De Numeris Amicabilibus.

Definitio.

§. I.

Bini Numeri vocantur amicabiles, si ita sint comparati, ut summa partium aliquotarum unius æqualis sit alteri numero, & vicissim summa partium aliquotarum alterius priori numero æquetur.

Sic isti numeri 220 & 284 sunt amicabiles; prioris enim 220 partes aliquotæ junctim sumtæ: 1+2+4+5+10+11+20+22+44+55+110 faciunt 284: & hujus numeri 284 partes aliquotæ: 1+2+4+71+142 producunt priorem numerum 220.

Scholion.

§. II. Stifelius, qui primus hujusmodi numerorum mentionem fecit, casu hos duos numeros 220 & 284 contemplatus ad hanc speculationem deductus videtur; analysin enim ineptam existimat, cujus ope plura istiusmodi numerorum paria inveniantur. Cartesius vero analysin ad hoc negotium accommodare est conatus, regulamque tradidit, qua tria talium numerorum paria eruit, neque præter ea Schotenius, qui multum in hac investigatione desudasse videtur, plura eruere valuit. Post hæc tempora nemo fere Geometrarum ad hanc quæstionem magis evolvendam operam impendisse reperitur. Cum autem nullum sit dubium quin analysis quoque ex hac parte incrementa non contemnenda sit consecutura, si methodus aperiatur, qua multo plura hujusmodi numerorum paria investigare liceat, haud abs re fore arbitror, si methodos quasdam huc spectantes, in quas forte incidi, communicavero. In hunc finem autem sequentia præmittere necesse est.

Hy-

Hypothesis.

§. III. Si n denotet numerum quemcunque integrum positivum, cujusmodi numeri hic semper sunt intelligendi, omnium ejus divisorum summam hoc signo $\int n$ indicabo, ita ut character $\int$ numero cuipiam præfixus summam omnium ejusdem numeri divisorum denotet; sic erit $\int 6 = 1+2+3+6 = 12$.

Corollarium. 1.

§. IV. Quoniam inter divisores cujusvis numeri hic ipse numerus refertur, partes aliquotæ autem censentur divisores ipso numero excepto, manifestum est summam partium aliquotarum numeri n exprimi per $\int n - n$.

Coroll. 2.

§. V. Quoniam numerus primus nullos alios divisores admittit præter unitatem & se ipsum, si n sit numerus primus erit $\int n = 1 + n$. Cum autem casu $n = 1$ sit $\int 1 = 1$, patet unitatem non recte numeris primis annumerari.

Lemma. 1.

§. VI. Si m & n fuerint numeri inter se primi, ut præter unitatem nullum habeant divisorem communem, tum erit $\int mn = \int m . \int n$, seu summa divisorum producti mn æqualis est producto ex summis divisorum utriusque numeri m & n.

Productum enim mn primo habet singulos divisores utriusque factoris m & n, tum vero insuper divisibile est per producta ex singulis divisoribus numeri m in singulos divisores numeri n. Hi vero omnes ipsius mn divisores junctim prodeunt, si $\int m$ per $\int n$ multiplicetur.

Coroll. 1.

§. VII. Si numerorum m & n uterque sit primus, ideoque $\int m = 1 + m$ & $\int n = 1 + n$, erit summa divisorum producti $\int mn =$

(1+m)

$(1+m)(1+n) = 1+m+n+mn$. Si præterea p sit numerus primus diversus ab m & n, erit $\int mnp = \int mn.\int p = \int m.\int n.\int p = (1+m)(1+n)(1+p)$. Hincque summa divisorum cujusque numeri, qui est productum ex quotcunque numeris primis diversis, facile assignabitur.

Coroll. 2.

§. VIII. Si m, n, & p non quidem sint numeri primi, sed tamen ejusmodi, ut præter unitatem nullum habeant divisorem communem, tum mn & p erunt numeri inter se primi, ac propterea $\int mnp = \int mn.\int p$. Cum autem sit $\int mn = \int m.\int n$: erit $\int mnp = \int m.\int n.\int p$.

Scholion.

§. IX. Nisi factores $m.\ n.\ p$ sint numeri inter se primi, summa divisorum producti, prout per lemma indicatur, non est justa. Cum enim secundum lemma singuli divisores factorum $m.\ n.\ p$ inter divisores producti mnp referantur, si haberent divisorem communem, is inter divisores producti bis numeraretur; at dum quæstio de summa divisorum cujuspiam numeri instituitur, nullum divisorem bis numerare oportet. Hinc si m & n sint numeri primi ac $m = n$, non erit $\int nn = \int n.\int n = (1+n)^2 = 1+2n+nn$, sed habebitur $\int m = 1+n+nn$, neque divisorem n bis poni convenit. Cum igitur per hoc lemma summa divisorum cujusque numeri, qui est productum ex quotcunque numeris primis diversis, recte assignetur, residuum est, ut pro factoribus æqualibus regula tradatur, cujus ope summa divisorum producti definiri queat.

Lemma. 2.

§. X. Si n sit numerus primus, erit $\int n^2 = 1+n+n^2$, $\int n^3 = 1+n+n^2+n^3$; $\int n^4 = 1+n+n^2+n^3+n^4$, & generatim erit

$$\int n^k = 1+n+n^2+\ldots\ldots+n^k = \frac{n^{k+1}-1}{n-1}.$$

Coroll. 1.

§. XI. Cum sit $\int n = 1 + n$, erit $\int n^2 = \int n + n^2$, vel etiam $\int n^2 = 1 + n\int n$. Simili modo erit $\int n^3 = \int n^2 + n^3$, vel etiam $\int n^3 = 1 + n\int n^2$; porroque $\int n^4 = \int n^3 + n^4$ seu $\int n^4 = 1 + \int n^3$ & ita porro. Sicque ex cognita summa divisorum cujusque potestatis n^k facile summa divisorum potestatis sequentis n^{k+1} assignatur, cum sit $\int n^{k+1} = \int n^k + n^{k+1}$ seu $\int n^{k+1} = 1 + n\int n^k$.

Coroll. 2.

§. XII. Quo summæ divisorum facilius per factores exprimi queant, notandum est esse $\int n^3 = (1+n)(1+n^2) = (1+n^2)\int n$; $\int n^5 = (1+n^2+n^4)\int n$; $\int n^7 = (1+n^2+n^4+n^6)\int n = (1+n^2)(1+n^4)\int n$: sicque summæ divisorum potestatum imparium semper per factores exhiberi possunt: at potestatum parium summæ divisorum quandoque erunt numeri primi.

Coroll. 3.

§. XIII. Hinc igitur facile tabula condi poterit, qua non solum numerorum primorum, sed etiam potestatum ipsorum summæ divisorum exhibeantur. Cujusmodi Tabulam hic adjicere visum est, in qua omnium numerorum primorum millenario non majorum, eorumque potestatum ad tertiam usque & altiores pro minoribus numeris summæ divisorum per factores expressæ traduntur.

