donec in gibba abdominis regione ſuperius ad umbilicum ſe prodiderint veſiculæ quædam, gangrænæ futuræ præſagia, quibus ruptis, ingens humorum prodiit copia, ac funiculi umbilicalis portio ſiderata ſummo cum fœtore, infauſti rei augurio, fuit. Poſtquam deinceps ſiniſtra fœtus manus ad cubiti flexum per integumenta corporis communia, jam gangræna & putredine ſqualida, ſibi exitum quæſiverat; Noſter, fœtum extracturus, ſub exporrecta ejus manu brachioque, ſiniſtræ ſuæ abdomini ingerit ſibi conductorem indicem, ac validiſſimo ſcalpello primum corporis integumenta diſcindit, mox muſculos, tandem peritonæum, pertundit, hacque ſectione peracta inuncta oleo raparum manum immittit vulneri, unumque & alterum extrahit pedem, poſthæc utrumque brachium, poſtremo autem totum non mediocris magnitudinis, & fœtentem, integrum tamen, eximit cæſonem. Sed progrediamur ad reliquas
Pag. 70. obſervationes, inter quas & obveniunt peſſus ob uteri lapſum per ſeptendecim annos geſtatus, e vulva extractus, molarum cauſæ indagatio, in qua Noſter exiſtimat, eas nunc ſine prægreſſo coitu generari, eo ſcilicet modo, ut ovum ſterile, ex ovario dimiſſum, ſuis floccis uteri tunicis appoſitum, incrementa capiat, & poſtea in molam degeneret, & idcirco a virginibus etiam atque viduis ſine ulla labis ſuſpicione excludi queant, nunc vero a ſecundinis, fœtu, per abortum excluſo, relictis, concreſcere, ſarcomatis indolem acquirere, nec ulla cavitate, quæ in mola, ab ovo ſterili exorta, obſervetur,
77. 80. inſtructum eſſe, porro hydropis ovarii hiſtoria, exulcerati
90. 92. perinæi hiſtoria, ulceris fiſtuloſi pertinaciſſimi hiſtoria, herpe-
94. tis eſthiomeni, ſive corroſivi, exemplum memorabile, tumor
96. oculi & palpebræ cancroſus, lentis cryſtallinæ ex oculo expul-
98. ſio, conſpicui tumoris, ſuperveniente gangræna in fœtidum
99. ulcus converſi, hiſtoria, ulceris cum labiis duris, oris inverſis,
101. 103. & vix credibili intumeſcentia, exemplum, inſignis tumoris car-
105 ſeq. cinomatoſi, e variis tuberculis conglomerati, cæſura, varia ſpinæ
125. ventoſæ exempla, & animadverſiones in illam, meliceria *Celſi*,
134. anchyle, ſive anchyloſis, ejusque diſcrimina, hydropis articulorum

culorum quatuor exempla, funeſti pedis ſphaceli genu trans- Pag. 138.
gredientis hiſtoria, ſphaceli diriſſimi exemplum, alius ſphaceli 141.
curati hiſtoria, & denique linguæ ſtupendæ magnitudinis facul- 137.
tatisque exemplum. Ex his obſervationibus nonnullæ iconibus illuſtrantur, quæ, elegantiſſime in æs inciſæ, non ſecus ac chartæ typorumque nitor, librum hunc non mediocriter commendant.

SOLUTIO PROBLEMATIS, IN NOV. ACTOrum Erud. Menſe Novembri A. 1743 propoſiti, *Autore* LEONH. EULERO.

1. Quamvis hoc Problema ad methodum tangentium inverſam, jam pridem ſatis pertractatam & excultam, pertinere videatur; tamen ob plurimas inſignes proprietates, quæ de Curvis, Problemati ſatisfacientibus, in ipſa propoſitione ſunt commemoratæ, dignum profecto haberi debet, ad quod ſolvendum Geometræ ſtudium atque operam impendant. Eſt enim plane ſingularis indolis, quod non ſolum innumerabiles Curvæ algebraicæ ſolutioni inſerviant, ſed etiam quilibet linearum ordo unicam curvam ſatisfacientem ſuppeditet. Circa reliquas Curvas ſatisfacientes notabile quoque eſt, quod eæ ad duas claſſes referantur, quarum altera conſtructionem ope quadraturæ circuli poſtulat, altera vero per nullas ſolitas quadraturas perfici queat. Hujusmodi autem Problemata inprimis ſunt attentione digna, propterea quod eorum ſolutio plerumque cum non contemnenda finium analyſeos amplificatione ſoleat eſſe conjuncta. Problema igitur, cujus ſolutionem hic ex-
ponam, huc redit: *Invenire lineam curvam AMB, in cujus axe* TAB. II
AB duo dentur ejusmodi puncta C & D, ut, ductis inde ad quod- Fig. 1.
vis peripheriæ punctum M rectis CM & DM, areæ ACM ex puncto C reſectæ perpetuo ſint proportionales angulis ADM, ad alterum punctum D formatis.

2. Ad hoc Problema ſolvendum ponatur diſtantia punctorum $CD = a$, ac, demiſſo ex puncto M ad axem AB perpendiculo MP, vocentur coordinatæ $DP = x$, $PM = y$. Ponatur

tur $\frac{x}{y} = t$, erit t cotangens, ſeu $\frac{1}{t}$ tangens anguli ADM poſito ſinu toto $= 1$. Hinc ipſe angulus erit $= \int \frac{-dt}{1+tt}$. Deinde, ſi ponatur $AP = p$, erit area $APM = \int y\,dp$, ad quam ſi addatur triangulum $CPM = \frac{1}{2}(x-a)y$, habebitur area $ACM = \int y\,dp + \frac{1}{2}xy - \frac{1}{2}ay$. Cum vero ſit $p + x =$ conſt. AD, erit $dp + dx = 0$, ideoque $dp = -dx$. Quoniam igitur area ACM debet eſſe proportionalis angulo ADM, ponatur ad homogeneitatem conſervandam $ACM = \frac{1}{2}cc.\ ADM$. quod in ſymbolis dat:

$$\int y\,dp + \tfrac{1}{2}xy - \tfrac{1}{2}ay = \tfrac{1}{2}cc\int\frac{-dt}{1+tt},$$

quæ differentiata dabit ob $dp = -dx$ hanc æquationem:

$$-ydx + xdy - ady = \frac{-ccdt}{1+tt}.$$

3. Variatis ergo ſignis, habebimus hanc æquationem, in qua ſolutio Problematis continebitur:

$$ydx - xdy + ady = \frac{ccdt}{1+tt};$$

in qua, etſi tres inſunt variabiles, x, y, & t, tamen ope æquationis ante aſſumtæ $\frac{x}{y} = t$, una pro lubitu eliminabitur. Commodiſſime autem ejicietur x, ope æquationis $x = ty$, quæ dat $dx = tdy + ydt$, unde emerget hæc æquatio:

$$ady + yydt = \frac{ccdt}{1+tt}.$$

Quæ cum ad formam famoſæ æquationis *Riccatianæ* ſit referenda, omnia quoque artificia, quæ in illius reſolutione ſunt adhibita, hic in uſum vocari debent.

