DETERMINATIO CALORIS ET FRIGORIS GRADVVM PRO SINGVLIS TERRAE LOCIS AC TEMPORIBVS.

§. 1.

Tab. I. Hoc loco in caloris et frigoris gradus a priori inquirere constitui, quatenus a sola actione solis proficiscuntur: et hanc ob rem neque ad ventos, neque ad tempestatum diuersitatem, quibus actio solis vehementer turbatur, respicio. In hoc itaque potissimum ero occupatus, vt remotis istis impedimentis pro quouis terrae loco et tempore gradum caloris frigorisue definiam a sola actione solis oriundum. Cum igitur ad hoc expediendum necesse sit gradum caloris quendam fixum et constantem adhibere, huius loco eum assumo, qui in ipsa solis superficie perpetuo viget, quemque constanter littera c designabo.

§. 2. Ad effectum autem solis in calore producendo definiendum concipio corpus plana superficie praeditum perpetuo soli ita expositum esse, vt radii solares normaliter in eius superficiem planam incidant. Sensim igitur istud corpus a sole calefiet, atque continuo maiorem caloris gradum recipiet, donec tandem summum gradum acquisierit, quem si semel fuerit consecutum, eum in aeternum sit conseruaturum. Gradum istum caloris appellabo vltimum siue naturalem caloris gradum, quem his circumstantiis exerere valet.

§. 3. Cum igitur calor a radiis solis proficiscatur hique diuergant ita, vt in duplicata ratione distantiarum a sole fiant rariores; etiam vltimus caloris gradus, quem sol

sol superficiei planae directe oppositae inducere potest, tenebit rationem reciprocam duplicatam distantiarum a sole. Quare si distantia huius corporis a sole ponatur $= s$, et semidiameter solis $= 1$, erit vltimus caloris gradus $= \frac{c}{ss}$; quoniam, si corpus in ipsa solis superficie seu in distantia 1 ab eius centro esset positum, eundem caloris gradum acciperet, qui in superficie solis existit.

§ 4. At si radii solis oblique in superficiem corporis cadant, tum vtique tantus calor non generabitur, quam praecedente casu, quo radii solis normaliter in superficiem incidebant. Primo quidem satis perspicuum est, si solis radii tantum lambant superficiem, seu si angulus incidentiae radiorum euanescat, tum effectum omnino nullum a sole produci posse. Ex quo satis tuto assumi posse videtur, gradum caloris vltimum, quem superficies recipit esse sinui anguli incidentiae proportionalem; quando quidem superficies illuminatur. Nam si sol infra horizontem versetur, tum sinus incidentiae fieret negatiuus non autem videtur, effectum solis vnquam in negatiuum abire posse. Quare, si sol sub horizonte existit, eius effectus aliter spectari nequit, ac si ipsum horizontem occuparet, hoc est vltimus gradus caloris semper erit $= 0$, quantumuis profunde sol sub horizonte sit submersus.

§. 5. Si ergo sumatur in terra locus quicunque, definiri poterit gradus caloris, quem radii solis in data obliquitate incidentes tandem inducerent, si quidem sol perpetuo eundem situm respectu istius loci retineret. Namque sit altitudinis solis supra horizontem illius loci sinus $= v$, posito sinu toto $= 1$, erit vltimus caloris gradus, cuius hic locus est capax $= \frac{cv}{ss}$, denotante semper s di-

 stantiam

ſtantiam ſolis a terra, exiſtente ſemidiametro ſolis $= 1$; ſiue etiam ſpectari poteſt s tanquam cotangens ſemidiametri ſolis apparentis. Vel ſi tangens ſemidiametri ſolis apparentis ponatur $= \varkappa$, erit vltimus gradus caloris $= \epsilon \varkappa^2 v$: ſi quidem ſol verſetur ſupra horizontem: at ſi ſol ſub horizonte lateat, gradus iſte erit $= 0$, qui eſt extremus ſummusque frigoris gradus.

§ 6. Hoc igitur modo vbique terrarum calor foret comparatus, ſi ſol reſpectu terrae quieſceret, ac perpetuo eundem ſitum retineret. Scilicet in iis regionibus in quibus ſol appareret, foret aliquis caloris gradus, isque eo maior, quo magis ſol fuerit eleuatus. In altero autem hemisphaerio, a ſole auerſo, ſummum perpetuo regnaret frigus, niſi quatenus calor partis oppoſitae influeret. Hic vero ab iſtiusmodi circumſtantiis cogitationes prorſus abſtineo, neque alium caloris fontem praeter ſolem, conſidero. Hancque ob rem omnem materiam terreſtrem cuiusvis gradus caloris et frigoris aeque ſuſceptibilem pono, ita vt quouis tempore in eum ſtatum conſtituatur, quem regulae requirunt.

§. 7. Si igitur terrae regio iam illum ipſum habeat caloris gradum, quem ſol ipſi communicare conetur, quemque actu tandem induceret, niſi eum iam haberet; tum nulla eueniet mutatio caloris in ea regione, ſed ille ipſe gradus conſeruabitur. At ſi praeſens regionis calor vel maior ſit vel minor, quam calor naturalis ſitui ſolis reſpondens, tum paullatim ille calor vel diminuetur vel augebitur, donec tandem calori ſolis naturali aequalis fiat. Veriſimile autem videtur, caloris illius vel augmenta vel decrementa, quae dato tempore gignuntur, differentiae

calorum

calorum ſolis ſcilicet naturalis et proprii, quem corpus habet, eſſe proportionalia. Cum enim aequalitas amborum caloris graduum tanquam finis ſit propoſita, eo fortior ad eam obtinendam erit actio, quo maior fuerit inaequalitas.

§. 8. Si ergo ponamus in regione terrae quadam caloris praeſentis gradum eſſe z; atque ſolem in altitudine ſupra horizontem verſari, cuius ſinus ſit $= v$; erit calor naturalis a ſole oriundus $= c\varkappa^2 v$, denotante c gradum caloris in ſuperficie ſolis et $\varkappa$ tangentem ſemidiametri apparentis ſolis. Tempusculo igitur dt calor regionis z incrementum accipiet proportionale exceſſui $c\varkappa^2 v - z$, ſiquidem fuerit $c\varkappa^2 v > z$, contrario enim caſu calor z decrementum patietur. Hinc ergo erit $dz = \alpha dt (c\varkappa^2 v - z)$: ac ſi ſol perpetuo iſtum ſitum obtineret, foret integrando $\alpha t = l \frac{C}{c\varkappa^2 v - z} = l \frac{c\varkappa^2 v - f}{c\varkappa^2 v - z}$, ſi f denotet gradum caloris, qui in ea regione fuit, principio a quo tempus t computatur.