Num.	Summa Divisorum.
2	3
2^2	7
2^3	3.5
2^4	31
2^5	$3^2.7$
2^6	127
2^7	3.5.17.
2^8	7.73
2^9	3.11.31
2^{10}	23.89
2^{11}	$3^2.5.7.13$
2^{12}	8191
2^{13}	3.43.127
2^{14}	7.31.151
2^{15}	3.5.17.257
2^{16}	131071
2^{17}	$3^3.7.19.73$
2^{18}	524287
2^{19}	$3.5^2.11.31.41$
2^{20}	$7^2.127.337$
2^{21}	3.23.89.683
2^{22}	47.178481
2^{23}	$3^2.5.7.13.17.241$
2^{24}	31.601.1801
2^{25}	3.2731.8191
2^{26}	7.73.262657
2^{27}	3.5.29.43.113.127
2^{28}	233.1103.2089
2^{29}	$3^2.7.11.31.151.331$
2^{30}	2147483647
2^{31}	3.5.17.257.65537
2^{32}	7.23.89.599479
2^{33}	3.43691.131071
2^{34}	31.71.127.122921
2^{35}	$3^3.5.7.13.19.37.73.109$
2^{36}	223.616318177.

Numeri	Summa Divisorum.
3	2^2
3^2	13
3^3	$2^3.5$
3^4	11^2
3^5	$2^2.7.13$
3^6	1093
3^7	$2^4.5.41$
3^8	13.757
3^9	$2^2.11^2.61$
3^{10}	23.3851
3^{11}	$2^3.5.7.13.73$
3^{12}	797161
3^{13}	$2^2.547.1093$
3^{14}	$11^2.13.4561$
3^{15}	$2^5.5.17.41.193$
5	2.3
5^2	31
5^3	$2^2.3.13$
5^4	11.71
5^5	$2.3^2.7.31$
5^6	19531
5^7	$2^3.3.13.313$
5^8	19.31.829
5^9	2.3.11.71.521
7	2^3
7^2	3.19
7^3	$2^4.5^2$
7^4	2801
7^5	$2^3.3.19.43$
7^6	29.4733
7^7	$2^5.5^2.1201$
7^8	$3^2.19.37.1063$
7^9	$2^3.11.191.2801$
7^{10}	329554457.

Numeri	Summa Divisorum.
11	$2^2.3$
11^2	7.19
11^3	$2^3.3.61$
11^4	5.3221
11^5	$2^2.3^2.7.19.37$
11^6	43.45319
11^7	$2^4.3.61.7321$
11^8	7.19.1772893
11^9	$2^2.3.5.3221.13421$
13	2.7
13^2	3.61
13^3	$2^2.5.7.17$
13^4	30941
13^5	2.3.7.61.157
13^6	5229043
13^7	$2^3.5.7.17.14281$
17	2.3^2
17^2	307
17^3	$2^2.3^2.5.29$
17^4	88741
17^5	$2.3^2.7.13.307$
19	$2^2.5$
19^2	3.127
19^3	$2^3.5.181$
19^4	151.911
19^5	$2^2.3.5.7^3.127$
23	$2^3.3$
23^2	7.79.
23^3	$2^4.3.5.53$
23^4	292561

Num,

Num	Sum Divisorum	Num.	Sum, Divisorum	Num	Summa Divisor.	Num.	Sum Divisorum.
29	2.3.5	67	2^2.17	109	2.5.11.	163	2^2.41
29^2	13.67	67^2	3.7^2.31	109^2	3.7.571	163^2	3.7.19.67
29^3	2^2.3.5.421	67^3	2^3.5.17.449	109^3	2^2.5.11.13.457	163^3	2^3.5.41.2657
31	2^5	71	2^3.3^2	113	2.3.19	167	2^3.3.7
31^2	3.331	71^2	5113	113^2	13.991	167^2	28057
31^3	2^6.13.37	71^3	2^4.3^2.2521	113^3	2^2.3.5.19.1277	167^3	2^4.3.5.7.2789
37	2.19	73	2.37	127	2^7	173	2.3.29
37^2	3.7.67	73^2	3.1801	127^2	3.5419	173^2	67.449
37^3	2^2.5.2603	73^3	2^2.5.13.37.41	127^3	2^8.5.1613	173^3	2^2.3.5.29.41.73
41	2.3.7	83	2^2.3.7	131	2^2.3.11	179	2^2.3^2.5
41^2	1723	83^2	19.367	131^2	17293	179^2	7.4603
41^3	2^2.3.7.29^2	83^3	2^3.3.5.7.13.53.	131^3	2^3.3.11.8581.	179^3	2^3.3^2.5.37.433
43	2^2.11	89	2.3^2.5	137	2.3.23	181	2.7.13
43^2	3.631	89^2	8011	137^2	7.37.73	181^2	3.79.139
43^3	2^3.5^2.11.37	89^3	2^2.3^2.5.17.233.	137^3	2^2.3.5.23.1877.	181^3	2^2.7.13.16381
47	2^4.3	97	2.7^2	139	2^2.5.7	191	2^6.3
47^2	37.61	97^2	3.3169	139^2	3.13.499	191^2	7.13^2.31
47^3	2^5.3.5.13.17	97^3	2^2.5.7^2.941	139^3	2^3.5.7.9661	191^3	2^7.3.17.29.37
53	2.3^3	101	2.3.17	149	2.3.5^2	193	2.97
53^2	7.409	101^2	10303	149^2	7.31.103	193^2	3.7.1783
53^3	2^2.3^3.5.281	101^3	2^2.3.17.5101.	149^3	2^2.3.5^2.11101.	193^3	2^2.5^3.97.149
59	2^2.3.5	103	2^3.13	151	2^3.19	197	2.3^2.11
59^2	3541	103^2	3.3571	151^2	3.7.1093	197^2	19.2053
59^3	2^3.3.5.1741	103^3	2^4.5.13.1061	151^3	2^4.13.19.877.	197^3	2^2.3^2.5.11.3881
61	2.31	107	2^2.3^3	157	2.79	199	2^3.5^2
61^2	3.13.97	107^2	7.13.127	157^2	3.8269	199^2	3.13267
61^3	2^2.31.1861	107^3	2^3.3^3.5^2.229	157^3	2^2.5^2.17.29.79	199^3	2^4.5^2.19801

Num.