4. Integratio atque conſtructio hujus æquationis potiſſimum pendet a ratione, quam conſtantes a & c inter ſe tenent.

Facile

Facile autem, qui in æquatione *Riccatiana* reſolvenda elaboraverit, agnoſcet, in æquatione hic inventa innumerabiles curvas tam algebraicas, quam tranſcendentes, contineri. Quo autem illas primum faciliùs eruere queam, transformo æquationem differentialem inventam in aliam differentialem ſecundi gradus, ponendo $y = \frac{adz}{zdt}$, & ſumto elemento dt conſtante, ſuperior æquatio transmutabitur in hanc:

$$aaddz = \frac{cczdt^2}{1+tt} \text{ ſeu } aaddz\,(1+tt) = cczdt^2.$$

5. Hæc æquatio jam iſtud habet commodum, ut variabilis z unica tantum dimenſio in ea occurrat, quæ conditio perquam eſt idonea ad valorem ipſius z per ſeriem infinitam eruendum, id quod ſequenti modo efficio. Ponatur:

$$z = f + gt + \alpha t^2 + \beta t^3 + \gamma t^4 + \delta t^5 + \varepsilon t^6 + \zeta t^7 + \&c.$$

eritque bis differentiando

$$\frac{ddz}{dt^2} = 2\alpha + 6\beta t + 12\gamma t^2 + 20\delta t^3 + 30\varepsilon t^4 + 42\zeta t^5 + \&c.$$

Cum igitur ſit $ccz = \frac{aaddz}{dt^2} + \frac{aattddz}{dt^2}$, erit

$$ccf + ccgt + cc\alpha t^2 + cc\beta t^3 + cc\gamma t^4 + cc\delta t^5 + \&c. =$$
$$2aa\alpha + 6aa\beta t + 12aa\gamma t^2 + 20aa\delta t^3 + 30aa\varepsilon t^4 + 42aa\zeta t^5 + \&c.$$
$$+ 2aa\alpha t_2 + 6aa\beta t^3 + 12aa\gamma t^4 + 20aa\delta t^5 + \&c.$$

ubi coefficiẽntes α, β, γ, &c. ita determinari debent, ut æqualitas utrinque producatur.

6. Coæquando autem terminos homogeneos, ſequentes inveniuntur valores:

$$\alpha = \frac{cc}{2aa}f$$

$$\beta = \frac{cc}{6aa}g$$

$\gamma =$

$$\gamma = \frac{(cc-2aa)\alpha}{12aa} = \frac{cc(cc-1.2aa)}{1.2.3.4a^4} f$$
$$\delta = \frac{(cc-6aa)\beta}{20aa} = \frac{cc(cc-2.3aa)}{1.2.3.4.5a^4} g$$
$$\varepsilon = \frac{(cc-12aa)\gamma}{30aa} = \frac{cc(cc-1.2aa)(cc-3.4aa)}{1.2.3.4.5.6a^6} f$$
$$\zeta = \frac{(cc-20aa)\delta}{42aa} = \frac{cc(cc-2.3aa)(cc-4.5aa)}{1.2.3.4.5.6.7a^6} g$$
&c.

His igitur valoribus in æquatione assumta
$z = f + gt + \alpha t^2 + \beta t^3 + \gamma t^4 + \delta t^5 + \varepsilon t^6 + \zeta t^7 +$ &c. substitutis, obtinebitur æquatio integralis completa æquationis differentio-differentialis $aaddz\,(1+tt) = cczdt^2$, id quod indicant binæ constantes arbitrariæ f & g, quæ per duplicem integrationem ingressæ sunt censendæ.

7. Inventa est ergo sequens æquatio generalis, solutionem propositi Problematis complectens:

$$z = \begin{cases} f + \dfrac{cc}{1.2aa} ft^2 + \dfrac{cc(cc-1.2aa)}{1.2.3.4a^4} ft^4 + \\ \qquad \dfrac{cc(cc-1.2aa)(cc-3.4aa)}{1.2.3.4.5.6a^6} ft^6 + \text{\&c.} \\ + gt + \dfrac{cc}{1.2.3aa} gt^3 + \dfrac{cc(cc-2.3aa)}{1.2.3.4.5a^4} gt^5 + \\ \qquad \dfrac{cc(cc-2.3aa)(cc-4.5aa)}{1.2.3.4.5.6.7a^6} gt^7 + \text{\&c.} \end{cases}$$

ex qua si formetur $y = \frac{adz}{zdt}$, & $x = ty$, orietur æquatio generalis algebraica inter x & y, omnes lineas curvas Problemati satisfacientes comprehendens, quæ autem cum terminis numero infinitis constet, re vera curvas transcendentes æque complectitur, atque algebraicas. Quodsi autem vel f, vel g, ponatur $= 0$, binæ æquationes resultabunt particulares hæ:

$$\text{I. } z = 1 + \frac{cc}{1.2aa}t^2 + \frac{cc(cc-1.2aa)}{1.2.3.4a^4}t^4 + \frac{cc(cc-1.2aa)(cc-3.4aa)}{1.2.3.4.5.6a^6}t^6 + \text{\&c.}$$

$$\text{II. } z = t + \frac{cc}{2.3aa}t^3 + \frac{cc(cc-2.3aa)}{2.3.4.5a^4}t^5 + \frac{cc(cc-2.3aa)(cc-4.5aa)}{2.3.4.5.6.7a^6}t^7 + \text{\&c.}$$

8. Hæ æquationes etsi in infinitum progrediuntur, tamen certis casibus numerus terminorum fit finitus, ulterioribus terminis evanescentibus omnibus. Perspicuum enim est, si cujuspiam termini coefficiens fiat $= 0$, tum sequentes terminos omnes simul evanituros. His igitur casibus æquationes istæ suggerent curvas algebraicas quæsito satisfacientes; hocque adeo eveniet, si cc ad aa ita fuerit comparatum, ut coefficiens cujuspiam termini evanescat; ex quo curvæ algebraicæ obtinebuntur sequentibus casibus: si

1. $cc = 0$
2. $cc = 1.2aa$
3. $cc = 2.3aa$
4. $cc = 3.4aa$
5. $cc = 4.5aa$
6. $cc = 5.6aa$

& ita porro.