§. 9. His nunc praemiſſis hypotheſibus pro quouis terrae loco gradum caloris diſfinire conabor ad quamlibet horam dati diei. Sit igitur HOR horizon loci propoſiti, Z zenith, P polus mundi borealis, et AOB parallelus, in quo ſol die propoſito mouetur. Sit eleuationis poli PR ſinus $=$ P coſinus $= p$, poſito ſinu toto $= 1$; ſinus declinationis borealis ſolis $=$ Q, coſinus $= q$, ſinusque Q abibit in ſui negatiuum, ſi declinatio ſolis ſit auſtralis. Ponamus ſolem in S verſari, angulumque APS eſſe $= t$, qui angulus exprimit tempus, quo ſol poſt tranſitum per meridianum ex A in S peruenit; anTab. I. fig. 2.

gulique APS ſinus ſit $= x$, coſinus $= y$; erit $dt = \frac{dx}{y} = \frac{-dy}{x}$ ob $xx + yy = 1$

§. 10. Quoniam nunc in triangulo ſphaerico PZS dantur primo angulus ZPS, cuius ſinus eſt x et coſinus y; deinde latus PZ, cuius ſinus eſt p, coſinus P; et tertio latus PS cuius ſinus eſt q, coſinus Q, reperietur lateris ZS coſinus $= pqy + PQ$; qui ſimul eſt ſinus altitudinis ſolis ſuper horizonte, dum in S verſatur. Solis igitur occaſus continget in puncto O, exiſtente anguli APO coſinu $= \frac{-PQ}{pq}$. Ponamus autem eſſe angulum ipſum APO $= g$, quo ſemiſſis temporis diurni deſignabitur atque poſito angulo 180. grad. $= \pi$, erit $\pi - g$ tempus dimidiae noctis.

§. 11. Inquiramus nunc primum in varietatem caloris regionis propoſitae, quamdiu ſol ſupra horizontem verſatur, ſitque tempore t poſt meridiem, quo ſol in S reperitur, gradus caloris in loco propoſito $= z$. Quoniam vero hoc tempore eſt ſinus altitudinis ſolis $= pqy + PQ$, erit calor ſolis naturalis $= c\varkappa^2 (pqy + PQ)$. Quamobrem tempuſculo dt, quod per angulum SPY repraeſentatur, calore z incrementum capiet dz tantum, vt ſit $dz = \alpha dt\, (c\varkappa^2 pqy + c\varkappa^2 PQ - z)$, in qua aequatione α quantitatem quandam conſtantem denotat, per obſeruationes determinandam.

§. 12. Aequatio differentialis inuenta $dz = \alpha dt\, (c\varkappa^2 pqy + c\varkappa^2 PQ - z)$ reducatur ad hanc formam $dz + \alpha z dt = \alpha c \varkappa^2 dt\, (pqy + PQ)$ quae multiplicata per $e^{\alpha t}$ denotante e numerum, cuius logarithmus hyperbolicus eſt $= 1$, fiet integrabilis: erit enim integrale $e^{\alpha t} z =$

$\alpha c \varkappa^2$

$\alpha c \varkappa^2 \int e^{\alpha t} dt (pqy + PQ) = c\varkappa^2 PQ e^{\alpha t} + \alpha c \varkappa^2 pq \int e^{\alpha t} dx$ ob $y dt = dx$. Cum autem sit $t = \int \frac{dx}{\sqrt{(1-xx)}}$ erit $\int e^{\alpha t} dx = \int e^{\int \frac{\alpha dx}{\sqrt{(1-xx)}}} dx = \frac{e^{\alpha t}(x + \alpha \sqrt{(1-xx)})}{1+\alpha\alpha}$. Addita ergo indefinita constante habebitur $e^{\alpha t} z = C c\varkappa^2 pq + c\varkappa^2 PQ e^{\alpha t} + \frac{\alpha c \varkappa^2 pq e^{\alpha t}(x + \alpha y)}{1 + \alpha\alpha}$, atque calor quaesitus $z = e^{-\alpha t} C c \varkappa^2 pq + c\varkappa^2 PQ + \frac{\alpha c \varkappa^2 pq(x+\alpha y)}{1+\alpha\alpha}$.

§. 13. Hinc igitur reperietur calor meridianus loci propositi faciendo $t = 0$, quo facto erit $x = 0$, et $y = 1$. Quare tempore meridiei erit calor $= C c\varkappa^2 pq + c\varkappa^2 PQ + \frac{\alpha^2 c \varkappa^2 pq}{1+\alpha\alpha}$. Si detur calor meridianus, isque ponatur $= f$, determinabitur inde constans quantitas C, quae per integrationem est ingressa, eritque $C c\varkappa^2 pq = f - c\varkappa^2 PQ - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}$; vnde ex dato calore meridiano definietur calor pro quauis hora eiusdem diei, ita vt sole in S existente post meridiem sit calor regionis propositae $z = e^{-\alpha t} f + c \varkappa^2 PQ(1 - e^{-\alpha t}) + \frac{\alpha c\varkappa^2 pq(x+\alpha y)}{1+\alpha\alpha} - \frac{\alpha^2 c \varkappa^2 pq e^{-\alpha t}}{1+\alpha\alpha}$.

