ac cœliacis arteriis obſtructiones ſæpius vere, quam in aliis, conting[illegible] nemini tamen, hujus rei cauſam, probabitur, in maſſulis ſanguinis, diametro vaſculorum majoribus factis, quærendam eſſe. Qua occaſione *Doraſcentius Michelottum* taxat, quod craſſa fluida dixerit non ea eſſe, quæ particulas maximas habeant, ſed quorum particulæ difficulter trajiciantur. Ab hac doctrina de obſtructione in univerſum, ad illam de ſcirrhis tranſit, quorum diſtinctionem in legitimos, ſeu exquiſitos, & non exquiſitos, ſine addita cauſa, conſtantem eſſe negat, &, de qua re nemo dubitat, ſedem ſcirrhi in glandulis eſſe, non novis exemplis oſtendit, ſed jam-diu notis, atque ex aliis Autoribus repetitis. Sunt vero illa exempla de interiorum partium ſcirrhis. Sequitur & cancri deſcriptio, quem non, ut alii exiſtimarunt, in ſolis mulierum mammis, & virorum ſcapulis, verum in aliis quoque corporis partibus, enaſci, frequentia exempla docent, ſeque, ſuperiore anno, in pede dextro artificis cujusdam, Romæ cancrum vidiſſe, Autor ſcribit. Quibus præmiſſis, ad chalybem tranſit, de qua pariter ac de ferro, ſeu marte, poſtquam aliqua nobis parum intellecta protulit, ac Methodicorum de ſtricto laxoque doctrinam laudavit, chalybem eatenus obſtructiones tollere oſtendit, quatenus ſanguini teretes illas minores maſſulas reſtituat, acidum peregrinum abſorbeat, & vaſa laxa acido-auſteris particulis roboret. Cui ſententiæ ut pondus adderet, eos Autores, qui chalybi eandem fere præcipueque hanc virtutem adſcripſerunt, aperiendi ſale, terreiſque particulis roborandi, magno numero excitavit, eorumque verba adduxit. De Mercurio autem propterea nihil dicimus, quod *Doraſcentium* deprehendimus hoc medicamenti genus vix tribus verbis attigiſſe.

Pag. 24 ſeq.

16.

27.

28.

29.

31 ſeq.

32-38.

L. E. SOLUTIO PROBLEMATIS CATOPTRICI, in Novis Actis Eruditorum Lipſienſibus pro Menſe Novembri A. 1745 propoſiti, continuata.

Conf. *Nova Acta Menſ. Januar.* hujus *Anni pag. 27.*

Problema V.

TAB. II. XXXIX. Fig. 1. Si radii incidentes CM fuerint axi AB paralleli, invenire curvam continuam $FMBmf$, ut ſinguli radii incidentes CM, poſtquam in curva duplicem reflexionem in M & m fuerint paſſi, axi denuo fiant paralleli, ſeu ut radius bis reflexus mc ſit incidenti MC parallelus.

Solutio.

Ad axem AB in quopiam puncto fixo A ducatur perpendicularis Ee, &, quoniam radius incidens CM poſt duplicem reflexionem abit in mc, manifeſtum eſt reciproce, ſi cm fuerit radius incidens, eum poſt duplicem reflexionem dirigi in MC. Si ergo M fuerit punctum prioris reflexionis in quæſita curva, & m punctum, in quo fit reflexio poſterior: viciſſim, ſi m ſit punctum reflexionis prioris, erit M punctum reflexionis poſterioris, ita ut hæc relatio inter bina puncta M & m ſit reciproca. Si igitur concipiamus, curvam FBf per functiones quantitatis cujuſpiam variabilis ω determinari, ita ut aliis atque aliis valoribus ipſi ω tribuendis, alia continuo curvæ puncta M definiantur, curva utique continua reſultabit, quod eſt primarium Problematis requiſitum. Oriatur ergo curvæ punctum M, ſi variabili illi ω tribuatur valor μ; punctum autem m prodeat, ſi ponatur $\omega = \nu$: atque hos binos valores μ & ν ita inter ſe affectos eſſe oportet, ut, quemadmodum μ ſe habet ad ν, ita viciſſim ſe habeat ν ad μ: quod evenit, ſi ſit vel $\mu = \nu$, vel $\mu = -\nu$: at, quia puncta M & m ſunt diverſa, aſſumi non licet $\mu = \nu$; erit ergo $\mu = -\nu$. Ex quo efficitur, ut, quemedmodum curvæ punctum M determinatur per functiones quantitatis ω, eodem prorſus modo punctum m determinetur per ſimiles functiones ipſius $-\omega$. Si igitur ponamus intervallum $AR = r$, angulum $ARM = \phi$, atque ſin. $\phi = s$ & coſ. $\phi = u$, quatenus punctum curvæ M ſpectatur, erit pro puncto curvæ m, ſi cm tanquam radius incidens conſideretur, idem intervallum in axe $AR = r$, angulus autem, qui ante erat $= \phi$, nunc fiet $ARm = 180 - \phi$, &, quia in plagam contrariam vergit, cenſendus eſt $= -180° + \phi$, ejusque ergo ſinus

erit

TAB. II ad Nov. Act. Erud. A. 1748. Menſ. Februar. pag. 62 ſeq.