9. Primo quidem casu, quo $cc = 0$, fit vel $z = 1$, vel $z = t$. Ex priori oritur $y = 0$, qua æquatione indicatur linea recta cum axe AB congruens; ex posteriori fit $y = \frac{a}{t}$ & $x = a$, quæ æquatio pariter est pro linea recta, quæ vero axem AB in puncto C normaliter trajiciet. Quin, posito $cc = 0$, etiam æquatio generalis fit finita, eritque $z = f + gt$; hinc $y = \frac{ag}{f+gt}$ & ob $t = \frac{x}{y}$ erit $y = \frac{agy}{fy+gx}$ seu $ag = fy$

$= fy + gx$, quæ eſt pro linea recta quacunque, axem AB in puncto C ſecante. Facillime autem patet, quemadmodum hujusmodi linea recta ſatisfaciat. Area enim ACM perpetuo erit $= 0$, ac propterea æqualis angulo CDM per $\frac{1}{2}cc = 0$ multiplicato. Jam itaque adepti ſumus lineam algebraicam ex primo ordine, rectam ſcilicet, quæ Problema reſolvit.

10. Ponamus nunc pro caſu ſecundo $cc = 1.2aa$, ſeu $c = a\sqrt{2}$, atque æquatio I induet hanc formam finitam $z = 1 + tt$, unde oritur $y = \frac{2at}{1+tt} = \frac{2axy}{xx+yy}$ ob $t = \frac{x}{y}$.

TAB. II Fig. 2. Hinc fluit iſta æquatio $xx + yy = 2ax$; quæ eſt pro circulo, qui quemadmodum ſatisfaciat, ex elementis planum eſt. Cadet enim punctum C in hujus circuli centrum, & D in ejus peripheriam, radio exiſtente $= CD = a$; ubi cum angulus CDM ſit ſemiſſis anguli ACM, erit quoque ſectori ACM proportionalis. Præterea vero, cum ſit area $ACM = \frac{1}{2}a.AM$, & angulus $CDM = \frac{1}{2}AM:a$, fiet area $ACM =$ ang. $CDM.aa =$ ang. $CDM.\frac{1}{2}cc$. Circulus igitur eſt linea ſecundi ordinis ſatisfaciens.

11. Sit pro tertio caſu $cc = 6aa$, atque æquatio ſecunda dabit $z = t + t^3$, unde fit $y = \frac{a(1+3tt)}{t+t^3} = \frac{a(y^3+3xxy)}{x(xx+yy)}$, ſeu $x^3 + xyy = ayy + 3axx$, quæ præbet lineam tertii ordinis, ad cujus naturam inveſtigandam ſpectetur æquatio:

$$yy = \frac{(3a-x)xx}{x-a}.$$

Fig. 3. Hæc igitur curva AM ab axe ACD in duas partes æquales dividitur; in D habebit punctum conjugatum, & normalis ad axem ex puncto C educta CE erit curvæ aſymptotos, vertexque curvæ cadet in A, ut ſit $DA = 3CD$. Hæc curva a *Neutono* in Enumeratione linearum tertii ordinis vocatur conchoidalis, atque ad ſpeciem 44 refertur. Erit ergo quoque in hac curva area ACM angulo ADM proportionalis.

12. Sit

12. Sit porro pro caſu quarto $cc = 12aa$, atque æquatio I dabit $z = 1 + 6tt + 5t^4 = (1+tt)(1+5tt)$, unde fit $y = \frac{a(12t+20t^3)}{(1+tt)(1+5tt)} = \frac{4ay(3xyy+5x^3)}{(xx+yy)(yy+5xx)}$, hincque $(xx+yy)(yy+5xx) = 12axyy + 20ax^3$, quæ eſt æquatio pro linea quarti ordinis AMD, cujus diameter erit linea ACD, eritque $AD = 4CD$. Ceterum hæc curva eſt in ſe rediens, punctumque D in ejus perimetro collocatur. TAB. II Fig. 4.

13. Simili modo, ſi pro caſu quarto ponatur $cc = 20aa$, æquatio II dabit $z = t + \frac{10}{3}t^3 + \frac{7}{3}t^5 = \frac{1}{3}t(1+tt)(3+7tt)$, unde fit $y = \frac{3a+30att+35at^4}{t(1+tt)(3+7tt)}$, ex qua ponendo $t = \frac{x}{y}$ naſcitur ſequens æquatio pro linea quinti ordinis:

$$x(yy+xx)(3yy+7xx) = 3ay^4 + 30axxyy + 35ax^4.$$

Hæc curva, ut ea, quæ ex tertio ordine eſt inventa, habebit punctum conjugatum in D, in C vero aſymptoton ad axem normalem, at verticem in A, ut ſit $DA = 5CD$.

14. Ex his jam intelligitur, ſi ponatur $cc = 30aa$, prodituram eſſe lineam ordinis ſexti in ſe redeuntem, in qua ſit $AD = 6CD$; tum vero, poſito $cc = 42aa$, linea prodibit ſeptimi ordinis, in qua ſit $AD = 7CD$, & applicata in C in aſymptoton abibit. Atque hoc modo ulterius progrediendo ex ſingulis ſequentibus linearum ordinibus una curva ſatisfaciens elicietur. Ex ordinibus quidem paribus curva illa erit in ſe rediens, ex imparibus vero aſymptoton habebit ad axem in puncto C normalem. Et, quoniam hoc pacto omnes curvæ algebraicæ reperiuntur, ſimul patet, ex quolibet linearum ordine plures una linea ſatisfaciente non dari.

15. Apparet ergo, curvas algebraicas exhiberi poſſe, quoties $\frac{cc}{aa}$ fuerit numerus pronicus, ſeu $\frac{cc}{2aa}$ numerus trigonalis. Quo autem facilius ſingulis caſibus æquationes algebraicæ

inveniri

inveniri queant, ponamus, esse $\frac{cc}{aa} = (2n-1)\,2n$, sicque casus ad æquationem I pertinebit, unde orietur:

$$z = 1 + \frac{2n(2n-1)}{1 \cdot 2}tt + \frac{(2n-2)(2n+1)}{3 \cdot 4}Pt^4 + \frac{(2n-4)(2n+3)}{5 \cdot 6}Pt^6 + \&c.$$

ubi notandum, P significare ubique coefficientem termini præcedentis totum. Quæ expressio cum semper factorem habeat $1 + tt$, erit:

$$z = (1+tt)(1 + \frac{(2n-2)(2n+1)}{1 \cdot 2}tt + \frac{(2n-4)(2n+3)}{3 \cdot 4}Pt^4 + \frac{(2n-6)(2n+5)}{5 \cdot 6}Pt^6 + \&c.)$$

16. Simili modo ex æquatione II si ponatur $\frac{cc}{aa} = 2n(2n+1)$ erit:

$$z = t + \frac{2n(2n+1)}{2 \cdot 3}t^3 + \frac{(2n-2)(2n+3)}{4 \cdot 5}Pt^5 + \frac{(2n-4)(2n+5)}{6 \cdot 7}Pt^7 + \&c.$$

&, quia hæc expressio pariter per $1 + tt$ est divisibilis, erit

$$z = (1+tt)(t + \frac{(2n-2)(2n+3)}{2 \cdot 3}t^3 + \frac{(2n-4)(2n+5)}{4 \cdot 5}Pt^5 + \frac{(2n-6)(2n+7)}{6 \cdot 7}Pt^7 + \&c.)$$

ubi, ut ante, P coefficientem totum termini præcedentis denotat. Ex his ergo formulis ex quovis ordine linearum facile erit curvam satisfacientem invenire. Si enim sit $\frac{cc}{aa} = 2n(2n-1)$, curva erit linea ordinis $2n$, &, si sit $\frac{cc}{aa} = 2n(2n+1)$, curva erit

erit ex linearum ordine $2n+1$. Perpetuo scilicet, si statuatur $\frac{cc}{aa} = m(m+1)$, curva algebraica exhiberi poterit pertinens ad linearum ordinem $m+1$.

