Sunt ergo hinc numeri amicabiles:

$$\left\{\begin{matrix} 4\cdot 17\cdot 43 \\ 4\cdot 5\cdot 131 \end{matrix}\right) \quad \& \quad \left\{\begin{matrix} 4\cdot 13\cdot 107 \\ 4\cdot 5\cdot 251 \end{matrix}\right)$$

Exempl. 3.

§. LXXIX. Sit $f=7$, erit $ff=gh=8$; $e=28-24=4$. & $PQ=16.8+16.6=224$.

Sit ergo primo $g=2$, $h=4$ erit

$$x=\frac{P+8}{4};\ y=\frac{Q+16}{4};\ p=4x-1;\ q=2y-1;$$
$$r=xy-1.$$

P	4	8	28	56
Q	56	28	8	4
x	3	4	9	16
y	18	11	6	5
$4x-1$	11	15*	35*	63*
$2y-1$	35*	21*	11	9*
$xy-1$	53	43	53	79

Sit secundo $g=1$, $h=8$; erit $x=\frac{P+4}{4}$; $y=\frac{Q+32}{4}$ & $p=8x-1$; $q=y-1$; $r=xy-1$.

P	4	8	28	56
Q	56	28	8	4
x	2	3	8	15
y	22	15	10	9
$8x-1$	15*	23	63*	119*
$y-1$	21	14	9	8
$xy-1$	43	44*	79	134*

Hinc ergo nulli prodeunt numeri amicabiles.

Exem-

Exemplum. 4.

§. LXXX. Sit $f = 11$, erit $gh = 12$, $e = 8$. $PQ = 16.12 + 32.10 = 512$, vel erit $(8x - 4g)(8y - 4h) = 512$, quae aequatio deprimitur ad $(2x - g)(2y - h) = 32$, qua resoluta erit $p = hx - 1$; $q = gy - 1$, & $r = xy - 1$. Sive autem hic ponatur $g = 1$, $h = 12$; sive $g = 2$, $h = 6$; sive $g = 3$, $h = 4$, nulli prodeunt numeri primi pro p, q & r.

Exemplum. 5.

§. LXXXI. Sit $f = 13$, erit $gh = 14$; $e = 10$; $PQ = 224 + 40.12 = 704$, & $(10x - 4g)(10y - 4h) = 704$, quae deprimitur ad $(5x - 2g)(5y - 2h) = 176$. Hinc autem nulli alii numeri amicabiles obtinentur nisi $\begin{Bmatrix} 4.5.251 \\ 4.13.107 \end{Bmatrix}$, qui jam ante (§.78.) sunt inventi. Simul vero jam patet, etiamsi pro f majores numeri primi statuantur, nullos novos numeros amicabiles prodire, quoniam vel p vel q sortietur valorem minorem, qui pro f assumi potuisset.

Exempl. 6.

§. LXXXII. Sit $f = 5.13$, erit $gh = 6.14 = 84$; $e = 8$; $PQ = 16.84 + 32.64 = 64.53$ & $(8x - 4g)(8y - 4h) = 64.53$ seu $(2x - g)(2y - h) = 4.53$. Hincque invenitur in numeris primis: $p = 43$; $q = 2267$, & $r = 1187$; unde erunt numeri amicabiles $\begin{Bmatrix} 4.43.2267 \\ 4.5.13.1187 \end{Bmatrix}$

Casus. II.

§. LXXXIII. Sit $a = 2^3 = 8$, erit $b = 8$, $c = 1$, tum positis numeris amicabilibus $8pq$ & $8fr$, & $ff = gh$ erit $e = 8f - 7gh$, atque

$$(ex - 8g)(ey - 8h) = 64gh + 8e(f + 1)$$

unde casus sunt dignoscendi, quibus fiunt numeri primi

$$p = hx - 1;\ q = gh - 1,\ \&\ r = xy - 1.$$

Exemplum. 1.

§. LXXXIV. Sit $f = 11$ erit $gh = 12$, $e = 4$, atque
$(4x - 8g)(4y - 8h) = 64.\,12 + 32.\,10 = 64.\,17$ seu
$(x - 2g)(y - 2h) = 4.\,17 = 68.$
Hine autem nulli numeri amicabiles reperiuntur.

Exempl. 2.

§. LXXXV. Sit $f = 13$, erit $gh = 14$; $e = 6$ atque
$(6x - 8g)(6y - 8h) = 64.\,14 + 48.\,12 = 64.\,23$ seu
$(3x - 4g)(3y - 4h) = 16.\,23$, verum etiam haec hypothesis est inutilis.

Exempl. 3.

§. LXXXVI. Sit $f = 17$, erit $gh = 18$; $e = 10$, atque
$(10x - 8g)(10y - 8h) = 64.\,17 + 40.\,16 = 64.\,38$ seu
$(5x - 4g)(5y - 4h) = 32.\,19$, hincque prodeunt numeri amicabiles. $\begin{cases} 8.\,23.\,59 \\ 8.\,17.\,79 \end{cases}$

Exempl. 4.