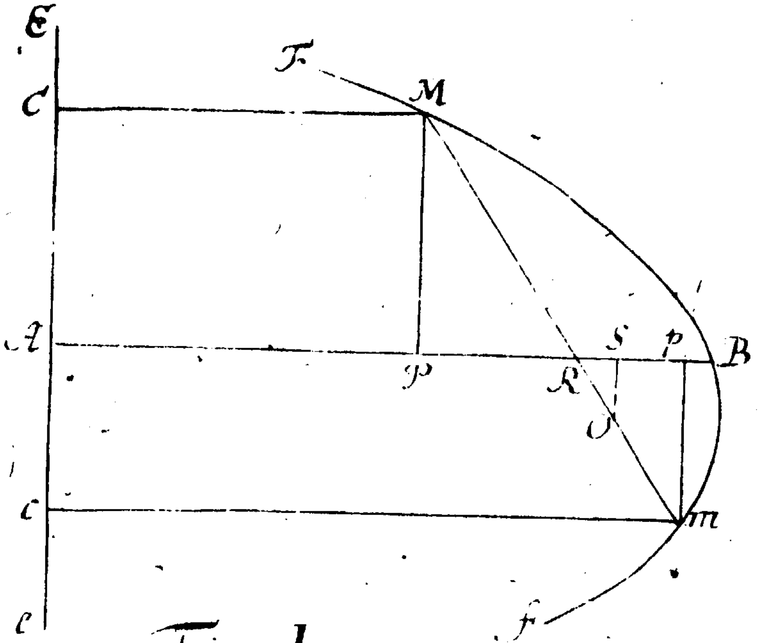


Fig. I

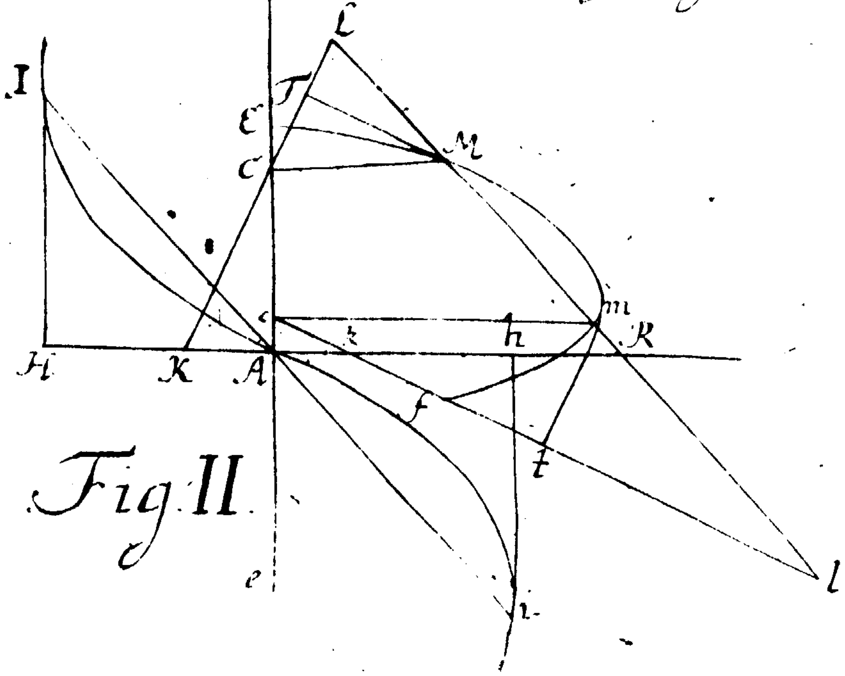
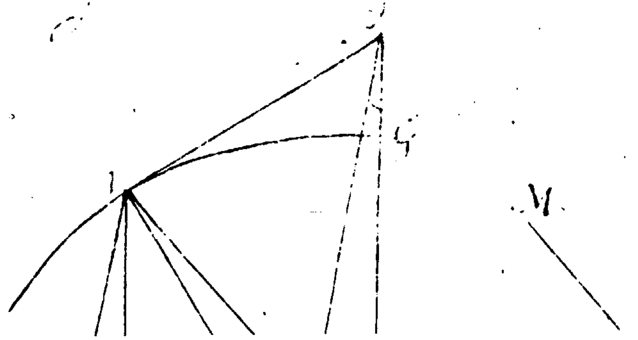
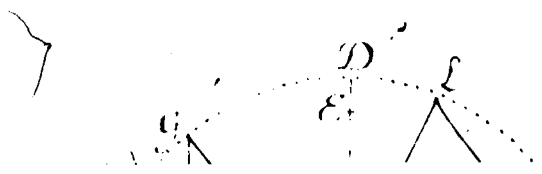


Fig. II





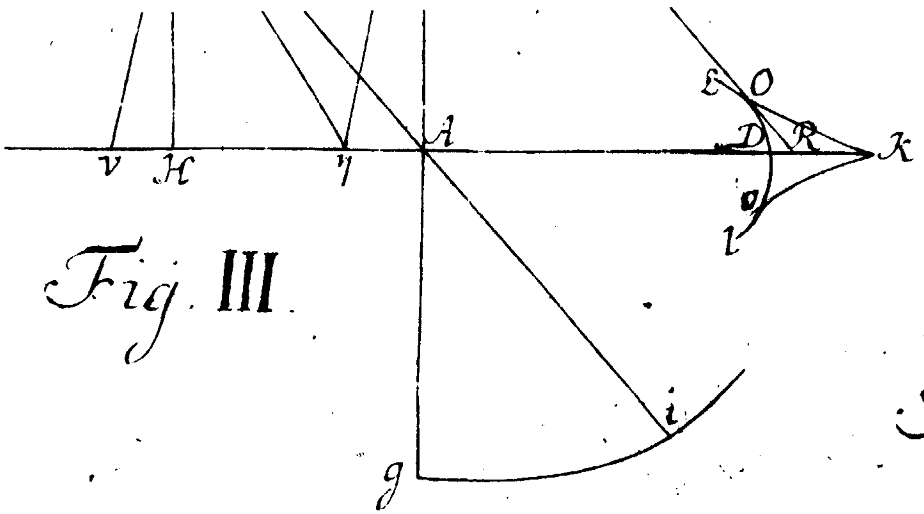


Fig. III.