17. Quod deinde ratio inter partes axis AD & CD pendeat a ratione $cc:aa$, hæc est proprietas non solum ad curvas algebraicas, sed etiam ad omnes omnino curvas problemati satisfacientes, patens. Quodsi enim generaliter fuerit $cc = naa$, erit quoque $AD : CD = 1 + \sqrt{(1+4n)} : 2$. Nam concipiatur punctum M proxime ad A admoveri, &, quia curva ad A normaliter axi insistere considerari potest, ponatur arcus minimus $AM = s$, erit area $ACM = \frac{1}{2}AC.s$; angulus vero ADM erit $= \frac{s}{AD}$. Cum igitur sit $ACM = \frac{1}{2}cc.\,ADM$, fiet $AC.AD = cc = n.CD^2$, & $CD:AC = AD:n.CD$. Vel ob $AC = AD - CD$ erit $AD^2 = AD.CD + n.CD^2$, seu $\frac{AD}{CD} = \frac{1}{2} + \sqrt{(\frac{1}{4} + n)}$. Sin autem curva axi in A oblique insistat, tum s in data ratione minui debebit, sicque eadem, quæ ante, ratio resultabit. In curvis itaque algebraicis, cum $4n+1$ fiat numerus quadratus, ratio inter AD & CD semper rationaliter exprimetur, & quidem ipso numero ordinis indicabitur.

18. Hæc eadem proprietas colligitur ex æquatione $ady + yydt = \frac{ccdt}{1+tt}$. Ponatur enim applicata y evanescens, atque anguli ADM, qui hoc casu pariter evanescit, tangens sit $= s$, fiet $\frac{y}{AD} = s$, & $t = \frac{1}{s}$; ac, posita $DA = b$, erit $y = bs$. His valoribus substitutis, habebitur $abds - bbds = -\frac{ccds}{1+ss}$, quæ ob $s = 0$ abit in $bb = ab + cc$, & dat $b = \frac{1}{2}a + \sqrt{(\frac{1}{4}aa + cc)}$. Quare, si sit $cc = naa$, erit $\frac{AD}{CD} = \frac{1+\sqrt{(1+4n)}}{2}$:

uti

uti ante invenimus; ita, ut, si detur ratio inter cc & aa, simul punctum A innotescat, ubi curva axem intersecabit.

19. Cum igitur curvas algebraicas, quæ Problemati satisfaciunt, exhibuerim; progredior ad alteram curvarum classem, quæ ita transcendentes esse in propositione indicantur, ut per quadraturam circuli construi queant. Primum autem observo, has curvas ex iisdem conditionibus valoris cc inveniri, ex quibus modo curvas algebraicas erui, seu hæ curvæ obtinebuntur ponendo, ut ante, $cc = m(m+1)a^2$, existente m numero quocunque integro. Quanquam enim, si statuatur $cc = m(m+1)aa$, valor algebraicus pro z invenitur, qui propterea æquationi $aaddz(1+tt) = cczdt^2$ satisfacit, tamen is non integrale completum constituit, sed præterea dantur infiniti alii valores, qui pro z substituti æquationem æque resolvunt.

20. Quod quo clarius perspiciatur, consideremus tantum casum $cc = 2aa$, pro quo invenimus $z = 1 + tt$, qui autem valor tantum ex æquatione I §. 7 est ortus; æquatio igitur II pro eodem casu alium præbebit æque satisfacientem, qui erit

$$z = t + \tfrac{1}{3}t^3 - \frac{1}{3\cdot5}t^5 + \frac{1}{5\cdot7}t^7 - \frac{1}{7\cdot9}t^9 + \frac{1}{9\cdot11}t^{11} -$$

&c. Atque æquatio generalis, loco citato exhibita, præbebit

$$z = f(1+tt) + g\left(t + \frac{1}{1\cdot3}t^3 - \frac{1}{3\cdot5}t^5 + \frac{1}{5\cdot7}t^7 - \frac{1}{7\cdot9}t^9 + \&c.\right)$$

hæcque est integralis completa æquationis $aaddz(1+tt) = 2aazdt^2$, seu hujus $ddz(1+tt) = 2zdt^2$, quæ idcirco non solum circulum, facto $g = 0$, sed etiam infinitas alias lineas curvas, complectitur.

21. Reliquæ istæ lineæ curvæ, æquationem $ddz(1+tt) = 2zdt^2$ resolventes, ex expressione generali infinita inveniri possunt. Cum enim sit

$$z = f(1+tt) + g\left(t + \frac{1}{1\cdot3}t^3 - \frac{1}{3\cdot5}t^5 + \frac{1}{5\cdot7}t^7 - \frac{1}{7\cdot9}t^9 + \&c.\right)$$

ponatur $T = t + \frac{1}{1\cdot3}t^3 - \frac{1}{3\cdot5}t^5 + \frac{1}{5\cdot7}t^7 - \&c.$ ut sit $z = f(1+tt) + gT$. Resolvantur singuli termini seriei, qua T definitur, in binos hoc modo:

$$T = t + \tfrac{1}{2}\left\{\begin{array}{l} +t^3 - \tfrac{1}{3}t^5 + \tfrac{1}{5}t^7 - \tfrac{1}{7}t^9 + \&c. \\ -\tfrac{1}{3}t^3 + \tfrac{1}{5}t^5 - \tfrac{1}{7}t^7 + \tfrac{1}{9}t^9 - \&c. \end{array}\right.$$

Erit ergo $T = t + \frac{1}{2}t^3 - \frac{1}{2}(1+tt)(\frac{1}{3}t^3 - \frac{1}{5}t^5 + \frac{1}{7}t^7 - \frac{1}{9}t^9 + \&c.)$