Num.	Summa Divisor.	Num.	Summa Divisorum	Num.	Summa Divisorum	Num.	Summa Divisorum
211	$2^2.53$	263	$2^3.3.11$	313	2.157	373	$2.11.17$
211^2	$3.13.31.37$	263^2	$7^2.13.109$	313^2	3.181^2	373^2	$3.7^2.13.73$
211^3	$2^3.53.113.197$	263^3	$2^4.3.5.11.6917.$	313^3	$2^2.5.97.101.157.$	373^3	$2^2.5.11.17.13913$
223	$2^5.7$	269	$2.3^3.5$	317	$2.3.53$	379	$2^2.5.19$
223^2	3.16651	269^2	$13.37.151$	317^2	7.14401	379^2	$3.61.787$
223^3	$2^6.5.7.4973.$	269^3	$2^2.3^3.5.97.373.$	317^3	$2^2.3.5.13.53.773$	379^3	$2^3.5.19.71821$
227	$2^2.3.19$	271	$2^4.17$	331	$2^2.83$	383	$2^7.3$
227^2	73.709	271^2	3.24571	331^2	$3.7.5233$	383^2	147073
227^3	$2^3.3.5.19.5153.$	271^3	$2^5.17.36721$	331^3	$2^3.29.83.1889.$	383^3	$2^8.3.5.14669$
229	$2.5.23$	277	2.139	337	2.13^2	389	$2.3.5.13$
229^2	$3.97.181$	277^2	$3.7.19.193$	337^2	$3.43.883$	389^2	7.21673
229^3	$2^2.5.13.23.2017$	277^3	$2^2.5.139.7673$	337^3	$2^2.5.13^2.41.277.$	389^3	$2^2.3.5.13.29.2609$
233	$2.3^2.13$	281	$2.3.47$	347	$2^2.3.29$	397	2.199
233^2	7.7789	281^2	109.727	347^2	$7.13.1327$	397^2	$3.31.1699$
233^3	$2^2.3^2.5.13.61.89$	281^3	$2^2.3.13.47.3037$	347^3	$2^3.3.5.29.12041$	397^3	$2^2.5.199.15761$
239	$2^4.3.5$	283	$2^2.71$	349	$2.5^2.7$	401	$2.3.67$
239^2	19.3019	283^2	$3.73.367$	349^2	$3.19.2143$	401^2	7.23029
239^3	$2^5.3.5.13^4$	283^3	$2^3.5.71.8009$	349^3	$2^2.5^2.7.60901$	401^3	$2^2.3.37.41.53.67$
241	2.11^2	293	$2.3.7^2$	353	$2.3.59$	409	$2.5.41$
241^2	3.19441	293^2	$86143.$	353^2	19.6577	409^2	3.55897
241^3	$2^2.11^2.113.257$	293^3	$2^2.3.5^2.7^2.17.101$	353^3	$2^2.3.5.17.59.733$	409^3	$2^2.5.41.83641$
251	$2^2.3^2.7$	307	$2^2.7.11$	359	$2^3.3^2.5$	419	$2^2.3.5.7$
251^2	43.1471	307^2	$3.43.733$	359^2	$7.37.499$	419^2	13.13537
251^3	$2^3.3^2.7.17^2.109$	307^3	$2^3.5^3.7.11.13.29$	359^3	$2^4.3^2.5.13.4957$	419^3	$2^3.3.5.7.41.2141$
257	$2.3.43$	311	$2^3.3.13$	367	$2^4.23$	421	2.211
257^2	61.1087	311^2	19.5107	367^2	$3.13.3463$	421^2	3.59221
257^3	$2^2.3.5^2.43.1321$	311^3	$2^4.3.13.137.353$	367^3	$2^5.5.23.13469$	421^3	$2^2.13.17.211.401$

Num.	Summa Divisorum	Num.	Summa Divisor.	Num.	Summa Divisorum	Num.	Summa Divisorum
431	$2^{4}.3^{3}$	479	$2^{5}.3.5$	547	$2^{2}.137$	601	$2.7.43$
431^{2}	$7.67.397$	479^{2}	43.5347	547^{2}	$3.163.613$	601^{2}	$3.13.9277$
431^{3}	$2^{6}.3^{3}.293.317$	479^{3}	$2^{6}.3.5.89.1289.$	547^{3}	$2^{3}.5.137.29921.$	601^{3}	$2^{2}.7.43.313.577$
433	$2.7.31$	487	$2^{3}.61$	557	$2.3^{2}.31$	607	$2^{5}.19$
433^{2}	$3.37.1693$	487^{2}	$3.7.11317$	557^{2}	$7^{2}.6343$	607^{2}	$3.13.9463$
433^{3}	$2^{2}.5.7.31.18749$	487^{3}	$2^{4}.5.37.61.641$	557^{3}	$2^{2}.3^{2}.5^{2}.17.31.73.$	607^{3}	$2^{6}.5^{2}.19.7369$
439	$2^{3}.5.11$	491	$2^{2}.3.41$	563	$2^{2}.3.47$	613	2.307
439^{2}	$3.31^{2}.67$	491^{2}	37.6529	563^{2}	31.10243	613^{2}	3.125461
439^{3}	$2^{4}.5.11.173.557.$	491^{3}	$2^{2}.3.41.145.809$	563^{3}	$2^{3}.3.5.29.47.1093$	613^{3}	$2^{2}.5.53.307.709$
443	$2^{2}.3.37$	499	$2^{2}.5^{3}$	569	$2.3.5.19$	617	$2.3.103$
443^{2}	7.28099	499^{2}	$3.7.109^{2}$	569^{2}	$7^{2}.6619$	617^{2}	$97.3931.$
443^{3}	$2^{2}.3.5^{4}.37.157$	499^{3}	$2^{3}.5^{3}.13.61.157$	569^{3}	$2^{2}.3.5.19.161881$	617^{3}	$2^{2}.3.5.103.38069$
449	$2.3^{2}.5^{2}$	503	$2^{3}.3^{2}.7$	571	$2^{2}.11.13$	619	$2^{2}.5.31$
449^{2}	97.2083	503^{2}	13.19501	571^{2}	$3.7.103.151$	619^{2}	$3.19.6733$
449^{3}	$2^{2}.3^{2}.5^{2}.100801.$	503^{3}	$2^{4}.3^{2}.5.7.25301.$	571^{3}	$2^{3}.11.13.163021.$	619^{3}	$2^{2}.5.13.31.14737$
457	2.229	509	$2.3.5.17$	577	2.17^{2}	631	$2^{3}.79$
457^{2}	$3.7.9967$	509^{2}	43.6037	577^{2}	$3.19.5851$	631^{2}	$3.307.433$
457^{3}	$2^{2}.5^{2}.229.4177.$	509^{3}	$2^{2}.3.5.17.281.461$	577^{3}	$2^{2}.5.13^{2}.17^{2}.197.$	631^{3}	$2^{4}.79.199081$
461	$2.3.7.11$	521	$2.3^{2}.29$	587	$2^{2}.3.7^{2}$	641	$2.3.107$
461^{2}	373.571	521^{2}	$31^{2}.283$	587^{2}	547.631	641^{2}	7.58789
461^{3}	$2^{2}.3.7.11.106261$	521^{3}	$2^{2}.3^{2}.29.135721.$	587^{3}	$2^{3}.3.5.7^{2}.34457.$	641^{3}	$2^{2}.3.107.205441$
463	$2^{4}.29$	523	$2^{2}.131$	593	$2.3^{3}.11$	643	$2^{2}.7.23$
463^{2}	$3.19.3769$	523^{2}	$3.13.7027$	593^{2}	163.2161	643^{2}	$3.97.1423$
463^{3}	$2^{5}.5.13.17.29.97$	523^{3}	$2^{3}.5.17.131.1609$	593^{3}	$2^{2}.3^{3}.5^{2}.11.13.541$	643^{3}	$2^{3}.5^{2}.7.23.8269$
467	$2^{2}.3^{2}.13$	541	2.271	599	$2^{3}.3.5^{2}$	647	$2^{3}.3^{4}$
467^{2}	19.11503	541^{2}	$3.7.13963$	599^{2}	7.51343	647^{2}	211.1987
467^{3}	$2^{3}.3^{2}.5.13.113.193$	541^{3}	$2^{2}.13.271.11257$	599^{3}	$2^{4}.3.5^{2}.17.61.173$	647^{3}	$2^{4}.3^{4}.5.41.1021$

N[illegible] 6[illegible] 6[illegible] 6[illegible] 6[illegible] 65[illegible] 65[illegible] 65[illegible] 66 65 66[illegible] 673 673 673 677 677 677 683 683^{2} 683^{3} 691 691^{2} 691^{3} 701 701^{2} 701^{3} 709 709^{2} 709^{3}

Num.

