitas xime

siue

Vnde

Vnde oblinemus:

$$M = a^3 \left(\frac{3x^2}{2y^3} \sin 2 \eta + \frac{3x^2}{8y^4} (\sin \eta + 5 \sin 3 \eta) \right)$$

$$N = \frac{a^3 (\delta + \mathcal{D})}{\Theta} \cos \psi^2 \left(\frac{1}{x x} - \frac{1}{b b} \right) - \frac{A}{x x}$$

$$- a^3 \left(\frac{x^2}{2y^3} (1 + 3 \cos 2 \eta) + \frac{3x^2}{8y^4} (3 \cos \eta + 5 \cos 3 \eta) \right)$$

XI.

Cum sit proxime $\cos \psi^2 = 1 - \frac{1}{2} \tan g^2 - \frac{1}{2} \tan g^2 \cos 2(\phi - \pi)$, eius valor unitate erit minor, atque ex parte constans, et parte variabili constabit, quae illa multo erit minor. Ponatur ergo

$$\cos \psi^2 = \lambda + \Pi; \text{ vt fit } \Pi = 1 - \lambda - \frac{1}{2} \tan g^2 - \frac{1}{2} \tan g^2 \cos 2(\phi - \pi)$$

vbi λ denotat partem constantem unitate proxime aequalem, Π vero partem variabilem.

Erit ergo:

$$N = \frac{\lambda a^3 (\delta + \mathcal{D})}{\Theta} \left(\frac{1}{x x} - \frac{1}{b b} \right) - \frac{A}{x x} + \frac{a^3 (\delta + \mathcal{D})}{\Theta} \Pi \left(\frac{1}{x x} - \frac{1}{b b} \right)$$

$$- a^3 \left(\frac{x^2}{2y^3} (1 + 3 \cos 2 \eta) + \frac{3x^2}{8y^4} (3 \cos \eta + 5 \cos 3 \eta) \right)$$

Structuratur nunc $A = \frac{\lambda a^3 (\delta + \mathcal{D})}{\Theta}$; vt fiat

$$N = - \frac{A}{b b} + A \Pi \left(\frac{1}{x x} - \frac{1}{b b} \right)$$

$$- a^3 \left(\frac{x^2}{2y^3} (1 + 3 \cos 2 \eta) + \frac{3x^2}{8y^4} (3 \cos \eta + 5 \cos 3 \eta) \right)$$

N n 2

ac

ac ponatur breuiter gratia: $p = b(1 + \xi)$

$$\text{erit } \frac{\sqrt{Ap}}{pp} = \sqrt{\frac{A}{b^3(1+\xi)^3}} = (1 - \frac{2}{3}\xi + \frac{1}{3}\xi^2) \sqrt{\frac{A}{b^3}}$$

ob ξ prae 1 vehementer paruum, sitque porro:

$$\sqrt{\frac{A}{b^3}} = \sqrt{\frac{\lambda^2 b^3 (\delta + \gamma)}{\ominus}} = m,$$

atque habebitur $d\phi = m d\omega (1 - \frac{2}{3}\xi + \frac{1}{3}\xi^2) (1 - q \operatorname{cof} \omega)^2$

XII.

Substituatur nunc pro x et y valores $\frac{p}{1 - q \operatorname{cof} \omega}$

et $\frac{c}{1 - e \operatorname{cof} \omega}$, erique

$$M = a^3 \left(\frac{3p(1 - e \operatorname{cof} \omega)^2}{2e^2(1 - q \operatorname{cof} \omega)^2} \sin 2\eta + \frac{3pp(1 - e \operatorname{cof} \omega)^4}{8e^4(1 - q \operatorname{cof} \omega)^2} (\sin \eta + 5 \sin 3\eta) \right)$$

$$N = -\frac{A}{bb} + A \Pi \left(\frac{(1 - q \operatorname{cof} \omega)^4}{pp} - \frac{1}{bb} \right),$$

$$-a^3 \left(\frac{p(1 - e \operatorname{cof} \omega)^2}{2e^2(1 - q \operatorname{cof} \omega)^2} (1 + 3 \operatorname{cof} 2\eta) + \frac{3pp(1 - e \operatorname{cof} \omega)^4}{8e^4(1 - q \operatorname{cof} \omega)^2} (3 \operatorname{cof} \eta + 5 \operatorname{cof} 3\eta) \right)$$

vbi quidem quoque terminus $\frac{A \Pi}{bb}$ prae termino $\frac{A}{bb}$

omitti potest. Nunc ut hinc valores $\frac{M}{A} \sqrt{Ap}$ et $\frac{N}{A} \sqrt{Ap}$

commode exprimentur, erit

$$\frac{a^2 b}{A e^2} \sqrt{Ab} = \frac{a^3}{m e^3} = \frac{1}{m(1 - e^2)^2} = \frac{1 + 3e^2}{m}$$

quoniam in his terminis minimis pro $1 - e^2$ scribere licet 1.

Tum vero fit $\frac{b}{c} = n$, erique \ast fractio valde parua.

XIII.

XIII.

Facilis ergo his substitutionibus, ob $p = b(1 + \xi)$ habebimus:

$$\frac{M}{A} \sqrt{Ap} = \frac{3(1 + 3e^2)(1 - e \operatorname{cof} \omega)^2}{2m(1 - q \operatorname{cof} \omega)} (1 + \frac{2}{3}\xi) \sin 2\eta$$

$$+ \frac{3m(1 - e \operatorname{cof} \omega)^4}{8m(1 - q \operatorname{cof} \omega)^2} (1 + \frac{2}{3}\xi) (\sin \eta + 5 \sin 3\eta)$$

Pro altera valore $\frac{N}{A} \sqrt{Ap}$ statuetur terminus minimus:

$$\frac{\sqrt{Ab}}{bb} = \sqrt{\frac{\lambda^2 b^3 (\delta + \gamma)}{\ominus b^4}} = i; \text{ erique}$$

$$\frac{N}{A} \sqrt{Ap} = -\frac{(1 + 3e^2)(1 - e \operatorname{cof} \omega)^2}{2m(1 - q \operatorname{cof} \omega)} (1 + \frac{2}{3}\xi) (1 + 3 \operatorname{cof} 2\eta)$$

$$- \frac{3m(1 - e \operatorname{cof} \omega)^4}{8m(1 - q \operatorname{cof} \omega)^2} (1 + \frac{2}{3}\xi) (3 \operatorname{cof} \eta + 5 \operatorname{cof} 3\eta)$$

$$+ m(1 - q \operatorname{cof} \omega)^2 (1 - \frac{2}{3}\xi) \Pi - i$$

vbi notari oportet, terminos per \ast multiplicatos ratione praecedentium esse minimos; tum vero quantitates ξ et Π atque multo magis i esse fractiones prae vnitatem fere euanescentes.

XIV.

Quoniam hi ipsi termini quantitates M et N involuentes sunt valde parui, in his sine errore altiores potestates virtusque excentricitatis q et e negligi possunt. In terminis ergo primis simpliciter per m diuisis excentricitates tantum ad duas dimensiones intro-

$N n^3$

ducan-

III.

II.

\sqrt{Ap}

$\frac{A}{bb}$

$\eta)$

$\eta)$

$\operatorname{cof} \omega$

$(\omega)^2$

ducantur, in terminis autem per $\frac{n}{m}$ multiplicatis pen-
tus omitantur, quia fractio n iam fere quadrato excen-
tricitatis q aequivaler. In termino autem littera mini-
ma II affecto, quia is per numerum m factis magnum,
verpote 13 fere, est multiplicatus, excentricitas q vnius
dimensionis reinestatur.

His observatis habebimus:

$$\frac{M}{A} \sqrt{Ap} = \left[\begin{aligned} & + \frac{3}{2m} (1 + \frac{2}{3} ee + \frac{1}{3} qq) \sin 2\eta + \frac{3q}{4m} \sin (2\eta - v) \\ & + \frac{3q}{4m} \sin (2\eta + v) - \frac{9e}{4m} \sin (2\eta - u) - \frac{9e}{4m} \sin (2\eta + u) \\ & + \frac{3q^2}{8m} \sin (2\eta - 2v) + \frac{3q^2}{8m} \sin (2\eta + 2v) \\ & + \frac{9ee}{8m} \sin (2\eta - 2u) + \frac{9ee}{8m} \sin (2\eta + 2u) \\ & - \frac{9eq}{8m} \sin (2\eta - v + u) - \frac{9eq}{8m} \sin (2\eta + v - u) \\ & - \frac{9eq}{8m} \sin (2\eta - v - u) - \frac{9eq}{8m} \sin (2\eta + v + u) \\ & + \frac{9}{4m} \xi \sin 2\eta + \frac{9}{8m} q \xi \sin (2\eta - v) + \frac{9}{8m} q \xi \sin (2\eta + v) \\ & - \frac{27}{8m} e \xi \sin (2\eta - u) - \frac{27}{8m} e \xi \sin (2\eta + u) \\ & + \frac{3e}{8m} \sin \eta + \frac{15e}{8m} \sin 3\eta \end{aligned} \right]$$

$$\frac{N}{A} \sqrt{Ap} =$$

$$\frac{N}{A} \sqrt{Ap} = \left[\begin{aligned} & - \frac{1}{2m} (1 + \frac{2}{3} ee + \frac{1}{3} qq) - \frac{3}{2m} (1 + \frac{2}{3} ee + \frac{1}{3} qq) \cos 2\eta \\ & - \frac{q}{2m} \cos v + \frac{3e}{2m} \cos u \\ & - \frac{3q}{4m} \cos (2\eta - v) - \frac{3q}{4m} \cos (2\eta + v) \\ & + \frac{9e}{4m} \cos (2\eta - u) + \frac{9e}{4m} \cos (2\eta + u) \\ & - \frac{9q}{4m} \cos 2v + \frac{3eq}{4m} \cos (v - u) + \frac{3eq}{4m} \cos (v + u) \\ & - \frac{3ee}{4m} \cos 2u \\ & - \frac{3q^2}{8m} \cos (2\eta - 2v) - \frac{3q^2}{8m} \cos (2\eta + 2v) \\ & - \frac{9ee}{8m} \cos (2\eta - 2u) - \frac{9ee}{8m} \cos (2\eta + 2u) \\ & + \frac{9eq}{8m} \cos (2\eta - v + u) + \frac{9eq}{8m} \cos (2\eta + v - u) \\ & + \frac{9eq}{8m} \cos 2\eta - v - u) + \frac{9eq}{8m} \cos (2\eta + v + u) \\ & - \frac{3}{4m} \xi - \frac{9}{4m} \xi \cos 2\eta - \frac{3q}{8m} \xi \cos v \\ & - \frac{9q}{8m} \xi \cos (2\eta - v) - \frac{9q}{8m} \xi \cos (2\eta + v) \\ & + \frac{9e}{4m} \xi \cos u + \frac{27e}{8m} \xi \cos (2\eta - u) \\ & + \frac{27e}{8m} \xi \cos (2\eta + u) - \frac{9m}{8m} \cos \eta - \frac{15m}{8m} \cos 3\eta \\ & + m \text{ II} - 2m q \text{ II} \cos v - \frac{1}{2} m \xi \text{ II} - i \end{aligned} \right]$$

XV.

Quæramus igitur valores evolutos nostrorum differentialium ad elementum temporis applicatorum : ac primo quidem habebimus :

$$\frac{d\theta}{d\omega} = m \left(1 + \frac{1}{2} gq \right) - 2mq \cos v + \frac{1}{2} mqq \cos 2v - \frac{1}{2} m \left(1 + \frac{1}{2} gq \right) \xi + 3mq\xi \cos v - \frac{1}{2} mqq\xi \cos 2v + \frac{1}{2} m^2 \xi^2$$

$$\frac{d\eta}{d\omega} = 1 + 2ce - 2e \cos v + \frac{1}{2} ee \cos 2v; \text{ vnde concludimus}$$

$$\frac{d\eta}{d\omega} = m \left(1 + \frac{1}{2} gq \right) - 1 - 2ce - 2mq \cos v + 2e \cos v + \frac{1}{2} mqq \cos 2v - \frac{1}{2} ee \cos 2v - \frac{1}{2} m \left(1 + \frac{1}{2} gq \right) \xi + 3mq\xi \cos v - \frac{1}{2} mqq\xi \cos 2v + \frac{1}{2} m^2 \xi^2$$

$$\text{Deinde cum sit } \frac{dp}{d\omega} = -2x \cdot \frac{M}{A} \sqrt{Ap} = -\frac{2b(1+\xi)}{1-q\cos v} \cdot \frac{M}{A} \sqrt{Ap},$$

$$\text{ob } p = b(1+\xi) \text{ erit } \frac{d\xi}{d\omega} =$$

$$\left(-2 \left(1 + \frac{1}{2} gq \right) - 2q \cos v - qq \cos 2v - 2\xi - 2q\xi \cos v \right) \frac{M}{A} \sqrt{Ap} \text{ ac valorem pro } \frac{M}{A} \sqrt{Ap} \text{ inuentum substituendo, obtinebi-}$$

mus sequentes formulas :

$$\frac{d\xi}{d\omega} =$$

$$\frac{d\xi}{d\omega} = \left\{ \begin{array}{l} -\frac{3}{m} \left(1 + \frac{1}{2} gq \right) \sin 2v - \frac{3q}{m} \sin(2v-v) - \frac{3q}{m} \sin(2v+v) \\ + \frac{9e}{2m} \sin(2v-v) + \frac{9e}{2m} \sin(2v+v) \\ - \frac{9qq}{4m} \sin(2v-2v) - \frac{9qq}{4m} \sin(2v+2v) \\ - \frac{9ee}{4m} \sin(2v-2v) - \frac{9ee}{4m} \sin(2v+2v) \\ + \frac{9e}{2m} \sin(2v-v) + \frac{9e}{2m} \sin(2v+v) \\ + \frac{5e}{2m} \sin(2v-v) + \frac{9e}{2m} \sin(2v+v) \\ - \frac{15}{2m} \xi \sin 2v - \frac{15q}{2m} \xi \sin(2v-v) - \frac{15q}{2m} \xi \sin(2v+v) \\ + \frac{45e}{4m} \xi \sin(2v-v) + \frac{45e}{4m} \xi \sin(2v+v) \\ - \frac{3x}{4m} \sin v - \frac{15x}{4m} \sin 3v \end{array} \right.$$

XVI.

$$\text{Porro cum sit } \frac{q \sin v^2}{1-q \cos v} = \frac{q-y \cos 2v}{2(1-q \cos v)} =$$

$$\frac{1}{2} q - \frac{1}{2} q \cos 2v + \frac{1}{2} qq \cos v - \frac{1}{2} qq \cos 3v; \text{ erit}$$

$$\frac{dq}{d\omega} = (2e \cos v - \frac{1}{2} q \cos 2v - \frac{1}{2} qq \cos v + \frac{1}{2} qq \cos 3v) \frac{M}{A} \sqrt{Ap} + \frac{N}{A} \sqrt{Ap}$$

Facta ergo substitutione valorum pro $\frac{M}{A} \sqrt{Ap}$ et $\frac{N}{A} \sqrt{Ap}$ inuentorum, habebitur :

00

$$\frac{dq}{d\omega} =$$

$$\begin{aligned}
& + \frac{9}{4m} (1 + 2ce + \frac{5}{2} qg) \sin(2v - w) + \frac{3}{4m} (1 + 2ce + \frac{5}{2} qg) \\
& \sin(2v + w) - \frac{1}{2m} (1 + 2ce + \frac{5}{2} qg) \sin v \\
& + \frac{3g}{4m} \sin 2v + \frac{3g}{2m} \sin(2v - w) + \frac{3g}{4m} \sin(2v + w) - \frac{g}{4m} \sin 2v \\
& - \frac{27e}{8m} \sin(2v - w) - \frac{27e}{8m} \sin(2v + w) - \frac{9e}{8m} \sin(2v - w) \\
& - \frac{9e}{8m} \sin(2v + w) + \frac{3e}{4m} \sin(w - w) + \frac{3e}{4m} \sin(w + w) \\
& + \frac{15g}{16m} \sin(2v - 3v) + \frac{9g}{16m} \sin(2v + 3v) - \frac{9g}{8m} \sin 3v \\
& - \frac{9e}{8m} \sin(2v - w) - \frac{9e}{8m} \sin(2v + w) - \frac{9e}{4m} \sin(2v - 2v + w) \\
& - \frac{9e}{4m} \sin(2v - 2v - w) - \frac{9e}{8m} \sin(2v + 2v - w) - \frac{9e}{8m} \sin(2v + 2v + w) \\
& + \frac{27ce}{16m} \sin(2v - v - 2w) + \frac{27ce}{16m} \sin(2v - v + 2w) \\
& + \frac{9ce}{16m} \sin(2v + v - 2w) + \frac{9ce}{16m} \sin(2v + v + 2w) \\
& + \frac{3ce}{8m} \sin(2v - w) + \frac{3ce}{8m} \sin(2v + w) \\
& - \frac{3ce}{8m} \sin(w - 2w) - \frac{3ce}{8m} \sin(w + 2w) \\
& + \frac{27}{8m} \xi \sin(2v - v) + \frac{9}{8m} \xi \sin(2v + v) - \frac{3}{4m} \xi \sin v \\
& + \frac{9g}{8m}
\end{aligned}$$

$$\begin{aligned}
& + \frac{9g}{8m} \xi \sin 2v + \frac{9g}{4m} \xi \sin(2v - 2v) + \frac{9g}{8m} \xi \sin(2v + 2v) \\
& - \frac{3g}{8m} \xi \sin 2v + \frac{9e}{8m} \xi \sin(w - w) + \frac{9e}{8m} \xi \sin(w + w) \\
& - \frac{81e}{16m} \xi \sin(2v - w - w) - \frac{81e}{16m} \xi \sin(2v - w + w) \\
& - \frac{27e}{16} \xi \sin(2v + w - w) - \frac{27e}{16m} \xi \sin(2v + w + w) \\
& + \frac{15m}{16m} \sin(v - v) - \frac{3m}{16m} \sin(v + v) \\
& + \frac{45m}{16m} \sin(3v - v) + \frac{15m}{16m} \sin(3v + v) \\
& + m \text{ II } \sin v - m g \text{ II } \sin 2v - \frac{3}{2} m \xi \text{ II } \sin v - j \sin v
\end{aligned}$$

XVII.