22. Cum jam sit arcus, cujus tangens $= t$, seu $A\,\text{tag}\,t = t - \frac{1}{3}t^3 + \frac{1}{5}t^3 - \frac{1}{7}t^7 + \frac{1}{9}t^9 - \&c.$ erit $T = t + \frac{1}{2}t^3 - \frac{1}{2}(1+tt)(t - A\,\text{tag}\,t)$, sive $T = \frac{1}{2}t + \frac{1}{2}(1+tt)A\,\text{tag}\,t$, & $z = f(1+tt) + gt + g(1+tt)\,A\,\text{tag}\,t$, valore ipsius g duplicato. Hinc fit $dz = 2ftdt + 2gdt + 2gtdt\,A\,\text{tag}\,t$, & $y = \frac{adz}{zdt} = \frac{2a(g + ft + gt\,A\,\text{tag}\,t)}{f(1+tt) + gt + g(1+tt)A\,\text{tag}\,t}$, atque $x = ty = \frac{2at(g + ft + gt\,A\,\text{tag}\,t)}{f(1+tt) + gt + g(1+tt)A\,\text{tag}\,t}$, vel ob $t = \frac{x}{y}$ erit $f(yy+xx) + gxy + g(yy+xx)\,A\,\text{tag}\,\frac{x}{y} = 2agy + 2afx + 2agx\,A\,\text{tag}\,\frac{x}{y}$, quæ est æquatio pro omnibus curvis, quæ Problemati satisfaciunt casu $cc = 2aa$.

23. Uti hæc æquatio integralis pro casu $cc = 2aa$ eruta est ex summatione seriei infinitæ, generaliter valorem ipsius z exprimentis; ita simili modo pro reliquis casibus, quibus cc curvas algebraicas præbuit, æquationes, finita forma constantes, reperiri possunt. Pendebit autem perpetuo summatio seriei infinitæ a quadratura circuli, qua concessa, omnes prorsus curvæ, quibus Problema resolvitur casibus $cc = m(m+1)aa$, construi poterunt.

24. Hac autem ratione mox in calculos inextricabiles illaberemur. Quam ob causam aliam methodum exponam hanc alteram

alteram curvarum satisfacientium classem evolvendi. Sit igitur $cc = m(m+1)aa$, atque vocetur T valor algebraicus, quem ante pro z invenimus; erit igitur T functio algebraica ipsius t, quæ loco z substituta æquationem $(1+tt)\,ddz = m(m+1)\,zdt^2$ identicam reddit. Ponatur nunc valor ipsius z generalis $= Tu$, erit $ddz = u\,ddT + 2\,du\,dT + T\,ddu$, qui in æquatione substitutus dat $u(1+tt)\,ddT + 2(1+tt)\,du\,dT + T(1+tt)\,ddu = m(m+1)\,Tudt^2$.

25. Cum autem valor T pro z substitutus satisfaciat, erit $(1+tt)\,ddT = m(m+1)\,Tdt^2$, ideoque $m(m+1)\,Tudt^2 = u(1+tt)\,ddT$, unde illa æquatio induet hanc formam simplicem:

$$2\,du\,dT + T\,ddu = 0, \text{ seu } \frac{2\,dT}{T} + \frac{ddu}{du} = 0,$$

cujus integralis est $TTdu = Cdt$; porroque integrando fit $u = C\int\frac{dt}{TT}$, ideoque $z = CT\int\frac{dt}{TT}$. Pendet ergo valor ipsius z completus ab integratione formulæ $\int\frac{dt}{TT}$. Quæ quemadmodum per quadraturam circuli absolvi debeat, mox docebo.

26. Quoniam invenimus $z = CT\int\frac{dt}{TT}$, erit $dz = C\,dT\int\frac{dt}{TT} + \frac{Cdt}{T}$. Unde, cum sit $y = \frac{a\,dz}{z\,dt}$, fiet $y = \frac{a\,dT}{Tdt} + \frac{a}{TT\int\frac{dt}{TT}}$, & $x = \frac{a\,t\,dT}{Tdt} + \frac{a\,t}{TT\int\frac{dt}{TT}}$.

Concessa ergo integratione $\int\frac{dt}{TT}$ curva quæsita facile construi poterit.

27. Si m fuerit numerus impar, puta $m = 2n - 1$, habebit valor ipsius z algebraicus, quem ponimus T, hujusmodi valorem ex §. 15:

$$T = (1+tt)(1 + \alpha tt + \beta t^4 + \gamma t^6 + \delta t^8 + \&c.), \text{ existente}$$

$$\alpha = \frac{(m-1)(m+2)}{1 \cdot 2}$$

$$\beta = \frac{(m-3)(m+4)}{3 \cdot 4}\,\alpha$$

$$\gamma = \frac{(m-5)(m+6)}{5 \cdot 6}\,\beta$$

$$\delta = \frac{(m-7)(m+8)}{7 \cdot 8}\,\gamma \ \&c.$$

Sin autem sit m numerus par $= 2n$, erit ex §. 16.

$$T = (1+tt)(t + \alpha t^3 + \beta t^5 + \gamma t^7 + \delta t^9 + \&c.), \text{ existente}$$

$$\alpha = \frac{(m-2)(m+3)}{2 \cdot 3}$$

$$\beta = \frac{(m-4)(m+5)}{4 \cdot 5}\,\alpha$$

$$\gamma = \frac{(m-6)(m+7)}{6 \cdot 7}\,\beta$$

$$\delta = \frac{(m-8)(m+9)}{8 \cdot 9}\,\gamma \ \&c.$$

28. Ad integrale ergo $\int\frac{dt}{TT}$ inveniendum, incipiamus a casibus simplicissimis, sitque primo $m = 1$, seu $cc = 2aa$, eritque $T = 1 + tt$, $\frac{dT}{Tdt} = \frac{2t}{1+tt}$, & $\int\frac{dt}{TT} = \int\frac{dt}{(1+tt)^2} = \text{Const.} + \frac{\frac{1}{2}t}{1+tt} + \frac{1}{2}A\,\mathrm{tag}\,t$. Ponatur $A\,\mathrm{tag}\,t = 90° - \varphi$, ut exprimat φ angulum ADM, erit $t = \cot\varphi = \frac{\cos\varphi}{\sin\varphi}$, & $1 + tt = \frac{1}{\sin\varphi^2}$. Hinc fiet $\frac{dT}{Tdt} = 2\sin\varphi\cos\varphi = \sin 2\varphi$,

$\sin 2\varphi$, & $\int \frac{dt}{TT} = \text{Conſt.} + \frac{1}{4}\sin 2\varphi + 45^\circ - \frac{1}{2}\varphi$. Sit Conſt. $+ 45^\circ = \frac{1}{2}\theta$, erit $\int \frac{dt}{TT} = \frac{1}{2}\theta - \frac{1}{2}\varphi + \frac{1}{4}\sin 2\varphi$, & $TT = \frac{1}{\sin\varphi^4}$. Unde $TT\int \frac{dt}{TT} = \frac{2(\theta-\varphi)+\sin 2\varphi}{4\sin\varphi^4}$.