Num.	Summa Divisorum	Num.	Summa Divisorum	Num.	Summa Divisor.	Num.	Summa Divisorum
653	2.3.109	719	$2^4.3^2.5$	773	$2.3^2.43$	839	$2^3.3.5.7$
653^2	7.[illegible]	719^2	487.1063	773^2	598303	839^2	704761
659^3	$2^2.3.5.109.42641$	719^3	$2^5.3^2.5.53.4877$	773^3	$2^2.3^2.5.43.59753$	839^3	$2^4.3.5.7.109.3229$
659	$2^2.3.5.11$	727	$2^3.7.13$	787	$2^2.197$	853	2.7.61
659^2	13.33457	727^2	3.176419	787^2	$3.37^2.151$	853^2	3.43.5647
659^3	$2^2.3.5.11.17.53.241$	727^3	$2^4.5.7.13.73109$	787^3	$2^3.5.197.241.257$	853^3	$2^2.5.7.13.29.61.193$
661	2.331	733	2.367	797	2.3.7.19	857	2.3.11.13
661^2	3.145861	733^2	3.19.9439	797^2	157.4051	857^2	735307
661^3	$2^2.331.218461$	733^3	$2^2.5.73.367.4733$	797^3	$2^2.3.5.7.19.63521$	857^3	$2^2.3.5^2.11.13.37.397$
673	2.337	739	$2^2.5.37$	809	$2.3^4.5$	859	$2^2.5.43$
673^2	3.151201	739^2	3.7.26041	809^2	7.13.19.379	859^2	3.246247
673^3	$2^2.5.337.45293$	739^3	$2^3.5.37.27$[illegible]	809^3	$2^2.3^4.5.229.1429$	659^3	$2^3.5.43.137.2693$
677	2.3.113	743	$2^3.3.31$	811	2.7.29	863	$2^5.3^3$
677^2	459007	743^2	552793	811^2	3.31.73.97	863^2	$7^2.15217$
677^3	$2^2.3.5.113.45833$	743^3	$2^4.3.5^2.31.61.181$	911^3	$2^3.7.13.29.41.67$	863^3	$2^6.3^3.5.13.17.337$
683	$2^2.3^2.19$	751	$2^4.47$	821	2.3.137	877	2.439
683^2	7.66739	751^2	3.7.26893	821^2	7.229.421	877^2	3.7.37.991
683^3	$2^2.3^2.5.19.46649$	751^3	$2^4.47.282001$	821^3	$2^2.3.137.337021$	877^3	$2^2.5.439.76913$
691	$2^2.173$	757	2.379	823	$2^3.103$	881	$2.3^2.7^2$
691^2	3.19.8389	757^2	3.13.14713	823^2	3.7.43.751	881^2	19.40897
691^3	$2^2.173.191.1217$	757^3	$2^2.5^2.73.157.379$	823^3	$2^4.5.103.67733$	881^3	$2^2.3^2.7^2.388081$
701	$2.3^3.13$	761	2.3.127	827	$2^2.3^2.23$	883	$2^2.13.17$
701^2	491101	761^2	579883	827^2	684757	883^2	3.260191
701^3	$2^2.3^3.13.17.$[illegible]	761^3	$2^2.3.17.$[illegible]	827^3	$2^3.3^2.5.23.$[illegible]	883^3	$2^3.5.13.17.77969$
709	2.5.71	769	[illegible]	829	2.5.83	887	$2^3.3.37$
709^2	3.7.[illegible]	769^2	[illegible]	829^2	3.211.1087	887^2	13.60589
709^3	[illegible]	769^3	[illegible]	829^3	[illegible]$.5.7^2.29.41.83$	887^3	$2^4.3.5.29.37.2713$

Num.	Summa Diviforum	Num.	Summa Diviforum.
907	$2^2.227$	971	$2^2.3^5$
907^2	$3.7.39217$	971^2	$13.79.919$
907^3	$2^3.5^2.227.16453$	971^3	$2^3.3^5.197.2393$
911	$2^4.3.19$	977	$2.3.163$
911^2	830833	977^2	7.136501
911^3	$2^5.3.19.29.41.349$	977^3	$2^2.3.5.53.163.1801$
919	$2^3.5.23$	983	$2^3.3.41$
919^2	$3.7.13.19.163$	983^2	103.9391
919^3	$2^4.5.23.37.101.113$	983^3	$2^4.3.5.13.41.7433$
929	$2.3.5.31$	991	$2^5.31$
929^2	157.5503	991^2	$3.7.13^2.277$
929^3	$2^2.3.5.31.431521$	991^3	$2^6.31.491041$
937	$2.7.67$	997	2.499
937^2	3.292969	997^2	$3.13.31.823$
937^3	$2^2.5.7.67.87797$	997^3	$2^2.5.499.99401$
941	$2.3.157$		
941^2	811.1093		
941^3	$2^2.3.13.157.34057$		
947	$2^2.3.79$		
947^2	$7.277.463$		
947^3	$2^3.3.5.79.89681$		
953	$2.3^2.53$		
953^2	181.5023		
953^3	$2^2.3^2.5.53.90821$		
967	$2^3.11^2$		
967^2	$3.67.4657$		
967^3	$2^4.5.11^2.19.7193$		

Schu-

Scholion.

§. XIV. Usus hujus tabulæ est amplissimus in quæstionibus circa divisores & partes aliquotas versantibus resolvendis. Ejus enim ope cujusque numeri propositi summa divisorum facili negotio inveniri potest, qua reperta, si inde ipse numerus propositus auferatur, remanebit ejus summa partium aliquotarum. Ex quo statim constat, hujus tabulæ subsidio numeros amicabiles, quos sum traditurus, facile explorari posse, utrum sint justi nec ne? Quemadmodum autem ope hujus tabulæ cujusvis numeri summa divisorum cognosci possit, in sequenti lemmate explicabo.

Lemma. 3.

§. XV. Proposito quocunque numero ejus summa divisorum sequenti modo colligitur.

Cum omnis numerus sit vel primus vel productum ex primis, resolvatur numerus propositus in suos factores primos, & qui inter se fuerint æquales, conjunctim exprimantur. Hoc modo numerus propositus semper ad hujusmodi formam redigetur $m^\alpha . n^\beta . p^\gamma . q^\delta$. &c. existentibus m, n, p, q, &c. numeris primis. Posito ergo numero proposito $= N$ cum sit $N = m^\alpha . n^\beta . p^\gamma . q^\delta$ &c. & factores m^α, n^β, p^γ, q^δ &c. inter se primi; erit $\int N = \int m^\alpha . \int n^\beta . \int p^\gamma . \int q^\delta$. &c. & valores $\int m^\alpha$, $\int n^\beta$, $\int p^\gamma$, $\int q^\delta$ &c. ex tabula adjuncta patebunt.

1. Exempl. *Sit numerus propositus* $N = 360$.

Resoluto hoc numero in suos factores primos erit $N = 2^3 . 3^2 . 5$, ideoque $\int 360 = \int 2^3 \int 3^2 . \int 5 = 3 . 5 . 13 . 2 . 3$, ob $\int 2^3 = 3 . 5$; $\int 3^2 = 13$; $\int 5 = 2 . 3$.

Unde his factoribus ordinatis fiet $\int 360 = 2 . 3^2 . 5 . 13 = 1170$.

2. Exempl. *Explorentur numeri 2620 & 2924 utrum sint amicabiles nec ne?*

Cum sit $2620 = 2^2 . 5 . 131$ & $2924 = 2^2 . 17 . 43$, examen ita instituetur.

Numeri propositi	2620	2924
per factores expressi	$2^2 . 5 . 131$	$2^2 . 17 . 43$
summæ divisorum	7. 6.132	7. 18.44
sive	5544	5544
Summæ partium aliquotarum	2924	2620

Cum igitur summæ partium aliquotarum sint numeris reciproce æquales, patet propositos numeros esse amicabiles.