§. LXXXVII. Magis foecunda est hypothesis $f = 11.\,23$ minor enim valor pro f in compositis substitui nequit, erit $gh = 12.\,24$, $e = 8$ unde

(8x

$$(8x - 8g)(8y - 8h) = 64.12.24 + 64.252$$

seu $(x - g)(y - h) = 540$. Hinc autem reperiuntur sequentes numeri amicabiles.

$$\begin{Bmatrix} 8.383.1907 \\ 8.11.23.2543 \end{Bmatrix} \quad \begin{Bmatrix} 8.467.1151 \\ 8.11.23.1871 \end{Bmatrix} \quad \begin{Bmatrix} 8.647.719 \\ 8.11.23.1619 \end{Bmatrix}$$

Hujusmodi numeris compositis pro f ponendis multi insuper alii inveniuntur numeri amicabiles.

Scholion.

§. LXXXVIII. Ingens combinationum numerus, qui in hoc exemplo locum habet, ansam mihi præbuit solutionem in aliam formam redigendi commodiorem. Scilicet cum sit; $e = bf - (b - c)gh$;

$$PQ = bbgh + be(f - 1) = (ex - bg)(ey - bh)$$

ex formulis $x = \frac{P + bg}{e}$ & $y = \frac{Q + bh}{e}$ efficiuntur valores

$$p = \frac{hP + bgh}{e} - 1; \quad q = \frac{gQ + bgh}{e} - 1; \quad r = \frac{PQ + b(hP + gQ) + bbgh}{ee} - 1$$

Sit ergo ob $gh = ff$;

$$e = bf - (b - c)ff; \quad L = bbff + be(f - 1)$$

& $MN = Lff$ erit

$$p = \frac{M + bff}{e} - 1; \quad q = \frac{N + bff}{e} - 1; \quad r = \frac{L + b(M + N) + bbff}{ee} - 1$$

& nunc quæstio eo reducitur, ut numerus Lff resolvatur in duos factores M & N, quorum uterque quantitate bff auctus fit divifibilis per n, & ut quoti hinc resultantes unitate minuti fint numeri primi. Denique oportet ut fit $r+1 = \frac{(p+1)(q+1)}{ff}$ & r numerus primus. Hunc ergo calculum in nonnullis casibus illustrabo.

Casus. III.

§. LXXXIX. Sit $a = 2^4 = 16$; erit $b = 16$; $c = 1$; atque

$$e = 16f - 15ff;\ L = 256ff + 16e(f-1)$$
$$\&\ MN = Lff$$

Numeri igitur primi esse debent:

$$p = \frac{M+16ff}{e} - 1;\ q = \frac{N+16ff}{e} - 1;\ r = \frac{L+256ff+16(M+N)}{ee} - 1$$

quibus inventis erunt numeri amicabiles;

$16pq$ & $16fr$.

Exempl. I.

§. LXXXX. Sit $f = 17$ erit $ff = 18$; $e = 2$; $L = 1024.5$ & $MN = 1024.5.18 = 2^{11}.3^2.5$

$$p = \frac{M+288}{2} - 1;\ q = \frac{N+288}{2} - 1;\ r = \frac{512.19+16(M+N)}{4} - 1$$

seu sit $M = 2m$; $N = 2n$ ut fit

$$mn = 2^9.3^2.5 \quad \text{erit}$$

$$p = m+143;\ q = n+143;\ \&\ r = 8(m+n)+2431$$

qui tres numeri debent esse primi, ut numeri amicabiles fint $16pq$ & $16.17.r$.

Hoc

Hoc autem succedit duobus modis, primo si $m = 24$, $n = 960$ & secundo si $m = 96$ & $n = 240$; unde numeri amicabiles prodeunt:

$$\begin{Bmatrix} 16.167.1103 \\ 16.17.10303 \end{Bmatrix} \quad \begin{Bmatrix} 16.383.239 \\ 16.17.5119 \end{Bmatrix}$$

Exempl. 2.

§ LXXXXI. Sit $f = 19$, erit $\int f = 20$, $r = 4$; $L = 128.49$ & $MN = 512.5.49 = 2^9.5.7^2$. Ergo

$$p = \frac{M+320}{4} - 1;\quad q = \frac{N+320}{4} - 1;\quad r = \frac{128.59 + 16(M+N)}{16} - 1$$

seu sit $M = 4m$ & $N = 4n$ ut sit

$$mn = 32.5.49 = 2^5.5.7^2 \text{ erit}$$

$$p = m + 79;\ q = n + 79 \ \& \ r = 4(m+n) + 711.$$

Hinc si $m = 70$, $n = 112$ prodeunt numeri amicabiles

$$\begin{Bmatrix} 16.149.191 \\ 16.19.1439 \end{Bmatrix}$$

Exempl. 3.