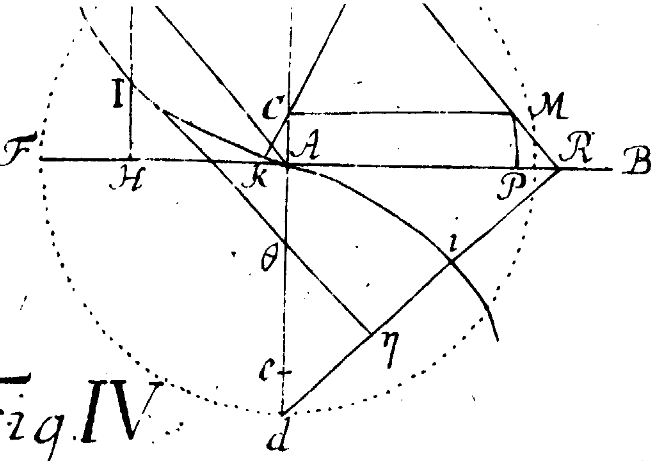


Fig IV.

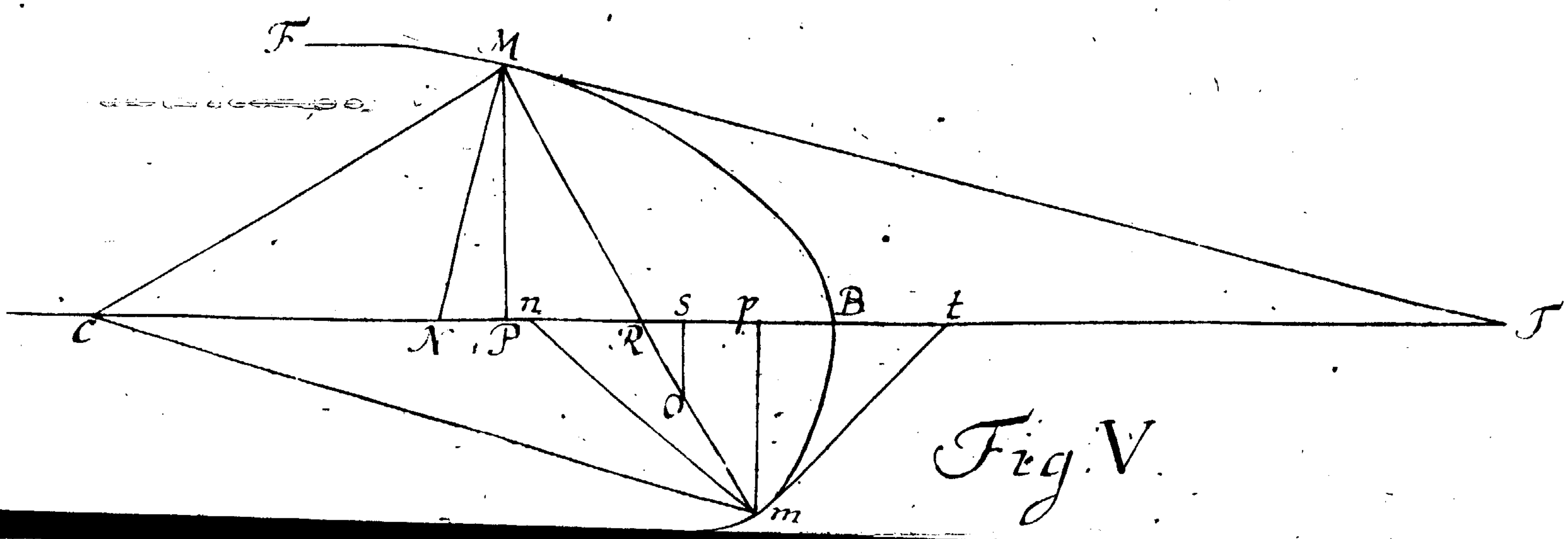


Fig. V.

erit $= -s$ & coſinus $= -u$. Hinc r ejusmodi functio eſſe debet ipſius ω, ut eundem valorem retineat, etiamſi pro ω ponatur $-\omega$, cujusmodi proprietatem habent poteſtates exponentium parium ipſius ω, & functiones utcunque ex iis compoſitæ: unde iſtiusmodi functiones ipſius ω, quæ immutatæ manent, poſito $-\omega$ pro ω, functiones pares vocare ſoleo. Quamobrem intervallum $AR = r$ exprimi debebit per functionem parem ipſius ω. Deinde, quia quantitates s & u, ſi translatio ab M ad m fiat, abeunt in $-s$ & $-u$, neceſſe eſt, ut s & u ſint ejusmodi functiones ipſius ω, quæ poſito $-\omega$ loco ω valores obtineant ſui negativos, nempe $-s$ & $-u$: cujusmodi functiones impares vocare ſoleo, quia poteſtates exponentium imperium, & functiones ex iis compoſitæ hanc proprietatem habent; unde conſequens eſt, quantitates s & u functiones impares ipſius ω eſſe oportere. Problemati ergo ſatisfiet, ſi pro r functio par ipſius ω quæcunque, & pro s ac u functiones impares ejusdem ω, accipiantur, hincque per Problema III curva FBf difiniatur: ubi quidem notandum eſt, eſſe debere $ss + uu = 1$. Functiones autem impares hac gaudent proprietate, ut, ſi P ſit functio impar ipſius Q, viciſſim Q ſit functio impar ipſius P. Hanc ob rem ω erit functio impar tam ipſius s, quam ipſius u; ideoque conjunctim ω erit functio impar ipſarum s & u, talis ſcilicet, ut, ſi loco s & u ſcribantur $-s$ & $-u$, ipſa ω abeat in $-\omega$. Cum igitur r debeat eſſe functio par ipſius ω, erit quoque functio par ipſarum s & u: ita ut eundem valorem retineat, etiamſi pro s & u ponantur $-s$ & $-u$: in functionibus igitur rationalibus intervallum r per s & u ita exprimetur, ut ſit

$$\text{vel } r = \frac{A + Bs^2 + Csu + Du^2 + Es^4 + Fs^3u + Gs^2u^2 + \&c.}{\alpha + \beta s^2 + \gamma su + \delta u^2 + \varepsilon s^4 + \zeta s^3u + \eta s^2u^2 + \&c.}$$

$$\text{vel } r = \frac{As + Bu + Cs^3 + Ds^2u + Esu^2 + Fu^3 + Gs^5 + \&c.}{\alpha s + \beta u + \gamma s^3 + \delta s^2u + \varepsilon su^2 + \zeta u^3 + \eta s^5 + \&c.}$$

Quæcunque igitur hujusmodi relatio inter r, & s, u accipiatur, ſemper per Problema III obtinebitur curva, Problemati ſatisfaciens. *Q. E. I.*

Coroll.