Scholion.

§. XVI. His igitur præmissis, quæ ad inventionem divisorum cujusque numeri pertinent, ipsum problema de investigatione numerorum amicabilium aggrediar, atque scrutabor, quemadmodum hujusmodi numeros ratione summæ divisorum inter se comparatos esse oporteat, quo deinceps facilius eorum inventio per regulas post tradendas suscipi queat.

Problema generale.

§. XVII. *Invenire numeros amicabiles, hoc est duos numeros hujus indolis, ut alter æqualis sit summæ partium aliquotarum alterius.*

Solutio.

Sint m & n duo hujusmodi numeri amicabiles, & per hypothesin $\int m$ & $\int n$ summæ divisorum eorumdem. Erit numeri m summa partium aliquotarum $= \int m - m$, & numeri n summa partium aliquotarum $= \int n - n$. Hinc ex natura numerorum amicabilium nascentur hæ duæ æquationes:

$\int m$

$$\int m - m = n \ \& \ \int n - n = m$$
$$\text{five} \quad \int m = \int n = m + n.$$

Numeri ergo amicabiles m & n primo habere debent eandem fummam diviforum, tum vero oportet, ut hæc communis diviforum fumma æqualis fit aggregato ipforum numerorum $m+n$.

Coroll. 1.

§. XVIII. Problema ergo huc reducitur, ut quærantur duo ejusmodi numeri, qui habeant eandem diviforum fummam, hæcque æqualis fit aggregato ipforum numerorum.

Coroll. 2.

§. XIX. Ipfa quidem problematis ratio exigit, ut bini numeri quæfiti fint inter fe inæquales: fin autem defiderentur æquales, ut fit $m=n$, fiet $\int n = 2n$ & $\int n - n = n$: hujus fcilicet numeri geminati n fumma partium aliquotarum ipfi fiet æqualis, quæ eft proprietas numeri perfecti. Ergo quilibet numerus perfectus repetitus numeros exhibet amicabiles.

Coroll. 3.

§. XX. Sin autem numeri amicabiles m & n, ut natura quæftionis poftulat, fint inæquales, manifeftem eft, alterum effe redundantem alterum deficientem; fumma fcilicet partium aliquotarum alterius ipfo erit major, alterius vero ipfo minor.

Scholion.

§. XXI. Ex hac quidem generali proprietate parum adjumenti confequimur ad numeros amicabiles inveniendos, eo quod ifta analyfeos fpecies, cujus ope æquationem $\int m = \int n = m+n$ evolvere liceat, etiamnunc penitus fit inculta. Ob quem defectum formulas magis particulares contemplari cogimur, ex quarum indole regulas fpeciales pro inventione numerorum amicabilium derivare

rivare liceat; quorsum etiam pertinet regula Cartesiana a Schotenio commemorata. Ac primo quidem, etiamsi non constet, utrum dentur numeri amicabiles inter se primi nec ne? formulas generales ita restringam, ut numeri amicabiles factorem communem obtineant.

Problema Particulare.

§. XXII. *Invenire indolem numerorum amicabilium, qui communem habeant factorem.*

Solutio.

Sit a communis factor numerorum amicabilium, quorum alter ponatur $= am$, alter $= an$; sint vero tam m & a, quam n & a numeri inter se primi, ut utriusque divisorum summa per praecepta data reperiri queat. Cum igitur primo utriusque eadem esse debeat divisorum summa, fiet $\int a . \int m = \int a . \int n$, ideoque $\int m = \int n$. Deinde vero necesse est ut sit $\int a . \int m$ seu $\int a . \int n$ ipsorum numerorum aequalis aggregato $am + an$, unde habetur $\frac{a}{\int a} = \frac{\int m}{m+n} = \frac{\int n}{m+n}$. Positis ergo numeris amicabilibus am & an, primo esse oportet $\int n = \int n$, tum vero requiritur ut sit $a(m+n) = \int a . \int m$.

Coroll. 1.

§. XXIII. Si ergo pro m & n ejusmodi numeri jam fuerint eruti ut sit $\int m = \int n$: tum numerus a investigari debet, ut sit $\frac{a}{\int a} = \frac{\int m}{m+n}$, seu ex ratione, quam numerus ad summam divisorum suorum tenere debet, ipse numerus a erit investigandus.

Coroll. 2.

§. XXIV. Si factor communis a fuerit datus, quaestio ad inventionem numerorum m & n reducitur, qui prouti vel primi

vel

vel compositi ex duobus pluribusve primis assumuntur, quoniam tum divisorum summae actu exhiberi possunt, regulae speciales ad eos inveniendos tradi poterunt.

Coroll. 3.

§. XXV. Statim autem perspicitur utrumque numerum m & n primum esse non posse: quare casus simplicissimus extat, si alter primus, alter vero productum ex duobus numeris primis assumatur. Tum uterque productum ex duobus, pluribusve numeris primis statui poterit, unde innumerae regulae speciales pro inveniendis numeris amicabilibus derivari poterunt.

Scholion.

§. XXVI. Diversae ergo numerorum amicabilium formae, quae hinc nascuntur, sequenti modo repraesentari poterunt. Sit a utriusque communis factor, & p, q, r, s &c. numeri primi, quorum nullus sit divisor communis factoris a: atque numerorum amicabilium formae erunt:

Forma Prima - - -	$\begin{cases} apq \\ ar \end{cases}$
Forma Secunda - - -	$\begin{cases} apq \\ ars \end{cases}$
Forma Tertia - - -	$\begin{cases} apqr \\ as \end{cases}$
Forma Quarta - - -	$\begin{cases} apqr \\ ast \end{cases}$
Forma Quinta - - -	$\begin{cases} apqr \\ astu \end{cases}$
&c.	

Quanquam numerus harum formarum in infinitum augeri potest, rett, minime tamen hinc concludere licet, in his formis omnes numeros amicabiles contineri. Primum enim, dum hic litterae p, q, r, s, t, &c. numeros primos diversos significant, non verisimile est, nullos dari numeros amicabiles, in quibus non occurrant potestates ejusdem numeri primi. Deinde pariter non constat, utrum non dentur numeri amicabiles, qui vel nullum habeant factorem communem a, vel in quibus facto hic non prorsus sit idem: veluti si darentur numeri amicabiles hujus formae $m^{\alpha} P$ & $m^{\beta} Q$, in quibus exponentes α & β essent diversi; quae forma propterea in superioribus non contineretur, etiamsi P & Q essent producta ex meris numeris primis inter se diversis. Ex his perspicitur quaestionem de numeris amicabilibus latissime patere; eamque ob hoc ipsum tam esse difficilem, ut solutio completa vix sit expectanda. Solutionibus igitur particularibus equidem tantum incumbam, & varias methodos aperiam, quarum ope ex formulis traditis plures numeros amicabiles mihi elicere licuit. Quaelibet autem forma duplicem mihi suspeditavit methodum, prout factor communis a vel datus assumitur, vel ipse quaeritur; basque methodos in sequentibus problematibus exponam.

Problema. 1.

§. XXVII. Invenire numeros amicabiles prima forma apq & ar, si factor communis a sit datus.

Solutio.