§. LXXXXII. Sit $f = 23$ erit $\int f = 24$; $r = 8$, $L = 256.5.7$ & $MN = 2048.3.5.7 = 2^{11}.3.5.7$

$$p = \frac{M+16.24}{8} - 1;\quad q = \frac{N+16.24}{8} - 1;$$

$$r = \frac{256.59 + 16(M+N)}{64} - 1$$

seu sit $M = 8m$; $N = 8n$; & $mn = 2^5.3.5.7$ erit

$$p = m + 47;\ q = n + 47;\ \& \ r = 2(m+n) + 235$$

Hinc tres casus oriuntur; $\begin{cases} m = 56; \\ n = 60; \end{cases}$ $\begin{cases} m = 42; \\ n = 80; \end{cases}$ $\begin{cases} m = 6 \\ n = 560 \end{cases}$

& numeri amicabiles funt:

$$\left\{\begin{matrix}16.103.107\\16.23.467\end{matrix}\right)\quad\left\{\begin{matrix}16.89.127\\16.23.479\end{matrix}\right)\quad\left\{\begin{matrix}16.53.607\\16.23.1367\end{matrix}\right)$$

Exemplum. 4.

§. LXXXIII. Sit $f = 31$; erit $\int f = 32$; $L = 512.31$ & $MN = 2^{14}.31$; $p = \frac{M+16.32}{16} - 1$; $q = \frac{N+16.32}{16} - 1$;

$$r = \frac{16(M+N)+512.47}{256} - 1$$

Sit ergo $M = 16m$; $N = 16n$ ut fit $mn = 2^{6}.31$ erit

$$p = m+31;\; q = n+31;\; r = m+n+93$$

Hinc autem nulli prodeunt numeri amicabiles.

Exempl. 5.

§. LXXXIV. Sit $f = 47$, $\int f = 48$ erit $e = 32$ & $L = 1024.5.7$ & $MN = 2^{14}.3.5.7$ unde

$$p = \frac{M+16.48}{32} - 1;\; q = \frac{N+16.48}{32} - 1;\; \&$$

$$r = \frac{16(M+N)+1024.47}{1024} - 1$$

Sit $M = 32m$ & $N = 32n$; ut fit $mn = 2^{4}.3.5.7$ erit $p = m+23$; $q = n+23$; $r = \frac{1}{2}(m+n) + 46$. Ergo $m+n$ debet effe numerus impariter par, ut $\frac{1}{2}(m+n)$ fiat impar, quod evenit fi vel m vel n fit impariter par. Sit

$m = 30$; $n = 56$ erunt N. Amic. $\left\{\begin{matrix}16.53.79\\16.47.89\end{matrix}\right)$

Exem-

Exempl. 6.

§. XCIV. Sit $f = 17.137$, erit $ff = 18.138 = 4.27.23 = 2484$; $\epsilon = 4$; $L = 256.2484 + 64.2328 = 512.3.7.73$ & $MN = 1048.81.7.23.73$

$$p = \frac{M + 16.2484}{4} - 1;\quad q = \frac{N + 16.2484}{4} - 1;$$

$$r = \frac{512.2775 + 16(M+N)}{16} - 1$$

Sit $M = 4m$; $N = 4n$ erit $mn = 128.81.7.23.73$ & $p = m + 9935$; $q = n + 9935$ & $r = 4(m+n) + 88799$

Sed hic semper prodit valor ipsius r major quam 100000, ita ut difficile sit discernere, utrum sit primus nec ne.

Exempl. 7.

§. XCV. Sit $f = 17.151$ erit $ff = 18.152 = 16.9.19 = 2736$, $\epsilon = 32$; & $L = 1024.1967 = 1024.7.281$. atque $MN = 2^{14}.9.7.19.281$.

Sit $M = 32m$; $N = 32n$ erit $mn = 16.9.7.19.281$ & $p = m + 1367$; $q = n + 1367$; $r = \frac{1}{2}(m+n) + 2650$

Sit $m = 2\mu$, $n = 8\nu$, erit $\mu\nu = 9.7.19.281$ &. $p = 2\mu + 1367$; $q = 8\nu + 1367$; $r = \mu + 4\nu + 2650$.

Hinc primum patet neque μ neque ν esse posse numerum formae $3\alpha + 2$; tum μ non posse desinere in 9 nec ν in 1; quibus observatis sequentes tantum resolutiones locum habent.

	*		*		*	*	
μ	3.281	7.19	21.281	21	63.281	3	1
ν	21.19	9.281	57	57.281	19	399.281	1197.281

quo-

quorum ii, qui asteriscis sunt notati, excluduntur ideo, ne p, q, vel r fiat per 7 divisibile. Quarta resolutio dabit hos numeros amicabiles $\left\{\begin{array}{l} 16.\,1409.\,129503 \\ 16.\,17.\,151.\,66739 \end{array}\right\}$, si modo hic numerus 129503 est primus.

Exemplum. 8.

§. XCVI. Sit $f = 17.\,167$, erit $\int f = 18.\,168 = 16.\,27.\,7 = 3024$, $e = 64$: $L = 2048.\,1797 = 2048.\,3.\,599$ &

$$MN = 2^{15}.\,3^4.\,7.\,599$$

sit $M = 64\,m$; $N = 64\,n$, erit $mn = 2^3.\,3^4.\,7.\,599$ &

$$p = m + 755;\quad q = n + 755;\quad r = \tfrac{1}{2}(m+n) + \frac{2173}{2}$$

sit $m = 2\mu$; $n = 4\nu$, erit $\mu\nu = 3^4.\,7.\,599$ &

$$p = 2\mu + 755;\quad q = 4\nu + 755;\quad r = \nu + \frac{\mu+1}{2} + 1086$$

Ubi patet esse oportere $\mu = 4\alpha - 1$, ne r fiat numerus par: nec $\mu = 3\alpha + 2$, nec $\nu = 3\alpha + 1$. Hinc prodeunt numeri amicabiles $\left\{\begin{array}{l} 16.\,809.\,51071 \\ 16.\,17.\,167.\,13679 \end{array}\right\}$

Casus. IV.