Coroll. 1.

XL. Si igitur ex puncto curvæ M ad axem AB demittatur perpendiculum MP, & vocentur coordinatæ $AP = x$, $PM = y$ erit, ut ſupra (§. 27) invenimus, $CM + MR = p = a + \int u\,dr$, hincque invento valore ipſius p habebitur $MR = \frac{p - r}{1 - u}$ atque $AP = x = \frac{r - up}{1 - u}$ & $PM = y = \frac{s(p - r)}{1 - u}$.

Coroll. 2.

XLI. Si ex altero reflexionis puncto m pariter ad axem AB demittatur perpendiculum mp, vocenturque $Ap = x'$ & $pm = -y'$, quia infra axem cadit, erit primo $cm + mR = p' = a - \int u\,dr$. Cum enim r ſit functio par ipſius u, erit $\int u\,dr$ functio impar, fietque idcirco pro puncto m negativa, unde erit $mR = \frac{p' - r}{1 + u}$ & $Ap = x' = \frac{r + up'}{1 + u}$, &

$$pm = -y' = \frac{+s(p' - r)}{1 + u}.$$

Coroll. 3.

XLII. Si igitur pro p & p' valores aſſignati ſubſtituantur, erit $MR = \frac{a - r + \int u\,dr}{1 - u}$. $AP = \frac{r - au - u\int u\,dr}{1 - u}$; & $PM = \frac{s(a - r + \int u\,dr)}{1 - u}$. Tum vero $Rm = \frac{a - r - \int u\,dr}{1 + u}$;

$$AP = \frac{r + au - u\int u\,dr}{1 + u} \text{ \& } pm = \frac{s(a - \int u\,dr - r)}{1 + u}.$$

Coroll. 4.

XLIII. Cum autem ſit $CM + MR = a + \int u\,dr$, & $cm + mR = a - \int u\,dr$, erit ſumma $CM + Mm + mc = 2a$. Radios ergo a recta Ee computando erit perpetuo ſumma radiorum incidentis CM, primum reflexi Mm & ſecundo reflexi mc conſtans $= 2a$.

Coroll. 5.

XLIV. Erit vero radius $Mm = MR + Rm = \frac{2a - 2r + 2u\int u\,dr}{1 - uu}$, & ſumma radiorum $CM +$

$cm =$

$cm = \frac{2r - 2auu - 2u\int udr}{1 - uu}$: unde denuo patet, ſummam omnium $CM + Mm + cm$ fore $= 2a$.

Coroll. 6.

XLV. Si O ſit punctum, ubi radius reflexus Mm cauſticam tangit, atque demiſſo ex O ad axem AB perpendiculo OS vocentur pro cauſtica coordinatæ orthogonales $AS = X$ & $SO = Y$ erit primo $RO = \frac{-ssdr}{du} = \frac{sudr}{ds}$: tum vero habebitur $AS = X = r \frac{-ssudr}{du} = r + \frac{suudr}{ds}$, & $SO = Y = \frac{-s^3dr}{du} = \frac{ssudr}{ds}$. Longitudo vero cauſticæ eſt $= CM + MO \pm C = a + \int udr \frac{-ssdr}{du} \pm C$.

Coroll. 7.

XLVI. Cauſtica ergo curvæ quæſitæ ſemper eſt curva algebraica, ſi quidem quantitas r fuerit functio algebraica ipſarum s & u. Fieri autem omnino poteſt, non obſtante conditione ſupra præſcripta, ut r ſit functio tranſcendens ipſarum s & u, ut mox videbimus.

Conſtructio Curvæ.

TAB. II Fig. 2.

XLVII. Circa punctum A deſcribatur curva IAi, partes alternatim poſitas AI & Ai ſimiles & æquales habens, ita ut quævis recta per A ducta habeat partes AI & Ai æquales. Cujusmodi curvæ IAi centro A præditæ appellari poſſunt: quarum æquationes inter coordinatas orthogonales $AH = \zeta$ & $HI = \eta$ ita erunt comparatæ, ut variabiles ζ & η in ſingulis terminis dimenſiones numero vel pares, vel impares, conſtituant. Manifeſtum enim eſt utroque caſu, ſi loco ζ & η ponantur $-\zeta$ & $-\eta$, æquationem non mutari, eandemque propterea intercedere relationem inter coordinatas Ah & hi atque inter AH & HI. Pro hujusmodi ergo curvis, ſi ſint algebraicæ, æquationum forma erit:

vel $o = A + B\zeta^2 + C\zeta\eta + D\eta^2 + E\zeta^4 + F\zeta^3\eta + G\zeta^2\eta^2 +$ &c.
vel $o = A\zeta + B\eta + C\zeta^3 + D\zeta^2\eta + E\zeta\eta^2 + F\eta^3 + G\zeta^5 +$ &c.
Aſſumta itaque hujusmodi curva IAi, ſi angulus HAI vocetur $= \varphi$, recta AI valorem idoneum præbebit pro r, quoniam abeunte angulo $HAI = \varphi$ in HAi, cujus ſinus & coſinus fiunt negativi, prodit recta $Ai = AI = r$: unde poſito $HAI = \varphi$ & $AI = r$ talem hæc curva relationem inter r & φ præbet, qualem ſolutio inventa poſtulat. Quæratur igitur valor $\int u\,dr$ ex hac curva, ſeu cum poſitis $AH = \zeta$ & $HI = \eta$ ſit $r = \sqrt{(\zeta\zeta + \eta\eta)}$ & $u = \frac{\zeta}{r}$ erit $dr = \frac{\zeta d\zeta + \eta d\eta}{r}$ &

$$\int u\,dr = \int \frac{\zeta dr}{r} = \int \frac{\zeta(\zeta d\zeta + \eta d\eta)}{\zeta\zeta + \eta\eta},$$