Deinde cum sit $\frac{q \sin v \cos v}{1 - q \cos v} = \frac{q \sin 2v}{2(1 - q \cos v)} =$
 $= \frac{1}{2} q \sin 2v + \frac{1}{2} q q \sin v + \frac{1}{2} q q \sin 3v$; erit
 pro motu elementari apogei:
 $\frac{q(d\phi - dv)}{d\omega} = (2 \sin v + \frac{1}{2} q \sin 2v + \frac{1}{2} q q \sin v$
 $+ \frac{1}{2} q q \sin 3v) \frac{M}{A} V A p - \cos v. \frac{N}{A} V A p$
 ac facta substitutione obtinebitur:

O o 2

$$\frac{q(d\phi - dv)}{d\omega} =$$

$$\frac{d(\Phi - \psi)}{dv} = \frac{9}{4m} (1 + \frac{3}{2} \epsilon \epsilon + \frac{3}{2} q q) \operatorname{col}(2\eta - v) - \frac{3}{4m} (1 + \frac{3}{2} \epsilon \epsilon + \frac{3}{2} q q) \operatorname{col}(2\eta + v) + \frac{1}{2m} (1 + \frac{3}{2} \epsilon \epsilon + \frac{3}{2} q q) \operatorname{col} v$$

$$+ \frac{3q}{4m} \operatorname{col} 2\eta + \frac{3q}{2m} \operatorname{col}(2\eta - 2v) - \frac{3q}{4m} \operatorname{col}(2\eta + 2v)$$

$$+ \frac{q}{4m} \operatorname{col} 2v + \frac{q}{4m} - \frac{3\epsilon}{4m} \operatorname{col}(v - u) - \frac{3\epsilon}{4m} \operatorname{col}(v + u)$$

$$- \frac{27\epsilon}{8m} \operatorname{col}(2\eta - v - u) - \frac{27\epsilon}{8m} \operatorname{col}(2\eta - v + u)$$

$$+ \frac{9\epsilon}{8m} \operatorname{col}(2\eta + v - u) + \frac{9\epsilon}{8m} \operatorname{col}(2\eta + v + u)$$

$$+ \frac{15qq}{16m} \operatorname{col}(2\eta - 3v) - \frac{9qq}{16m} \operatorname{col}(2\eta + 3v) + \frac{9q}{8m} \operatorname{col} 3v$$

$$- \frac{9\epsilon q}{8m} \operatorname{col}(2\eta - u) - \frac{9\epsilon q}{8m} \operatorname{col}(2\eta + u) - \frac{3\epsilon q}{4m} \operatorname{col} u$$

$$- \frac{3\epsilon q}{8m} \operatorname{col}(2v - u) - \frac{3\epsilon q}{8m} \operatorname{col}(2v + u)$$

$$- \frac{9\epsilon q}{4m} \operatorname{col}(2\eta - 2v + u) - \frac{9\epsilon q}{4m} \operatorname{col}(2\eta - 2v - u)$$

$$+ \frac{9\epsilon q}{8m} \operatorname{col}(2\eta + 2v - u) + \frac{9\epsilon q}{8m} \operatorname{col}(2\eta + 2v + u)$$

$$+ \frac{27\epsilon \epsilon}{16m} \operatorname{col}(2\eta - v - 2u) + \frac{27\epsilon \epsilon}{16m} \operatorname{col}(2\eta - v + 2u)$$

$$- \frac{9\epsilon \epsilon}{16m} \operatorname{col}(2\eta + v - 2u) - \frac{9\epsilon \epsilon}{16m} \operatorname{col}(2\eta + v + 2u)$$

$$+ \frac{3\epsilon \epsilon}{8m} \operatorname{col}(v - 2u) + \frac{3\epsilon \epsilon}{8m} \operatorname{col}(v + 2u)$$

+ 27 / 8m

Euolu
et cum f
Substitu
V Ap =
Negle

17 / m

$$+ \frac{27}{8m} \xi \operatorname{col}(2\eta - v) - \frac{9}{8m} \xi \operatorname{col}(2\eta + v) + \frac{3}{4m} \xi \operatorname{col} v$$

$$+ \frac{9q}{8m} \xi \operatorname{col} 2\eta + \frac{9q}{4m} \xi \operatorname{col}(2\eta - 2v) - \frac{9q}{8m} \xi \operatorname{col}(2\eta + 2v)$$

$$+ \frac{3q}{8m} \xi + \frac{3q}{8m} \xi \operatorname{col} 2v$$

$$- \frac{81\epsilon}{16m} \xi \operatorname{col}(2\eta - v - u) + \frac{27\epsilon}{16m} \xi \operatorname{col}(2\eta + v - u)$$

$$- \frac{81\epsilon}{16m} \xi \operatorname{col}(2\eta - v + u) + \frac{27\epsilon}{16m} \xi \operatorname{col}(2\eta + v + u)$$

$$- \frac{9\epsilon}{8m} \xi \operatorname{col}(v - u) - \frac{9\epsilon}{8m} \xi \operatorname{col}(v + u)$$

$$+ \frac{15\epsilon}{16m} \operatorname{col}(v - v) + \frac{3\epsilon}{16m} \operatorname{col}(v + v)$$

$$+ \frac{45m}{16m} \operatorname{col}(3\eta - v) - \frac{15m}{16m} \operatorname{col}(3\eta + v)$$

$$- m \text{ II } \operatorname{col} v + m q \text{ II } \operatorname{col} 2v + \frac{3}{2} m \xi \text{ II } \operatorname{col} v + \epsilon \operatorname{col} v$$

XVIII.

Euoluamus simili modo valorem differentialium dx et dy,
et cum fit $\frac{y}{x^3} - \frac{1}{y} = \frac{3x}{y^3} \operatorname{cfl} \eta + \frac{3xx}{4y^3} (3\eta \operatorname{col} \eta) \operatorname{er} d\Phi = \frac{dx}{x^2} V Ap \operatorname{er} t$
 $dx = - \frac{x^2 x dy}{V Ap} (\frac{3x}{y^3} \operatorname{col} \eta + \frac{3xx}{4y^3} (3 + 5 \operatorname{col} 2\eta)) \operatorname{fin}(\theta - \eta) \operatorname{fin}(\Phi - \eta)$
Substitutis autem valoribus $x = \frac{p}{1 - q \operatorname{col} v}$, $y = \frac{e}{1 - \epsilon \operatorname{col} v}$, $p = b(1 + \frac{1}{2} \xi)$,
 $V Ap = m V b^2 p = m b^2 (1 + \frac{1}{2} \xi)$, $\frac{e^3}{e^3} = \frac{1}{(1 - \epsilon \operatorname{col} v)^3} = 1 + 3 \epsilon \operatorname{col} v + \frac{3}{2} \epsilon^2 \operatorname{col}^2 v$, erit
 $dx = - \frac{dx \operatorname{fin}(\theta - \eta) \operatorname{fin}(\Phi - \eta)}{m} \left[\frac{3(1 + \frac{1}{2} \xi)(1 + 3 \epsilon \operatorname{col} v)^2}{(1 - q \operatorname{col} v)^2} \operatorname{cfl} \eta + \frac{3}{2} m (3\eta \operatorname{col} \eta) \right]$
Negle

Neglectis igitur terminis, qui nullum valorem sensibilem continent, habebimus

$$\frac{dn}{du} = \frac{-\frac{3}{4m}(1+ge+\frac{1}{2}gg) - \frac{3}{4m}(1+ge+\frac{1}{2}gg)\cos 2\eta}{+\frac{3}{4m}(1+\frac{1}{2}ee+\frac{1}{2}gg)\cos(\Phi-\pi) + \frac{3}{4m}(1+\frac{1}{2}ee+\frac{1}{2}gg)\cos(\theta-\pi)} - \frac{3g}{2m}\cos v - \frac{3g}{4m}\cos(2\eta-v) - \frac{3g}{4m}\cos(2\eta+v) + \frac{9e}{4m}\cos u + \frac{9e}{8m}\cos(2\eta-u) + \frac{9e}{8m}\cos(2\eta+v) + \frac{3g}{4m}\cos(2\Phi-2\pi-v) + \frac{3g}{4m}\cos(2\Phi-2\pi+v) + \frac{3g}{4m}\cos(2\theta-2\pi-v) + \frac{3g}{4m}\cos(2\theta-2\pi+v) - \frac{9e}{8m}\cos(2\Phi-2\pi-u) - \frac{9e}{8m}\cos(2\Phi-2\pi+u) - \frac{9e}{8m}\cos(2\theta-2\pi-u) - \frac{9e}{8m}\cos(2\theta-2\pi+u) - \frac{9}{8m}\xi - \frac{9}{8m}\xi\cos 2\eta + \frac{9}{8m}\xi\cos(2\Phi-2\pi) + \frac{9}{8m}\xi\cos(2\theta-2\pi) - \frac{11\pi}{16m}\cos \eta + \frac{3\pi}{8m}\cos(\Phi+\theta-2\pi) - \frac{5\pi}{16m}\cos 3\eta + \frac{5\pi}{16m}\cos(3\Phi-\theta-2\pi) + \frac{5\pi}{16m}\cos(3\theta-\Phi-2\pi)$$

XIX.

XIX.

Simili autem modo praecedentem valorem per tang $(\Phi-\pi)$ dividendo prodibit differentiale logarithmi tangens inclinationis ρ , erit enim

$$\frac{d \text{tang} \rho}{du} = \frac{+\frac{3}{4m}(1+ge+\frac{1}{2}gg)\sin 2\eta - \frac{3}{4m}(1+ge+\frac{1}{2}gg)\sin 2(\Phi-\pi)}{\sin 2(\Phi-\pi) - \frac{3}{4m}(1+ge+\frac{1}{2}gg)\sin 2(\theta-\pi)} + \frac{3g}{4m}\sin(2\eta-v) + \frac{3g}{4m}\sin(2\eta+v) - \frac{9e}{8m}\sin(2\eta-u) - \frac{9e}{8m}\sin(2\eta+u) - \frac{3g}{4m}\sin(2\Phi-2\pi-v) - \frac{3g}{4m}\sin(2\Phi-2\pi+v) - \frac{3g}{4m}\sin(2\theta-2\pi-u) - \frac{3g}{4m}\sin(2\theta-2\pi+v) + \frac{9e}{8m}\sin(2\Phi-2\pi-u) + \frac{9e}{8m}\sin(2\Phi-2\pi+u) + \frac{9e}{8m}\sin(2\theta-2\pi-u) + \frac{9e}{8m}\sin(2\theta-2\pi+u) + \frac{9}{8m}\xi\sin 2\eta - \frac{9}{8m}\xi\sin(2\Phi-2\pi) - \frac{9}{8m}\xi\sin(2\theta-2\pi) + \frac{\pi}{16m}\sin \eta - \frac{3\pi}{8m}\sin(\Phi+\theta-\pi) + \frac{5\pi}{16m}\sin 3\eta - \frac{5\pi}{16m}\sin(3\Phi-\theta-2\pi) - \frac{5\pi}{16m}\sin(3\theta-\Phi-2\pi)$$

XX.

XX.

Quo iam facilius has formulas admodum complicatas evolvere queamus, quadruplicis generis terminos distingui conuenit. Primum scilicet genus eos complectitur terminos, qui tantum ab excentricitate orbis lunaris pendunt, neque excentricitatem solis, neque parallaxin solis seu litteram ω , neque inclinationem orbis lunaris seu litteram Π inuoluunt. Ad secundum genus refero terminos, qui ad primum genus insuper excentricitatem solis adiungunt. Ad tertium autem eos, qui praeterea parallaxin solis seu litteram ω inducunt. In quarto autem eas inaequalitates, quae insuper ab obliquitate orbis lunaris proueniunt, complexurus sum. Ab inaequalitatibus ergo primi generis exordiar, ideoque cum excentricitatem solis e , cum eius parallaxin, cum quoque obliquitatem orbis lunaris relictam

INVESTIGATIO INAEQUALITATUM

LUNAR PRIMI GENERIS.

XXI.

Neglectis ergo excentricitate solis cum eius parallaxi et obliquitate orbis lunaris, has habebimus aequationes:

$$\frac{d\xi}{d\omega} =$$

$$\frac{d\xi}{d\omega} = \left[\begin{array}{l} -\frac{3}{m} (1 + \frac{1}{2} q^2) \sin 2\eta - \frac{3q}{m} \sin (2\eta - \omega) - \frac{3q}{m} \sin (2\eta + \omega) \\ -\frac{9q^2}{4m} \sin (2\eta - 2\omega) - \frac{9q^2}{4m} \sin (2\eta + 2\omega) \\ -\frac{15}{2m} \xi \sin 2\eta - \frac{15q}{2m} \xi \sin (2\eta - \omega) - \frac{15q}{2m} \xi \sin (2\eta + \omega) \\ +\frac{9}{4m} (1 + \frac{1}{2} q^2) \sin (2\eta - \omega) + \frac{3}{4m} (1 + \frac{1}{2} q^2) \sin (2\eta + \omega) \\ -\frac{1}{2m} (1 + \frac{1}{2} q^2) \sin \omega - i \sin \omega \\ +\frac{3q}{4m} \sin 2\eta + \frac{3q}{2m} \sin (2\eta - 2\omega) + \frac{3q}{4m} \sin (2\eta + 2\omega) \\ -\frac{q}{4m} \sin 2\omega \\ +\frac{15q^2}{16m} \sin (2\eta - 3\omega) + \frac{9q^2}{16m} \sin (2\eta + 3\omega) - \frac{9q}{8m} \sin 3\omega \\ +\frac{27}{8m} \xi \sin (2\eta - \omega) + \frac{9}{8m} \xi \sin (2\eta + \omega) - \frac{3}{4m} \xi \sin \omega \\ +\frac{9q}{8m} \xi \sin 2\eta + \frac{9q}{4m} \xi \sin (2\eta - 2\omega) + \frac{9q}{8m} \xi \sin (2\eta + 2\omega) \\ -\frac{3q}{8m} \xi \sin 2\omega \end{array} \right]$$

P p

$$\frac{q^2(d^2\xi/d\omega^2)}{d\omega} =$$

$$\frac{dP}{d\omega} = \left[\begin{aligned} & + \frac{9}{4m} (1 + \frac{1}{2}gg) \operatorname{cof}(2\eta - v) - \frac{3}{4m} (1 + \frac{1}{2}gg) \operatorname{cof}(2\eta + v) \\ & + \frac{1}{2m} (1 + \frac{1}{2}gg) \operatorname{cof} v + i \operatorname{cof} v \\ & + \frac{3g}{4m} \operatorname{cof} 2\eta + \frac{3g}{2m} \operatorname{cof}(2\eta - 2v) - \frac{3g}{4m} \operatorname{cof}(2\eta + 2v) \\ & + \frac{g}{4m} \operatorname{cof} 2v + \frac{g}{4m} \\ & + \frac{15gg}{16m} \operatorname{cof}(2\eta - 3v) - \frac{9gg}{16m} \operatorname{cof}(2\eta + 3v) + \frac{gg}{8m} \operatorname{cof} 3v \\ & + \frac{27}{8m} \xi \operatorname{cof}(2\eta - v) - \frac{9}{8m} \xi \operatorname{cof}(2\eta + v) + \frac{3}{4m} \xi \operatorname{cof} v \\ & + \frac{9g}{8m} \xi \operatorname{cof} 2\eta + \frac{9g}{4m} \xi \operatorname{cof}(2\eta - 2v) - \frac{9g}{8m} \xi \operatorname{cof}(2\eta + 2v) \\ & + \frac{3g}{8m} \xi + \frac{3g}{8m} \xi \operatorname{cof} 2v \end{aligned} \right]$$

$$\frac{dP}{d\omega} = m \left(1 + \frac{1}{2}gg \right) - 2mg \operatorname{cof} v + \frac{1}{2}mgg \operatorname{cof} 2v - \frac{1}{2}m \left(1 + \frac{1}{2}gg \right) \xi$$

$$+ 3mg \xi \operatorname{cof} v - \frac{1}{2}mgg \xi \operatorname{cof} 2v + \frac{1}{2}m \left(1 + \frac{1}{2}gg \right) \xi$$

$$+ 3mg \xi \operatorname{cof} v - 1 - 2mg \operatorname{cof} v + \frac{1}{2}mgg \operatorname{cof} 2v - \frac{1}{2}m \left(1 + \frac{1}{2}gg \right) \xi$$

$$+ 3mg \xi \operatorname{cof} v - \frac{1}{2}mgg \xi \operatorname{cof} 2v + \frac{1}{2}m \left(1 + \frac{1}{2}gg \right) \xi$$

XXII.