Cum p, q, & r sint numeri primi, atque $\int r = \int p . \int q$ seu $r+1 = (p+1)(q+1)$, ponatur $p+1 = x$ & $q+1 = y$, fietque $r = xy - 1$, ideoque x & y ejusmodi esse oportet numeros, ut tam $x-1$, & $y-1$ quam $xy-1$ sint numeri primi. Deinde ut $a(x-1)(y-1)$ & $a(xy-1)$ sint numeri amicabiles, oportet ut eorum aggregatum $a(2xy-x-y)$ aequale sit summae divisorum

rum alterutrius $xy\int a$: unde nanciscimur hanc aequationem $xy\int a = 2axy - ax - ay$ seu $y = \frac{ax}{(2a - \int a)x - a}$. Sit brevitatis gratia $\frac{a}{2a - \int a} = \frac{b}{c}$, & $\frac{b}{c}$ sit valor fractionis $\frac{a}{2a - \int a}$ ad minimos terminos reductae, eritque $y = \frac{bx}{cx - b}$ seu $cy = \frac{bcx}{cx - b} = b + \frac{bb}{cx - b}$, unde habebimus $(cx - b)(cy - b) = bb$. Cum igitur $cx - b$ & $cy - b$ sint factores ipsius bb, quadratum cognitum bb in ejusmodi binos factores resolvi debet, quorum uterque numero b auctus fiat per c divisibilis, & quoti x & y inde emergentes ita sint comparati, ut $x - 1$, $y - 1$, & $xy - 1$ evadant numeri primi. Quae conditio quoties obtineri poterit, quod quidem pro quovis valore ipsius a assumto statim dispicitur, toties obtinebuntur numeri amicabiles, qui erunt $a(x - 1)(y - 1)$ & $a(xy - 1)$ Q. E. I.

Coroll.

§. XXVIII. Prout igitur pro a alii aliique numeri accipiuntur, unde valores b & c innotescant, regulae emergent particulares, quarum ope numeri amicabiles, si qui in eo genere dentur, facile eruentur.

Regula 1.

§. XXIX. Sit factor communis a potestas quaecunque binarii, puta $a = 2^n$ erit $\int a = 2^{n+1} - 1$, ideoque $2a - \int a = 1$, unde erit $\frac{a}{2a - \int a} = 2^n$, & propterea $b = 2^n$ & $c = 1$. Hinc oritur $(x - 2^n)(y - 2^n) = 2^{2n}$.

Quare

Quare cum 2^{2n} alios non habeat factores nisi potestates binarii, erit:

$$x - 2^n = 2^{n+k} \qquad x = 2^{n+k} + 2^n$$
$$y - 2^n = 2^{n-k} \text{ seu } \qquad y = 2^{n-k} + 2^n$$

Quocirca dispiciendum est, an ejusmodi valor pro k detur, ut sequentes tres numeri

$$x - 1 = 2^{n+k} + 2^n - 1$$
$$y - 1 = 2^{n-k} + 2^n - 1$$
$$xy - 1 = 2^{2n+1} + 2^{2n+k} + 2^{2n-k} - 1$$

sint numeri primi. Quod si succedat erunt numeri amicabiles:

$$2^n(2^{n+k} + 2^n - 1)(2^{n-k} + 2^n - 1)$$
$$2^n(2^{2n+1} + 2^{2n+k} + 2^{2n-k} - 1)$$

Vel sit $n - k = m$ seu $n = m + k$, fietque

$$x - 1 = 2^m(2^{2k} + 2^k) - 1 = q$$
$$y - 1 = 2^m(1 + 2^k) - 1 = p$$
$$xy - 1 = 2^{2m}(2^{2k+1} + 2^{3k} + 2^k) - 1 = r$$

qui numeri, quoties fuerint primi, praebebunt numeros amicabiles.

Casus 1.

§. XXX. Sit $k = 1$; & numeri amicabiles obtinebuntur, quoties sequentes tres numeri fuerint primi:

3.2

$3.2^m - 1$; $6.2^m - 1$; & $18.2^{2m} - 1$

Tum enim positis:

$$p = 3.2^m - 1;\ q = 6.2^m - 1;\ \&\ r = 18.2^{2m} - 1$$

numeri amicabiles erunt: $2^{m+1}pq$ & $2^{m+1}r$, ob $n = m + k = m + 1$. Hæcque est regula Cartesii a Schotenio tradita.

Exemplum 1.

§. XXXI. Sit $m = 1$; eritque

$p = 3.2 - 1 = 5$ numerus primus.
$q = 6.2 - 1 = 11$ numerus primus.
$r = 18.4 - 1 = 71$ numerus primus.

hinc ergo oriuntur numeri amicabiles;

$$2^2.5.11 \quad \& \quad 2^2.71$$

Sive 220 & 284, qui sunt minimi omnium, qui exhiberi possunt.

Exempl. 2.

§. XXXII. Sit $m = 2$, eritque $2^m = 4$ & $2^{2m} = 16$ atque

$p = 3.4 - 1 = 11$ numerus primus.
$q = 6.4 - 1 = 23$ numerus primus.
$r = 18.16 - 1 = 287$ numerus non-primus.

hincque adeo nulli numeri amicabiles oriuntur.

Exempl. 3.

§. XXXIII. Sit $m = 3$, eritque $2^m = 8$ & $2^{2m} = 64$ atque

$p = 3.8 - 1 = 23$ primus
$q = 6.8 - 1 = 47$ primus
$r = 18.64 - 1 = 1151$ primus.

Ergo

Ergo hinc numeri amicabiles erunt:

$2^4 . 23 . 47$ & $2^4 . 1151$
five 17296 & 18416.

Exempla feqq.

§. 34. Hæc exempla cum fequentibus, in quibus exponenti m majores valores tribuuntur, commodius uno confpectu ita repræfentari poterunt.

Sit $m =$	1	2	3	4	5	6	7	8
erit $p =$	5	11	23	47	95*	191	383	767*
$q =$	11	23	47	95*	191	383	767*	1535*
$r =$	71	287*	1151	4607*	18431*	73727	294911	1179647

Ubi numeri non-primi afterifcis funt notati: unde hinc tantum terni numeri amicabiles obtinentur, nempe

I. $\begin{cases} 2^2 . 5 . 11 \\ 2^2 . 71 \end{cases}$ II. $\begin{cases} 2^4 . 23 . 47 \\ 2^4 . 1151 \end{cases}$ III. $\begin{cases} 2^7 . 191 . 383 \\ 2^7 . 73727 \end{cases}$

Ulterius autem progredi non licet, quoniam valores ipfius r nimis funt magni, quam ut dignofci poffit, utrum fint primi nec ne? Tabulæ namque numerorum primorum adhuc conftructæ vix ultra 100000 porriguntur.

Cafus. II.

§. XXXV. Sit $k = 2$ & valores litterarum p, q, r, quæ debent effe primi, erunt:

$$p = 5 . 2^m - 1$$

$$q = 20 . 2^m - 1$$

$$r = 100 . 2^{2m} - 1$$

quo-

quorum cum postremus semper sit per ternarium divisibilis, ob $2^{2m} = 3\alpha+1$ & $r = 300\alpha+99$, hinc nulli numeri amicabiles consequuntur.

Casus. III.

§. XXXVI. Ponatur $k = 3$, eritque

$$p = 9 \cdot 2^{m} - 1$$

$$q = 72 \cdot 2^{m} - 1$$

$$r = 648 \cdot 2^{2m} - 1$$

quorum cum nullus necessario videatur divisorem admittere, valores ipsorum p, q, r, ex valoribus simplicioribus ipsius m oriundos hic conjunctim repraesentabo.