Atque ex his colligitur

$$\frac{y}{a} = \sin 2\varphi + \frac{4\sin\varphi^4}{2(\theta-\varphi)+\sin 2\varphi} \quad \&$$

$$\frac{x}{a} = 2\cos\varphi^2 + \frac{4\sin\varphi^3 . \cos\varphi}{2(\theta-\varphi)+\sin 2\varphi} \quad \text{atque}$$

$$\frac{DM}{a} = 2\cos\varphi + \frac{4\sin\varphi^3}{2(\theta-\varphi)+\sin 2\varphi}.$$

Ex longitudine autem DM cum angulo $ADM = \varphi$ curva facillime conſtruitur.

29. Ponamus nunc $m = 2$, ſeu $cc = 6aa$, erit $T = t(1+tt) = t + t^3$; & $\frac{dT}{Tdt} = \frac{1+3tt}{t(1+tt)}$, atque $\int \frac{dt}{TT} = \int \frac{dt}{tt(1+tt)^2} = \text{Conſt.} - \frac{1}{t} - \frac{\frac{1}{2}t}{1+tt} - \frac{3}{2}A\,\text{tag}\,t$. Sit iterum $A\,\text{tag}\,t = 90 - \varphi$; erit $t = \frac{\cos\varphi}{\sin\varphi}$; $1 + tt = \frac{1}{\sin\varphi^2}$; & $T = \frac{\cos\varphi}{\sin\varphi^3}$; $\frac{dT}{Tdt} = \frac{\sin\varphi^3 + 3\sin\varphi\cos\varphi^2}{\cos\varphi}$, atque $\int \frac{dt}{TT} = \text{Conſt.} - \frac{\sin\varphi}{\cos\varphi} - \frac{1}{2}\sin\varphi\cos\varphi - \frac{3}{2}.90^\circ + \frac{3}{2}\varphi = \frac{3}{2}(\theta+\varphi) - \frac{\sin\varphi}{\cos\varphi} - \frac{1}{2}\sin\varphi\cos\varphi$. Unde fit

$$\frac{y}{a} = \begin{cases} \dfrac{\sin\varphi^3 + 3\sin\varphi\cos\varphi^2}{\cos\varphi} \\ + \dfrac{2\sin\varphi^6}{3(\theta+\varphi)\cos\varphi^2 - 2\sin\varphi\cos\varphi - \sin\varphi\cos\varphi^3} \end{cases}$$

$\frac{x}{a} =$

$$\frac{x}{a} = \sin\varphi^2 + 3\cos\varphi^2 + \frac{2\sin\varphi^5}{3(\theta+\varphi)\cos\varphi - 2\sin\varphi - \sin\varphi\cos\varphi^2}$$

$$\& \frac{DM}{a} = \begin{cases} \dfrac{\sin\varphi^2 + 3\cos\varphi^2}{\cos\varphi} + \\ \dfrac{2\sin\varphi^5}{3(\theta+\varphi)\cos\varphi^2 - 2\sin\varphi\cos\varphi - \sin\varphi\cos\varphi^3} \end{cases}$$

30. Ponatur ulterius $m = 3$, ſeu $cc = 12aa$, erit $T = (1+tt)(1+5tt) = 1 + 6tt + 5t^4$, unde fit $\frac{dT}{Tdt} = \frac{12t + 20t^3}{(1+tt)(1+5tt)}$, atque $\int \frac{dt}{TT} = \int \frac{dt}{(1+tt)^2(1+5tt)^2} = \text{Conſt.} + \frac{t}{32(1+tt)} + \frac{25t}{32(1+5tt)} + \frac{3}{16}A\,\text{tag}\,t$. Poſito nunc $A\,\text{tag}\,t = 90^\circ - \varphi$: erit $t = \frac{\cos\varphi}{\sin\varphi}$; $1 + tt = \frac{1}{\sin\varphi^2}$; $1 + 5tt = \frac{1+4\cos\varphi^2}{\sin\varphi^2}$; $3 + 5tt = \frac{3+2\cos\varphi^2}{\sin\varphi^2}$. Ergo habebitur $\frac{dT}{Tdt} = \frac{4\sin\varphi\cos\varphi(3+2\cos\varphi^2)}{1+4\cos\varphi^2}$; $\int \frac{dt}{TT} = \frac{3}{16}(\theta-\varphi) + \frac{1}{32}\sin\varphi\cos\varphi + \frac{25\sin\varphi\cos\varphi}{32(1+4\cos\varphi^2)}$; hincque

$$\frac{DM}{a} = \frac{4\cos\varphi(3+2\cos\varphi^2)}{1+4\cos\varphi^2} + 32\sin\varphi^7 : (6(\theta-\varphi)(1+4\cos\varphi^2)^2 + \sin\varphi\cos\varphi(1+4\cos\varphi^2)^2 + 25\sin\varphi\cos\varphi(1+4\cos\varphi^2)).$$

31. Ex his tribus caſibus pertractatis luculenter jam intelligitur, in reliquis quoque caſibus integrationem $\int \frac{dt}{TT}$ conceſſa ſola circuli quadratura abſolvi poſſe, neque opus eſſe exiſtimo, ut ulterius hanc inveſtigationem proſequar. Hinc itaque colligo, ſi fuerit $cc = m(m+1)aa$, curvas Problemati ſatisfacientes vel eſſe algebraicas, & quidem ordinis $m+1$; vel eſſe tranſcendentes ope quadraturæ circuli conſtruibiles.

Tt 32. Su-

32. Superſunt reliqui Problematis caſus inveſtigandi, quibus non eſt $cc = m(m+1)aa$, vel quando m ſignificat numerum quemcunque non integrum, ſive fractum, ſive irrationalem. Atque his quidem caſibus formulæ, ſupra pro z inventæ, perpetuo in infinitum excurrunt. Interim tamen, ſi ponatur

$$T = 1 + \frac{m(m+1)}{1 \cdot 2} tt + \frac{(m-1)(m+2)}{3 \cdot 4} P t^4 + \frac{(m-3)(m+4)}{5 \cdot 6} P t + \&c.$$

ſeu

$$T = (1+tt)\left(1 + \frac{(m-1)(m+2)}{1 \cdot 2} tt + \frac{(m-3)(m+4)}{3 \cdot 4} P t^4 + \frac{(m-5)(m+6)}{5 \cdot 6} P t^6 + \&c.\right)$$

$$\& \; V = t + \frac{m(m+1)}{2 \cdot 3} t^3 + \frac{(m-2)(m+3)}{4 \cdot 5} P t^5 + \frac{(m-4)(m+5)}{6 \cdot 7} P t^7 + \&c.$$

ſeu

$$V = (1+tt)\left(t + \frac{(m-2)(m+3)}{2 \cdot 3} t^3 + \frac{(m-4)(m+5)}{4 \cdot 5} P t^5 + \frac{(m-6)(m+7)}{6 \cdot 7} P t^7 + \&c.\right)$$

erit $z = fT + gV$. Quare, ſi iſtæ expreſſiones infinitæ utcunque ad formas finitas perduci poſſent, quæſito foret ſatisfactum.