§. XCVII. Sit vel $a = 3^3.\,5$ vel $a = 3^2.\,7.\,13$, ut sit $b = 9$, $c = 2$ erit $e = 9f - 7\int f$: $L = 81.\int f + 9e(f-1)$ & $MN = L\int f$ erit

$$p = \frac{M + 9\int f}{e} - 1;\quad q = \frac{N + 9\int f}{e} - 1;$$

$$r = \frac{9(M+N) + L + 81\int f}{ee} - 1$$

qui

qui numeri p, q, r si fuerint primi erunt numeri amicabiles.

$$\begin{Bmatrix} apq \\ afr \end{Bmatrix}$$

Exemplum.

§. XCVIII. Sit $f=7$; $\int f=8$, erit $c=7$, $L=2.27.19$; $MN=16.27.19$, erit $p=\frac{M+72}{7}-1$; $q=\frac{N+72}{7}-1$;

$$r=\frac{9(M+N)+2.27.31}{49}-1$$

Unde posito $M=54$, $N=152$ oriuntur numeri amicabiles

$a.\ 17.\ 31$ $\quad$ seu $\quad$ $\begin{Bmatrix} 3^3.5.\ 17.\ 31 \\ 3^3.5.\ 7.\ 71 \end{Bmatrix}$
$a.\ 7.\ 71$

Problema. 4.

§. XCIX. *Invenire numeros amicabiles hujus formae:* $agpq$, & ahr, *ubi* p, q, r *sint numeri primi, at* g & h *sive primi sive compositi dati, una cum factore communi* a.

Solutio.

Ex factore communi a quaeratur in minimis terminis fractio $\frac{b}{c}=\frac{a}{2a-\int a}$; deinde sit $\frac{\int g}{\int h}=\frac{m}{n}$; & ex prima proprietate numerorum amicabilium erit $(p+1)(q+1)\int g=(r+1)\int h$ seu $r+1=\frac{m}{n}(p+1)(q+1)$. Altera vero proprietas praebet: $(r+1)\int a.\int h=a(gpq+hr)$, vel ob $\frac{\int a}{a}=\frac{2b-c}{b}$ erit $(r+1)(2b-c)\int h=b(gpq+hr)$ & pro r substituto valore

$$m(2b-c)(p+1)(q+1)fh = b(ngpq + mh(p+1)(q+1) - nh)$$

Sit brevitatis gratia $p+1 = x$; $q+1 = y$ erit:

$$m(2b-c)xyfh = b(mhxy + ngxy - ngx - ngy + ng - nh) \text{ vel}$$

$$\begin{array}{l} \quad mbh \\ \quad nbg \; xy - nbgx - nbgy = nb(h-g) \\ -2mbfh \\ +mcfh \end{array}$$

Ponatur brevitatis gratia $e = b(mh+ng) - (2b-c)mfh$ eritque $eexy - nbgex - nbey + nnbbgg = nnbbgg + nb(h-g)e$ seu $(ex - nbg)(ey - nbg) = nnbbgg + nb(h-g)e$

Ponatur ergo $nnbbgg + nb(h-g)e = MN$ fietque

$$x = \frac{M+nbg}{e} \;\&\; y = \frac{N+nbg}{e} \quad \text{seu}$$

$$p = \frac{M+nbg}{e} - 1, \;\&\; q = \frac{N+nbg}{e} - 1; \;\&\; r = \frac{m}{n}xy - 1$$

Qui tres numeri p, q, & r si fuerint primi, erunt numeri amicabiles $agpq$ & ahr, dummodo utriusque factores sint primi inter se.

Coroll.

§. C. Si sint g & h numeri primi: erit $\frac{m}{n} = \frac{g+1}{h+1}$; sit ergo $g = km - 1$ & $h = kn - 1$; erit $fh = kn$, unde fiet $e = b(2kmn - m - n) - (2b-c)kmn = ckmn - b(m+n)$ $MN = nb(nb(km-1)^2 + k(n-m)e) = (ex - bn(km-1))(ey - bn(km-1))$ & $p = x - 1$; $q = y - 1$ atque $r = \frac{m}{n} xy - 1$;

Ca-

Casus. I.

§. CI. Sit $m = 1$; $n = 3$; ergo $g = k - 1$; $h = 3k - 1$; eritque $e = 3ck - 4b$; & $MN = 3b(3b(k-1)^2 + 2ke)$ ideoque

$$x = \frac{M + 3b(k-1)}{e}; \quad y = \frac{N + 3b(k-1)}{e}$$

ac denique $p = x - 1$; $q = y - 1$; & $r = \frac{1}{3}xy - 1$.