$m =$	1	2	3	4	5
$p =$	17	35*	71	143*	287*
$q =$	143*	287*	575*	1151	2303*
$r =$	2591	10367*	41471*	165887	663551

Hinc ergo, quoniam ulterius progredi non licet, nulli numeri amicabiles inveniuntur.

Casus. IV.

§. XXXVII. Ponatur $k = 4$, & sequentes tres numeri debent esse primi.

$$p = 17 \cdot 2^{m} - 1$$

$$q = 272 \cdot 2^{m} - 1$$

$$r = 4624 \cdot 2^{2m} - 1$$

Ubi cum r semper sit multiplum ternarii, patet hinc nullos prodire numeros amicabiles.

Casus V.

§. XXXVIII. Ponatur $k = 5$, & sequentes tres numeri debebunt esse primi.

$$p = 33 \cdot 2^m - 1$$
$$q = 1056 \cdot 2^m - 1$$
$$r = 34848 \cdot 2^{2m} - 1$$

Ubi statim patet casum $m = 1$ esse inutilem, cum det $p = 65$. Sit ergo $m = 2$, fietque

$p = 131$, $q = 4223^*$, $r = 557567$

ubi cum q non sit primus, & majores valores pro m ob defectum tabularum numerorum primorum examini subjici nequeant, neque hinc etiam novi numeri amicabiles eruuntur. At vero ob eandem rationem majores valores ipsi k tribuere non licet.

Scholion.

§. XXXIX. Quoniam potestates binarii pro a positae valorem ipsius c in fractione $\frac{b}{c} = \frac{a}{2a - \int a}$ unitati aequalem reddiderunt, hincque solutiones obtinere licuit, alios valores pro a, qui pariter ipsi c valorem $= 1$ inducant, ponam. Inter hos autem imprimis sunt notandi, qui ex hac forma $a = 2^n(2^{n+1} + e)$ nascuntur, si quidem $2^{n+1} + e$ sit numerus primus, tum enim fit $2a -$

$\int a$

$fa = e+1$, & $\frac{b}{c} = \frac{2^n(2^{n+1}+e)}{e+1}$: si igitur $e+1$ sit divisor numeratoris $2^n(2^{n+1}+e)$ valor ipsius c fiet itidem $=1$.

Regula. II.

§. XL. Sit factor communis $a = 2^n(2^{n+1}+2^k-1)$, at $2^{n+1}+2^k-1$ numerus primus, erit ob $e+1=2^k$, fractio $\frac{c}{b} = \frac{2^n(2^{n+1}+2^k-1)}{2^k} = 2^{n-k}(2^{n+1}+2^k-1)$, si quidem non sit $k>n$. Hac ergo hypothesi habebimus $b=2^{n-k}(2^{n+1}+2^k-1)$ & $c=1$. Quadratum ergo bb in duos ejusmodi factores $(x-b)(y-b)$ resolvendum est, ex quibus non solum valores numerorum $x-1=p$ & $y-1=q$, sed etiam $xy-1=r$ fiant numeri primi. Cujusmodi casus si eruere liceat, erunt numeri amicabiles apq & ar. Verum hic notandum est eos casus rejiciendos esse, in quibus aliquis numerorum primorum p, q, r prodit divisor ipsius a, seu aequalis $2^{n+1}+2^k-1$, quia a per nullum alium numerum primum est divisibile.

Sit $n-k=m$, seu $n=m+k$, erit $a=2^{m+k}(2^{m+k+1}+2^k-1)$ & $b=2^m(2^{m+k+1}+2^k-1.)$ Jam quia $2^{m+k+1}+2^k-1$ debet esse numerus primus, ponatur $2^{m+k+1}+2^k-1=f$ seu $f=$

$2^k(2^{m+1}+1)-1$, ut fit $a = 2^{m+k}f$ & $b = 2^m f$; erit $bb = 2^{2m}ff = (x-b)(y-b)$. Nunc ob f numerum primum, numerus $2^{2m}ff$ duplici modo in genere in duos factores resolvetur. Priori modo fiet $(x-b)(y-b) = 2^{m-\alpha}f \cdot 2^{m+\alpha}f$, ideoque

$$x = 2^{m-\alpha}f + 2^m f; \qquad p = (2^{m-\alpha} + 2^m)f - 1$$

$$y = 2^{m+\alpha}f + 2^m f; \qquad q = (2^{m+\alpha} + 2^m)f - 1$$

$$\& \; r = (2^{2m+1} + 2^{2m+\alpha} + 2^{2m-\alpha})ff - 1$$

qui tres numeri p, q, r debent esse primi. Posteriori modo resolutio fiet ita:

$$(x-b)(y-b) = 2^{m\pm\alpha} \cdot 2^{m\mp\alpha}ff, \text{ unde fit}$$

$$x = 2^{m\pm\alpha} + 2^m f; \qquad p = 2^{m\pm\alpha} + 2^m f - 1$$

$$y = 2^{m\mp\alpha}ff + 2^m f; \qquad q = (2^{m\mp\alpha}f + 2^m)f - 1$$

$$\& \; r = (2^{2m+1}f + 2^{2m\pm\alpha} + 2^{2m\mp\alpha}ff)f - 1$$

& quoties p, q, r hoc modo prodeunt numeri primi, inde oriuntur numeri amicabiles apq & ar.

Casus. 1.

§. XLI. Sit $k = 1$, erit $a = 2^{m+1}(2^{m+2}+1)$, $b =$

$2^m(2^{m+2}+1)$ atque $f = 2^{m+2}+1$, qui numerus debet esse primus. Cum ergo sit $(x-b)(y-b) = 2^{2m}ff$, erit

vel	vel
$p = (2^{m-n}+2^m)f-1$	$p = 2^{m\pm n}+2^m f-1$
$q = (2^{m+n}+2^m)f-1$	$q = (2^{m+n}f+2^m)f-1$
$r = (2^{2m+1}+2^{2m+n}+2^{2m-n})ff-1$	$r = (2^{2m+1}f+2^{2m\pm n}+2^{2m+n}ff)f-1$

Notandum autem est, ut $2^{m+2}+1$ sit numerus primus, exponentem $m+2$ esse oportere potestatem binarii: valores ergo ipsius m erunt: 0, 2, 6, 14, &c. At casus $m=0$ rejici debet, ob nullum valorem ipsius n assignabilem.

Exemplum. 1.

§. XLII. Sit ergo $m=2$, ut sit $a = 8.17$ & $b = 4.17 = 68$ atque $f = 17$. Cum igitur esse debeat $(x-b)(y-b) = 4^2.17$, erit resolutione in factores instituenda:

$x-68=$	2	4	8	34
$y-68=$	8.17^2	1156	578	136
$x=$	70	72	76	102
$y=$	2380	1224	646	204
$p=$	69*	71	75*	101
$q=$	2379*	1223	645*	203*
$r=$	166599*	88127*	49095*	20807

Hinc ergo nulli numeri amicabiles obtinentur.

Exem-

Exemplum. 2.

§. XLIII. Sit $m = 6$, ut $a = 2^7 . 257$; $b = 2^6 . 257$ & $f = 257$. Cum igitur sit

$$(x - b)(y - b) = 2^{12} . 257^2$$

Resolutio ita institui debebit:

$x - 16448 =$	$32 . 257$
$y - 16448 =$	$128 . 257$
$x =$	24672
$y =$	49344
$p =$	24671
$q =$	49343^*
$r =$	- - - -

Valores ex reliquis factoribus oriundi adhuc magis fiunt magni quam ut, an primi sint nec ne, judicari possit.