33. Cum igitur totum negotium ad ſummationem ſerierum reducatur, exponam hic ſingularem methodum, cujus ope jam olim æquationis *Riccatianæ* conſtructionem generalem exhibui, & quæ etiam in præſenti negotio magnam afferet utilitatem. Conſidero ſcilicet hanc expreſſionem $\int \frac{u^{\alpha-1} du}{(1-uu)^\mu} \int u^{\beta-1} du (1+kuu)^\nu$, cujus integrale utique per quadraturas exhiberi poterit. At poſt integrationem pono $u = 1$, quo variabilis u ex calculo egrediatur, & quantitatem ſic reſultantem voco

voco $= Q$, quæ ergo pariter per quadraturas aſſignari poteſt. Nunc exponentes α, β, μ, ν, cum coefficiente k ita definiri debebunt, ut ex quantitate Q ſuperiores expreſſiones T & V determinentur.

34. Cum igitur ſit $Q = \int \frac{u^{\alpha-1} du}{(1-uu)^\mu} \int u^{\beta-1} du (1+kuu)^\nu$ poſito poſt integrationem $u = 1$, iſtum ipſius Q valorem quoque per ſeriem infinitam exprimamus. Primo igitur eſt

$$(1+kuu)^\nu = 1 + \frac{\nu}{1} kuu^2 + \frac{\nu . \nu-1}{1 \cdot 2} k^2 u^4 + \frac{\nu . \nu-1 . \nu-2}{1 \cdot 2 \cdot 3} k^3 u^6 + \&c.$$

ideoque erit $\int u^{\beta-1} du (1+kuu)^\nu =$

$$\frac{u^\beta}{\beta} + \frac{\nu . k}{1.(\beta+2)} u^{\beta+2} + \frac{\nu . \nu-1 . k^2}{1 . 2(\beta+4)} u^{\beta+4} + \frac{\nu . \nu-1 . \nu-2 . k^3}{1 . 2 \cdot 3(\beta+6)} u^{\beta+6} + \&c.$$

Hinc Q exprimetur per ſequentem ſeriem formularum integralium ſimplicium:

$$Q = \frac{1}{\beta} \int \frac{u^{\alpha+\beta-1} du}{(1-uu)^\mu} + \frac{\nu . k}{1(\beta+2)} \int \frac{u^{\alpha+\beta+1} du}{(1-uu)^\mu} + \frac{\nu . \nu-1 . k^2}{1 . 2(\beta+4)} \int \frac{u^{\alpha+\beta+3} du}{(1-uu)^\mu} + \&c.$$

35. Formulæ iſtæ autem ita ſunt comparatæ, ut cujusvis integratio pendeat ab integratione præcedentis; eſt enim generaliter

$$\int \frac{u^{\delta+2} du}{(1-uu)^\mu} = \frac{-u^{\delta+1}}{(\delta-2\mu+3)(1-uu)^{\mu-1}} + \frac{\delta+1}{\delta-2\mu+3} \int \frac{u^\delta du}{(1-uu)^\mu}$$

quod

quod differentianti statim patebit. Quoniam autem eum tantum casum hic spectamus, quo $u = 1$, fieret membrum algebraicum $= \infty$, si $\mu > 1$. Quare, ne hoc incommodum eveniat, necesse est, ut sit $\mu < 1$, ut membrum algebraicum evanescat, fietque nostro casu

$$\int \frac{u^{\delta+2}\, du}{(1-uu)^{\mu}} = \frac{\delta+1}{\delta-2\mu+3} \int \frac{u^{\delta}\, du}{(1-uu)^{\mu}}.$$

36. Concessa ergo primi membri integratione, sequentia omnia facile integrabuntur pro casu $u = 1$. Ponatur ergo

$$\int \frac{u^{\alpha+\beta-1}\, du}{(1-uu)^{\mu}} = B,$$ si quidem post integrationem ponatur $u = 1$; atque integralia singulorum membrorum ita se habebunt:

$$\int \frac{u^{\alpha+\beta-1}\, du}{(1-uu)^{\mu}} = B$$

$$\int \frac{u^{\alpha+\beta+1}\, du}{(1-uu)^{\mu}} = \frac{\alpha+\beta}{\alpha+\beta-2\mu+2}\, B$$

$$\int \frac{u^{\alpha+\beta+3}\, du}{(1-uu)^{\mu}} = \frac{(\alpha+\beta)(\alpha+\beta+2)}{(\alpha+\beta-2\mu+2)(\alpha+\beta-2\mu+4)}\, B$$

$$\int \frac{u^{\alpha+\beta+5}\, du}{(1-uu)^{\mu}} = \frac{(\alpha+\beta)(\alpha+\beta+2)(\alpha+\beta+4)}{(\alpha+\beta-2\mu+2)(\alpha+\beta-2\mu+4)(\alpha+\beta-2\mu+6)}\, B$$

&c.

37. His igitur valoribus substitutis, tandem obtinebitur

$$Q = \frac{B}{\beta} + \frac{\nu(\alpha+\beta)}{1(\alpha+\beta-2\mu+2)(\beta+2)}\, kB + \frac{\nu(\nu-1)(\alpha+\beta)(\alpha+\beta+2)}{1.2(\alpha+\beta-2\mu+2)(\alpha+\beta-2\mu+4)(\beta+4)}\, k^2 B + \&c.$$

vel, si P denotet ubique coefficientem termini præcedentis, erit

$\frac{\beta Q}{B}$

$$\frac{\beta Q}{B} = 1 + \frac{\nu(\alpha+\beta)\beta}{1(\alpha+\beta-2\mu+2)(\beta+2)}\, k + \frac{(\nu-1)(\alpha+\beta+2)(\beta+2)}{2(\alpha+\beta-2\mu+4)(\beta+4)}\, Pk^2 + \frac{(\nu-2)(\alpha+\beta+4)(\beta+4)}{3(\alpha+\beta-2\mu+6)(\beta+6)}\, Pk^3 + \&c.$$

Hæc expressio jam aliquot modis ad formam serierum T & V propius reduci potest. Primo scilicet poni debet $k = tt$; quanquam enim k ut constans est spectata, tamen nihil impedit, quo minus ipsi, postquam integratio formulæ Q eo, quo præceptum est, modo fuerit peracta, valor variabilis tribuatur.