§. 14. Inuestigemus nunc gradum caloris, qui fuit eodem die, cum sol est ortus; poni debebit $t = -g$, eritque pariter ac in occasu $y = \frac{-PQ}{pq}$ at $x = -\frac{\sqrt{(p^2q^2 - P^2Q^2)}}{pq}$, ob angulos ante meridiem negatiuos. Ex quibus prodibit calor, quando sol oritur $= e^{\alpha g} f - c\varkappa^2 PQ (e^{\alpha g} - 1) - \frac{\alpha c \varkappa^2(\alpha PQ + \sqrt{(p^2q^2 - P^2Q^2)})}{1+\alpha\alpha} - \frac{\alpha^2 c\varkappa^2 pq e^{\alpha g}}{1+\alpha\alpha} = e^{\alpha g}(f - c\varkappa^2 PQ - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) + \frac{c\varkappa^2 PQ - \alpha c \varkappa^2 \sqrt{(p^2q^2 - P^2Q^2)}}{1+\alpha\alpha}$. At si anguli g sinus ponatur $= m$, cosinus $= n$; erit calor tempore ortus solis $= e^{\alpha g}(f + c\varkappa^2 npq - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) - \frac{c\varkappa^2 npq - \alpha c\varkappa^2 mpq}{1+\alpha\alpha}$.

§. 15.

§. 15. Perueniet nunc ſol ad occaſum in O fietque $t = g$; $y = n = \frac{-PQ}{pq}$ et $x = m = \frac{\sqrt{(p^2q^2 - P^2Q^2)}}{pq}$, ideoque $PQ = -npq$. Hinc igitur tempore occaſus ſolis prodibit calor loci propoſiti $= e^{-\alpha g}(f + c\varkappa^2 npq - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) - \frac{c\varkappa^2 npq + \alpha c\varkappa^2 mpq}{1+\alpha\alpha}$. Maximus autem hoc die calor erit tum, quando fit $z = c\varkappa^2 pqy + c\varkappa^2 PQ$; quippe quo tempore erit $dz = 0$. Subſtituto autem hoc valore in aequatione inuenta prodibit $e^{-\alpha t}(f - c\varkappa^2 PQ \frac{-\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) = \frac{c\varkappa^2 pq(y - \alpha x)}{1+\alpha\alpha}$; ex qua valor anguli t erutus et in tempus conuerſus praebebit momentum maximi caloris a meridie computando.

§. 16. Cum igitur calor tempore ſolis occaſus ſit repertus, quem ponamus breuitatis gratia $= h$, quantum iſte calor ſequente nocte diminuatur, inueſtigemus. Verſetur itaque ſol poſt occaſum in V, et ponatur angulus $OPV = t$, et calor tum ſit $= z$, dum ergo ſol per arculum Vv progreditur erit decrementum caloris $dz = -\alpha z\, dt$, hincque $l\frac{h}{z} = \alpha t$; ex quo fiet $z = e^{-\alpha t}h$. Sequente igitur die, cum ſol iterum orietur, prodibit calor $= e^{-2\alpha(\pi - g)}h$. Per totam ergo noctem calor diminuetur, vnde quauis nocte frigus ſummum erit, quando ſol oritur.

§. 17. Cum igitur primo die tempore ortus ſolis gradus caloris fuiſſet $= e^{\alpha g}(f + c\varkappa^2 npq - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) - \frac{c\varkappa^2 pq(n + \alpha m)}{1+\alpha\alpha}$; die ſequente calor tempore ortus ſolis erit $= e^{\alpha g - 2\alpha\pi}(f + c\varkappa^2 npq - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) - \frac{e^{-2\alpha(\pi - g)} c\varkappa^2 pq(n - \alpha m)}{1+\alpha\alpha}$; ſi quidem ſol ponatur interea eandem declinationem conſeruare. At ſi hoc eodem ſequente die calor tempore meridiei ponatur Φ, debebit calor tempore ortus eiusdem diei eſſe $= e^{\alpha g}(\Phi + c\varkappa^2 npq - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) - \frac{c\varkappa^2 pq(n + \alpha m)}{1+\alpha\alpha}$ vnde prodibit

$\Phi +$

$$\Phi + c\varkappa^2 npq - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha} - \frac{e^{-\alpha g} c\varkappa^2 pa(n+\alpha m)}{1+\alpha\alpha} = e^{-2\alpha\pi}$$
$$(f + c\varkappa^2 npq - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) - \frac{e^{\alpha g - 2\alpha\pi} c\varkappa^2 pq(n-\alpha m)}{1+\alpha\alpha}$$

§. 18. Si ſol perpetuo eandem declinationem conſeruaret, tum quoque perpetuo eadem diei hora idem gradus caloris haberetur. Hoc igitur caſu foret $\Phi = f$, atque calor tempore ortus ſolis primo die aequalis calori ſequentis diei eodem tempore. Hinc igitur erit $e^{\alpha g}(1 - e^{-2\alpha\pi})(f + c\varkappa^2 npq - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) = \frac{c\varkappa^2 pq(n + \alpha m - e^{-2\alpha(\pi - g)} n\, \alpha m))}{1+\alpha\alpha}$ atque calor meridianus, qui erit conſtans, definietur $f = \frac{c\varkappa^2 pq(e^{-\alpha g}(n+\alpha m) - e^{\alpha g - 2\alpha\pi}(n-\alpha m))}{(1-e^{-2\alpha\pi})(1+\alpha\alpha)} - c\varkappa^2 npq + \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}$. Sub aequatore ergo, ob $P = 0$, et $p = 1$, itemque $m = 1$ et $n = 0$, hincque $g = \frac{\pi}{2}$ foret conſtans calor meridianus $f = \frac{\alpha c\varkappa^2 q e^{-\frac{\alpha\pi}{2}}}{(1+\alpha^2)(1-e^{-\alpha\pi})} + \frac{\alpha^2 c\varkappa^2 q}{1+\alpha\alpha}$.

§. 19. Formulae iſtae tantopere ſunt compoſitae et perplexae; vt ex iis vix quicquam concludi queat. Huius autem incommodi ratio in eo eſt poſita, quod amplitudo ortiua et occidua in eas ingrediatur, atque lex continuitatis ſit interrupta: quoniam pro nocte alio ſum vſus calculo alio pro die. Quamobrem vt aliquid ad vtilitatem deriuare queamus, neceſſe eſt aliquantum a veri ſimilitudine recedere, atque continuam legem aſſumere, ſecundum quam gradus caloris mutentur. Ita cum calores ſolis ſupra horizontem exiſtentis ſint ſinibus altitudinum proportionales; eadem lege oportebit calorem ſolis naturalem ſub horizonte latentis negatiuum ſtatuere, ac ſinui depreſſionis ſub horizonte proportionalem.