Exempl. 1.

§. CII. Sit $a = 4$; $b = 4$; $c = 1$; erit $e = 3k - 16$; & $MN = 12(12(k-1)^2 + 2ke)$ & $x = \frac{M + 12(k-1)}{e}$ & $y = \frac{N + 12(k-1)}{e}$. Hic poni potest

I. $k = 6$, fietque $g = 5$, $h = 17$, & $e = 2$, sed hinc nihil efficitur

II. $k = 8$, fietque $g = 7$, $h = 23$; & $e = 8$, $MN = 12(12 \cdot 49 + 128)$ seu $MN = 16 \cdot 3 \cdot 179 = (8x - 84)(8y - 84)$ ideoque $3 \cdot 179 = (2x - 21)(2y - 21$ unde nihil pariter sequitur:

Exempl. 2.

§. CIII. Sit $a = 8$; $b = 8$, $c = 1$; erit $e = 3k - 32$; $MN = 24(24(k-1)^2 + 2ke)$ seu $MN = 48(15kk - 56k + 12) = (ex - 24(k-1))(ex - 24(k-1))$

Verum ne hinc quoque quicquam concludere licet.

Casus. II.

§. CIV. Sit $m = 3$; $n = 1$, erit $e = 3ck - 4b$; & $g = 3k - 1$; $h = k - 1$

$MN = b(b(3k-1)^2 - 2ke) = (ex - b(3k-1))(ey - b(3k-1))$ atque $p = x - 1$; $q = y - 1$, & $r = 3xy - 1$.

Exemplum. 1.

§. CV. Sit $a = 10$, $b = 5$, $c = 1$, erit $e = 3k - 20$, &
$5(5(3k-1)^2 - 2ke) = (ex - 5(3k-1))(ey - 5(3k-1))$
Si hic ponatur $k = 8$, fiet $5 . 29 . 89 = (4x - 115)(4y - 115)$.

Unde prodit $x = 30$, $y = 674$, $3xy = 60660$, & numeri amicabiles erunt;

$$\left\{ \begin{array}{l} 10 . 23 . 29 . 673 \\ 10 . 7 . 60659 \end{array} \right]$$

Exempl. 2.

§. CVI. Sit $a = 3^2 . 5$, $b = 9$, $c = 2$, erit $e = 6k - 36$; &
$(9(3k-1)^2 - 3ke) = (\frac{1}{2}ex - 3(3k-1))(\frac{1}{2}ey - 3(3k-1))$

Jam fiat $k = 8$, erit $e = 12$; & $3 . 1523 = (4x - 69)(4y - 69)$ hincque oritur $x = 18$, $y = 398$, $3xy = 21492$ eruntque numeri primi; $g = 23$; $h = 7$; $p = 17$; $q = 397$; $r = 21491$ & numeri amicabiles:

$$\left\{ \begin{array}{l} 3^2 . 5 . 23 . 17 . 397 \\ 3^2 . 5 . 7 . 21491 \end{array} \right]$$

Scholion.

§. CVII. Ex his exemplis usus hujus problematis in inveniendis numeris amicabilibus satis luculenter perspicitur; sed ob ipsam nimiam fingendi libertatem non parum molestum est secundum praecepta hic tradita omnes casus percurrere. Cum igitur sufficiat hanc methodum tradidisse, ejusque usum monstrasse, ei prolixius non immoror, sed ad ultimam methodum, cujus ope numeros amicabiles eruere liceat, qua quidem sum usus, exponendam progredior. Nititur ea autem singularibus proprietatibus, quibus numeri ratione summae divisorum gaudent, quas oblata occasione explicabo, ne plurium lemmatum praemissio taedium creet.

Iis

Iis autem expositis non difficile erit plura alia problemata ad hoc genus pertinentia resolvere.

Problema 5.

§. CVIII. *Invenire numeros amicabiles hujus formae:* zap *&* zbq, *ubi factores* a *&* b *sint dati,* p *&* q *numeri primi, & factor communis* z *investigari debeat.*

Solutio.

Sit $\int a : \int b = m : n$; & cum esse debeat $\int a.(p+1) = \int b.(q+1)$, erit $m(p+1) = n(q+1)$. Ponatur $p+1 = nx$ & $q+1 = mx$, eruntque numeri amicabiles $za(nx-1)$ & $zb(mx-1)$. Ubi quidem requiritur ut $mx-1$ & $nx-1$ sint numeri primi. Cum jam utriusque numeri eadem sit summa divisorum $= nx\int a.\int z = mx\int b\int z$ oportet ut ea sit aequalis summae numerorum $z((na+mb)x-a-b)$. Unde obtinetur ista aequatio $\frac{z}{\int z} = \frac{nx\int a}{(na+mb)x-a-b}$. Quo jam ex hac aequatione valor ipsius z elici queat, fractio $\frac{nx\int a}{(na+mb)x-a-b}$ ad minimos terminos reducatur, quae sit $= \frac{r}{s}$, ita ut habeatur $\frac{z}{\int z} = \frac{r}{s}$: hincque sequentia sunt notanda. Primo esse z vel ipsi r aequale, vel ejus multiplo cuipiam puta kr. Priori casu si $z = r$ erit $\int z = s$ ac propterea $s = \int r$. Posteriori casu, si $z = kr$ erit $\int z = ks = \int kr$: Verum quicquid sit k, erit $\frac{\int kr}{\int r} > k$, nam $\int kr$ continet omnes divisores ipsius r singulos per k multiplicatos, & insuper eos divisores ipsius r, qui non sunt per k divisibiles; eritque ergo $\int kr >$

$k\int r$.