Hic autem primo patet valores litterarum ξ et g sine cognitis rationibus $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ definiiri non posse, has autem vicissim ipsas quantitates ξ et g inuolueri. Cum autem ad valores ξ et g inueniendos non opus sit rationes

iones $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ eo praecisionis gradu posse, quo ipsi illi valores desiderantur; Patet si valores ξ et g prope tantum veri consentent, his in rationibus $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ adhibitis, eosdem multo exactiores reperiri. Cum igitur, si motus esset regularis, foret $\xi = 0$ et $g = \operatorname{cosec}$ anguli, hinc primam hypothesein constituamus. Sic ergo $\xi = 0$ et $g = g$

et neglectis terminis, qui ob harum litterarum errores affici possent, vixore valde paruis praereliquis, habebimus proxime

$$\frac{dP}{d\omega} = m \left(1 + \frac{1}{2}gg \right) - 2mg \operatorname{cof} v; \quad \frac{d\eta}{d\omega} = \left(1 + \frac{1}{2}gg \right) - 1 - 2mg \operatorname{cof} v$$

$$\text{et } \frac{dP - d\eta}{d\omega} = \frac{9}{4mg} \left(1 + \frac{1}{2}gg \right) \operatorname{cof}(2\eta - v) - \frac{3}{4mg} \left(1 + \frac{1}{2}gg \right) \operatorname{cof}(2\eta + v)$$

$$+ \frac{1}{2mg} \left(1 + \frac{1}{2}gg \right) \operatorname{cof} v + \frac{1}{g} \operatorname{cof} v + \frac{1}{4m}$$

ideoque

$$\frac{dv}{d\omega} = m \left(1 + \frac{1}{2}gg \right) - \frac{1}{4m} - (2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g}) \operatorname{cof} v$$

$$- \frac{9}{4mg} \left(1 + \frac{1}{2}gg \right) \operatorname{cof}(2\eta - v) + \frac{3}{4mg} \left(1 + \frac{1}{2}gg \right) \operatorname{cof}(2\eta + v)$$

XXIII.

Ponamus ad has formulas abbreviandas:

$$m \left(1 + \frac{1}{2}gg \right) - 1 = a; \quad 2mg = \gamma$$

$$m \left(1 + \frac{1}{2}gg \right) - \frac{1}{4m} = \delta; \quad 2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g} = \theta$$

et neglectis quadratis gg in reliquis terminis, habebimus has formulas simpliciores :

$$\frac{dy}{dx} = \alpha - \gamma \cos v$$

$$\frac{dv}{dx} = \epsilon - \delta \cos v - \frac{9}{4mg} \cos(2y-v) + \frac{3}{4mg} \cos(2y+v)$$

Tum vero pro valoribus ξ et q propius inveniendis has aequationes :

$$\frac{d\xi}{dx} = -\frac{3}{m} (1 + \frac{1}{2} g\xi) \sin 2y - \frac{3\delta}{m} \sin(2y-v) - \frac{3\delta}{m} \sin(2y+v)$$

$$\frac{dq}{dx} = +\frac{9}{4m} (1 + \frac{1}{2} g\xi) \sin(2y-v) + \frac{3}{4m} (1 + \frac{1}{2} g\xi) \sin(2y+v)$$

$$- \frac{1}{2m} (1 + \frac{1}{2} g\xi) \sin v - \delta \sin v$$

$$+ \frac{3\delta}{4m} \sin 2y + \frac{3\delta}{2m} \sin(2y-2v) + \frac{3\delta}{4m} \sin(2y+2v) - \frac{\delta}{4m} \sin 2v$$

XXIV.

Fingamus ergo primo :

$$\xi = \mathfrak{A} \cos 2y + \mathfrak{B} \cos(2y-v) + \mathfrak{C} \cos(2y+v)$$

vbi notandum est terminos binos posteriores, vii in differentiis, multo esse minores primo. Quare cum etiam in differentialibus dy et dx duplicis generis termini occurrant, quorum posteriores prae primis sine valde parvi, in differentiatione solus primi termini totum differentialis dy valorem pono, in duobus vero reliquis tantum valorem principalem ; sic prodibit

$$\frac{d\xi}{dx} = -2\alpha \mathfrak{A} \sin 2y + \gamma \mathfrak{A} \sin(2y-v) + \gamma \mathfrak{A} \sin(2y+v)$$

$$- (2\alpha - \epsilon) \mathfrak{B} - (2\alpha + \epsilon) \mathfrak{C}$$

Collato ergo hoc differentiis cum forma proposita obtineatur :

bimus

$$\mathfrak{A} = \frac{3}{2mg} (1 + \frac{1}{2} g\xi)$$

$$(2\alpha - \epsilon) \mathfrak{B} = \gamma \mathfrak{A} + \frac{3\delta}{m} \quad \text{ergo} \quad \mathfrak{B} = \frac{3(\gamma + 2\alpha g)}{2mg(2\alpha - \epsilon)}$$

$$(2\alpha + \epsilon) \mathfrak{C} = \gamma \mathfrak{A} + \frac{3\delta}{m} \quad \text{ergo} \quad \mathfrak{C} = \frac{3(\gamma + 2\alpha g)}{2mg(2\alpha + \epsilon)}$$

XXV.

Simili modo fingatur :

$$y = g + A \cos(2y-v) + B \cos(2y+v) + C \cos v$$

$$+ D \cos 2y + E \cos(2y-2v) + F \cos(2y+2v) + G \cos 2v$$

$$+ H \cos 4y + J \cos(4y-2v) + K \cos(4y+2v)$$

vbi linea prior continet terminos multo maiores, quam binae inferiores. Hinc ergo sit differenciando secundum regulam supra datam :

$$\frac{dy}{dx} = - (2\alpha - \epsilon) A \sin(2y-v) - (2\alpha + \epsilon) B \sin(2y+v) - \epsilon C \sin v$$

$$+ \left(\frac{1}{2} (2\gamma - \delta) A + \frac{1}{2} (2\gamma + \delta) B + \frac{3}{2mg} C - 2\alpha D \right) \sin 2y$$

$$+ \left(\frac{1}{2} (2\gamma - \delta) A - \frac{9}{8mg} C - 2(\alpha - \epsilon) E \right) \sin(2y-2v)$$

$$+ \left(\frac{1}{2} (2\gamma + \delta) B - \frac{3}{8mg} C - 2(\alpha + \epsilon) F \right) \sin(2y+2v)$$

$$+ \left(\frac{1}{2} \delta C - \frac{3A + 9B}{8mg} - 2\epsilon G \right) \sin 2v$$

$$+ \left(\frac{3A + 9B}{8mg} - 4\alpha H \right) \sin 4y$$

$$+ \left(-\frac{9A}{8mg} - 2(2\alpha - \epsilon) J \right) \sin(4y-2v)$$

$$+ \left(-\frac{3B}{8mg} - 2(2\alpha + \epsilon) K \right) \sin(4y+2v)$$

R p 3

Hinc

Hincque elicientur sequentes coefficientium valores :

$$(2\alpha-\beta) A = -\frac{9}{4m} (1 + \frac{1}{2}gg)$$

$$(2\alpha+\beta) B = -\frac{3}{4m} (1 + \frac{1}{2}gg)$$

$$gC = \frac{1}{2m} (1 + \frac{1}{2}gg) + i$$

$$2\alpha D = \frac{1}{2} (2\gamma-\delta) A + \frac{1}{2} (2\gamma+\delta) B + \frac{3}{2mg} C - \frac{3g}{4m}$$

$$2(\alpha-\beta) E = \frac{1}{2} (2\gamma-\delta) A - \frac{9}{8mg} C - \frac{3g}{2m}$$

$$2(\alpha+\beta) F = \frac{1}{2} (2\gamma+\delta) A - \frac{3}{8mg} C - \frac{3g}{4m}$$

$$2gG = -\frac{3A+9}{8mg} B + \frac{1}{2} \delta C + \frac{g}{4m}$$

$$4\alpha H = \frac{3A+9\beta}{8mg} ; 2(2\alpha-\beta) J = -\frac{9}{8mg} A$$

$$2(2\alpha+\beta) K = -\frac{3}{8mg} B$$

XXVI.

Cum igitur his inuenitis valoribus sit multo verius:

$$\xi = \frac{1}{2} \text{cof } 2\gamma \text{ et } \eta = g + A \text{cof}(2\gamma-x) + B \text{cof}(2\gamma+v) + C \text{cof } \alpha$$

ubi terminos minores data opera adhuc omitto, quia forsalle correctione egent, praecedentes operationes multo accuratius influere atque ad ordinem terminorum vltiorem progredi poterimus. Obtinebimus ergo:

$$\frac{d\phi}{d\omega} = m (1 + \frac{1}{2}gg - C) - 2mg \text{cof } v$$

$$-m(\frac{1}{2}g(A+B) + \alpha \text{cof}(2\gamma-x) - mB \text{cof}(2\gamma+2v) + m(\frac{1}{2}gg - C) \text{cof } 2\alpha$$

$$\text{hincque } \frac{d\eta}{d\omega} = \frac{d\phi}{d\omega} - 1.$$

Porro

Porro ob $\frac{1}{g} = \frac{1}{g} - \frac{A}{gg} \text{cof}(2\gamma-v) - \frac{B}{gg} \text{cof}(2\gamma+v) - \frac{C}{gg} \text{cof } \alpha$

erit

$$\frac{d\phi-dv}{d\omega} = \frac{9}{4m} (1 + \frac{1}{2}gg) \text{cof}(2\gamma-v) - \frac{3}{4mg} (1 + \frac{1}{2}gg) \text{cof}(2\gamma+v)$$

$$+ \frac{1}{2mg} (1 + \frac{1}{2}gg) \text{cof } \alpha + \frac{1}{g} \text{cof } v + \frac{1}{4m} (1 - \frac{9A+3B-2C}{2gg})$$

$$+ \frac{1}{4m} (3 - \frac{A-B-3C}{gg}) \text{cof } 2\gamma + \frac{1}{4m} (6 - \frac{2A-9C}{2gg}) \text{cof}(2\gamma-2v)$$

$$- \frac{1}{4m} (3\gamma - \frac{2B-3C}{2gg}) \text{cof}(2\gamma+2v) + \frac{1}{4m} (1 + \frac{3A-9B-2C}{2gg}) \text{cof } 2\alpha$$

$$+ \frac{3A-9B}{8mg} \text{cof } 4\gamma - \frac{9A}{8mg} \text{cof}(4\gamma-2v) + \frac{3B}{8mg} \text{cof}(4\gamma+2v)$$

XXVII.

Ponatur ad abbreviandum :

$$m (1 + \frac{1}{2}gg - C) - 1 = a ; 2mg = \gamma$$

$$m (3\eta + A + B) = \epsilon ; \text{vt sit}$$

$$\frac{d\eta}{d\omega} = a - \gamma \text{cof } v - \epsilon \text{cof } 2\eta$$

$$-m A \text{cof}(2\eta-2v) - m B \text{cof}(2\eta+2v) + m(\frac{1}{2}gg - C) \text{cof } 2\alpha$$

Porro fit

$$m (1 + \frac{1}{2}gg - C) - \frac{1}{4m} (1 - \frac{9A+3B-2C}{2gg}) = \zeta$$

$$2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{1}{g} = \delta$$

$$+ m (\frac{1}{2}g(A+B) + \frac{1}{4m} (3 - \frac{A-B-3C}{gg})) = \xi$$

m A

$$m A + \frac{1}{4m} \left(6 - \frac{2A-9C}{2gg} \right) = 1$$

$$m B - \frac{1}{4m} \left(3 + \frac{2B-3C}{2gg} \right) = \theta$$

$$m \left(C - \frac{1}{2} gg \right) + \frac{1}{4m} \left(1 + \frac{3A-9B-2C}{2gg} \right) = k$$

vt habeatur

$$\frac{dv}{d\omega} = 6 - \delta \cos v - \frac{9}{4mg} \cos(2v-v) + \frac{3}{4mg} \cos(2v+v)$$

$$- \zeta \cos(2v-v) \cos(2v-2v) - \theta \cos(2v+2v) - \kappa \cos 2v$$

$$- \frac{3A+9B}{8mg} \cos 4v + \frac{9A}{8mg} \cos(4v-2v) - \frac{3B}{8mg} \cos(4v+2v)$$

vbi censeatur, ne coefficientes η , θ , cum angulis cognominibus confundantur.

XXVIII.

Opus plane non est, vt valores litterarum ξ et η accuratius determinemus, atque ad plures terminos, quam ante inuenimus, expediamus; verum hos ipsos terminos, quos ante inuenimus, accuratius obtinebimus, si litteris a et δ eos valores tribuemus, quos nunc eis conuenire collegimus. Pluribus autem terminis non indigemus tam ad longitudinem lunae ϕ , quam ad eius anomaliam veram v satis exacte definendam. Verum ad hoc ipsam negotium valores differentiales $\frac{d\phi}{d\omega}$ et $\frac{dv}{d\omega}$, ac praecipue hunc posterorem, quo motus apogei continetur, accuratius euoluere oportet, quoniam imprimis in motu medio apogei minutae particulae ingentis momenti esse possunt.

XXIX.

XXIX.

Cum igitur accuratius quam adhuc assumimus sit

$$\xi = \mathcal{X} \cos 2\eta + \mathcal{Y} \cos(2\eta-v) + \mathcal{Z} \cos(2\eta+v) \quad \text{et}$$

$$\eta = g + A \cos(2\eta-v) + B \cos(2\eta+v) + C \cos v$$

$$+ D \cos 2\eta + E \cos(2\eta-2v) + F \cos(2\eta+2v) + G \cos v$$

$$+ H \cos 4\eta + J \cos(4\eta-2v) + K \cos(4\eta+2v)$$

erit terminus ad quartum vsque ordinem extensus

I.

II.

$$\frac{d\phi}{d\omega} = m \left(1 + \frac{1}{2} g g - C \right) - (2mg - \frac{1}{2} mg C + m G) \cos v$$

III.

$$- m \left(\frac{1}{2} \mathcal{X} + A + B \right) \cos 2\eta - m A \cos(2\eta-2v) - m B \cos(2\eta+2v) - m \left(C - \frac{1}{2} gg \right) \cos 2v$$

IV.

$$+ m \left(\frac{1}{2} g \mathcal{X} - \frac{1}{2} \mathcal{Y} + g A + \frac{1}{2} g B - D - E \right) \cos(2\eta-v) \\ + m \left(\frac{1}{2} g \mathcal{X} - \frac{1}{2} \mathcal{Z} + g B + \frac{1}{2} g A - D - F \right) \cos(2\eta+v) \\ + m \left(\frac{1}{2} g A - E \right) \cos(2\eta-3v) + m \left(\frac{1}{2} g B - F \right) \cos(2\eta+3v) \\ + m \left(\frac{1}{2} g C - G \right) \cos 3v \\ - m (H+J) \cos(4\eta-v) - m (H+K) \cos(4\eta+v) - m K \cos(4\eta+3v)$$

vnde cum esset ante $\gamma = 2mg$, nunc accuratius erit

$$\gamma = 2mg - \frac{1}{2} mg C + m G$$

Qq

XXX.

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IX.

XXX.

Deinde cum nunc quoque fit accuratius :

$$\frac{1}{g} = \left(\frac{1}{g} + \frac{AA + BB + CC}{2g^3} \right)$$

II.

III.

$$-\frac{A}{gg} \operatorname{cof}(2\gamma - \nu) - \frac{B}{gg} \operatorname{cof}(2\gamma + \nu) - \frac{C}{gg} \operatorname{cof} \nu$$

IV.

$$\begin{aligned} & + \left(\frac{A+B}{g^3} - \frac{D}{gg} \right) \operatorname{cof} 2\gamma + \left(\frac{AC}{g^3} - \frac{E}{gg} \right) \operatorname{cof}(2\gamma - 2\nu) \\ & + \left(\frac{BC}{g^3} - \frac{F}{gg} \right) \operatorname{cof}(2\gamma + 2\nu) + \left(\frac{2AB+CC}{2g^3} - \frac{G}{gg} \right) \operatorname{cof} 2\nu \\ & + \left(\frac{AB-H}{g^3} - \frac{I}{gg} \right) \operatorname{cof} 4\gamma + \left(\frac{AA}{2g^3} - \frac{J}{gg} \right) \operatorname{cof}(4\gamma - 2\nu) \\ & + \left(\frac{BB}{g^3} - \frac{K}{gg} \right) \operatorname{cof}(4\gamma + 2\nu) \end{aligned}$$

Hinc quoque ad terminos quarti ordinis vsque valor formulae $\frac{d\Phi-d\nu}{d\omega}$ definiti posset, sed expressio prodiret tantopere complicata, vt eius euolucio summam requireret patientiam; neque tamen hic labor vilius foret vsus, nisi forte in motu apogei exactius eruendo: ipsae enim inaequalitates nullius forent momenti; propterea quod error in anomalia commissus multo minorem errorem in longitudine producit.