§. 20. Hac autem admissa hypothesi facile perspicitur conclusiones inde deducendas veritati minus fore consentaneas, eo quod soli hoc pacto, quando sub horizonte versatur, non solum omnis effectus ad calorem generandum adimatur, sed etiam vis contraria frigorifica tribuatur. Qua quidem in re experientia aduersari videtur, cum sol sub horizonte calori praesenti plus nocere non possit, quam si prorsus ab esset; at dum ipsum horizontem occupat, iam perinde est, ac si prorsus foret sublatus. Quicquid autem sit, conueniet ex hac hypothesi consequentias formare, quae nihilominus aliquam vtilitatem habere poterunt, cum iam ante pateat, qua in parte conclusiones ab experientia dissentire debeant.

§. 21. Quoniam hoc pacto continuitatis lex obseruatur, ex aequatione supra inuenta $z = e^{-\alpha t}(f - c\varkappa^2 PQ - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) + c\varkappa^2 PQ + \frac{\alpha c\varkappa^2 pq(x+\alpha y)}{1+\alpha^2}$, quae praebet calorem post meridiem tempore t, existente calore meridiano $= f$, etiam gradus caloris nocturni poterunt determinari. Ita sequente media nocte ob $t = \pi$ et $x = 0$, $y = -1$, erit gradus caloris $= e^{-\alpha\pi}(f - c\varkappa^2 PQ - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) + c\varkappa^2 PQ - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}$. Sequentis autem diei meridie erit calor $= e^{-2\alpha\pi}(f - c\varkappa^2 PQ - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) + c\varkappa^2 PQ + \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}$, ob $t = 2\pi$ et $x = 0$, $y = 1$.

§. 22. Incrementum ergo caloris, a meridie primi diei ad meridiem sequentem erit $= (1 - e^{-2\alpha\pi})(c\varkappa^2 PQ + \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha} - f)$. Si ergo sol perpetuo eandem declinationem retineret, foret etiam quotidie eadem semper caloris conditio, atque meridianus calor constans. Hoc itaque casu erit calor meridianus $f = c\varkappa^2(PQ + \frac{\alpha^2 pq}{1+\alpha\alpha})$. Singulis ergo

diebus

diebus, cum maximus calor incidat, quando eſt $e^{-\alpha t}(f - c\varkappa^2 PQ - \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha}) = \frac{c\varkappa^2 pq(y-\alpha x)}{1+\alpha\alpha}$ incidet noſtro caſu maximus calor poſt meridiem, quando fit $y = \alpha x$ ſeu $\frac{x}{y} = \frac{1}{\alpha}$: atque ipſe calor maximus erit $= c\varkappa^2(PQ + \frac{\alpha pq}{\sqrt{(1+\alpha\alpha)}}$; hoc eſt quando ſol iam tantum meridianum eſt transgreſſus, vt ſit anguli APS tangens $= \frac{1}{\alpha}$. Quare cum maximus calor circiter in horam tertiam pomeridianam incidere obſeruetur, idque tum quando aeſtas maxime eſt aequabilis, erit proxime $\alpha = 1$.

§. 23. Cum igitur, ſole eandem declinationem conſeruante, incrementum caloris meridiani vno die ſit $= (1 - e^{-2\alpha\pi})(c\varkappa^2 PQ + \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha} - f)$, erit incrementum duobus diebus acquiſitum $= (1 - e^{-4\alpha\pi})(c\varkappa^2 PQ + \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha} - f)$, atque generaliter incrementum n diebus acquiſitum erit $= (1 - e^{-2n\alpha\pi})(c\varkappa^2 PQ + \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha} - f)$. Cum igitur ſol tempore vnius diei circiter $\frac{1}{365}$ partem ecclipticae percurrat; n diebus conficiet $\frac{n}{365}$ partes; hoc eſt angulum $\frac{2\pi n}{365}$; denotante π angulum 180. graduum. Tempore ergo, quo ſol ecclipticae arculum ds percurrit, ob $n = \frac{365 ds}{2\pi}$, erit incrementum caloris meridiani $= (1 - e^{-365\alpha ds})(c\varkappa^2 PQ + \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha} - f)$; puta in eadem eleuatione poli, vbi tum eſt meridies.

§. 24. Quoniam autem propter exponentem $365\alpha ds$ infinite paruum eſt $e^{-365\alpha ds} = 1 - 365\alpha ds$, erit, dum ſol per arculum ecclipticae ds progreditur, exiſtente tota ecclipticae circumferentia $= 2\pi$, incrementum caloris meridiani $= 365\alpha ds(c\varkappa^2 PQ + \frac{\alpha^2 c\varkappa^2 pq}{1+\alpha\alpha} - f)$; in qua expreſſione denotat α numerum vnitati prixime aequalem; c

 gradum

gradum caloris conſtantem in ſuperficie ſolis; P eleuationis poli in loco propoſito ſinum, Q ſinum declinationis ſolis borealis; atque p et q coſinus ſinibus P et Q reſpondentes. Denique $\varkappa$ eſt tangens ſemidiametri ſolis apparentis, atque f denotat ipſum calorem meridianum tempore oblato.

Tab. I. fig. 2. §. 25. Denotet ergo AVB aequatorem, et EVL eccliptícam, interſectio V aequinoctium vernum atque P polum loci propoſiti. Verſetur ſol in S poſt aequinoctium vernum, et ponatur arcus VS $= s$ exiſtente tota eccliptica $= 2\pi$; atque ſit arcus VS ſinus $= u$; angulique S VT, qui eſt 23° 29′, ſinus $= m$; calor meridianus autem, dum ſol in S verſatur $= r$. His poſitis erit Q $= mu$, et $q = \sqrt{(1 - m^2 uu)}$; ac dum ſol per arculum ds progreditur erit incrementum caloris meridiani $dr = 365\,\alpha ds\,(c\varkappa^2 m P u + \frac{\alpha^2 c \varkappa^2 p \sqrt{(1-m^2 uu)}}{1+\alpha\alpha} - r)$ exiſtente $ds = \frac{du}{\sqrt{(1-uu)}}$. Ponatur breuitatis gratia n pro numero $365\,\alpha$ erit $dr + nr\,ds = (c\varkappa^2 m P u + \frac{\alpha^2 c\varkappa^2 p \sqrt{(1-m^2 uu)}}{1+\alpha\alpha})\, n\,ds$.