quod integrale ponatur $= \underline{Q}$. Tum in axe HA producto capiatur $AR = AI = r$, & ducatur ipſi AI parallela LRL, in qua primo ſurſum capiatur $RL = a + Q$, tum vero deorſum $Rl = a - Q$, denotante a quantitatem conſtan[illegible] pro lubitu aſſumendam. Deinde in axe ſumtis $RK = RL$ et $Rk = Rl$ jungantur rectæ KL & kl, quæ, ſi opus eſt, productæ ſecent rectam Ee axi HR in O normaliter junctæ in punctis C & c, per quæ axi parallelæ agantur CM & cm rectam Ll ſecantes in M & m, eruntque M & m duo curvæ quæſitæ puncta. Si denique anguli CML & cml biſecentur rectis MT & mt, hæ erunt tangentes curvæ quæſitæ in punctis M & m: hocque modo deſcribetur curva Problemati ſatisfaciens $EMmf$, in qua $CMmc$ repræſentat viam radii ad curvam in M & m bis reflexi.

Coroll. 8.

XLVIII. Manifeſtum ergo eſt, ex eadem linea curva IAi infinitas lineas curvas $EMmf$, Problemati ſatisfacientes, conſtrui poſſe. Non ſolum enim quantitas conſtans a pro labitu variari poteſt, ſed etiam quamvis aliam rectam, per A tranſeuntem, pro axe aſſumere licet.

Conſtructio Causticæ.

XLIX. Quoniam invenimus $RO = \frac{sudr}{ds}$; & per coordinatas

ordinatas $AH = \zeta$ & $HI = \eta$ curvæ IAi, qua ante ad curvas Problemati ſatisfacientes inveniendas ſumus uſi, eſt $dr = \frac{\zeta d\zeta + \eta d\eta}{r}$; $u = \frac{\zeta}{r}$ & $s = \frac{\eta}{r}$ erit $ds = \frac{\text{[illegible]} d\eta - \eta dr}{rr} = \frac{\zeta(\zeta d\eta - \eta d\zeta)}{r^3}$ & $sudr = \frac{\zeta\eta(\zeta d\zeta + \eta d\eta)}{r^3}$, unde fit $RO = \frac{\eta(\zeta d\zeta + \eta d\eta)}{\zeta d\eta - \eta d\zeta}$. Ducatur ad curvæ aſſumtæ punctum I tangens $I\theta$ rectæ AG ad axem AR normali occurrens in θ, atque normalis $I\eta$, ipſi axi occurrens in η, erit $A\theta = \eta - \frac{\zeta d\eta}{d\zeta}$ & $A\eta = \zeta + \frac{\eta d\eta}{d\zeta}$, unde fit $\frac{A\eta}{A\theta} = \frac{\zeta d\zeta + \eta d\eta}{\eta d\zeta - \zeta d\eta}$; ideoque $RO = - \frac{HI.A\eta}{A\theta}$. Ducatur igitur recta $\theta\eta$, eique per I parallela $I\nu$ erit $H\nu = \frac{HI.A\eta}{A\theta}$ ideoque $RO = - H\nu$. Quare in axe ſumto intervallo $AR = AI$, ductaque RM parallela ipſi AI, in ea ob ſignum − ſurſum capiatur $RO = H\nu$, eritque punctum O in cauſtica & recta RO ejus tangens: unde cauſtica $DOLK$ ex data curva GI geometrice conſtruetur.

TAB. II Fig. 3.

Coroll. 9.

L. Conſtructa autem cauſtica vel per ejus rectificationem, vel ope fili ei circumplicati per evolutionem, eo modo, qui in doctrina de cauſticis tradi ſolet, curvæ Problemati ſatisfacientes deſcribentur.

Scholion.

LI. Notatu dignæ ſunt figuræ cauſticarum, quæ hoc modo conſtruuntur. Si enim curva aſſumta Ii in infinitum extendatur, cauſtica pariter habebit ramos in infinitum expanſos; ſin autem curva Ii in ſe redeat, tum cauſtica quoque tota in ſpatio finito includetur, & quidem cuſpidibus erit prædita L, l, K, uti in figura exhibetur: niſi curva Ii fuerit circulus, quo caſu cauſtica in punctum abit. Ponamus, curvam Ii eſſe ellipſin, cujus natura exprimatur hac æquatione: $\eta\eta = bb - n\zeta\zeta$: erit $r = \sqrt{(bb + (1-n)\zeta\zeta)}$: $s = \frac{\eta}{r}$ & $u = \frac{\text{[illegible]}}{r}$ atque $\eta d\eta = -n\zeta d\zeta$; unde $d\eta : d\zeta = n\zeta : -\eta$, ideoque $RO = \frac{\eta(n\zeta\eta - \zeta\eta)}{n\zeta\zeta + \eta\eta} = \frac{(n-1)\zeta\eta\eta}{bb}$, ſeu $RO = \frac{(n-1)\zeta(bb - n\zeta\zeta)}{bb}$. Num autem ob $X = r + \frac{\zeta}{r}$. RO & $V = \frac{\eta}{r}$. RO erit $rX = bb + (1-n)\zeta\zeta + (n-1)\zeta\zeta - n(n-1)\zeta^4 : bb$, ideoque $X = \frac{b^4 - n(n-1)\zeta^4}{bb\sqrt{(bb + (1-n)\zeta\zeta)}}$ & $Y = \frac{(n-1)\zeta(bb - n\zeta\zeta)\sqrt{(bb - n\zeta\zeta)}}{bb\sqrt{(bb + (1-n)\zeta\zeta)}}$ hincque $XX + YY = bb + n(n-1)\zeta\zeta - 2nn(n-1)\zeta^4 : bb + nn(n-1)^2\zeta^6 : b^4$. Erit ergo $RO = 0$ duobus caſibus, altero, quo $\zeta = 0$, altero, quo $\eta = 0$; maximum vero erit RO, ſi ſit $bb = 3n\zeta\zeta$, ſeu $\zeta = \frac{b}{\sqrt{3n}}$. Quo caſu ſit $\eta = b\sqrt{\frac{2}{3}}$; & $r = b\sqrt{\frac{2n+1}{3n}}$, atque $RO = \frac{2(n-1)b}{3\sqrt{3n}}$; tum vero $s = \sqrt{\frac{2n}{2n+1}}$ & $u = \sqrt{\frac{1}{2n+1}}$. His caſibus autem cauſticæ cuſpides L & l deſignantur. Verum de his figuris facilius judicare poterimus, ſi curvas algebraicas Problemati ſatisfacientes inveſtigaverimus.