XXXI.

XXXI.

Ponatur ergo longitudo apogei :

 $\Phi - \nu = \text{Const.}$

$$\begin{aligned} & + A' \sin(2\gamma - \nu) + B' \sin(2\gamma + \nu) + C' \sin 2\nu \\ & + \Delta\omega + D' \sin 2\gamma + E' \sin(2\gamma - 2\nu) + F' \sin(2\gamma + 2\nu) + G' \sin 2\nu \\ & + H' \sin 4\gamma + J' \sin(4\gamma - 2\nu) + K' \sin(4\gamma + 2\nu) \end{aligned}$$

et erit differentiando :

$$\frac{d\Phi-d\nu}{d\omega} =$$

$$\begin{aligned} & (2\alpha - \epsilon) A' \operatorname{cof}(2\gamma - \nu) + (2\alpha + \epsilon) B' \operatorname{cof}(2\gamma + \nu) + \epsilon C' \operatorname{cof} \nu \\ & + \Delta - \frac{1}{2} \delta C' + \frac{2A'}{8mg} + \frac{3B'}{8mg} - \epsilon D' - (mA - \eta) E' \\ & - (mB + \theta) F' - \alpha G' - \frac{2AJ'}{8mg} - \frac{3BK'}{3mg} \\ & \operatorname{cof} 2\nu \left(-\frac{1}{2}(2\gamma - \delta) A' - \frac{1}{2}(2\gamma + \delta) B' - \frac{3C'}{4mg} + 2\alpha D' \right. \\ & \operatorname{cof}(2\gamma - 2\nu) \left(-\frac{1}{2}(2\gamma - \delta) A' - \frac{9C'}{8mg} + 2(\alpha - \epsilon) E' \right. \\ & \operatorname{cof}(2\gamma + 2\nu) \left(-\frac{1}{2}(2\gamma + \delta) B' + \frac{3C'}{8mg} + 2(\alpha + \epsilon) F' \right. \\ & \operatorname{cof} 2\nu \left(-\frac{1}{2} \delta C' - \frac{3A'}{8mg} - \frac{9B'}{8mg} + 2\epsilon G' \right. \\ & \operatorname{cof} 4\gamma \left(-\frac{3A'}{8mg} - \frac{9B'}{8mg} + 4\alpha H' \right. \\ & \operatorname{cof}(4\gamma - 2\nu) \left(\frac{9A'}{8mg} + 2(2\alpha - \epsilon) J' \right. \\ & \left. \left. \operatorname{cof}(4\gamma + 2\nu) \left(\frac{3B'}{8mg} + 2(2\alpha + \epsilon) K' \right) \right) \right) \end{aligned}$$

Qq 2

XXXIII.

XXXIII.

Calculo autem praecipue in primis terminis accuratius expedito erit:

$$\frac{d\Phi-dv}{d\omega} =$$

$$+\frac{1}{4mg} \operatorname{cof}(2\gamma-v) \left(9 + 2^2 \frac{gg}{gg} + \frac{27AA + 18BB + 15CC}{4gg} \right.$$

$$\left. - \frac{3AB + 2AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g} \right)$$

$$+\frac{1}{4mg} \operatorname{cof}(2\gamma+v) \left(-3 - 2^2 \frac{gg}{gg} - \frac{6AA - 9BB + 15CC}{4gg} \right.$$

$$\left. + \frac{9AB + AC + 2BC}{gg} - \frac{2D - 2E - 9G - 9H + 3K}{2g} \right)$$

$$+\frac{1}{2mg} \operatorname{cof} v \left(1 + 2^2 \frac{gg}{gg} + \frac{2AA + 2BB + 3CC}{4gg} \right.$$

$$\left. + \frac{AB + 6AC + 6BC}{2gg} - \frac{3D - 3E - 3F - G}{2g} + 2mi \right)$$

$$+\frac{1}{4m} \left(1 - \frac{9A + 3B - 2C}{2gg} \right)$$

$$+\frac{1}{4m} \left(3 - \frac{A - B - 3C}{gg} \right) \operatorname{cof} 2\gamma + \frac{1}{4m} \left(1 + \frac{3A - 9B - 2C}{2gg} \right) \operatorname{cof} 2v$$

$$+\frac{1}{4m} \left(6 - \frac{2A - 9C}{2gg} \right) \operatorname{cof}(2\gamma - 2v) - \frac{1}{4m} \left(3 + \frac{2B - 3C}{2gg} \right) \operatorname{cof}(2\gamma + 2v)$$

$$+\frac{3A - 9B}{8mg} \operatorname{cof} 4\gamma - \frac{9A}{8mg} \operatorname{cof}(4\gamma - 2v) + \frac{3B}{8mg} \operatorname{cof}(4\gamma + 2v)$$

Simili

Simili autem modo ex valore ipsius $\frac{d^2 q}{d\omega^2}$ accuratius erit

$$(2a - \epsilon) A = -\frac{1}{4m} (9 + 2^2 \frac{gg}{gg} + 3C)$$

$$(2a + \epsilon) B = -\frac{1}{4m} (3 + 2^2 \frac{gg}{gg} + 3C)$$

$$\epsilon C = \frac{1}{2m} (1 + 2^2 \frac{gg}{gg} + 2^2 A + 2mi)$$

XXXIII.

Comparatione autem inflicita reperitur:

$$(2a - \epsilon) A' = \frac{1}{4mg} \left(9 + 2^2 \frac{gg}{gg} + \frac{27AA + 18BB + 15CC}{4gg} \right.$$

$$\left. - \frac{3AB + 2AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g} \right)$$

$$\text{feu}$$

$$(2a - \epsilon) A' = -\frac{1}{g} (2a - \epsilon) A + \frac{1}{4mg} \left(\frac{1}{2} \frac{gg}{gg} 3C + \frac{27AA + 18BB + 15CC}{4gg} \right.$$

$$\left. - \frac{3AB + 2AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g} \right)$$

$$(2a + \epsilon) B' = +\frac{1}{g} (2a + \epsilon) B + \frac{1}{4mg} \left(\frac{1}{2} \frac{gg}{gg} 3C - \frac{6AA - 9BB + 15CC}{4gg} \right.$$

$$\left. + \frac{9AB + AC + 2BC}{gg} - \frac{2D - 2E - 9G - 9H + 3K}{2g} \right)$$

$$\epsilon C' = +\frac{1}{g} \epsilon C + \frac{1}{2mg} \left(\frac{1}{2} \frac{gg}{gg} - 2^2 A + \frac{2AA + 2BB + 3CC}{4gg} \right.$$

$$\left. + \frac{AB + 6AC + 6BC}{2gg} - \frac{3D - 3E - 3F - G}{2g} \right)$$

Q 9 3

Quibus

Quibus valoribus substituitis obtinebitur pro apogei motu medio, qui in termino $\Delta\omega$ continetur:

$$\Delta = \frac{1}{4m} + \frac{2mg\delta-1}{4m\delta g} C$$

$$+ \frac{\delta}{4\delta mg} \left(\frac{1}{2} g\delta - \frac{1}{2} A + \frac{2AA + 2BB + 3CC}{4\delta g} \right)$$

$$+ \frac{AB + 6AC + 6BC}{2\delta g} - \frac{3D - 3E - 3F - G}{2g}$$

$$\frac{9}{32(2\alpha-\delta)m\delta g} \left(\frac{1}{2} g\delta - \frac{1}{2} C + \frac{27AA + 18BB + 15CC}{4\delta g} \right)$$

$$- \frac{3AB + 2AC + BC}{\delta g} - \frac{2D - 2E + 3G + 3H - 9J}{2g}$$

$$\frac{3}{32(2\alpha+\delta)m\delta g} \left(\frac{1}{2} g\delta + \frac{1}{2} C - \frac{6AA - 9BB + 15CC}{4\delta g} \right)$$

$$+ \frac{9AB + AC + 2BC}{\delta g} - \frac{2D - 2F - 9G - 9H + 3K}{2g}$$

$$+ \epsilon D' + (m\alpha - \delta)E' + (m\beta + \delta)F' + *G' + \frac{9A'J'}{8m\delta g} + \frac{3BK'}{8m\delta g}$$

Quae expressio, cum omnino sit similis illi, quae methodo praecedente est inuenta, nullum etiam dubium relinquit, quin et hinc motus apogei prodicurus sit observationibus conformis; ideoque littera illa δ omissa poterit.

XXXIV.

Hinc igitur patet ad motum apogei definiendum valores litterarum A, B, C et A', B', C' summa accuratiorne investigari debere, qui cum consentiant partibus duplicis ordinis, etiam si partes posterioris ordinis praesertim

no-

CC

J)

CC

K)

IK'

JG

10-

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rae

mo

primo admodum videantur parvae, eas tamen omni cura evolvere oportet, propterea quod pro motu apogei partes primi ordinis se destruant. Quod cum in determinatione reliquorum coefficientium vni non eueniat, in his quoque non erit opus, ut partes istae minores in computum ducantur, sed sufficerit partibus principalibus vti. Scilicet est determinatio litterae Δ maxime est laboriosa, argue a reliquorum coefficientium exactissimis valoribus pender, reliqui tamen coefficientes tantam solertiam minime requirunt, sed satis exacte sine tanta opera definiiri possunt.

XXXV.

Valores ergo reliquorum coefficientium sequenti modo neglectis exiguis particulis ita se habebunt,

$$(2\alpha - \delta) A' = \frac{9}{4mg}; \quad (2\alpha + \delta) B' = -\frac{3}{4mg}; \quad \epsilon C' = \frac{1}{2mg}.$$

$$2\alpha D' = \frac{1}{2}(2\gamma - \delta)A' + \frac{1}{2}(2\gamma + \delta)B' + \frac{3C'}{4mg} + \frac{1}{4m} \left(3 - \frac{A-B-3C}{\delta g} \right)$$

$$2(\alpha - \delta)E' = \frac{1}{2}(2\gamma - \delta)A' + \frac{9C'}{8mg} + \frac{1}{4m} \left(6 - \frac{2A-9C}{2\delta g} \right)$$

$$2(\alpha + \delta)F' = \frac{1}{2}(2\gamma + \delta)B' - \frac{3C'}{8mg} - \frac{1}{4m} \left(3 + \frac{2B-3C}{2\delta g} \right)$$

$$2\epsilon G' = \frac{1}{2} \delta C' + \frac{3A' + 9B'}{8mg} + \frac{1}{4m} \left(1 + \frac{3A-9B-2C}{2\delta g} \right)$$

$$4\epsilon H' = \frac{3A' + 9B'}{8mg} + \frac{8A-9B}{8m\delta g} = 0$$

$$2(2\alpha - \delta)J' = \frac{9A'}{8mg} - \frac{9A}{8m\delta g} = 2(2\alpha + \delta)K' = \frac{3H'}{8mg} + \frac{3B}{8m\delta g} = 0$$

$$\text{ob } A' = -\frac{A}{g}; \quad B' = \frac{B}{g} \quad \text{et } C' = \frac{C}{g} \text{ proxime.}$$

XXXVI.

XXXVI.

Cum autem sit proxime: $\gamma = 2mg$; $\delta = 2mg + \frac{1}{2mg}$;
his valoribus quoque substitutis fiet:

$$(2\alpha - \epsilon)A' = \frac{9}{4mg}; (2\alpha + \epsilon)B' = -\frac{3}{4mg}; \epsilon C' = \frac{1}{2mg}; \text{fiue:}$$

$$A' = -\frac{A}{g}; B' = \frac{B}{g}; C' = \frac{C}{g};$$

$$2\alpha D' = \frac{3}{4m} - m A + 3m B;$$

$$2(\alpha - \epsilon)E' = \frac{3}{2m} - m A$$

$$2(\alpha + \epsilon)F' = -\frac{3}{4m} + 3m B$$

$$2\delta G' = \frac{1}{4m} + m C$$

et reliqui coefficientes H', J', K' pro evanescentibus
sunt habendi. Valores autem litterarum A, B, C, ecc.
§. 25. sunt exhibiti.

XXXVII.

Queramus nunc quoque longitudinem lunae ϕ ,
huncque in finem fingamus:

$$\phi = \text{Const.}$$

$$+ \mathcal{M}'\omega + \mathcal{N}'\sin v + \mathcal{O}'\sin 2v + \mathcal{P}'\sin(2v - 2v) + \mathcal{Q}'\sin(2v + 2v) + \mathcal{R}'\sin 2v \\ + \mathcal{S}'\sin(2v - v) + \mathcal{T}'\sin(2v + v) + \mathcal{U}'\sin(2v - 3v) + \mathcal{V}'\sin(2v + 3v) + \mathcal{W}'\sin 3v \\ + \mathcal{X}'\sin(4v - v) + \mathcal{Y}'\sin(4v + v) + \mathcal{Z}'\sin(4v - 3v) + \mathcal{A}'\sin(4v + 3v)$$

ac

ac differentiatione inflexura obinebimus:

$$\frac{d\phi}{d\omega} = + \mathcal{M}' - \frac{1}{2} \delta \mathcal{N}'$$

$$+ \cos v \left(\epsilon \mathcal{N}' - \frac{1}{2} \kappa \mathcal{N}' + \frac{9\mathcal{D}'}{4mg} + \frac{3\mathcal{E}'}{4mg} - \delta \mathcal{F}' \right)$$

$$+ \cos 2v \left(-\frac{3\mathcal{N}'}{4mg} + 2\alpha \mathcal{G}' \right)$$

$$+ \cos(2v - 2v) \left(-\frac{9\mathcal{N}'}{8mg} + 2(\alpha - \epsilon) \mathcal{D}' \right)$$

$$+ \cos(2v + 2v) \left(+ \frac{3\mathcal{N}'}{8mg} + 2(\alpha + \epsilon) \mathcal{E}' \right)$$

$$+ \cos 2v \left(-\frac{1}{2} \delta \mathcal{N}' + 2 \epsilon \mathcal{F}' \right)$$

$$+ \cos(2v - v) \left(-\frac{1}{2} \mathcal{N}' - \frac{1}{2} \kappa \mathcal{N}' - \gamma \mathcal{E}' - (\gamma - \delta) \mathcal{D}' + \frac{3\mathcal{H}'}{4mg} (2\alpha - \epsilon) \mathcal{G}' \right)$$

$$+ \cos(2v + v) \left(-\frac{1}{2} \mathcal{N}' - \frac{1}{2} \kappa \mathcal{N}' - \gamma \mathcal{E}' - (\gamma + \delta) \mathcal{E}' - \frac{9\mathcal{H}'}{4mg} + (2\alpha + \epsilon) \mathcal{J}' \right)$$

$$+ \cos(2v - 3v) \left(-\frac{1}{2} \kappa \mathcal{N}' - (\gamma - \delta) \mathcal{D}' - \frac{9\mathcal{H}'}{4mg} + (2\alpha - 3\epsilon) \mathcal{I}' \right)$$

$$+ \cos(2v + 3v) \left(-\frac{1}{2} \kappa \mathcal{N}' - (\gamma + \delta) \mathcal{E}' + \frac{3\mathcal{H}'}{4mg} + (2\alpha + 3\epsilon) \mathcal{K}' \right)$$

$$+ \cos 8v \left(-\frac{1}{2} \kappa \mathcal{N}' - \frac{3\mathcal{D}'}{4mg} - \frac{9\mathcal{E}'}{4mg} - \delta \mathcal{F}' + 3 \epsilon \mathcal{G}' \right)$$

$$+ \cos(4v - v) \left(-\frac{3A + 9B}{16mg} \mathcal{N}' + \frac{9A}{16mg} \mathcal{N}' - \frac{3\mathcal{D}'}{4mg} + (4\alpha - \epsilon) \mathcal{M}' \right)$$

$$+ \cos(4v + v) \left(-\frac{3A + 9B}{16mg} \mathcal{N}' - \frac{3B}{16mg} \mathcal{N}' - \frac{9\mathcal{E}'}{4mg} + (4\alpha + \epsilon) \mathcal{N}' \right)$$

$$+ \cos(4v - 3v) \left(+ \frac{9A}{16mg} \mathcal{N}' + \frac{9\mathcal{D}'}{4mg} + (4\alpha - 3\epsilon) \mathcal{O}' \right)$$

$$+ \cos(4v + 3v) \left(-\frac{3B}{16mg} \mathcal{N}' + \frac{3\mathcal{E}'}{4mg} + (4\alpha + 3\epsilon) \mathcal{P}' \right)$$

R r

XXXVIII.