§. 26. Poſtrema haec aequatio integrata dabit $c^{ns} r = c\varkappa^2 n \int e^{ns} ds\,(m P u + \frac{\alpha^2 p \sqrt{(1-m^2 uu)}}{1+\alpha\alpha}) = \frac{c\varkappa^2 m n P}{1+nn} e^{ns}\,(nu - \sqrt{(1-uu)}) + \frac{\alpha^2 c \varkappa^2 n p}{1+\alpha\alpha} \int e^{ns} ds \sqrt{(1 - m^2 u^2)}$. Ad integrale poſterioris membri inueniendum oportet noſſe, eſſe $\int e^{ns} u^{\nu}$

$$ds = \frac{e^{ns} u^{\nu-1}\,(nu - \nu \sqrt{(1-uu)}) + \nu\,(\nu-1) \int e^{ns} u^{\nu-2}\, ds}{nn + \nu\nu};$$

vnde eſt $\int e^{ns} ds = \frac{e^{ns}}{n}$; $\int e^{ns} u^2 ds = \frac{e^{ns} u\,(nu - 2\sqrt{(1-uu)}) + \frac{2}{n} e^{ns}}{4 + nn}$

$$= \frac{e^{ns}\,(2 + n^2 u^2 - 2nu \sqrt{(1-uu)})}{n\,(4+nn)};\ \int e^{ns} u^4\, ds =$$

e^{ns}

$$\frac{e^{ns}u^3(uu-4\surd(1-uu))+\frac{12e^{ns}}{n(4+nn)}(2+n^2u^2-2nu\surd(1-uu))}{16+nn}$$

$$=\frac{e^{ns}(24+12n^2u^2+4n^2u^4+n^4u^4-4nu(6+4uu+nnuu)\surd(1-uu))}{n(4+nn)(16+nn)}$$

atque ita porro.

§. 27. Quoniam ergo est $\surd(1-m^2u^2)=1-\frac{m^2u^2}{2}-\frac{m^4u^4}{8}-$ etc. erit $\int e^{ns}ds\surd(1-m^2u^2)=e^{ns}\Big(\frac{1}{n}-\frac{mm(2+nnuu-2nu\surd(1-uu))}{2n(4+nn)}-\frac{m^4}{8}\cdot\frac{(24+12n^2u^2+4n^2u^4+n^4u^4-4nu(6+4u+nnuu)\surd(1-uu))}{n(4+nn)(16+nn)}-$ etc.$\Big)+$ const. Ponamus $s=0$, erit $u=0$ et $\surd(1-uu)=1$; fietque calor meridianus in aequinoctio verno $r=C-\frac{c\varkappa^2mnP}{1+nn}+\frac{\alpha^2c\varkappa^2np}{1+\alpha\alpha}\Big(\frac{1}{n}-\frac{mm}{n(4+nn)}-\frac{3m^4}{n(4+nn)(16+nn)}-\frac{45m^6}{n(4+nn)(16+nn)(36+nn)}-$ etc. At posito $s=2\pi$, r denotabit calorem meridianum in sequente aequinoctio verno, ita vt sit $e^{2n\pi}r=C-\frac{c\varkappa^2mnPe^{2n\pi}}{1+nn}+\frac{\alpha^2c\varkappa^2npe^{2n\pi}}{1+\alpha\alpha}\Big(\frac{1}{n}-\frac{m^2}{n(4+nn)}-\frac{3m^4}{n(4+nn)(16+nn)}-$ etc.

§. 28. Cum autem post innumerabiles reuolutiones solis perpetuo idem calor meridianus in aequinoctium vernum incidere debeat, oportebit quantitatem constantem C esse $=0$; si pro loco proposito ponatur calor meridianus in aequinoctio verno $=a$, erit $a=\frac{\alpha^2c\varkappa^2p}{1+\alpha\alpha}\Big(1-\frac{m^2}{4+n^2}-\frac{3m^4}{(4+n^2)(16+n^2)}-$ etc.$\Big)-\frac{c\varkappa^2mnP}{1+nn}$. Atque dum sol in loco quocunque S ecclipticae versatur, erit calor meridianus

ridianus $r = \frac{\alpha^2 c x^2 p}{1 + \alpha\alpha}\left(1 - \frac{m^2(2 + n^2u^2 - 2nu\sqrt{(1-uu)})}{2(4+nn)} - \frac{m^4(24 + 12n^2u^2 + 4n^2u^4 + n^4u^4}{8(4+nn)}\right.$ $\left. - \frac{4nu(6 + 4u + nnuu)\sqrt{(1-uu)}}{(16+nn)}\right) + \frac{cx^2 mnP(nu - \sqrt{(1-uu)})}{1+nn}$.

§. 29. Cum n ſit numerus valde magnus, quippe proxime aequalis numero 365, ob $\alpha = 1$, atque m numerus vnitate minor, ſcilicet $= 0,3982823$. pro dato terrae loco ad quemuis anni cuiusque diem proxime poterit definiri calor meridianus. Aequinoctio nimirum verno erit calor meridianus $\frac{\alpha^2 cx^2 p}{1+\alpha\alpha} - \frac{cx^2 mnP}{1+nn}$: aequinoctio vero autumnali erit calor meridianus $= \frac{\alpha^2 cx^2 p}{1+\alpha\alpha} + \frac{cx^2 mnP}{1+nn}$. At ſolſtitio aeſtiuo habebitur calor meridianus $= \frac{\alpha^2 cx^2 p}{1+\alpha\alpha}\left(1 - \frac{m^2n^2}{2(4+nn)} - \frac{m^4n^4}{8(4+nn)(16+nn)} - \text{etc.}\right) + \frac{cx^2 mn^2 P}{1+nn} = \frac{\alpha^2 cx^2 p\sqrt{(1-mm)}}{1+\alpha\alpha} + \frac{cx^2 mn^2 P}{1+nn}$; atque ſolſtitio hyemali erit calor meridianus $= \frac{\alpha^2 cx^2 p}{1+\alpha\alpha}\left(1 - \frac{m^2n^2}{2(4+nn)} - \text{etc.}\right) - \frac{cx^2 mn^2 P}{1+nn} = \frac{\alpha^2 cx^2 p\sqrt{(1-mm)}}{1+\alpha\alpha} - \frac{cx^2 mn^2 P}{1+nn}$, neglectis terminis nimium exiguis.