$k\int r$. Cum igitur sit $\int z > k\int r$, erit quoque $ks > k\int r$ seu $s > \int r$. Quare si in fractione $\frac{r}{s}$ fuerit $s = \int r$, erit $z = r$; sin autem sit $s > \int r$ erit z aequale multiplo cuipiam ipsius r. Unde patet si sit $s < \int r$ aequationem $\frac{z}{\int z} = \frac{r}{s}$ esse impossibilem, neque hoc casu numeros amicabiles inveniri posse. Deinde cum sit $\frac{\int z}{z} = \frac{na + mb}{n\int a} - \frac{a - b}{nx\int a} = \frac{a}{\int a} + \frac{b}{\int b} - \frac{a - b}{nx\int a}$; ob $\frac{a}{\int a} < 1$ & $\frac{b}{\int b} < 1$ erit $\frac{\int z}{z} < 2 - \frac{a - b}{nx\int a}$, ideoque multo magis $\frac{z}{\int z} > \frac{1}{2}$ ita ut z sit semper numerus deficiens. Hincque patet aequationem $\frac{z}{\int z} = \frac{r}{s}$ semper ita fore comparatam, ut sit $\frac{r}{s} > \frac{1}{2}$ seu $s < 2r$. Unde si sit $\int r = s$, erit $\int r < 2r$, &, si $s > \int r$ erit multo magis $\int r < 2r$. Utroque igitur casu r erit numerus deficiens. Quocirca si x tanquam numerus incognitus spectetur, proposita aequatione $\frac{z}{\int z} = \frac{nx\int a}{(na + mb)x - a - b}$, valorem ipsius x ita determinari oportet; ut reducta fractione $\frac{nx\int a}{(na + mb)x - a - b}$, ad minimos terminos $\frac{r}{s}$, fiat r numerus deficiens, & ut sit vel $s = \int r$ vel $s > \int r$. Quibus conditionibus animadversis, tam r quam s in suos factores simplices primos resolvatur, ut prodeat hujusmodi aequatio $\frac{z}{\int z} = \frac{A^{\alpha} B^{\beta} C^{\gamma}}{E^{\epsilon} F^{\zeta} G^{\eta}}$, tunc autem successive vel

vel A^{α} vel altior potestas ipsius A ponatur factor ipsius z, seu ponatur $z = P.A^{\alpha+\nu}$ erit $\int z = \int A^{\alpha+\nu} \int P$ & $\frac{z}{\int z} = \frac{PA^{\alpha+\nu}}{\int A^{\alpha+\nu} \int P}$,

ideoque $\frac{P}{\int P} = \frac{B^{\varepsilon} C^{\nu} \int A^{\alpha+\nu}}{A^{\ldots} E^{\ldots} F^{\zeta} G^{\ldots}}$. Similique modo ponatur ulterius $P = B^{\varepsilon+\mu} Q$, & hoc pacto procedatur, donec tandem perveniatur ad aequationem hujus formae $\frac{Z}{\int Z} = \frac{u}{\int u}$, ex qua habeatur $Z = u$. Saepe quidem haec operatio successu optato caret, sed pro quovis casu objecto facilius erit operationem hanc per exempla docere, quam per praecepta.

Exemplum. I.

§. CIX. Sit $a = 3$; $b = 1$ erit $\int a = 4$; $\int b = 1$; & $m = 4$; $n = 1$ ac numeri amicabiles erunt: $3(x-1)z$ & $(4x-1)z$, si sint $x-1$ & $3x-1$ numeri primi & $\frac{z}{\int z} = \frac{4x}{7x-4}$. Hic autem primo patet, si 4 ex numeratore non tollatur, fore $7x-4 < \int 4x$ ob $\int 4x = 7\int x$. Ergo necesse est ut sit $7x-4$ numerus par. Ponatur $x = 4p$; erit $\frac{z}{\int z} = \frac{4p}{7p-1}$. Nunc fiat $7p-1$ numerus par, ponendo $p = 2q+1$, erit $\frac{z}{\int z} = \frac{2(2q+1)}{7q+3}$; & $x = 8q+4$; atque $x-1 = 8q+3$; $4x-1 = 32q+15$.

