null
\[ \frac{d}{dx} \left( \frac{d}{dy} f(x, y) \right) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \]

\[ \int \left( \frac{d}{dx} \frac{d}{dy} f(x, y) \right) dx = \frac{d}{dy} \int \frac{d}{dx} f(x, y) dx - \frac{d}{dx} \int \frac{d}{dy} f(x, y) dy \]

\[ \int \left( \frac{d}{dy} \frac{d}{dx} f(x, y) \right) dy = \frac{d}{dx} \int \frac{d}{dy} f(x, y) dy - \frac{d}{dy} \int \frac{d}{dx} f(x, y) dx \]
Theorem 1: If \( \Delta ABC \) is a triangle with sides \( a, b, c \), then the area \( \Delta \) of the triangle is given by

\[
\Delta = \frac{1}{2} ab \sin \gamma
\]

where \( \gamma \) is the angle between sides \( a \) and \( b \).

Proof: Consider \( \Delta ABC \) with sides \( a, b, c \) and angle \( \gamma \) between sides \( a \) and \( b \). Draw a line \( CD \) parallel to \( AB \) through \( C \), and let \( D \) be the point where this line intersects \( AB \). Since \( CD \parallel AB \), \( \Delta ACD \) is similar to \( \Delta ABC \), and the ratio of their areas is

\[
\frac{\text{area of } \Delta ACD}{\text{area of } \Delta ABC} = \left( \frac{AC}{AB} \right)^2 = \left( \frac{a}{b} \right)^2
\]

Also, since \( DC \) is parallel to \( AB \), \( \angle ADC = \angle ABC \). Therefore, \( \Delta ACD \) is a right triangle, and its area is

\[
\text{area of } \Delta ACD = \frac{1}{2} \cdot DC \cdot AC = \frac{1}{2} b \cdot AC \cdot \sin \gamma
\]

Similarly, the area of \( \Delta BCD \) is

\[
\frac{1}{2} b \cdot BC \cdot \sin \gamma = \frac{1}{2} b \cdot c \cdot \sin \gamma
\]

Adding the areas of \( \Delta ACD \) and \( \Delta BCD \), we get

\[
\frac{1}{2} b \cdot AC \cdot \sin \gamma + \frac{1}{2} b \cdot c \cdot \sin \gamma = \frac{1}{2} b \cdot (AC + c) \cdot \sin \gamma
\]

Since \( \Delta ABC \) is the sum of \( \Delta ACD \) and \( \Delta BCD \), the area of \( \Delta ABC \) is

\[
\Delta = \frac{1}{2} b \cdot (AC + c) \cdot \sin \gamma
\]

Which is the desired result.
\[ \phi \frac{\partial u}{\partial x} = \frac{\partial \phi}{\partial x} \frac{1}{L} \]

Definition: The wave function $\phi$ is an eigenfunction of the operator $\frac{\partial^2}{\partial x^2}$. The wave number $k$ is related to the wavelength $\lambda$ by $k = \frac{2\pi}{\lambda}$.

**Example:**

\[ \phi(x) = e^{-x^2} \]

Find the first few eigenfunctions and their corresponding eigenvalues.

**Solution:**

The eigenfunctions are given by $\phi_n(x) = \sqrt{\frac{2}{\pi}} e^{-\frac{n^2 \pi^2 x^2}{4}}$, where $n$ is a non-negative integer. The corresponding eigenvalues are $\lambda_n = n \pi$.

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**Section 3:**

In the text, there is a discussion on the properties of the wave equation and its solutions. The text mentions the use of Green's functions to solve the equation in various contexts.

**Example:**

Consider the wave equation in one dimension:

\[ \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \]

where $c$ is the wave speed. The solutions to this equation can be found using the method of separation of variables.

**Solution:**

The general solution is given by $u(x,t) = \sum_{n=1}^{\infty} A_n \sin \left( n \pi \frac{x}{L} \right) \cos \left( n \pi \frac{ct}{L} \right)$, where $A_n$ are determined by the initial and boundary conditions.

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**Section 4:**

The text discusses the application of the wave equation in various physical contexts, including acoustics and electromagnetism.

**Example:**

In acoustics, the wave equation is used to model sound propagation in fluids and solids. The solutions to this equation can be used to predict the behavior of sound waves in different environments.

**Solution:**

For a wave in a fluid, the equation is modified to account for the absorption and dispersion of the wave. The solution can be found using numerical methods or by approximating the wave as a series of simpler waves.

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The text concludes with a discussion on the limitations of the wave equation and the need for more advanced models in certain situations.
\[
\frac{E + \epsilon}{1 + \epsilon} + \frac{\epsilon + \epsilon}{1 + \epsilon} = \frac{n(n-1)}{n^2} \int \frac{f}{n}.
\]

Consider the following integral:

\[
\int \frac{n(n-1)}{n^2} f(n, \eta, \xi) \, d\eta d\xi.
\]

Hint: Use integration by parts and the property of Green's functions.

Using the above expression, derive the following result:

\[
\int (n\eta + \xi) \, d\eta d\xi = \frac{L - g}{(L + w)(g - w)} + \frac{1}{(L + w)(\xi - w)} + \frac{1}{(L + w)(\xi - w)} + 1.
\]

Finally, evaluate the definite integral from 0 to 1.