§. 30. Sub aequatore ergo eſt calor meridianus in aequinoctio verno $= \frac{cx^2}{2}$, in ſolſtitio aeſtiuo $= \frac{cx^2}{2} - \frac{cx^2 m^2 n^2}{4(4+nn)}$; in aequinoctio autumnali $= \frac{cx^2}{2}$; et in ſolſtitio hyemali $= \frac{cx^2}{2} - \frac{cx^2 m^2 n^2}{4(4+nn)}$. At ſub eleuatione poli 60°; erit calor meridianus in aequinoctio verno $\frac{cx^2}{4} - \frac{cx^2 mn\sqrt{3}}{2(1+nn)}$; in ſolſtitio aeſtiuo $= \frac{cx^2}{4} - \frac{cx^2 m^2 n^2}{8(4+nn)} + \frac{cx^2 mn^2\sqrt{3}}{2(1+nn)} = \frac{cx^2\sqrt{1-mm}}{4} + \frac{cx^2 mn^2\sqrt{3}}{2(1+nn)}$; in aequinoctio autumnali $= \frac{cx^2}{4} + \frac{cx^2 mn\sqrt{3}}{2(1+nn)}$, et in ſolſtitio hyberno $= \frac{cx^2}{4} - \frac{cx^2 m^2 n^2}{8(4+nn)} - \frac{cx^2 mn^2\sqrt{3}}{2(1+nn)} = \frac{cx^2\sqrt{1-mm}}{4} - \frac{cx^2 mn^2\sqrt{3}}{2(1+nn)}$. Sub ipſo denique polo boreali erit calor in aequinoctio verno $= \frac{-cx^2 mn}{1+nn}$; in ſolſtitio aeſtiuo $= \frac{cx^2 mn^2}{1+nn}$; in aequinoctio autumnali $= \frac{cx^2 mn^2}{1+nn}$; atque in ſolſtitio hyberno $= \frac{-cx^2 mn^2}{1+nn}$: in quibus formulis denotat $n = 365$, cum poſuerimus $\alpha = 1$.

§. 31.

§. 31. Ex formula inuenta intelligitur, calorem meridianum non fore maximum ipſo ſolſtitio aeſtiuo, neque minimum ſolſtitio hyberno; ſed aliquantum poſt has tempeſtates incidere. Quod tempus vt definiatur pro quavis eleuatione poli, quaeratur, quando fiat $dr = 0$. Hoc autem euenit, cum fit $r = c\varkappa^2 m P u + \frac{\alpha^2 c\varkappa^2 p \sqrt{(1-m^2u^2)}}{+\alpha\alpha}$: erit ergo $c\varkappa^2 m P u + \frac{\alpha^2 c\varkappa^2 p \sqrt{(-m^2u^2)}}{1+\alpha\alpha} = \frac{\alpha^2 c\varkappa^2 p \sqrt{(1-m^2\ ^2)}}{1+\alpha\alpha} + \frac{c\varkappa^2 m n P(nu - \sqrt{(1-uu)})}{1+nn}$; eſt enim proxime $r = \frac{\alpha^2 c\varkappa^2 p \sqrt{(1-m^2u^2)}}{1+\alpha\alpha} + \frac{\alpha^2 c\varkappa^2 p m^2 u \sqrt{(1-uu)}}{(1+\alpha^2) n \sqrt{(1-m^2u^2)}} + \frac{c\varkappa^2 m n P(nu - \sqrt{(1-uu)})}{1+nn}$; generaliter ſumendis iis tantum terminis, in quibus n plurimas habet dimenſiones.

§. 32. Ad tempus igitur definiendum, quo calor meridianus eſt maximus, iſta habetur aequatio $u = -n\sqrt{(1-uu)}$ et $\frac{u}{\sqrt{(-uu)}} = -n$ ergo $u = \frac{n}{\sqrt{(1+nn)}}$ et $\sqrt{(1-uu)} = \frac{1}{\sqrt{(1+nn)}}$. Maximus ergo calor meridianus, pariter ac maximum frigus incident poſt ſolſtitia; at diſcrimen ne vnicum quidem diem adaequat, ita vt ſatis tuto ipſa ſolſtitia pro momentis, in quibus calor meridianus tum ſit maximus tum minimus, haberi queant. Quodſi cum obſeruationibus minus congruat, id hypotheſi a veritate nimium aberranti eſt tribuendum. Vi hypotheſis enim in ſolſtitio aeſtiuo calor per noctem minime diminuitur, per diem autem maxime augetur.

§. 33. Quoniam calor meridianus ſub aequatore non multum variatur per anni tempeſtates, ex eo commodiſſime calor in ſuperficie ſolis poterit determinari. Denotet enim i maximum calorem meridianum ſub aequatore obſeruatum, qui cum ſit $= \frac{c\varkappa^2}{2}$, erit $c = \frac{2i}{\varkappa^2}$. At cum $\varkappa$ ſit tangens ſemidiametri ſolis apparentis, ſi pro $\varkappa$ ſumatur

matur tangens $16'$, fiet $c = 92328\ i$; ita vt calor in ſuperficie ſolis propemodum ſit centies millies maior, quam ſummus calor tempore meridiei, ſub ipſo aequatore. Verum vt ratio multiplicationis caloris pateat, notandum eſt, ſummum calorem meridianum ſub aequatore duplo maiorem eſſe, quam medium gradum caloris, inter gradus caloris meridiani aequinoctialis verni et autumnalis, ſub eleuatione poli $60°$.

§. 34. Qui formulas datas contemplabitur, mox perſpiciet, ſummum calorem meridianum neque ſub aequatore neque ſub polis deprehenſum iri. Quamobrem operae pretium erit illas regiones terrae determinare, in quibus tempore ſolſtitii aeſtiui maximus obſeruari debeat calor. Calculo vero ſubducto reperietur harum regionum eleuatio poli 41. graduum: fieri enim debet $\frac{P}{p} = \frac{2m}{\sqrt{(1-mm)}}$. His vero regionibus erit calor meridianus in ſolſtitio aeſtiuo $= \frac{cx^2\sqrt{(1+mm)}}{2} = \frac{1,215\ cx^2}{2}$, cum ſummus calor meridianus ſub aequatore ſit $= \frac{cx^2}{2}$.