XXXVIII.

Comparata iam hac forma cum valore ipsius $\frac{dQ}{d\omega}$ in §. 29. exhibito, obtinebitur

$$\mathcal{N}' = \frac{1}{2} \delta \mathcal{N}' + m \left(1 + \frac{1}{2} g \mathcal{E} - C \right)$$

$$6 \mathcal{N}' = \frac{1}{2} \kappa \mathcal{N}' - \frac{9 \mathcal{D}' - 3 \mathcal{E}'}{4mg} + \delta \mathcal{S}' - 2mg + \frac{1}{2} mg C - m G$$

$$2\alpha \mathcal{E}' = \frac{3 \mathcal{B}'}{4mg} - m \left(\frac{1}{2} \mathcal{N}' + A + B \right)$$

$$2(\alpha - \mathcal{E}') \mathcal{D}' = \frac{9 \mathcal{B}'}{8mg} - m A$$

$$2(\alpha + \mathcal{E}') \mathcal{E}' = -\frac{3 \mathcal{B}'}{8mg} - m B$$

$$2 \mathcal{E} \mathcal{S}' = \frac{1}{2} \delta \mathcal{N}' - m \left(C - \frac{1}{2} g \mathcal{E} \right)$$

$$(2\alpha - \mathcal{E}') \mathcal{E}' = \frac{1}{2} (\xi + \eta) \mathcal{N}' + \gamma \mathcal{E}' + (\gamma - \delta) \mathcal{D}' - \frac{3 \mathcal{S}'}{4mg} + m \left(\frac{1}{2} g \mathcal{N}' - \frac{1}{2} \mathcal{N} + g A + \frac{1}{2} g B - D - E \right)$$

$$(2\alpha + \mathcal{E}') \mathcal{S}' = \frac{1}{2} (\xi + \theta) \mathcal{N}' + \gamma \mathcal{E}' + (\gamma + \delta) \mathcal{E}' + \frac{9 \mathcal{S}'}{4mg} + m \left(\frac{1}{2} g \mathcal{N}' - \frac{1}{2} \mathcal{N} + g B + \frac{1}{2} g A - D - F \right)$$

$$(2\alpha - 3\mathcal{E}') \mathcal{S}' = \frac{1}{2} \gamma \mathcal{N}' + (\gamma - \delta) \mathcal{D}' + \frac{9 \mathcal{S}'}{4mg} + m \left(\frac{1}{2} g A - E \right)$$

$$(2\alpha + 3\mathcal{E}') \mathcal{S}' = \frac{1}{2} \theta \mathcal{N}' + (\gamma + \delta) \mathcal{E}' - \frac{3 \mathcal{S}'}{4mg} + m \left(\frac{1}{2} g B - F \right)$$

$$3 \mathcal{E} \mathcal{E}' = \frac{1}{2} \kappa \mathcal{N}' + \frac{3 \mathcal{D}' + 9 \mathcal{E}'}{4mg} + \delta \mathcal{S}' + m \left(\frac{1}{2} g C - G \right)$$

$$(4\alpha - \mathcal{E}') \mathcal{D}' = -\frac{6A - 9B}{16mg} \mathcal{N}' + \frac{3 \mathcal{D}'}{4mg} - m (H + J)$$

$$(4\alpha + \mathcal{E}') \mathcal{D}' = +\frac{3A - 6B}{16mg} \mathcal{N}' + \frac{9 \mathcal{E}'}{4mg} - m (H + K)$$

$$(4\alpha - 3\mathcal{E}') \mathcal{D}' = -\frac{9A}{16mg} \mathcal{N}' - \frac{9 \mathcal{D}'}{4mg} - m J$$

$$(4\alpha + 3\mathcal{E}') \mathcal{D}' = +\frac{3B}{16mg} \mathcal{N}' - \frac{3 \mathcal{E}'}{4mg} - m K$$

XXXIX.

XXXIX.

Inuentis iam valoribus literarum $p = b(1 + \xi)$ et q vna cum anomalia vera ω , distantia curata lunae a terra

$$x = \frac{p}{1 - q \cos \omega} \text{ cognosceatur: ac si deinceps latitudinis lu-}$$

nae ψ ratio habeatur, erit distantia vera $= \frac{p}{(1 - q \cos \omega) \cos \psi}$.

In Astronomia autem non tam distantia lunae, quam eius diameter apparet et parallaxis requiri solet; quarum utraque cum sit distantiae lunae a terra reciproce proportionalis, erit tam diameter apparetis quam parallaxis ut $\frac{(1 - q \cos \omega) \cos \psi}{p}$; vnde si utriusque valor medius

ex observationibus fuerit definitus, ad quoduis tempus valor verus assignari poterit. Sic igitur siue diametri apparetis siue parallaxis horizontalis valor medius $= \sigma$, erit- que is pro tempore dato $= \frac{b\sigma}{p} (1 - q \cos \omega) \cos \psi$. Est au-

tem proxime $\cos \psi = 1 - \frac{1}{2} \tan g^2 \omega - \frac{1}{2} \tan g^2 \omega^2 \cos^2 \psi = \frac{1}{2} \frac{\lambda + \Pi}{3}$, et $\frac{b}{p} = 1 - \xi + \xi \xi$: vnde fit diameter seu parallaxis $= \frac{1}{2} \sigma (2 + \lambda + \Pi) (1 - \xi + \xi \xi) (1 - q \cos \omega)$, quae euoluitur in hanc expressionem: ob $\frac{2 + \lambda}{3} = 1$ proxime:

$$\frac{1}{2} (2 + \lambda) \sigma [1 - q \cos \omega - \xi + q \xi \cos \omega + \xi \xi + \frac{1}{2} \Pi - \frac{1}{2} q \Pi \cos \omega]$$

XL.

Pro praesenti ergo casu, quo parallaxis solis, eiusque excentricitatem vna cum inclinatione orbitae luna-

R r 2

ris

ris negligimus, erit lunæ diameter apprensus vel parallaxis horizontalis

$$= \frac{1}{2} (2 + \lambda) \sigma \text{ mult. per}$$

$$1 - \frac{1}{2} C - (g + \frac{1}{2} G) \cos v$$

$$- \frac{1}{2} (2\mathcal{H} + A + B) \cos 2v - \frac{1}{2} A \cos(2v - 2v) - \frac{1}{2} B \cos(2v + 2v) - \frac{1}{2} C \cos 2v$$

$$- \frac{1}{2} (2\mathcal{D} + D + E) \cos(2v - v) - \frac{1}{2} (2\mathcal{E} + D + F) \cos(2v + v)$$

$$- \frac{1}{2} E \cos(2v - 3v) - \frac{1}{2} F \cos(2v + 3v) - \frac{1}{2} G \cos 3v$$

$$- \frac{1}{2} (H + J) \cos(4v - v) - \frac{1}{2} (H + K) \cos(4v + v) - \frac{1}{2} J \cos(4v - 3v) - \frac{1}{2} K \cos(4v + 3v)$$

vbi quidem factor constans $\frac{1}{2} (2 + \lambda) \sigma$ omitti potest, si quidem tantum proportio vel diametri apprensus vel parallaxis horizontalis desideretur.

XII.

Si nunc hos valores in numeris euoluere velimus, ex observationibus primum colligimus has determinationes:

$$\mathcal{H}' = 13,3682; A = 0,1123 \text{ et proxime } g = 0,05445$$

ac postremo quidem valore ipsius g tantum in terminis minimis var, in maioribus ipsam litteram g relinquitur, vt deinceps ex collatione calculi cum observationibus accuratius fortasse determinari possit. Habemus ergo

$$13,3682 = \frac{1}{2} \delta \mathcal{H}' + m (1 + \frac{1}{2} g g - C)$$

vide ob \mathcal{H}' , g et C numeros admodum paruos, statim prope colligitur $m = 13,3682$. Tum vero est prope

$$\mathcal{H}' = m \frac{2mg}{g}; \delta = m \text{ et } \delta = 2mg + \frac{1}{2} m^2 \text{ seu } \delta = 13,3682; \delta = 2,$$

paral-

$$\delta = 2,1419; \text{ hinc } \frac{1}{2} \delta \mathcal{H}' = -0,1165, \text{ ergo accuratius}$$

$$m (1 + \frac{1}{2} g g - C) = 13,4847 = m + 1 \text{ et } m = 12,4847$$

$$\text{Porro ob } C = \frac{1}{2m^2} \text{ erit satis exacte } m = 13,5039$$

$$\text{vnde ex valoribus } A \text{ et } B \text{ proxime collectis fit } \delta = 13,0644$$

$$y = 26,9524g.$$

At valor ipsius δ duabus constat partibus, altera per g multiplicata altera diuisa, quibus separatim expressis erit

$$\delta = 27,0355 g + 0,0370 \frac{1}{g} = 2,1521 \text{ proxime.}$$

XIII.

Hinc iam computo insituto sequentes supra assumptorum coefficientium eruantur valores numerici:

$$\mathcal{H}' = 0,008931\mathcal{H} = 0,03895g; \mathcal{E} = 0,01219g; \mathcal{G} = 0,00064$$

$$\text{ideoque } \mathcal{H} = 0,008931 \cos 2v + 0,03895 g \cos(2v - v) + 0,01219 g \cos(2v + v)$$

Deinde reperitur:

$$A = -0,013995; B = -0,001460; C = +0,002834$$

$$D = -0,001213g + 0,00002198 \frac{1}{g} = -0,000280 \text{ proxime}$$

$$E = +0,25834g - 0,00001889 \frac{1}{g} = +0,014012 \text{ proxime}$$

$$F = -0,00225g - 0,00000207 \frac{1}{g} = -0,000161 \text{ proxime}$$

$$G = +0,000218g + 0,000001221 \frac{1}{g} = +0,000340 \text{ proxime}$$

$$H = -0,00001022 \frac{1}{g} = -0,0000184$$

$$J = +0,00004897 \frac{1}{g} = +0,000882$$

$$K = +0,00000053 \frac{1}{g} = +0,000009$$

ADDITAMENTUM.

XLIIL.

Hinc porro pro motu apogei eiusque inaequalita-
tibus colligitur : $\Delta = 0,1123$; qui quidem valor ex
obferuationibus est defumtus

- A' = -0,013995. $\frac{1}{g}$ = -0,25703 proxime
- B' = -0,001460. $\frac{1}{g}$ = -0,02682 proxime
- C' = +0,002834. $\frac{1}{g}$ = +0,05205 proxime
- D' = +0,007432 $\frac{1}{g}$ | F' = -0,002249
- E' = -0,259170 | G' = -0,002176

in minutis fecundis

- A' = - 53018'' = - 14°, 43', 38''
- B' = - 5532 = - 1°, 32', 12
- C' = + 10736 = + 2, 58, 56
- D' = + 1533 = + 0, 25, 33
- E' = - 53459 = - 14, 50, 59
- F' = - 464 = - 0, 7, 44
- G' = + 449 = + 0, 7, 29

Ergo longitudo apogei in minutis fecundis

$\phi - v = \text{Conf.}$

- + 0,1123^w — 53018'' $\text{fn}(2\phi - v)$ + 1533'' $\text{fn} 2\phi$
- 5532 $\text{fn}(2+v)$ — 53459 $\text{fn}(2\phi - 2v)$
- + 10736 $\text{fn} v$ — 464 $\text{fn}(2\phi + 2v)$
- + 449 $\text{fn} 2v$

XLIIV.

tabita-
or ex

ADDITAMENTUM.

XLIIV.

Iam pro longitudine ipsa inuenienda habentur pri-
mo ex §. 27. valores : $\gamma = 1,46756$

$\delta = + 2,15210$; $\epsilon = - 0,027791$; $\zeta = + 0,07138$

$\eta = - 0,06674$; $\theta = - 0,039809$; $\kappa = - 0,070920$

Deinde cum sit proxime

$\epsilon \mathfrak{B}' = - 2mg$ seu $\mathfrak{B}' = - 0,11256$

erit quoque proxime

$\mathfrak{C}' = - 0,003485$; $\mathfrak{D}' = - 0,014456$

$\mathfrak{E}' = + 0,001509$; $\mathfrak{F}' = - 0,005334$

Hinc ergo accuratius elicietur $\mathfrak{B}' = - 0,11019$, ideo-
que hic et reliqui confidentes eam absolute quam in
numeris fecundis erunt :

absolute

in minutis fecundis

$\mathfrak{B}' = - 0,11019$	$\mathfrak{B}' = - 22728'' = - 6°, 18', 4''$
$\mathfrak{C}' = - 0,00339$	$\mathfrak{C}' = - 700 = - 0, 11, 40$
$\mathfrak{D}' = - 0,01742$	$\mathfrak{D}' = - 3594 = - 0, 59, 54$
$\mathfrak{E}' = + 0,00149$	$\mathfrak{E}' = + 306 = + 0, 5, 6$
$\mathfrak{F}' = - 0,00524$	$\mathfrak{F}' = - 1081 = - 0, 18, 1$
$\mathfrak{G}' = - 0,01824$	$\mathfrak{G}' = - 3762 = - 1, 2, 42$
$\mathfrak{H}' = - 0,00056$	$\mathfrak{H}' = - 115 = - 0, 1, 55$
$\mathfrak{I}' = + 0,01368$	$\mathfrak{I}' = + 2823 = + 0, 47, 3$
$\mathfrak{K}' = + 0,00023$	$\mathfrak{K}' = + 47 = + 0, 0, 47$
$\mathfrak{L}' = - 0,00062$	$\mathfrak{L}' = - 128 = - 0, 2, 8$
$\mathfrak{M}' = - 0,00119$	$\mathfrak{M}' = - 246 = - 0, 4, 6$
$\mathfrak{N}' = + 0,00020$	$\mathfrak{N}' = + 41 = + 0, 0, 41$
$\mathfrak{O}' = + 0,00084$	$\mathfrak{O}' = + 379 = + 0, 6, 19$
$\mathfrak{P}' = - 0,00001$	$\mathfrak{P}' = - 2 = - 0, 0, 2$

XLIV.

XLV.

Hinc ergo si ad datum tempus iam cognita sit anomalia lunae vera v cum angulo η , longitudo lunae per aequationes in minutis secundis expressas erit

$$\phi = \text{Const.}$$

+13,3682 ω	— 22728''	fin v	— 700'	fin 2 η
— 1081	fin 2 ω	— 3594	fin (2 η -2 ω)	
— 128	fin 3 ω	+ 306	fin (2 η +2 ω)	
— 3762''	fin (2 η -v)	— 246''	fin (4 η -v)	
— 115	fin (2 η +v)	+ 41	fin (4 η +v)	
+ 2823	fin (2 η -3 ω)	+ 379	fin (4 η -3 ω)	
+ 47	fin (2 η +3 ω)	— 2	fin (4 η +3 ω)	

vbi Const. + 13,3682 ω denotat longitudinem mediam; in reliquis autem terminis continentur inaequalitates periodicae pro hac hypothesisi.

XLVI.

Inde iam vicissim anomalia vera lunae v colligitur,

v ω sit	— 33464''	fin v	— 2233''	fin 2 η
13,2559 ω	— 1530	fin 2 ω	+ 49864	fin (2 η -2 ω)
— 128	fin 3 ω	+ 770	fin (2 η +2 ω)	
+ 49256	fin (2 η -v)	— 246	fin (4 η -v)	
+ 5417	fin (2 η +v)	+ 41	fin (4 η +v)	
+ 2823	fin (2 η -3 ω)	+ 379	fin (4 η -3 ω)	
+ 47	fin (2 η +3 ω)	— 2	fin (4 η +3 ω)	

vbi primus terminus 13,2559 ω designat anomaliam mediam lunae, quae sit = ξ ; tum, ex ea primum quaeratur

fit anomae per

1 η	2 η -2 ω
	2 η +2 ω
	4 η -v
	4 η +v
	4 η -3 ω
	4 η +3 ω

igitur,

1	2 η -2 ω
	2 η +2 ω
	4 η -v
	4 η +v
	4 η -3 ω
	4 η +3 ω

anomalia Kepleriana, quae scilicet a sola excentricitate pendet, siquae ea = ξ , v ω sit

$\xi = \xi - 33464''$ fin ξ — 1530'' fin 2 ω — 128'' fin 3 ω vnde quidem facile tabulae construuntur. Tum servatur $v = \xi + z$, et quia angulus z est modicus, inde is factis prope poterit desiniri. Interim tamen expedire videtur aliquot operationibus iterandis istam anomaliam veram v determinari; dum scilicet primum valor non nimis a veritate abhorrens pro v aestimando assumitur, ex eoque deinceps exactior colligitur; qui si nimis ab assumto discrepare reperitur, ex hoc denovo exactior quaeratur, donec nulla amplius correctione fuerit opus.

XLVII.