Casus reliqui.

§. XLIV. Cum $f = 2^{m+k+1} + 2^k - 1$ debeat esse numerus primus, quaeramus primo casus simpliciores, quibus hoc evenit, cum casus nimis compositos evolvere non liceat. Sit ergo $k = 2$, & ob $f = 2^{m+3} + 3$, valores idonei pro m erunt: 1, 3, 4: sit $k = 3$, erit $f = 2^{m+4} + 7$, & valores idonei pro m erunt 1, 4, 8. Casu $k = 4$, est $f = 2^{m+5} + 15$, & m erit 1, vel 3 neque ulterius progredi licet.

Exempl. 1.

§. XLV. Ponamus ergo $k = 2$, & $m = 1$, erit $f = 19$ & $a = 8 . 19$ atque $b = 2 . 19 = 38$, unde fiet

(x-

$$(x-38)(y-38) = 2^2 \cdot 19^2 = 1444$$

& resolutio dabit.

$x - 38 =$	2	4	Neuter scilicet factor assumi potest impar
$y - 38 =$	722	361	
$x =$	40	..	
$y =$	760	imp:	
$p =$	39*		

Quia hic jam p non est primus, patet hinc nullos numeros amicabiles resultare.

Exempl. 2.

§. XLVI. Ponamus $k = 2$ & $m = 3$, ut sit $f = 67$ erit $a = 32 \cdot 67$ & $b = 8 \cdot 67 = 536$: und fit

$$(x-536)(y-536) = 2^6 \cdot 67^2$$

$x - 536$	268	16	reliqui valores pro p præbent numeros per 3 divisibiles, quos propterea omisi. Sequentia exempla ad nimis magnos numeros deducunt.
$y - 536$	1072	17956	
$x =$	804	552	
$y =$	1608	...	
$p =$	803*	551*	
$q =$	1607	...	

Regula. III.

§. XLVII. Sit ut ante $a = 2^n(2^{n+1} + 2^k - 1)$ & $2^{n+1} + 2^k - 1 = f$ numerus primus, et in fractione $\frac{b}{c} = \frac{2^n(2^{n+1} + 2^k - 1)}{2^k}$ sit $k > n$; eritque $b = 2^{n+1} + 2^k - 1$ & $c = 2^{k-n}$. Ponamus $k - n = m$, ut sit $k = m + n$, erit $a = 2^n(2^{n+1} + 2^{m+n} - 1)$; $b =$

$b = 2^{n+1} + 2^{m+n} - 1 = f$ & $c = 2^{2n}$; unde hæc habebitur æquatio

$$(2^m x - b)(2^m y - b) = bb.$$

Cum autem $b = f$ sit numerus primus, alia resolutio locum non invenit præter $1.\,bb$: ex qua fit

$$x = \frac{1+b}{2^m} \text{ \& } y = \frac{b(1+b)}{2^m} \text{ sive}$$

$$x = 2^n + 2^{n+1-m} \text{ \& } y = (2^{n+1} + 2^{m+n} - 1)(2^n + 2^{n+1-m})$$

Jam notandum est hos quatuor numeros esse oportere primos:

$$f = 2^{n+1} + 2^{m+n} - 1$$

$$p = x - 1;\ q = y - 1;\ \&\ r = xy - 1$$

atque necesse est ut sit $m < n+1$. Quibus conditionibus si satisfiat, erunt numeri amicabiles: apq & ar.

Casus. 1.

§. XLVIII. Sit $m = 1$, erit $f = 2^{n+2} - 1$; $x = 2^{n+1}$, & $p = 2^{n+1} - 1$, fieri autem nequit, ut simul & f & p sit numerus primus, nisi casu $n = 1$, quo vero fit $q = 27$. Ergo ex hypothesi $m = 1$ nulli oriuntur numeri amicabiles.

Casus. 2.

§. XLIX. Sit ergo $m = 2$, ut sit $f = 3.2^{n+1} - 1$; $x = 3.2^{n-1}$ & $y = 3.2^{n-1}(3.2^{n+1} - 1)$, atque $a = 2^n.f$.

Sequen-

Sequentes ergo quatuor numeri debent esse primi:

$f = 3 \cdot 2^{n+1} - 1$; $p = 3 \cdot 2^{n-1} - 1$; $q = 3 \cdot 2^{n-1} (3 \cdot 2^{n+1} - 1) - 1$ & $r = 9 \cdot 2^{2n-2} (3 \cdot 2^{n+1} - 1) - 1$, unde formantur hæc exempla;

$n =$	1	2	3	4	5
$f =$	11	23	47	95*	191
$p =$	2	5	11	- -	47
$q =$	32*	137	563	- -	9167*
$r =$	98*	827	6767*	- -	
		valet			

hincque ergo ex $n = 2$, & $a = 4 \cdot 23$ nascuntur numeri amicabiles:

$$\begin{cases} 4 \cdot 23 \cdot 5 \cdot 137 \\ 4 \cdot 23 \cdot 827 \end{cases}$$

Casus ceteri.

§. L. Si $m = 3$, iterum vel f vel p fit divisibile per 3, quod idem evenit si $m = 5$, vel 7, &c. Sit ergo $m = 4$; erit $f = 9 \cdot 2^{n+1} - 1$; $x = 9 \cdot 2^{n-3}$ & $y = 9 \cdot 2^{n-3} (9 \cdot 2^{n+1} - 1)$ & $a = 2^n \cdot f$, unde formantur hæc exempla:

$n =$	1	4	5	6
$f =$	35*	287*	575*	1151
$x =$	-	- - -	- - -	72
$y =$	-	- - -	- - -	82872
$p =$	-	- - -	- - -	71
$q =$	-	- - -	- - -	82871*
$r =$	-	- - -	- - -	- - -

Neque ergo hinc neque ex majoribus valoribus ipsi m tribuendis numeros amicabiles elicere licet.

Regula. IV.

§. LI. Possunt etiam aliae expressiones pro factore communi a inveniri, ex quibus fractionis $\frac{b}{c}$ denominator c vel unitati, vel potestati binarii fiat aequalis. Fingamus namque $a = 2^n (g-1)(h-1)$, ut sint $g-1$ & $h-1$ numeri primi; erit $\int a = (2^{n+1} - 1) gh = 2^{n+1} gh - gh$; at est $2a = 2^{n+1} gh - 2^{n+1} g - 2^{n+1} h + 2^{n+1}$ unde fit

$$2a - \int a = gh - 2^{n+1} g - 2^{n+1} h + 2^{n+1}$$

Ponatur $2a - \int a = d$, erit $gh - 2^{n+1}(g+h) + 2^{n+1} = d$ & $(g - 2^{n+1})(h - 2^{n+1}) = d - 2^{n+1} + 2^{2n+2}$: unde per resolutionem in factores ejusmodi valores pro g & h elici debent, ut $g-1$ & $h-1$ fiant numeri primi, eritque tum $a = 2^n (g-1)(h-1)$ & $\frac{b}{c} = \frac{a}{d}$.

I. Ponamus $n = 1$, erit $(g-4)(h-4) = d + 12$, ubi ut $d + 12$ duos obtineat factores pares, sequentes prodibunt valores:

Sit $d = 4$; erit $(g-4)(h-4) = 16 = 2.8$, unde $g = 6$, $h = 12$; $a = 2.5.11$ atque $\frac{b}{c} = \frac{2.5.11}{4}$ ergo $b = 5.11$ & $c = 2$.

Sit