§. 35. Hinc igitur ſequentem tabellam deduxi, in qua pro variis poli eleuationibus conſpiciuntur gradus caloris meridiani, tempore tam aequinoctiorum quam ſolſtitiorum.

Calor

Calor meridianus.

Eleuatio Poli.	Aeq. Verno	Solſtit. Aeſt.	Aeq. Aut.	Solſtit. hyb.
0°	0, 500	0, 459	0, 500	0, 459
10°	0, 492	0, 521	0, 492	0, 382
20°	0, 469	0, 567	0, 470	0, 295
30°	0, 432	0, 596	0, 434	0, 198
40°	0, 382	0, 607	0, 384	0, 095
50°	0, 321	0, 600	0, 322	-0, 010
60°	0, 249	0, 574	0, 251	-0, 115
70°	0, 170	0, 531	0, 172	-0, 218
80°	0, 085	0, 472	0, 088	-0, 313
90°	-0, 001	0, 398	0, 001	-0, 398

§. 36. Cum autem hac theoria pro quouis loco ſolis in ecccliptica detur calor meridianus, r ita vt ſit $r = \frac{c\kappa^2 p\sqrt{(1-m^2u^2)}}{2} + \frac{c\kappa^2 pm^2u\sqrt{(1-uu)}}{2n\sqrt{(1-m^2u^2)}} + \frac{c\kappa^2 mnP(nu-\sqrt{(1-uu)})}{1+nn}$. Hoc ergo die tempore t poſt meridiem erit calor $= e^{-\alpha t} c\kappa^2 \left(\frac{pm^2u\sqrt{(1-uu)}}{2n\sqrt{(1-m^2u^2)}} - \frac{mnP\sqrt{(1-uu)}}{1+nn}\right) + c\kappa^2 mPu + \frac{c\kappa^2 p(x+y)\sqrt{(1-m^2u^2)}}{2}$ ſeu neglectis terminis qui per n ſunt diuiſi erit calor iſte $Z = c\kappa^2 \left(mPu + \frac{p(x+y)\sqrt{(1-m^2u^2)}}{2}\right)$; vnde ſequitur maximum cuiusuis diei calorem incidere in horam tertiam pomeridianam: ideo quia poſuimus $\alpha = 1$. Nam ſi α maius foret, maximus calor propius ad meridiem incideret.

§. 37. Quamobrem ſi longitudinis ſolis ſinus ponatur $= u$; atque P deſignet ſinum eleuationis poli, p eius coſinum; ac m ſinum inclinationis eclipticae ad aequatorem: erit pro dato loco

	Calor.
Meridie	$c\varkappa^2\left(mPu + \frac{p\sqrt{(1-m^2u^2)}}{2}\right)$
hora 3	$c\varkappa^2\left(mPu + \frac{p\sqrt{(1-m^2u^2)}}{\sqrt{2}}\right)$
hora 6	$c\varkappa^2\left(mPu + \frac{p\sqrt{(1-m^2u^2)}}{2}\right)$
hora 9	$c\varkappa^2(mPu + 0)$
hora 12	$c\varkappa^2\left(mPu - \frac{p\sqrt{(1-m^2u^2)}}{2}\right)$
mane	
hora 3	$c\varkappa^2\left(mPu - \frac{p\sqrt{(1-m^2u^2)}}{\sqrt{2}}\right)$
hora 6	$c\varkappa^2\left(mPu - \frac{p\sqrt{(1-m^2u^2)}}{2}\right)$
hora 9	$c\varkappa^2(mPu - 0)$
hora 12	$c\varkappa^2\left(mPu + \frac{p\sqrt{(1-m^2u^2)}}{2}\right)$

§. 38. Hinc autem clariſſime inſufficientia huius hypotheſis elucet, cum ſub aequatore ipſo media nocte maius frigus deberet regnare, quam rigidiſſima hyeme ſub polis. Cauſa quidem huius abſurdi ſponte ſe offert; quoniam ſecundum theoriam ſub aequatore tempore nocturno calor maxime imminui debeat, eo quod ſoli ſub horizonte latenti adeo vim frigefaciendi tribuimus, eamque eo maiorem, quo profundius ſit ſubmerſus: in zona autem torrida profundiſſime ſubmergitur. Quocirca iſta theoria correctione indiget maxima; ſi quidem ad obſervationes accommodari debeat.

§. 39. Ob has difficultates tentabo rem expedire per priorem hypotheſin, quae veritati magis eſt conſentanea; manentibus ergo poſtremis denominationibus, praeterquam quod r denotet gradum caloris tempore ortus ſolis, reperietur haec aequatio $dr + nrds =$
$\frac{c\varkappa^2 ne^{-2\alpha\pi} ds(mPu(e^{2\alpha g}-1)+\alpha(1+e^{-\alpha g})\sqrt{(p^2-m^2u^2)}}{(1+\alpha\alpha)(1-e^{-2\alpha\pi})}$ vbi g de-

notat

notat arcum, cuius ſinus eſt $\frac{\sqrt{(p^2-m^2u^2)}}{p\sqrt{(1-m^2u^2)}}$. Ex qua aequatione, cum n ſit numerus tam magnus ſcilicet 365, erit proxime $r = \frac{c\varkappa^2}{1+\alpha\alpha} \frac{e^{-2\alpha\pi}(m\mathrm{P}u(e^{2\alpha g}-1)+\alpha(e^{2\alpha g}+1)\sqrt{(p^2-m^2u^2)})}{(1-e^{-2\alpha\pi})}$

vnde reperitur calor meridianus $f = \frac{c\varkappa^2 e^{-\alpha g}}{1+\alpha^2}$. $\left(\frac{(e^{2\alpha g-2\alpha\pi}-1)m\mathrm{P}u+(e^{2\alpha g-2\alpha\pi}+1)\alpha\sqrt{(p^2-m^2u^2)}}{1-e^{-2\alpha\pi}}\right.$ $+ c\varkappa^2 m\mathrm{P}u$ $+ \frac{\alpha^2 c\varkappa^2 p\sqrt{(1-m^2u^2)}}{1+\alpha\alpha}$. Calor in occaſu erit $=$ $\frac{c\varkappa^2 e^{-2\alpha g}((e^{2\alpha g}-1)m\mathrm{P}u+(e^{2\alpha g}+1)\alpha\sqrt{(p^2-m^2u^2)})}{(1+\alpha^2)(1-e^{-2\alpha\pi})}$, ergo quouis die ſe habet calor ortus ad calorem occaſus vt $e^{-2\alpha\pi}$ ad $e^{-2\alpha g}$ ſeu vt $e^{2\alpha g}$ ad $e^{2\alpha\pi}$; quo breuiores ergo ſunt dies, eo maior eſt differentia inter gradus caloris in ortu et occaſu.