Problema VI.

TAB. II Fig. 1.

LII. Iisdem propoſitis conditionibus, quæ in Problemate præcedente ſunt allatæ, invenire curvas algebraicas $FMBmf$, quæ radios incidentes CM axi parallelos poſt duplicem reflexionem eidem axi iterum reddant parallelos.

Solutio.

Jam ante invenimus, ſi ponatur intervallum $AR = r$, angulus

angulus $ARM = \phi$, ejus ſinus $= s$ & coſinus $= u$, æquationem pro curva ita comparatam eſſe oportere, ut r ſit functio parium dimenſionum ipſarum u & s. Hinc autem fieri vidimus $CM + MR = p = a + \int u\,dr$. Unde coordinatæ $AP = x$, $PM = y$ ita determinantur, ut ſit $x = \frac{r - up}{1 - u}$ & $y = \frac{s(p - r)}{1 - u}$. Quo igitur curva fiat algebraica, primo neceſſe eſt, ut r ſit functio algebraica ipſarum s & u, tum vero ut formula $\int u\,dr$ ſit integrabilis. Eſt vero $\int u\,dr = ur - \int r\,du$, &, cum r ſit functio par, erit tam ur, quam $\int r\,du$, functio impar ipſarum s & u. Denotet ergo in genere v functionem quamcunque algebraicam imparem ipſarum s & u, ac ſtatuatur $\int r\,du = v$, fietque $r = \frac{dv}{du}$: & $p = a + \int u\,dr = a + ur - v = a + \frac{u\,dv}{du} - v$, ita ut valor ipſius p hinc prodeat algebraicus. Erit ergo $p - r = a - \frac{dv}{du} + \frac{u\,dv}{du} - v = a - v - \frac{(1 - u)dv}{du}$; & $r - up = \frac{(1 - uu)\,dv}{du} - u(a - v)$, unde habebuntur coordinatæ:

$$x = \frac{(1 + u)dv}{du} - \frac{u(a - v)}{1 - u}\,; \quad \& \; y = \frac{s(a - v)}{1 - u} - \frac{s\,dv}{du}.$$

Quæcunque ergo functio algebraica impar ipſarum s & u pro v accipiatur, ſemper orietur æquatio algebraica pro curva $FMBmf$, quæ radios CM, axi AB parallele incidentes, poſt duplicem reflexionem axi iterum parallelos reddat. *Q. E. I.*

Coroll. 1.

LIII. Quoniam v debet eſſe functio impar ipſarum s & u, ea ſi fuerit rationalis, in altera ſequentium formularum debet eſſe contenta:

$$v = \frac{A + Bss + Csu + Duu + Es^4 + Fs^3u + Gs^2u^2 + Hsu^3 + \text{\&c.}}{\alpha s + \beta u + \gamma s^3 + \delta s^2 u + \varepsilon s u^2 + \zeta u^3 + \eta s^5 + \theta s^4 u + \text{\&c.}}$$

$$v = \frac{As + Bu + Cs^3 + Ds^2u + Esu^2 + Fu^3 + Gs^5 + Hs^4u + \&c}{\alpha + \beta ss + \gamma su + \delta uu + \varepsilon s^4 + \zeta s^3 u + \eta s^2 u^2 + \theta s u^3 + \&c.}$$

Coroll. 2.

LIV. Cum ſit $s = \sqrt{(1 - uu)}$, hæ formulæ ſequenti modo ſimpliciter exprimi poterunt, ut ſit

$$\text{vel } v = \frac{A + Buu + Cu^4 + Du^6 + \&c. + (Pu + Qu^3 + Ru^6 + \&c.)}{\alpha u + \beta u^3 + \gamma u^5 + \&c. + (\pi + \varrho u^2 + \sigma u^4 + \tau u^6 + \&c.)}$$

$$\frac{\sqrt{(1 - uu)}}{\sqrt{(1 - uu)}}$$

$$\text{vel } v = \frac{Au + Bu^3 + Cu^5 + \&c. + (P + Qu^2 + Ru^4 + Su^6 + \&c.)}{\alpha + \beta uu + \gamma u^4 + \delta u^6 + \&c. + (\pi u + \varrho u^3 + \sigma u^5 + \&c.)}$$

$$\frac{\sqrt{(1 - uu)}}{\sqrt{(1 - uu)}}$$

Coroll. 3.

LV. Si irrationalitas $\sqrt{(1 - uu)}$ ex denominatore utriusque harum formularum tollatur more conſueto, utraque formula in hanc unam formam abibit:

$$v = \frac{Au + Bu^3 + Cu^5 + Du^7 + \&c. + (P + Qu^2 + Ru^4 + Su^6 + \&c.)\sqrt{(1 - uu)}}{\alpha + \beta u^2 + \gamma u^4 + \delta u^6 + \varepsilon u^8 + \&c.}$$

Coroll. 4.

LVI. Æquatio etiam inter v & u & s quæcunque aſſumi poteſt, dummodo hæ tres quantitates in ſingulis terminis vel pares ubique, vel impares ubique, conſtituant dimenſiones. Ita tam hæc æquatio $v^3 + \alpha u v^2 + \beta s v^2 + \gamma u^3 + \delta s u u + \varepsilon v + \zeta s + \eta u = \theta$, quam hæc $v^2 + \alpha s v + \beta u v + \gamma u^2 + \delta s u + \varepsilon = o$ idoneam pro v functionem ſuppeditabit.