Unde

Unde q nequit esse multiplum ternarii, ne $x - 1$ fiat per 3 divisibile. Erit ergo vel $q = 3r + 1$, vel $q = 3r - 1$, priori casu fit $2q + 1 = 6r + 3$, et z deberet esse divisibile per 3, quod pariter fieri nequit, quia in altero numero quaesito $3(x - 1)z$ jam inest factor 3. Sit igitur $q = 3r - 1$, erit $\frac{z}{\int z} = \frac{2(6r-1)}{21r-4}$ atque $x = 24r - 4$; $x - 1 = 24r - 5$; & $4x - 1 = 96r - 17$. Cum autem z factorem 4 habere nequeat, nisi binarius ex numeratore $2(6r - 1)$ tollatur, z erit divisibile per 2, & posito $z = 2y$ fieret $\frac{2y}{\int 2y} = \frac{2(6r-1)}{21r-4}$, & $\frac{y}{\int y} = \frac{3(6r-1)}{21r-4}$: ideoque evaderet y & propterea quoque z per 3 divibile quod fieri nequit. Hanc obrem iste binarius ex numeratore tolli debet, ponendo $r = 2s$; ut fit $x - 1 = 48s - 5$; $4x - 1 = 192s - 17$ eritque $\frac{z}{\int z} = \frac{12s - 1}{21s - 2}$. Jam si s sit numerus impar ob z numerum imparem, fiet quoque $\int z = k(21s - 2)$ numerus impar, ex quo sequitur, numerum z fore quadratum; sin autem s sit numerus par, factor communis z non erit quadratus. Evolvantur ergo ii ipsius s valores, qui efficiunt $x - 1 = 48s - 5$ & $4x - 1 = 192s - 17$ numeros primos; & dispiciatur utrum aequationi $\frac{z}{\int z} = \frac{12s-1}{21s-2}$ satisfieri queat. Sit $s = 7$; erit $x - 1 = 331$, $4x - 1 = 1327$ & $\frac{z}{\int z} = \frac{83}{145}$. Jam cum z debeat esse quadratum, ponatur $z = 83^2 A$, erit $\int z = 367.\ 19 \int A$ & $\frac{A}{\int A} = \frac{367.\ 19.}{5.\ 29.\ 83}$. Nunc autem ipsius A factor statui nequit 19^2, ob $\int 19^2 =$

3.127

3.127, prodiret enim 3 factor ipsius A, altioribus vero potestatibus sumendis, mox devenitur ad numeros tam grandes, ut facile pateat opus succedere non posse.

Si $s = 12$; erit $x - 1 = 571$; $4x - 1 = 2287$ & $\frac{z}{\int z} = \frac{11.13}{2.125}$ quæ neque 11^2 neque 13 pro factoribus ipsius z assumendo resolvi potest.

Neque vero etiam ex majoribus valoribus pro s mihi quicquam præstare licuit.

Exempl. 2.

§. CX. Sit $a = 5$, $b = 1$; erit $\int a = 6$; $\int b = 1$; $m = 6$, $n = 1$ & numeri amicabiles erunt $5(x-1)z$ & $(6x-1)z$, habebiturque $\frac{z}{\int z} = \frac{6x}{11x - 6}$. Quæ æquatio ut sit possibilis ex numeratore $6x$ vel binarium, vel ternarium tollere oportet, quia alioquin numerator maneret numerus redundans. Habebimus ergo duos casus evolvendos.

1. Tollatur ex numeratore ternarius ponendo $x = 3p$, erit $\frac{z}{\int z} = \frac{6p}{11p-2}$, nunc vero porro ponatur $p = 3q + 1$, eritque $\frac{z}{\int z} = \frac{2(3q+1)}{11q+3}$ & ob $x = 9q + 3$ numeri primi esse debeat $x - 1 = 9q + 2$, & $6x - 1 = 54q + 17$, ubi patet q esse debere numerum imparem. Sit ergo $q = 2r - 1$; erit $x - 1 = 18r - 7$; $6x - 1 = 108r - 37$

$$\& \quad \frac{z}{\int z} = \frac{2(6r-2)}{22r-8} = \frac{2(3r-1)}{11r-4}.$$

Evolvantur jam casus quibus $18r - 7$ & $108r - 37$ fiunt numeri primi: qui sunt:

1). $r = 1$; erit $x - 1 = 11$; $6x - 1 = 71$; & $\frac{z}{\int z} = \frac{2.2.}{4} = \frac{4}{7}.$

Cum igitur hic sit $7 = \int 4$, erit $z = 4$, & numeri amicabiles erunt $\left\{ \begin{matrix} 4.5.11 \\ 4.71 \end{matrix} \right)$, quos quidem jam invenimus:

2). $r = 2$; erit $x - 1 = 29$, $6x - 1 = 179$ & $\frac{z}{\int z} = \frac{2.5.}{2.9.} = \frac{5}{9}$. At z factorem 5 habere nequit.

3). $r = 5$; erit $x - 1 = 83$, $6x - 1 = 503$ & $\frac{z}{\int z} = \frac{4.7}{3.17}$; at hic $3.17 < \int 4.7$.