§. 40. Poſt meridiem ergo tempore t quod in arcum aequatoris conuerſum dat angulum t, erit calor in regione propoſita $z = \frac{c\varkappa^2 e^{-\alpha(t+g)}}{(1+\alpha^2)(1-e^{-2\alpha\pi})}\left(\left((e^{2\alpha(g-\pi)}-1)\mathrm{P}u\right)+(e^{2\alpha(g-\pi)}+1)\alpha\sqrt{(p^2-m^2u^2)}\right)+c\varkappa^2 m\mathrm{P}u+\frac{\alpha c\varkappa^2 p(x+\alpha y)\sqrt{(1-m^2u^2)}}{1+\alpha^2}$, vbi u denotat ſinum longitudinis ſolis; x ſinum anguli t et y coſinum ipſius: g vero angulum cuius ſinus eſt $\frac{\sqrt{(p^2-m^2u^2)}}{p\sqrt{(1-m^2u^2)}}$ et coſinus $= \frac{-m\mathrm{P}u}{p\sqrt{(1-m^2u^2)}}$ ſeu $g = \frac{\pi}{2} + \mathrm{Ar.\ ſin.}\frac{m\mathrm{P}u}{p\sqrt{(1-m^2u^2)}}$ vel $dg = \frac{m\mathrm{P}du}{(1-m^2u^2)\sqrt{(p^2-m^2u^2)}}$.

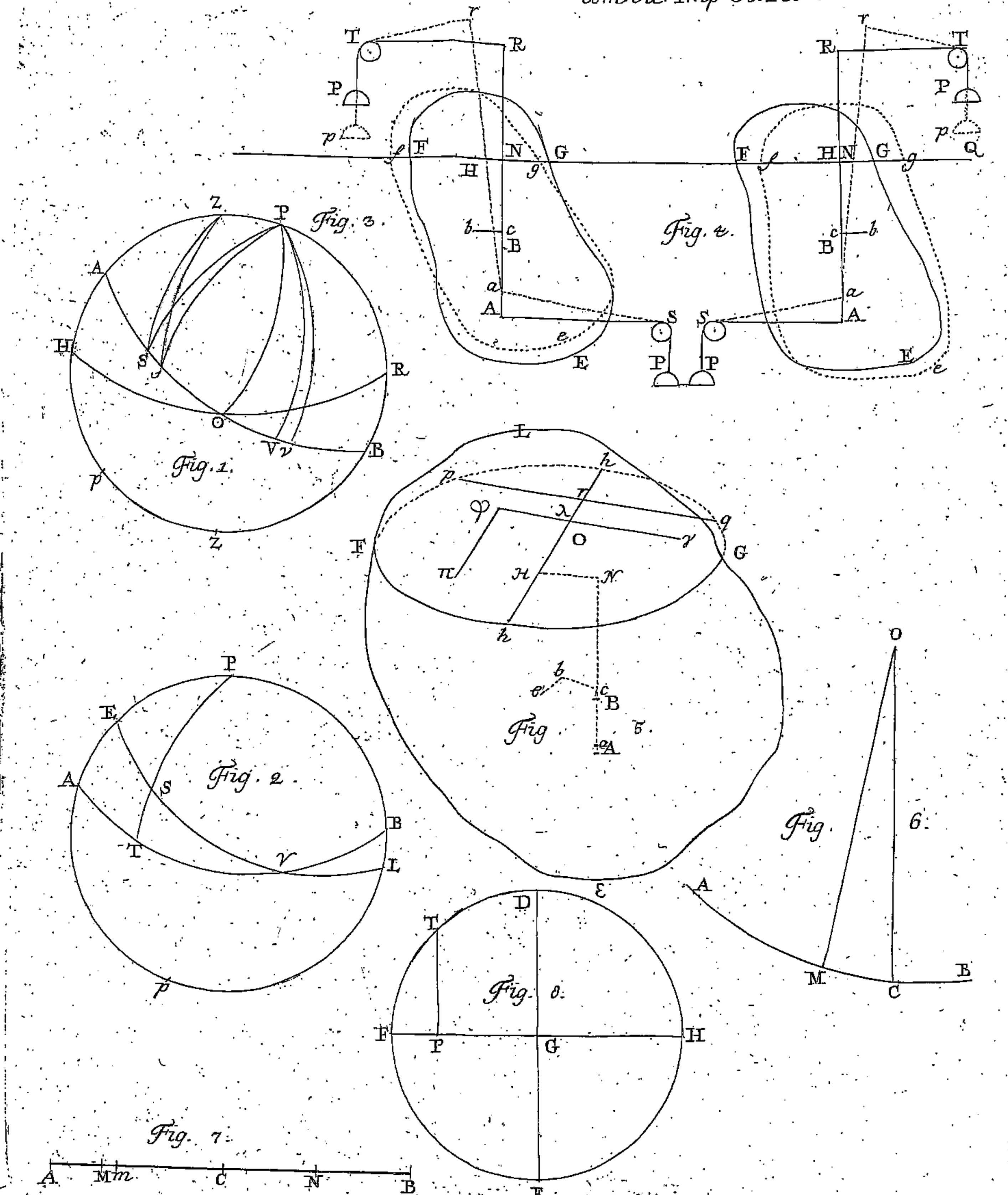
Fig. 1.
Fig. 2
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.