Coroll. 5.

LVII. Cum igitur ſit pro primo reflexionis puncto M:

$$AR = \frac{dv}{du} : RM = \frac{a - v}{1 - u} - \frac{dv}{du} ; PM = \frac{s(a - v)}{1 - u} - \frac{s\,dv}{du} ; PR = \frac{u(a - v)}{1 - u} - \frac{u\,dv}{du} \;\&\; AP = \frac{(1 + u)\,dv}{du} - \frac{u(a - v)}{1 - u}$$

$-\frac{u(a-v)}{1-u}$ erit pro altero puncto m; quo tam v, quam s & u, fiunt negativa: $AR=\frac{dv}{du}$; $Rm=\frac{a+v}{1+u}-\frac{dv}{du}$ $pm=\frac{s(a+v)}{1+u}-\frac{sdv}{du}$; $Rp=\frac{u(a+v)}{1+u}-\frac{udv}{du}$; & $Ap=\frac{(1-u)dv}{du}+\frac{u(a+v}{1+u}$.

Coroll. 6.

LVIII. Punctum autem O in cauſtica ob $r=\frac{dv}{du}$, & ſumto du conſtante $dr=\frac{ddv}{du}$, ita definietur, ut ſit $RO=\frac{-ssddv}{du^2}=\frac{-(1-uu)ddv}{du^2}$; hincque $OS=\frac{-s(1-uu)ddv}{du^2}$; $RS=\frac{-u(1-uu)ddv}{du^2}$ atque $AS=\frac{dv}{du}-\frac{u(1-uu)ddv}{du^2}$, unde curva cauſtica cognoſcetur.

Conſtructio Curvæ.

LIX. Circa punctum A deſcribatur curva algebraica IAi, ramis alternis AI & Ai æqualibus & ſimilibus prædita, & per A ducatur recta FAB pro axe habenda. Vocetur abſciſſa $AH=u$ & applicata $HI=v$, ſietque v functio impar ipſius u, uti ſolutio poſtulat. Tum axi AB in A normaliter conſtituatur DAd, cui tangens $I\theta$ occurrat in θ, erit $A\theta=\frac{udv}{du}-v$: unde ſi capiatnr $AE=Ae=a$ erit $E\theta=a+\frac{udv}{du}-v=p=a+\int udr$ & $e\theta=a-\int udr$. Deinde centro A radio arbitrario $AD=Ad=1$ deſcribatur circulus, cui

TAB. II Fig. 4.

cui applicata HI occurrat in G, erit $HG = \sqrt{(1-uu)} = s$, & FAG æquabitur angulo ϕ. Jam ex d ad $I\theta$ productam ducatur perpendicularis $d\eta R$, axi occurrens in R, erit $AR = \frac{dv}{du} = r$: tunc ex R radio AG ducatur parallela RL, in eaque capiatur $RL = E\theta = a + \int u\,dr$, & fiat $RK = RL$, junctaque KL per interſectionem C in recta Dd axi AB agatur parallela CM, erit M punctum in curva quæſita.

Exemplum 1.

TAB. II Fig. 1. LX. Cum v debeat eſſe functio impar ipſarum v & u, ponatur $v = bu$, erit $\frac{dv}{du} = b$, & $p = a$: atque $r = b$: ita ut omnes radii reflexi Mm in eodem puncto axem ſecent; ex quo jam perſpicuum eſt, curvam FBf fore parabolam, focum in R habentem. Quod idem æquatio ſtatim indicat; fit enim $AP = x = b(1+u) - \frac{u(a-bu)}{1-u} = \frac{b-au}{1-u}$ & $PM = y = \frac{s(a-bu)}{1-u} - bs = \frac{(a-b)s}{1-u} = (a-b)\sqrt{\frac{1+u}{1-u}}$, ut ſit $y^2 = (a-b)^2 . \frac{1+u}{1-u}$. At eſt $\frac{1+u}{1-u}\,\frac{a+b-2x}{a-b}$, unde fit $yy = (a-b)(a+b-2x)$. Vertex ergo hujus parabolæ prodit, ſi $y = 0$, hoc eſt, ſi $u = -1$, quo caſu fit $x = AB = \frac{a+b}{2}$ & parameter hujus parabolæ eſt $= 2(a-b)$. quadruplo major, quam $BR = AB - AR = \frac{a-b}{2}$. Per con- Fig. 4. ſtructionem autem præcedentem hæc parabola prodit, ſi IAi ſtatuatur linea recta; tum enim ob $A\theta = 0$ erit $RL = EA$ conſtans, ſimiliterque ob directionem $I\theta$ conſtantem erit quoque dR conſtans, ideoque punctum R fixum.

Exem-

Exemplum 2.

TAB. II Fig. 1.

LXI. Ponamus, eſſe $v = bu + cs$, ob $ds = \frac{-udu}{s}$, erit $\frac{dv}{du} = b - \frac{cu}{s} = r = AR$, & $p = a + bu \frac{-cuu}{s} - bu - cs$, ſeu $p = a - \frac{c}{s}$: unde fit $r - up = b - au$ & $p - r = a - b - \frac{c(1-u)}{s}$, ideoque $AP = x = \frac{b - au}{1-u} = \frac{a+b}{2} - \frac{(a-b)(1+u)}{2(1-u)}$ & $PM = y = -c + \frac{(a-b)s}{1-u} = -c + (a-b)\surd\frac{1+u}{1-u}$: unde $\frac{1+u}{1-u} = \frac{(y+c)^2}{(a-b)^2}$, qui valor ibi ſubſtitutus dabit $x = \frac{a+b}{2} - \frac{(y+c)^2}{2(a-b)}$: quæ æquatio eſt pro parabola quoque, cujus axis recta AB eſt parallelus, ab eoque intervallo $= a$ diſtat; parameter vero eſt $= 2(a-b)$. Patet ergo, hanc hypotheſin non aliam dare curvam atque præcedentem, cum ex utraque oriatur parabola inter omnes, quæ Problemati ſatisfaciunt, ſine dubio ſimpliciſſima.