Formula denique, cui tam diameter lunae apparens geocentrica quam parallaxis horizontalis est proportionalis, ex §. 40. reperitur

1	— 0,05470	col v	— 0,00120	col 2 η
— 0,00142	col 2 ω	+ 0,00700	col (2 η -2 ω)	
— 0,00017	col 3 ω	+ 0,00073	col (2 η +2 ω)	
— 0,00898	col (2 η -v)	— 0,00035	col (4 η -v)	
— 0,00042	col (2 η +v)	+ 0,00009	col (4 η +v)	
— 0,00701	col (2 η -3 ω)	— 0,00044	col (4 η -3 ω)	
+ 0,00008	col (2 η +3 ω)	— 0,00001	col (4 η +3 ω)	

quorum quidem terminorum plures, qui pro parallaxi infra aliquot minuta secunda subsistunt, tuto omitti poterunt. His igitur tribus formulis pro anomalia vera v, longitudine ϕ et parallaxi seu diametro apparente invenitur motus lunae consideretur, si quidem tam solis parallaxis

laxis quam eius excentricitas et inclinatio orbitae lunaris ad eclipticam negligantur. Hae autem sunt inaequalitates praecipuae, quae etiam ad reliquas eruendas adhiberi debent; unde nunc ad inaequalitates ab excentricitate folis oriundas progrediamur.

INVESTIGATIO INAEQUALITATUM

LUNAE SECUNDI GENERIS SEU AB EXCENTRICITATE SOLIS PENDENTIAM.

XLVIII.

Formulae nostrae differentiales, quatenus ab excentricitate orbitae solaris pendent, omittis terminis, quos iam constat esse minimos, erunt

$$\begin{aligned} \frac{d^2 \xi}{dt^2} &= \text{Praec.} + \frac{9^e}{2m} \sin(2\gamma - u) + \frac{9^e}{2m} \sin(2\gamma + u) \\ \frac{d\eta}{dt} &= \text{Praec.} + \frac{3^e}{4m} \sin(v - u) + \frac{3^e}{4m} \sin(v + u) \\ &\quad - \frac{27^e}{8m} \sin(2\gamma - v - u) - \frac{27^e}{8m} \sin(2\gamma - v + u) \\ &\quad - \frac{9^e}{8m} \sin(2\gamma + v - u) - \frac{9^e}{8m} \sin(2\gamma + v + u) \\ \frac{d(\Delta Q - \Delta \theta)}{dt} &= \text{Pr.} - \frac{3^e}{4m} \cos(v - u) - \frac{3^e}{4m} \cos(v + u) \\ &\quad - \frac{27^e}{8m} \cos(2\gamma - v - u) - \frac{27^e}{8m} \cos(2\gamma - v + u) \\ &\quad + \frac{9^e}{8m} \cos(2\gamma + v - u) + \frac{9^e}{8m} \cos(2\gamma + v + u) \end{aligned}$$

Quam-

Quoniam enim nunc tam ξ quam q etiam ab excentricitate e pendent, tamen in his formulis, in quas hae quantitates ingrediuntur, haec mutatio earum sine errore pro nihilo haberi potest; quoniam hi termini per se sunt minimi, et quia iam terminos ab e et q simul pendentes omittimus. Tum vero erit

$$\begin{aligned} \frac{d\theta}{dt} &= m(1 + \frac{1}{2}q) - 2mq \cos v + \frac{1}{2}mq \cos 2v - \frac{3}{2}m\xi + 3mq\xi \cos v \\ \text{et } \frac{d\mu}{dt} &= 1 + 2ee - 2e \cos \mu \end{aligned}$$

XLIX.

Ad formulae has integrandas seu tantum ad eas integralium partes inveniendas, quae ab excentricitate folis e pendent, opus est vt formularum $\frac{d\eta}{dt}$, $\frac{dv}{dt}$ et $\frac{d\mu}{dt}$ prius habeamus partes principales, tum vero etiam eas quae a simplici folis excentricitate e pendent: habebimus ergo primo

$$\begin{aligned} \frac{d\eta}{dt} &= m(1 + \frac{1}{2}eg) - 1 - 2ee - 2mg \cos v + 2e \cos v \\ \frac{dv}{dt} &= m(1 + \frac{1}{2}eg) - 2mg \cos v + \frac{3^e}{4mg} \cos(v - u) + \frac{3^e}{4mg} \cos(v + u) \\ &\quad - \frac{1}{4m} \left(1 - \frac{9A + 3B - 2C}{2gg} \right) + \frac{27^e}{8mg} \cos(2\gamma - v - u) + \frac{27^e}{8mg} \cos(2\gamma - v + u) \\ &\quad - \frac{9^e}{8mg} \cos(2\gamma + v - u) - \frac{9^e}{8mg} \cos(2\gamma + v + u) \end{aligned}$$

seu introducendis, vt supra §. 27. breuiteris gratis, litteris

$$\alpha = m(1 + \frac{1}{2}eg - C) - 2ee \quad ; \quad \gamma = 2mg$$

$$6 = m(1 + \frac{1}{2}eg - C) - \frac{1}{4m} \left(1 - \frac{9A + 3B - 2C}{2gg} \right) ; \quad \delta = 2mg + \frac{1}{2mg} + \frac{3g}{8m}$$

Ss 2 erit:

erit: $\frac{d^4}{dx^4} x - \gamma \operatorname{cof} v + 2\epsilon \operatorname{cof} u$

$$\frac{dv}{dx} = \epsilon - \delta \operatorname{cof} v - \frac{9}{4mg} \operatorname{cof}(2\eta - v) + \frac{3}{4mg} \operatorname{cof}(2\eta + v)$$

$$+ \frac{3\epsilon}{4mg} \operatorname{cof}(v - u) + \frac{3\epsilon}{4mg} \operatorname{cof}(v + u) + \frac{27\epsilon}{8mg} \operatorname{cof}(2\eta - v - u)$$

$$+ \frac{27\epsilon}{8mg} \operatorname{cof}(2\eta - v + u) - \frac{9\epsilon}{8mg} \operatorname{cof}(2\eta + v - u) - \frac{9\epsilon}{8mg} \operatorname{cof}(2\eta + v) u$$

et $\frac{d^2 u}{dx^2} = 1 - 2\epsilon \operatorname{cof} u$.

L.

Fingamus nunc primo:

$\xi = \mathfrak{M} \operatorname{cof} 2\eta + \mathfrak{B} \operatorname{cof}(2\eta - v) + \mathfrak{C} \operatorname{cof}(2\eta + v) + \mathfrak{H} \operatorname{cof}(2\eta - u) + \mathfrak{Q} \operatorname{cof}(2\eta + u)$
 ac differentiando eos tanrum sumamus terminos, qui
 formulae differentiali respondent, quandoquidem reli-
 quos iam invenimus: eritque

$$\frac{d\xi}{dx} = \frac{2\epsilon}{2m} \mathfrak{M} \operatorname{fin}(2\eta - u) - 2\epsilon \mathfrak{M} \operatorname{fin}(2\eta + u)$$

$$- (2\alpha - 1) \mathfrak{H} - (2\alpha + 1) \mathfrak{Q}$$

vnde colligitur:

$$(2\alpha - 1) \mathfrak{H} = -\frac{9\epsilon}{2m} - 2\epsilon \mathfrak{M}; \quad (2\alpha + 1) \mathfrak{Q} = -\frac{9\epsilon}{2m} - 2\epsilon \mathfrak{M}$$

Cum igitur $\epsilon = 0,0168$, erit in numericis:

$$\mathfrak{H} = -0,000247 \quad \text{et} \quad \mathfrak{Q} = -0,000227.$$

LL.

Fingatur porro:

$$\mathfrak{g} = \mathfrak{A} \operatorname{cof}(2\eta - v) + \mathfrak{B} \operatorname{cof}(2\eta + v) + \mathfrak{C} \operatorname{cof} v + \mathfrak{M} \operatorname{cof}(v - u) + \mathfrak{N} \operatorname{cof}(v + u) \\ + \mathfrak{P} \operatorname{cof}(2\eta - v - u) + \mathfrak{Q} \operatorname{cof}(2\eta - v + u) + \mathfrak{R} \operatorname{cof}(2\eta + v - u) + \mathfrak{S} \operatorname{cof}(2\eta + v + u)$$

ac differentiando obtinebitur: $\frac{d\mathfrak{g}}{dx} =$

$$-2\epsilon \mathfrak{A} \operatorname{fin}(2\eta - v - u) - 2\epsilon \mathfrak{A} \operatorname{fin}(2\eta - v + u) - 2\epsilon \mathfrak{B} \operatorname{fin}(2\eta + v) u \\ - (2\alpha - \epsilon - 1) \mathfrak{P} - (2\alpha - \epsilon + 1) \mathfrak{Q} - (2\eta \epsilon - 1) \mathfrak{R} - (2\eta \epsilon + 1) \mathfrak{S} \\ - (\epsilon - 1) \mathfrak{M} \operatorname{fin}(v - u) - (\epsilon + 1) \mathfrak{H} \operatorname{fin}(v + u)$$

Comparatione ergo infirma reperietur:

$$(\epsilon - 1) \mathfrak{M} = -\frac{3\epsilon}{4m}; \quad (\epsilon + 1) \mathfrak{N} = -\frac{3\epsilon}{4m}$$

$$(2\alpha - \epsilon - 1) \mathfrak{P} = \frac{27\epsilon}{8m} - 2\epsilon \mathfrak{A}; \quad (2\alpha - \epsilon + 1) \mathfrak{Q} = \frac{27\epsilon}{8m} - 2\epsilon \mathfrak{A}$$

$$(2\alpha + \epsilon - 1) \mathfrak{R} = \frac{9\epsilon}{8m} - 2\epsilon \mathfrak{B}; \quad (2\alpha + \epsilon + 1) \mathfrak{S} = \frac{9\epsilon}{8m} - 2\epsilon \mathfrak{B}$$

et in numericis

$$\mathfrak{M} = -0,00008; \quad \mathfrak{P} = +0,00042; \quad \mathfrak{R} = +0,00004$$

$$\mathfrak{N} = -0,00006; \quad \mathfrak{Q} = +0,00036; \quad \mathfrak{S} = +0,00004$$

LII.

Hic autem in differentiatione neglectimus partes
 ipsius $\frac{dv}{dx}$ ab excentricitate ϵ pendentes, quarum tamen
 eodem iure ratio haberi debuisset, argue partis in diffe-
 rentiali $\frac{d\mathfrak{g}}{dx}$; inde autem multo plures termini accedent
 ad valorum ipsius \mathfrak{g} , ponatur ergo ob hos terminos:

$$\mathfrak{g} = \mathfrak{g}$$

$$+ \mathfrak{A} \operatorname{cof}(2\eta - v) + \mathfrak{B} \operatorname{cof}(2\eta + v) + \mathfrak{C} \operatorname{cof} v + \mathfrak{M} \operatorname{cof}(v - u)$$

$$+ \mathfrak{N} \operatorname{cof}(v + u) + \mathfrak{D} \operatorname{cof}(2\eta - u) + \mathfrak{E} \operatorname{cof}(2\eta + u)$$

$$+ \mathfrak{P} \operatorname{cof}(2\eta - v - u) + \mathfrak{Q} \operatorname{cof}(2\eta - v + u) + \mathfrak{R} \operatorname{cof}(2\eta + v - u)$$

$$+ \mathfrak{S} \operatorname{cof}(2\eta + v + u) + \mathfrak{K} \operatorname{cof}(2v - u) + \mathfrak{L} \operatorname{cof}(2v + u)$$

S s 3

$$\begin{aligned}
 & + F \cos(2y-2v-u) + G \cos(2y-2v+w) + H \cos(2y+2v-u) \\
 & \quad + J \cos(2y+2v+w) + T \cos(4y-u) + V \cos(4y+w) \\
 & + W \cos(4y-2v-u) + X \cos(4y-2v+w) + Y \cos(4y+2v-u) \\
 & \quad + Z \cos(4y+2v+w)
 \end{aligned}$$

et sumto differentiali pleno reperitur:

$$\begin{aligned}
 \frac{dq}{dt} = & + \sin(2y-v-u) [-2eA - (2a-5-1)P] \\
 & + \sin(2y-v+u) [-2eA - (2a-5+1)Q] - (5-1)M \sin(v-u) \\
 & + \sin(2y+v-u) [-2eB - (2a+5-1)R] \\
 & + \sin(2y+v+w) [-2eB - (2a+5+1)S] - (5+1)N \sin(v+w) \\
 & + \sin(2y-u) \left(+ \frac{3eA}{8mg} - \frac{3eB}{8mg} - \frac{27eC}{16mg} - \frac{9eC}{16mg} - (2a-1)D \right) \\
 & + \sin(2y+w) \left(+ \frac{3eA}{8mg} - \frac{8eB}{8mg} - \frac{27eC}{16mg} - \frac{9eC}{16mg} - (2a+1)E \right) \\
 & + \sin(2y-2v+w) \left(+ \frac{3eA}{8mg} + \frac{27eC}{16mg} - (2a-25+1)G \right) \\
 & + \sin(2y-2v-u) \left(+ \frac{3eA}{8mg} + \frac{27eC}{16mg} - (2a-25-1)F \right) \\
 & + \sin(2y+2v-u) \left(- \frac{3eB}{8mg} + \frac{9eC}{16mg} - (2a+25-1)H \right) \\
 & + \sin(2y+2v+w) \left(- \frac{3eB}{8mg} + \frac{9eC}{16mg} - (2a+25+1)J \right) \\
 & + \sin u \left(+ \frac{27eA}{16mg} - \frac{27eA}{16mg} + \frac{9eB}{16mg} - \frac{9eB}{16mg} - \frac{3eC}{8mg} + \frac{3eC}{8mg} \right) \\
 & + \sin(4y-2v-u) \left(+ \frac{27eA}{16mg} - (4a-25-1)W \right) \\
 & + \sin(4y-2v+w) \left(+ \frac{27eA}{16mg} - (4a-25+1)X \right)
 \end{aligned}$$

+

$$\begin{aligned}
 & 2y+2v-u) \\
 & \cos(4y+w) \\
 & 4y+2v-u) \\
 & 4y+2v+w)
 \end{aligned}$$

$$\begin{aligned}
 & \sin(v-u) \\
 & \sin(v+w) \\
 & -1)D) \\
 & +1)E)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{3eC}{8mg}
 \end{aligned}$$

+

$$\begin{aligned}
 & + \sin(4y+2v-u) \left(+ \frac{9eB}{16mg} - (4a+25-1)Y \right) \\
 & + \sin(4y+2v+w) \left(+ \frac{9eB}{16mg} - (4a+25+1)Z \right) \\
 & + \sin(2v-u) \left(+ \frac{9eA}{16mg} - \frac{27eB}{16mg} - \frac{3eC}{8mg} - (25-1)K \right) \\
 & + \sin(2v+w) \left(+ \frac{9eA}{16mg} - \frac{27eB}{16mg} - \frac{3eC}{8mg} - (25+1)L \right) \\
 & + \sin(4y-u) \left(- \frac{9eA}{16mg} - \frac{27eB}{16mg} - (4a-1)T \right) \\
 & + \sin(4y+w) \left(- \frac{9eA}{16mg} - \frac{27eB}{16mg} - (4a+1)V \right)
 \end{aligned}$$

vide reperitur:

D =	-	0,000010	;	H =	+	0,000001
E =	-	0,000010	;	J =	+	0,000001
F =	+	0,000005	;	K =	-	0,000005
G =	+	0,000069	;	L =	-	0,000005
T =	+	0,000004	;	X =	-	0,000032
V =	+	0,000004	;	Y =	-	0,000009
W =	-	0,000023	;	Z =	-	0,000009

L'III.