4). $r = 8$; erit $x - 1 = 137$, $6x - 1 = 827$ & $\frac{z}{\int z} = \frac{23}{2.3.7}$;

Ponatur $z = 4.23P$, erit $\int z = 24\int P$ & $\frac{P}{\int P} = \frac{24}{23} \cdot \frac{z}{\int z} = \frac{4}{7}$; unde $P = 4$, & $z = 4.23$: quam operationem ita breviter repræsento $\frac{z}{\int z} = \frac{23}{2.3.7.} \left| \frac{23}{24} \right| \frac{4}{7} \left| \frac{4}{7} \right|$, unde fit $z = 4.23$ & numeri amicabiles erunt.

$$\left\{ \begin{matrix} 4.23.5.137 \\ 4.23.827 \end{matrix} \right)$$

Reliqui valores, quousque quidem examinavi nullos dant numeros amicabiles.

II. Tollatur ex numeratore binarius, ponendo $x = 2p$ erit $\frac{z}{\int z} = \frac{6p}{11p - 3}$; Nunc sit $p = 2q + 1$; erit $\frac{z}{\int z} = \frac{3(2q+1)}{11q+4}$: & numeri primi esse debebunt (ob $x = 4q + 2$); $x - 1 = 4q + 1$;

6x-

$6x-1 = 24q+11$: quare esse nequit $q = 3\alpha - 1$. Deinde cum z non esse debeat divisibile per 5, neque $2q+1$, neque $4q+1$, neque $24q+11$ per 5 debet esse divis: unde excluduntur casus $q = 5\alpha+2$; $q = 5\alpha+1$. Rejectis ergo his aliisque valoribus inutilibus ipsius q, qui pro $x-1$ & $6x-1$ non præbent numeros primos, calculus ita se habebit

q	$x-1$	$6x-1$	$\frac{z}{\int z} = \frac{3(2q+1)}{11q+4}$
3	13	83	$\frac{3\cdot7}{37}$ nihil dat
4	17	107	$\frac{3\cdot9}{48} = \frac{9}{16}\left[\frac{9}{13}\right]\frac{13}{16}\left[\frac{13}{14}\right]\frac{7}{8}\left[\frac{7}{8}\right]$; $z = 9\cdot7\cdot13$ vel $\frac{9}{16}\left[\frac{27}{40}\right]\frac{5}{6}\left[\frac{5}{6}\right]$ ergo $z = 27\cdot5$. Hic autem valor ob $a = 5$ est inutilis; erunt ergo numeri amicabiles $\left\{\begin{matrix} 9\cdot7\cdot13\cdot5\cdot17 \\ 9\cdot7\cdot13\cdot107 \end{matrix}\right)$
9	37	227	$\frac{3\cdot19}{103}$ nihil dat
10	41	251	$\frac{3\cdot21}{114} = \frac{3\cdot7}{2\cdot19}\left[\frac{7^2}{3\cdot19}\right]\frac{3^2}{2\cdot7}\left[\frac{3^2}{13}\right]\frac{13}{14}\left[\frac{13}{14}\right]$ Ergo $z = 3^2\cdot7^2\cdot13$ & numeri amicabiles erunt $\left\{\begin{matrix} 3^2\cdot7^2\cdot13\cdot5\cdot41 \\ 3^2\cdot7^2\cdot13\cdot251 \end{matrix}\right)$

18	73	443	$\frac{3\cdot 37}{202} = \frac{3\cdot 37}{2\cdot 101}$ nihil dat.
24	97	587	$\frac{3\cdot 49}{268} = \frac{3\cdot 49}{4\cdot 67}$ nihil dat.
28	113	683	$\frac{3\cdot 57}{312} = \frac{9\cdot 19}{8\cdot 39} = \frac{3\cdot 19}{8\cdot 13}$ nihil dat
34	137	827	$\frac{3\cdot 69}{378} = \frac{23}{2\cdot 21} = \frac{23}{2\cdot 3\cdot 7}\left\|\frac{23}{24}\right\|\frac{4}{7}\left\|\frac{4}{7}\right\|$; $z = 4\cdot 23$; ut ante
39	157	947	$\frac{3\cdot 79}{433}$ nihil dat.
45	181	1091	$\frac{3\cdot 91}{499} = \frac{3\cdot 7\cdot 13}{499}$
48	193	1163	$\frac{3\cdot 97}{532} = \frac{3\cdot 97}{4\cdot 7\cdot 19} = \frac{3\cdot 97}{4\cdot 133}\left\|\frac{97}{2\cdot 7^2}\right\|\frac{3\cdot 7}{2\cdot 19}\left\|\frac{7^2}{3\cdot 19}\right\|\frac{3^2}{2\cdot 7}$ $\left\|\frac{3^2}{13}\right\|\frac{13}{14}$
			Ergo $z = 3^2\cdot 7^2\cdot 13\cdot 97$ & numeri amicabiles sunt
			$\left\{\begin{array}{l} 3^2\cdot 7^2\cdot 13\cdot 97\cdot 5\cdot 193 \\ 3^2\cdot 7^2\cdot 13\cdot 97\cdot 1163 \end{array}\right)$
49	197	1187	$\frac{3\cdot 99}{543} = \frac{9\cdot 11}{181}$
60	241	1451	$\frac{3\cdot 121}{664} = \frac{3\cdot 11^2}{8\cdot 83}$

69 | 2
79 | 3
84 | 3
93 | 3
100 | 4
244 | 9
Hi