Exemplum 3.

LXII. Ponamus $v = bu + cu^3$, ut fit $\frac{dv}{du} = b + 3 - cuu = AR = r$ & $p = a + 2cu^3$; unde fit $r - up = b + 3cuu - au - 2cu^4$ & $p - r = a - b - 3cuu + 2cu^3$. Habebimus ergo $AP = x = \frac{b - au + 3cuu - 2cu^4}{1-u}$

$PM = y = (a - b - 3cuu + 2cu^3)\surd\frac{1+u}{1-u}$. Sit $a = b + c$, atque prodibunt ſequentes formulæ: $AP = x = b - cu + 2cuu + 2cu^3 = b - cu(1 - 2u - 2uu)$.

$PM = y = c(1 + u - 2uu)\sqrt{}(1 - uu) = c(1 - u)(1 + 2u)\sqrt{}(1 - uu)$. Ad curvam autem per plurima puncta conſtruendam præſtabit notaſſe, eſſe $AR = b + 3cuu$; coſ. $ARM = u$; & $RM + MC = a + 2cu^3$, ſeu $RM = \frac{a - b - 3cuu + 2cu^3}{1 - u}$: eritque $Rm + mc = a - 2cu^3$, ſeu $Rm = \frac{a - b - 3cuu - 2cu^3}{1 + u}$. Tum vero pro cauſtica erit $\frac{ddv}{du^2} = 6cu$ & $RO = - 6cu(1 - uu)$, unde fit $AS = X = b + 3cuu - 6cuu(1 - uu) = b - 3cuu + 6cu^4$ & $OS = Y = 6cu(1 - uu)\sqrt{}(1 - uu)$.

Exemplum 4.

LXIII. Ponamus $v = bu + \frac{c}{u}$, erit $\frac{dv}{du} = b - \frac{c}{uu} =$ $AR = r$ & $RM + MC = p = a + bu - \frac{c}{u} - bu - \frac{c}{u} = a - \frac{2c}{u}$: unde fit $r - up = b - \frac{c}{uu} - au + 2c$ & $p - r = a - b - \frac{2c}{u} + \frac{c}{uu}$, atque $AP = x = \frac{(b + 2c)uu - c - au^3}{uu(1 - u)}$ & $PM = y = (a - b - \frac{2c}{u} + \frac{c}{uu})\sqrt{}\frac{1 + u}{1 - u}$. Statuamus, quo hæ formulæ fiant ſimpliciores $a = b + c$, erit $AP = x = b - \frac{c}{uu} - \frac{c}{u} + c$, & $PM = y = c\left(\frac{1 - u}{uu}\right)\sqrt{}(1 - uu)$, vel $x = a - \frac{c}{u} - \frac{c}{uu}$; & $y = - c\left(\frac{1}{u} - \frac{1}{uu}\right)\sqrt{}(1 - uu)$.

Ponatur $t = a - x$, ut fit $t = \frac{c}{u} + \frac{c}{uu}$, & $y = c\left(\frac{1}{u} - \frac{1}{uu}\right)\sqrt{}(1 - uu)$, ex quarum illa oritur $u = \frac{c + \sqrt{}(cc + 4ct)}{2t}$

$\frac{1}{u} = \frac{-c+\sqrt{(cc+4ct)}}{2t}$ & $\frac{1}{uu} = \frac{2cc+4ct-2c\sqrt{(cc+4ct)}}{4cc}$ $= \frac{c+2t-\sqrt{(cc+4ct)}}{2c}$. Hinc fit $c\left(\frac{1}{u} - \frac{1}{uu}\right) = -c-t+ \sqrt{(c\,\text{[illegible]}\,4ct)}$; eſt vero $1-uu = \frac{4tt-2cc-4ct-2c\sqrt{(c^2+4ct)}}{4tt}$. ideoque $2t^2yy = (c+t-\sqrt{(cc+4ct)})^2(2tt-cc-2ct-c\sqrt{(cc+4ct)})$ ac proinde $2yy+cc-10ct-2tt = -(c+4t)\sqrt{(cc+4ct)}$ ſeu $y^4+(cc-10ct-2tt)yy+t^4-6ct^3+12cctt-8c^3t=0$. Habemus ergo lineam quarti ordinis quæſtioni ſatisfacientem, quæ hac æquatione commodius exprimitur:

$$16yy = (\sqrt{(c+4t)}-3\sqrt{c})^3(\sqrt{(c+4t)}+\sqrt{c})$$

vel ſi ponatur $c=4e$ & $t+e$ [illegible] x habebitur:

$$yy = (\sqrt{x}-3\sqrt{e})^3(\sqrt{x}+\sqrt{e}) = xx-3x\sqrt{ex}+18ex-27ee.$$

Ad cauſticam vero hujus curvæ inveniendam eſt $\frac{ddv}{du^2} = \frac{2c}{u^3}$ & $RO = \frac{-2c(1-uu)}{u^3}$: unde $AS = X = b - \frac{3c}{uu} + 2c$ & $OS = Y = \frac{-2c(1-uu)\sqrt{(1-uu)}}{u^3}$. Ponatur alia abſciſſa $b+2c-X=T$, erit $\frac{3c}{uu}=T$, & $uu = \frac{3c}{T}$, unde f[illegible] $\frac{3cY}{T}\sqrt{\frac{3c}{T}} = -2c\left(1-\frac{3c}{T}\right)^{\frac{3}{2}}$, ſeu $3Y\sqrt{3c} = -2(T-3c)^{\frac{3}{2}}$. Ponatur abſciſſarum initium denuo mutando $T-3c=X$, erit $3Y\sqrt{3c} = -2X\sqrt{X}$, & $27cYY = 4X^3$; eſt ergo cauſtica hæc parabola cubicalis Neoliana, diametrum habens radiis incidentibus parallelam.

JOANNIS HEUMANNI, JUR. PROFESSORIS Altorfini, Commentarii de re diplomatica Imperatorum ac