Ponamus nunc etiam pro motu spogei

$$\Phi - v = \text{Const.} + \Delta s$$

$$\begin{aligned}
 & + A/\sin(2v-u) + B/\sin(2v+w) + C/\sin v + M/\sin(v-u) + N/\sin(v+w) \\
 & + P/\sin(2y-v-u) + Q/\sin(2y-v+w) + R/\sin(2y+v-u) + S/\sin(2y+v+w) \\
 & + D/\sin(2y-u) + E/\sin(2y+w) + K/\sin(2v-u) + L/\sin(2v+w) + O/\sin u \\
 & + F/\sin(2y-2v-u) + G/\sin(2y-2v+w) + H/\sin(2y+2v-u) + J/\sin(2y+2v+w) \\
 & + W/\sin(4y-2v-u) + X/\sin(4y-2v+w) + Y/\sin(4y+2v-u) + Z/\sin(4y+2v+w) \\
 & + T/\sin(4y-u) + V/\sin(4y+w)
 \end{aligned}$$

et

et sumto differentiali pleno reperitur :

$$\frac{dQ-dv}{du} = \Delta + \text{cof}(2y-v-u) [2eA' + (2a-5-1)P] + \text{cof}(v-u) [2eA' + (2a-5+1)Q] + (5-1)M \text{cof}(v-u) \\ + \text{cof}(2y+v+u) [2eB' + (2a+5-1)R'] \\ + \text{cof}(2y+v+u) [2eB' + (2a+5+1)S'] + (5+1)N \text{cof}(v'u) \\ + c[(2y-u) \left(-\frac{3e}{8mg} A' + \frac{3e}{8mg} B' + \frac{27e}{16mg} C' - \frac{9e}{16mg} C''(2a-1)D' \right) \\ + c[(2y+u) \left(-\frac{3e}{8mg} A' + \frac{3e}{8mg} B' + \frac{27e}{16mg} C' - \frac{9e}{16mg} C''(2a+1)E' \right) \\ - \frac{1}{2} \text{cof}(2y-2v-u) \left(-\frac{3e}{8mg} A' + \frac{27e}{16mg} C' + (2a-25-1)F' \right) \\ + \text{cof}(2y-2v+u) \left(-\frac{3e}{8mg} A' + \frac{27e}{16mg} C' + (2a-25+1)G' \right) \\ + \text{cof}(2y+2v-u) \left(+\frac{3e}{8mg} B' + \frac{9e}{16mg} C' + (2a+25-1)H' \right) \\ + \text{cof}(2y+2v+u) \left(+\frac{3e}{8mg} B' + \frac{9e}{16mg} C' + (2a+25+1)J' \right) \\ + \text{cof} u \left(-\frac{27e}{16mg} A' - \frac{27e}{16mg} A'' - \frac{9e}{16mg} B' - \frac{9e}{16mg} B'' + \frac{3e}{8mg} C' + \frac{3e}{8mg} C'' \right) \\ + \text{cof}(4y-2v-u) \left(-\frac{27e}{16mg} A' + (4a-25-1)W' \right) \\ + \text{cof}(4y-2v+u) \left(-\frac{27e}{16mg} A' + (4a-25+1)X' \right) \\ + \text{cof}(4y+2v-u) \left(-\frac{9e}{16mg} B' + (4a+25-1)Y' \right) \\ + \text{cof}(4y+2v+u) \left(-\frac{9e}{16mg} B' + (4a+25+1)Z' \right)$$

+

cof(v-u)
cof(v'u)
-1)D'
4T)E'
F'
G'
H'
J'
O'

+

$$+ \text{cof}(2v-u) \left(+\frac{9e}{16mg} A' + \frac{27e}{16mg} B' + \frac{3e}{8mg} C' + (25-1)K' \right) \\ + \text{cof}(2v+u) \left(+\frac{9e}{16mg} A' + \frac{27e}{16mg} B' + \frac{3e}{8mg} C' + (25+1)L' \right) \\ + \text{cof}(4y-u) \left(+\frac{9e}{16mg} A' + \frac{27e}{16mg} B' + (2a-1)T' \right) \\ + \text{cof}(4y+u) \left(+\frac{9e}{16mg} A' + \frac{27e}{16mg} B' + (2a+1)V' \right)$$

LIV.

Singuli iam hi termini multiplicentur per q, cuius valor quidem erit = ε, quoniam hi termini in suo genere iam sunt minimi: sed quoniam valor $\frac{dQ-dv}{du}$ adhuc hos terminos praecipuos continet:

$$(2a-5) A' \text{cof}(2y-v) + (2a+5) B' \text{cof}(2y+v) + 5 C' \text{cof} v \\ \text{fi et hi per } q \text{ multiplicentur, inde nascetur quoque terminus angulum } u \text{ inuoluentem, erit autem pro his, summis partibus tantum praecipuis:}$$

$$q = \text{Praec.} + P \text{cof}(2y-v-u) + Q \text{cof}(2y-v+u)$$

Ergo ad illos terminos per q multiplicatos insuper accedent isti:

$$\text{cof} u \left[\frac{1}{2} (2a-5) P A' + \frac{1}{2} (2a-5) Q A' \right] + \frac{1}{2} 5 P C' \text{cof}(2y-u) \\ \text{cof}(4y-2v-u) \left[\frac{1}{2} (2a-5) P A' + \frac{1}{2} 5 P C' \right] + \frac{1}{2} 5 Q C' \text{cof}(2y+u) \\ \text{cof}(4y-2v+u) \left[\frac{1}{2} (2a-5) Q A' + \frac{1}{2} 5 Q C' \right] \\ \text{cof}(2v+u) \left[\frac{1}{2} (2a+5) P B' \right] + \text{cof}(4y-u) \left[\frac{1}{2} (2a+5) P B' \right] \\ \text{cof}(2v-u) \left[\frac{1}{2} (2a+5) Q B' \right] + \text{cof}(4y+u) \left[\frac{1}{2} (2a+5) Q B' \right]$$

LIV.

LIV.

Hinc ergo obtinentur sequentes determinaciones:

$$\begin{aligned}
2eg A' + (2a-6-1)g P' &= -\frac{27e}{8m} \\
2eg A' + (2a-6+1)g Q' &= -\frac{27e}{8m} \\
2eg B' + (2a+6-1)g R' &= +\frac{9e}{8m} \\
2eg B' + (2a+6+1)g S' &= +\frac{9e}{8m} \\
(6-1)g M' &= -\frac{3e}{4m} ; (6+1)g N' = -\frac{3e}{4m} \\
-\frac{3e}{8m}(A'-B) + \frac{9e}{8m}C' + (2a-1)g D' + \frac{1}{2}6PC' &= 0 \\
-\frac{3e}{8m}(A'-B) + \frac{9e}{8m}C' + (2a+1)g E' + \frac{1}{2}6QC' &= 0 \\
-\frac{3e}{8m}A' + \frac{27e}{16m}C' + (2a-26-1)g F' &= 0 \\
-\frac{3e}{8m}A' + \frac{27e}{16m}C' + (2a-26+1)g G' &= 0 \\
+\frac{3e}{8m}B' + \frac{9e}{16m}C' + (2a+26-1)g H' &= 0 \\
+\frac{3e}{8m}B' + \frac{9e}{16m}C' + (2a+26+1)g J' &= 0 \\
-\frac{27e}{8m}A' - \frac{9e}{8m}B' + \frac{3e}{8m}C' + g O' + \frac{1}{2}(2a-6)(P+Q)A' &= 0 \\
-\frac{27e}{8m}A' + (4a-26-1)g W' + \frac{1}{2}(2a-6)PA' + \frac{1}{2}6PC' &= 0 \\
-\frac{27e}{8m}A' + (4a-26+1)g X' + \frac{1}{2}(2a-6)QA' + \frac{1}{2}6QC' &= 0 \\
-\frac{9e}{16m}B' + (4a+26-1)g Y' &= 0 ; -\frac{9e}{16m}B' + (4a+26+1)g Z' &= 0
\end{aligned}$$

LVI.

$$\begin{aligned}
+\frac{9e}{16m}A' + \frac{27e}{16m}B' + \frac{3e}{8m}C' + (26-1)g K' + \frac{1}{2}(2a+6)QB' &= 0 \\
+\frac{9e}{16m}A' + \frac{27e}{16m}B' + \frac{3e}{8m}C' + (26+1)g L' + \frac{1}{2}(2a+6)PB' &= 0 \\
+\frac{9e}{16m}A' + \frac{27e}{16m}B' + (4a-1)g T' + \frac{1}{2}(2a+6)P'B' &= 0 \\
+\frac{9e}{16m}A' + \frac{27e}{16m}B' + (4a+1)g V' + \frac{1}{2}(2a+6)Q'B' &= 0
\end{aligned}$$

Valores ergo horum coefficientium iam ad minuta secunda reductorum erunt: O' = + 310''

- P' = -1285'' ; M' = -293'' ; F' = + 401''
- Q' = -1087 ; N' = -251 ; G' = + 5412
- R' = + 148 ; D' = - 52 ; H' = - 2
- S' = + 141 ; E' = - 45 ; J' = - 2
- W' = - 91'' ; K' = + 61''
- X' = - 95 ; L' = + 61
- Y' = - 1 ; T' = + 35
- Z' = - 1 ; V' = + 31

Unde ob excentricitatem orbitae solaris erit:

- ξ = + 0,008931 cof 2γ — 0,000247 cof (2γ - u) ;
- + 0,002090 cof (2γ - v) — 0,000227 cof (2γ + u) ;
- + 0,000640 cof (2γ + v) — 0,000280 cof 2z
- η = 0,013595 cof (2γ - v) — 0,000184 cof 4γ
- + 0,001460 cof (2γ + v) — 0,014012 cof (2γ - 2v) ;
- + 0,002834 cof v — 0,020162 cof (2γ + 2v) ;
- + 0,000340 cof 2v — 0,000420 cof (2γ - v - u) ;
- + 0,000882 cof (4γ - 2v) — 0,000360 cof (2γ - v + u) ;
- + 0,000000 cof (4γ + 2v) — 0,000000 cof (2γ + v + u) ;

Tr 2

$$\Phi - v = \text{Conf.} + 0,1123w$$

$$\begin{aligned} & -53018'' \sin(2v-v) + 1533'' \sin 2v - 1285'' \sin(2v-v-w) \\ & - 5532 \sin(2v+v) - 53549 \sin(2v-2v) - 1087 \sin(2v-v-w) \\ & + 10736 \sin v - 464 \sin(2v+2v) + 148 \sin(2v+v-w) \\ & + 449 \sin 2v + 141 \sin(2v+v+w) \\ & - 293'' \sin(v-w) + 401'' \sin(2v+2v-w) + 61'' \sin(2v-w) \\ & - 251 \sin(v+w) + 5412 \sin(2v-2v+w) + 61 \sin(2v+w) \\ & - 52 \sin(2v-w) - 91 \sin(4v-2v-w) + 35 \sin(4v-w) \\ & - 45 \sin(2v+w) - 95 \sin(4v-2v+w) + 31 \sin(4v+w) \\ & + 310 \sin w \end{aligned}$$

neglectis fallice terminis minimis.

LVII.

Denique pro longitudine lunae vera Φ invenienda,

$$\text{cum fit } \frac{d\Phi}{d\omega} = \text{Praec.}$$

$$\begin{aligned} & + mg. 0,00005 \text{ cof}(2v-v-w) - m. 0,00005 \text{ cof}(2v-w) \\ & + mg. 0,00002 \text{ cof}(2v-v+w) - m. 0,00002 \text{ cof}(2v+w) \\ & + mg. 0,00005 \text{ cof}(2v+v-w) - m. 0,00042 \text{ cof}(2v-2v-w) \\ & + mg. 0,00002 \text{ cof}(2v+v+w) - m. 0,00036 \text{ cof}(2v-2v+w) \\ & + mg. 0,00042 \text{ cof}(2v-3v-w) \\ & + mg. 0,00036 \text{ cof}(2v-3v+w) \end{aligned}$$

$$\Phi = \text{Conf.}$$

$$\text{ponatur } \Phi = \text{Conf.} + \mathfrak{D}' \sin(2v-2v) + \mathfrak{D}'' \sin(2v-v) + \mathfrak{D}''' \sin(2v-3v)$$

vna cum nouis terminis

$$\begin{aligned} & + a' \sin(2v-v-w) + e' \sin(2v-v) + g' \sin(2v-2v-w) + v' \sin w \\ & + b' \sin(2v-v+w) + f' \sin(2v+v) + h' \sin(2v-2v+w) + m' \sin(2v-w) \\ & + e' \sin(2v-v-w) + i' \sin(2v-3v-w) + n' \sin(2v+w) \\ & + b' \sin(2v+v+w) + j' \sin(2v-3v+w) \end{aligned}$$

+ $g' \sin(2v+2v-w) + q' \sin(v-w) + s' \sin(4v-w)$
+ $v' \sin(2v+2v+w) + r' \sin(v+w) + t' \sin(4v+w)$
pro reliquorum terminorum, quos forma differentialis
requirit, coefficientibus ponamus litteram l.

LVIII.

Differentiatione iam per regulas praecedentes infi-

tura erit:

$$\frac{d\Phi}{d\omega} = \text{Praec.}$$

$$\begin{aligned} & + \text{cof} w \left(\frac{3e}{8mg} \mathfrak{B}' + \frac{3e}{8mg} \mathfrak{B}' - \frac{27e}{16mg} \mathfrak{B}' - \frac{27e}{16mg} \mathfrak{B}' + v' \right) \\ & + \text{cof}(2v-w) \left(\frac{3e}{8mg} \mathfrak{B}' + \frac{9e}{16mg} \mathfrak{B}' - \frac{27e}{16mg} \mathfrak{B}' - \frac{27e}{16mg} \mathfrak{B}' + (2a+1) v' \right) \\ & + \text{cof}(2v+w) \left(\frac{3e}{8mg} \mathfrak{B}' + \frac{9e}{16mg} \mathfrak{B}' - \frac{18e}{16mg} \mathfrak{B}' + (2e+1) v' \right) \\ & + \text{cof}(2v-w) \left(+ \frac{27e}{16mg} \mathfrak{B}' - \frac{9e}{16mg} \mathfrak{B}' - \frac{3e}{8mg} \mathfrak{B}' + (2a-1) v' \right) \\ & + \text{cof}(2v+w) \left(+ \frac{27e}{16mg} \mathfrak{B}' - \frac{9e}{16mg} \mathfrak{B}' - \frac{3e}{8mg} \mathfrak{B}' + (2a+1) v' \right) \\ & + \text{cof}(2v-2v-w) \left(+ \frac{27e}{16mg} \mathfrak{B}' + 2v' - \frac{9e}{8mg} \mathfrak{B}' - \frac{9e}{8mg} \mathfrak{B}' \right) \\ & + (2a-2e-1) g' \end{aligned}$$

$$+ \text{cof}(2v-2v+w) \left(+ \frac{27e}{16mg} \mathfrak{B}' + 2e v' - \frac{3e}{8mg} \mathfrak{B}' - \frac{9e}{8mg} \mathfrak{B}' \right)$$

$$+ (2a-2e+1) v'$$

T e 3

$$\begin{aligned}
 & + \frac{27e}{16mg} \mathcal{Q}' + (4e-1) \mathcal{Q}' \\
 & + \cos(4v+u) \left(+ \frac{27e}{16mg} \mathcal{Q}' + (4e+1) \mathcal{V}' \right) \\
 & + \cos(2v+2v-u) \left(- \frac{9e}{16mg} \mathcal{Q}' + (2a+2e-1) \mathcal{V}' \right) \\
 & + \cos(2v+2v+u) \left(- \frac{9e}{16mg} \mathcal{Q}' + (2a+2e+1) \mathcal{V}' \right) \\
 & + \cos(2v-u) \left(- \frac{3e}{4mg} \mathcal{Q}' + 2e \mathcal{V}' + (2a-e-1) \mathcal{V}' \right) \\
 & + \cos(2v+3v+u) \left(- \frac{3e}{8mg} \mathcal{Q}' + 2e \mathcal{Q}' + (2a-3e+1) \mathcal{V}' \right) \\
 & + \cos(2v-u+u) \left(- \frac{3e}{4mg} \mathcal{Q}' + 2e \mathcal{Q}' + (2a-e+1) \mathcal{V}' \right) \\
 & + \cos(2v-3v-u) \left(- \frac{3e}{4mg} \mathcal{Q}' + 2e \mathcal{Q}' + (2a-3e-1) \mathcal{V}' \right) \\
 & + \cos(v-u) \left(- \frac{27e}{8mg} \mathcal{Q}' + (e-1) \mathcal{Q}' \right) \\
 & + \cos(v+u) \left(- \frac{27e}{8mg} \mathcal{Q}' + (e+1) \mathcal{V}' \right) \\
 & + \cos(4v-3v-u) \left(- \frac{27e}{8mg} \mathcal{Q}' + (4a-3e-1) \mathcal{I} \right) \\
 & + \cos(4v-3v+u) \left(- \frac{27e}{8mg} \mathcal{Q}' + (4a-3e+1) \mathcal{I} \right) \\
 & + \cos(3v-u) \left(+ \frac{9e}{8mg} \mathcal{Q}' + (3e-1) \mathcal{I} \right) \\
 & + \cos(3v+u) \left(+ \frac{9e}{8mg} \mathcal{Q}' + (3e+1) \mathcal{I} \right)
 \end{aligned}$$

$$\begin{aligned}
 & + \cos(4v-u) \left(+ \frac{9e}{8mg} \mathcal{Q}' + (4a-e-1) \mathcal{I} \right) \\
 & + \cos(4v-u+u) \left(+ \frac{9e}{8mg} \mathcal{Q}' + (4a-e+1) \mathcal{I} \right) \\
 & + \cos(4v-2v-u) \left(- \frac{27e}{16mg} \mathcal{Q}' + \frac{27e}{16mg} \mathcal{Q}' + (4a-2e-1) \mathcal{I} \right) \\
 & + \cos(4v-2v+u) \left(- \frac{27e}{16mg} \mathcal{Q}' + \frac{27e}{16mg} \mathcal{Q}' + (4a-2e+1) \mathcal{I} \right) \\
 & + \cos(4v-u) \left(+ \frac{9e}{16mg} \mathcal{Q}' + (4a-1) \mathcal{I} \right) \\
 & + \cos(4v+u) \left(+ \frac{9e}{16mg} \mathcal{Q}' + (4a+1) \mathcal{I} \right) \\
 & + \cos(2v-4v-u) \left(- \frac{9e}{8mg} \mathcal{Q}' + (2a-4e-1) \mathcal{I} \right) \\
 & + \cos(2v-4v+u) \left(- \frac{9e}{8mg} \mathcal{Q}' + (2a-4e+1) \mathcal{I} \right) \\
 & + \cos(4v-4v-u) \left(- \frac{81e}{3mg} \mathcal{Q}' + (4a-4e-1) \mathcal{I} \right) \\
 & + \cos(4v-4v+u) \left(- \frac{81e}{3mg} \mathcal{Q}' + (4a-4e+1) \mathcal{I} \right)
 \end{aligned}$$

LIX.

Collectis hinc valoribus coefficientium assumptis, obtinebitur longitudo lunae vt sequitur :

$$\Phi = C + 13,3682 u$$

— 22728//sin v	+ 2823//sin(2v-3v)	+ 100//sin u
— 1081 sin 2v	+ 47 sin(2v+3v)	— 23 sin(v-u)
— 128 sin 3v	— 246 sin(4v-v)	— 20 sin(v+u)
— 700 sin 2v	+ 41 sin(4v+v)	+ 22 sin(2v-u)
— 3594 sin(2v-2v)	+ 379 sin(4v-3v)	+ 21 sin(2v+u)
+ 306 sin(2v+2v)	— 2 sin(4v+3v)	+ 2 sin(3v-u)
— 3762 sin(2v-v)		+ 2 sin(3v+u)
— 115 sin(2v+v)		— 2 sin(4v-u)
		— 2 sin(4v+u)
+ 17//sin(2v-u)	—	2//sin(2v-4v-u)
+ 19 sin(2v+u)	—	1 sin(2v-4v+u)
+ 1 sin(4v-u)	—	11 sin(4v-2v-u)
+ 1 sin(4v+u)	—	10 sin(4v-2v+u)
+ 6 sin(2v-v-u)	—	28 sin(4v-3v-u)
+ 5 sin(2v-v+v)	—	23 sin(4v-3v+u)
+ 60 sin(2v-2v-u)	—	98 sin(4v-4v-u)
— 194 sin(2v-2v+u)	—	245 sin(4v-4v+u)
— 6 sin(2v+2v-u)	+ 2 sin(4v-v-u)	
— 6 sin(2v+2v+u)	+ 2 sin(4v-v+u)	
+ 6 sin(2v-3v-u)		
+ 8 sin(2v-3v+u)		

LX.

Plurimae igitur. Prodeunt inaequalitates ab excen-
tricitate folis pendentes, quarum nonnullae ita sunt ma-
gnae,

u
(v-u)
(v+u)
(2v-u)
(2v+u)
(3v-u)
(3v+u)
(4v-u)
(4v+u)

gnae, ut sine notabili errore omitti nequeant; cuiusmo-
di sunt imprimis, quae ab angulis $2v-2v+u$ et $4v-4v+u$
pendent. Sed in his fere idem incommodum visu venit,
quo methodus praecedens premebatur, quod magnitudo
harum inaequalitatum per Theoriam non satis accurate
describi queat. Cum enim pro his terminis invenien-
dis diversos $2a-26+1$ et $4a-46+1$ sint perquam exi-
gui, manifestum est in dividendis terminis minimos ne-
glectos non exigui fore momenti: praecipue cum pro
litteris g' et h' termini maiores fere se mutuo destru-
xissent. Vnde cum ex valore Φ tantum termini maio-
res \mathcal{B}' , \mathcal{D}' , \mathcal{G}' et \mathcal{H}' essent adhibiti, perspicuum est si
etiam reliqui minores fuissent introducti, ex iis insignem
mutationem in valore coefficientium g' et h' ortam
fuisse.

LXI.

De his autem inaequalitatibus tenendum est, eas
per satis notabile temporis spatium vix immutari; nam
inaequalitates ab angulo $2v-2v+u$, ob quantitatem
 $2a-26+1 = -0,1594$ periodum habent annorum circi-
ter $6 \frac{2}{3}$ annorum, et intervallo 19 annorum ter tantum
revolvuntur: et inaequalitas ab angulo $4v-4v+u$ pen-
dens spatio 29 annorum 19 periodos absolvit. Ex quo
cum istae inaequalitates per theoriam saltem propemo-
dum fuerint desinitae, eas deinceps per observationes
accuratius describi conveniet: nisi forte quis laborem in
se suscipere voluerit, calculum hic adumbratum multo
accuratius instituendi terminorumque hic omniforum ra-
tionem habendi; cum vero etiam valores ξ , η et $\Phi-v$

V v

multo

multo maiori studio, quam hic feci, euolui oporteret, quoniam in horum determinatione multa neglexi, quae in calculo tandem ad notabilem quantitatem excrescere potuissent.

LXII.

Inerim tamen hic notari conuenit, haec methodo eas tantum inaequalitates prodire incertas, quae satis longis periodis abfoluntur; quae incertitudo minus officit, cum per observationes facilius emendari possit: praecedente vero methodo etiam aliae inaequalitates minoribus periodis circumscriptae aliquantum incertae prodierunt, quod sane ingens erat incommodum. Vnde ex hac parte haec methodus posterior priori antefenda videtur: verum si ingentem inaequalitatum numerum spectemus, quibus non solum lunae longitudo afficitur, sed etiam longitudo apogei, calculus tantopere fit operosus, ut etiam si has formulas accuratissime euoluerem, tamen in praxi difficillimi foret visus. Quin etiam plurimae inaequalitates in motum apogei ingredi videntur, quarum effectum deinceps per alias longitudinis inaequalitates iterum destrui oportet, ita ut factus fuisset illas penitus omittere.

LXIII.

Multitudo autem harum inaequalitatum, quibus tam apogei, quam ipsius lunae longitudo turbatur, inde potissimum originem trahit, quod inaequalitates excentricitatis prae eius quantitate media admodum sunt notabiles, acque adeo quadrantem mediae quantitatis superent; ita ut prae ea negligi minime queant. Multo plures autem

oporteret, lexii, quae excrescere

methodo quae satis do minus ari possit: liates minime pro- i. Vnde si antefendum, numerum longitudo tantopere ime euolui. Quin i ingredi gitudinis iffer illas

ibus tantum inde potissimum excentricitatis notabiles superent; autem

tem adhuc inaequalitates essent accessuræ, si excentricitates lunae media adhuc esset minor, quo certe casu calculi difficultates insuperabiles euassissent: hoc vero ipso casu methodus prior multo tractabilior redderetur, cum enim pleræque inaequalitates ibi multo minores prodirent. Arque ob hanc causam minus expedire videretur, anomaliam lunae ita constitutæ, ut eius sinus tam pro maximis quam pro minimis distantis lunae a terra plane euanelcat, etiam si haec ratio naturae rei maxime consentanea videatur.

LXIV.

Cum igitur numerus inaequalitatum iam tantopere increuerit, facile perspicitur eum adhuc multo magis auctum iri, si eas inaequalitates, quae cum a parallaxi solis, tum ab eius inclinatione ad eclipicam essent euoluturæ, quo labore propterea, cum eius visus fere nullus futurus esset, superedebo. Inerim tamen hinc tantum colligere licet, inaequalitates ab angulis $2\frac{1}{2} - 2\frac{1}{2} \pm \frac{1}{2}$ et $4\frac{1}{2} - 4\frac{1}{2} \pm \frac{1}{2}$ oras, minime esse contemendas; quae cum methodo praecedente sint vel missae vel non satis accurate determinatae, sine dubio causam in se continent, quod etiam accuratissime obtulæ per observationes emendatae adhuc vltra $4'$ saepe a veritate aberrant.

LXV.

Sufficiat igitur methodum exposuisse, cuius ope inaequalitates lunae tam ratione apogei, quam longitudo nis ac latitudinis verae ex anomalia hic adhibita determinari queant; neque propterea laborem calculi reliqua-

quarum inaequalitatum, quae vel ex solis paralleli vel ex inclinatione orbitae lunaris ad eclipticam oriuntur, suscipio; quippe quarum numerus, siquidem omnes, quae alicuius momenti essent futurae, persequi vellem, in immensum excreveret. Non solum autem multitudine inaequalitatum hanc methodum omni utilitate in praxi privabit, sed etiam ingentes aequationes, quas determinatio apogei, atque anomaliae inde pendendis requirit, ita sunt comparatae, ut ipsae iam latius exactam tam longitudinis quam anomaliae cognitionem requirant; quae res est initio supponi possent, deinceps iterata eadem operatione accuratius definiendae, tamen quia correctio apogei ultra 30° gradus asurgere potest, calculus ob inaequalitatum multitudinem per se tædiosus, nimis crebro repeti debet, antequam de conclusione certi esse possimus.

APPLICATIO

FORMULARUM INVENTARUM

AD ALIOS CALCULOS LUNARES.

LXVII.

Cum igitur calculus inaequalitatum motus lunae habitenus duplici modo sit institutus, dum priori anomalia vera regulis Keplerianis conformis est assumta, posteriori vero ita constituta, ut eius sinus tam pro maximis lunae a terra distantis quam pro minimis profus evanesceret, quorum uterque vii vidimus incommodis non caret: ita etiam insidiosis aliis modis lunae inaequalitates representari poterunt, quos breviter exposuisse hanc abs-

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ADDITAMENTUM.

341

re fore arbitror. Nullum enim est dubium, quin inter hos infinitos modos quidam reperiantur, qui ipsi naturae rei magis sine consentaneis, neque iis incommodis laborent, quibus utrumque exposuimus non mediocriter impediti compertimus; etiam si adhuc difficillimum videatur, inter hanc infinitam multitudinem modum convenientissimum eligere.

LXVIII.

Postquam autem investigationem ab aequationibus differentio-differentialibus ad aequationes simpliciter differentiales produximus, etiam si ad hoc anomalia vera v cuius sinus in maximis ac minimis lunae a terra distantis evanescat, sinus vsi, tamen haec conditio iam iterum exui potest. Cum enim tam sinus quam cosinus ipsius v ubique per quantitatem q sit multiplicatus, loco harum duarum variabilium q et v iam alias duas variables in calculum introducere poterimus, quod commodissime fiet ponendo $q \cos v = r$ et $q \sin v = s$, ut sit

$$qq = rr + ss \text{ et } \operatorname{tang} v = \frac{s}{r}, \text{ cum enim vi formularum}$$

§. IX. exhibitarum habebimus istas aequationes:

$$d\phi = -s d\psi + \frac{2M}{A} ds \sqrt{A} p;$$

$$d\epsilon = r d\psi + \left(\frac{N}{A} - \frac{M'}{A(1-r^2)} \right) ds \sqrt{A} p.$$

V v 3

LXVIII.

LXX.

Quodsi iam pro r statueretur iste valor $k \cos v$, ita ut k esset quantitas constans, oriretur modus initio traditus inaequalitates lunae representandi, foret enim tum v anomalia vera Kepleri et k denotaret excentricitatem orbitae lunaris. Unde patet etiam inaequalitates lunae per methodum primam erutas, ex his formulis inveniri posse, neque ad hoc aequationibus secundi gradus esse opus. Reduceretur autem hoc casu indoles differentio-differentialium ad intentionem quantitatis r , quem in finem pro r assumi deberet series quaedam sinuum angulorum η , v , et $\Phi - \pi$ formatorum cum indefinitis coefficientibus, quos deinceps determinare liceret: hoc autem modo solutio primum tradita esset profutura.

LXXXI.

Cum sit $p = b(1 + \xi)$ et $V/A = m\sqrt{b^3}$, ob $dx = \frac{sd\omega}{p} VAp$, fiat $\frac{dx}{d\omega} = -\frac{mbs}{V(1+\xi^2)} = -mbs(1 - \frac{1}{2}\xi + \frac{3}{8}\xi^2)$, unde patet si eiusmodi anomalia v introducantur, ut sit $r = q \sin v$, siue q sit quantitas constans siue variabilis, tum hanc anomalian tam in maximis quam minimis distantis suava evanescentem esse habituram. Ac si pro q quantitas vel constans vel ex angulis cognitis composita assumatur, tum inde coefficientes assumi ac praeterea valor litterae r determinabitur. Sin autem pro q eiusmodi quantitas incognita assumatur, ut sit praeterea $r = q \cos v$, tum solutio ante exposta falsabitur.

LXXXII.

LXXII.

Semper autem usus astronomicus exigit, ut anomalia vera quaedam angulo v contenta introducantur, id quod infinitis modis fieri potest. Quo autem quantitas r variabilem distantiarum lunae a terra accuratius exprimat, et valor ipsius ξ quam minimas mutationes subbeat, necesse est, ut quantitas r huiusmodi continet terminum $k \cos v$, vbi k excentricitatem designet, qui sit quasi eius pars praecipua; hocque etiam locum habet, si pro r sumatur $q \cos v$, denotante q quantitatem variabilem, quippe cuius pars potior excentricitatem k praebere debet. Verum praeterea quantitas r alios terminos continere potest, qui ab angulo v vel pendent vel non pendent: ita poni possent: $r = k \cos v + A \cos 2\eta + B \cos 4\eta + C \cos(2\eta - v)$ etc. quo valore assumto litterae quoque r , ξ cum reliquis suis valores debitos obtinerent.

LXXXIII.

Hoc modo illud incommodum evitari potest, quo methodum in hoc additamento traditam laborare vidimus, si excentricitas orbitae lunaris esset nimis parva, vel adeo evanescentis; cum enim distantiae maximae et minimae non amplius ab anomalia penderent, sed potius ab angulo η , atque imprimis quidem a cosinu dupli anguli 2η . Casu ergo quo excentricitas plane evanescit, pro variabili r , cuius loco vicique nova variabilis introduci debet, non contentet anomalian v introducere, sed praestabit assumi seriem eorundem ex solis angulis 2η , et

LXXIII.

et $\Phi - \pi$ constantem, quorum coefficientes est sunt constantes, tamen quia terminorum numerus in infinitum excurrit, vicem novae variabilis sustinebunt. Tum autem valor ipsius r ex simili serie sinuum eorundem angulorum constabit.

LXXIV.

Quodsi ergo rem generatim pro quacunque eccentricitate expedire velimus, poterimus ad hos terminos, qui ex hypothesei eccentricitatis evanescendis prodent, adhuc adiungere terminos ex anomalia v formatos. Ita neglectis tam inaequalitatibus parallacticis, quam his quae cum ab eccentricitate orbitae solaris, tum ab inclinatione orbitae lunaris ad ellipticam pendent, poni conveniet:

$$r = k \cos v + A \cos 2v + B \cos(2v - \omega) + C \cos(2v + \omega) \\ + D \cos 4v + E \cos(4v - \omega) \text{ etc.} \\ r = \Delta k \sin v + \mathcal{A} \sin 2v + \mathcal{B} \sin(2v - \omega) + \mathcal{C} \sin(2v + \omega) \\ + \mathcal{D} \sin 4v + \mathcal{E} \sin(4v - \omega) \text{ etc.} \\ \xi = \dots \cos v + \dots \cos 2v + \dots \cos(2v - \omega) + \dots \cos(2v + \omega) \\ + \dots \cos 4v + \dots \cos(4v - \omega) \text{ etc.}$$

Argue si hoc modo omnes angulorum $2v$ et v combinationes adhibeantur, hique valores in aequationibus supra datis substituantur, primo inde elicetur ratio $dv : d\omega$, ac deinceps coefficientes determinationes suas nanciscantur.

LXXV.

Manebunt autem coefficientes vnius seriei velut ipsius r indeterminati, propterea quod ipsae series haec

constantem, excurrit, valor ipsi constabit.

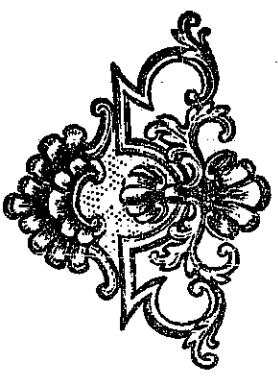
e excen- terminos, prodent, tos. Ita i iis quae :inatione leniet:

$$of(2v + \omega) \\ 1 - \omega) \text{ etc.} \\ n(2v + \omega) \\ 1 - \omega) \text{ etc.} \\ 1)(2v + \omega) \\ 1 - \omega) \text{ etc.}$$

mbinatio- nus supra : : $d\omega$, ac ilicentur.

iei velut rics haec

ab arbitrio nostro pendet, dum pro r vel solum primum terminum $k \cos v$, vel quorquos libuerit, assumere potuissimus. Hinc autem id commodi consequetur, ut istos coefficientes ad scopum quam convenientissime definire valeamus: scilicet eos ita definiti conveniet, ut primo nullius reliquorum coefficientium determinatio laboriosa et incerta evadat, vii in utraque methodo expostita visu venit: deinde vero ut nulli coefficientes fiant nimis magni praeter necessitatem, ita ut eorum effectus per alios terminos iterum destrui necesse sit. Fateri quidem cogor calculum hoc modo instituendum admodum futurum esse prolixum, verum fortasse in ipsa operatione non contemnenda se offerent compendia; vnde confido hanc speculationem, etiam si mihi ipsi eam suscipere non vacet, visu non esse curiuram.



BEROLINI, EX OFFICINA MICHAELIS.

(17-1)