38. Cum igitur in seriebus T & V coefficientes binis factoribus crescant, in nostra serie idem erit efficiendum. Obtinebitur hoc primum, si ponatur $\beta = 2$; eritque

$$\frac{2Q}{B} = 1 + \frac{2\nu(\alpha+2)}{4(\alpha-2\mu+4)}\, tt + \frac{(2\nu-2)(\alpha+4)}{6(\alpha-2\mu+6)}\, Pt^4 + \&c.$$

quæ vero ad formas propositas reduci nequit. Commodissime vero hæc reductio nostro scopo inserviet, qua fit $k = tt$ & $\alpha = 2$, eritque

$$\frac{\beta Q}{B} = 1 + \frac{2\nu.\beta}{2(\beta-2\mu+4)}\, tt + \frac{(2\nu-2)(\beta+2)}{4(\beta-2\mu+6)}\, Pt^4 + \frac{(2\nu-4)(\beta+4)}{6(\beta-2\mu+8)}\, Pt^6 + \&c.$$

39. Comparemus hanc seriem cum prima, cujus summationem quærimus, quæ ex §. 32 est:

$$T = 1 + \frac{(m+1)m}{1.2}\, tt + \frac{(m-1)(m+2)}{3.4}\, Pt^4 + \frac{(m-3)(m+4)}{5.6}\, Pt^6 + \&c.$$

Fiat ergo $\beta = m$; $\nu = \frac{m+1}{2}$; & $\beta - 2\mu + 4 = m - 2\mu + 4 =$ 1; erit $\mu = \frac{m+3}{2}$. Quare, cum hoc casu fiat $T = \frac{mQ}{B}$, erit

Tt 3 $T = m$

$$T = m\int \frac{u\,du}{(1-uu)^{\frac{m+3}{2}}} \int u^{m-1}\,du\,(1+ttuu)^{\frac{m+1}{2}} : \int \frac{u^{m+1}\,du}{(1-uu)^{\frac{m+3}{2}}}.$$

40. Si ſumatur altera ſeries, qua T exprimitur, erit

$$\frac{T}{1+tt} = 1 + \frac{(m-1)(m+2)}{1\cdot 2}\,tt + \frac{(m-3)(m+4)}{3\cdot 4}\,Pt^4 + \&c.$$

quæ ex ſuperiori oritur ſumendo $k = tt$; $\alpha = 2$; $\nu = \frac{m-1}{2}$; $\beta = m+2$, & $\beta - 2\mu + 4 = m - 2\mu + 6 = 1$, unde fit $\mu = \frac{m+5}{2}$. Hinc itaque erit

$$\frac{T}{1+tt} = (m+2)\int \frac{u\,du}{(1-uu)^{\frac{m+5}{2}}} \int u^{m+1}\,du\,(1+ttuu)^{\frac{m-1}{2}} : \int \frac{u^{m+3}\,du}{(1-uu)^{\frac{m+5}{2}}}.$$

41. Ad valorem ipſius V inveniendum, cum ſit:

$$\frac{V}{t} = 1 + \frac{m(m+1)}{2\cdot 3}\,tt + \frac{(m-2)(m+2)}{4\cdot 5}\,Pt^4 + \&c.$$

fieri debet ut hactenus $k = tt$ & $\alpha = 2$; eritque $\nu = \frac{m}{2}$, $\beta = m+1$, $\beta - 2\mu + 4 = m - 2\mu + 5 = 3$, & $\mu = \frac{m+2}{2}$. Unde obtinebitur:

$$V = (m+1)\,t\int \frac{u\,du}{(1-uu)^{\frac{m+2}{2}}} \int u^{m}\,du\,(1+tt\,uu)^{\frac{m}{2}} : \int \frac{u^{m+2}\,du}{(1-uu)^{\frac{m+2}{2}}};$$

ubi

ubi quidem incommodum accidit, quod $\mu > 1$, unde utrumque integrale evadit infinitum.

42. Huic autem incommodo medela afferetur, ſi pro Q alia forma integralis accipiatur. Statuatur igitur

$$Q = \int \frac{v^{\alpha-1}\,dv}{(1+vv)^{\mu}} \left(1 + \frac{ttvv}{1+vv}\right)^{\frac{\nu}{2}}$$ poſito poſt integrationem $v = \infty$, ſitque eodem caſu $\int \frac{v^{\alpha-1}\,dv}{(1+vv)^{\mu}} = B$. Jam ergo fiet

$$Q = \int \frac{v^{\alpha-1}\,dv}{(1+vv)^{\mu}} \left(1 + \frac{\nu}{2}\cdot\frac{ttvv}{1+vv} + \frac{\nu(\nu-2)}{2\cdot 4}\cdot\frac{t^4v^4}{(1+vv)^2} + \frac{\nu(\nu-2)(\nu-4)}{2\cdot 4\cdot 6}\cdot\frac{t^6v^6}{(1+vv)^3} + \&c.\right),$$

At eſt:

$$\int \frac{v^{\delta+1}\,dv}{(1+vv)^{\mu+1}} = \frac{-1}{2\mu}\cdot\frac{v^{\delta}}{(1+vv)^{\mu}} + \frac{\delta}{2\mu}\int \frac{v^{\delta-1}\,dv}{(1+vv)^{\mu}}.$$

Quare, cum hic eum tantum caſum perpendamus, quo $v = \infty$, evaneſcit membrum algebraicum, dummodo ſit $\delta < 2\mu$; proinde erit

$$\int \frac{v^{\delta+1}\,dv}{(1+vv)^{\mu+1}} = \frac{\delta}{2\mu}\int \frac{v^{\delta-1}\,dv}{(1+vv)^{\mu}}.$$

43. His ergo præmiſſis, erit:

$$\frac{Q}{B} = 1 + \frac{\nu\cdot\alpha}{2\cdot 2\mu}\,tt + \frac{(\nu-2)(\alpha+2)}{4(2\mu+2)}\,Pt^4 + \&c.$$

Facto ergo $v = m+1$; $\alpha = m$ & $\mu = \frac{1}{2}$ erit $T =$

$$\frac{Q}{B} = \int \frac{v^{m-1}\,dv}{\sqrt{(1+vv)}} \left(1 + \frac{tt\,vv}{1+vv}\right)^{\frac{m+1}{2}} : \int \frac{v^{m-1}\,dv}{\sqrt{(1+vv)}}$$

poſito poſt integrationem $v = \infty$, hæcque expreſſio nullo laborat incommodo, dummodo ſit $m < 1$ ſimulque $m > 0$. Simili modo ſi ponatur $\nu = m$; $\alpha = m+1$, & $\mu = \frac{3}{2}$ fiet

$$\frac{V}{t} = \frac{Q}{B},$$ ideoque erit

$$V = t\int \frac{v^m\,dv}{(1+vv)^{\frac{3}{2}}} \left(1 + \frac{tt\,vv}{1+vv}\right)^{\frac{m}{2}} : \int \frac{v^m\,dv}{(1+vv)^{\frac{3}{2}}}.$$

poſito poſt integrationem $v = \infty$; hocque valet, ſi m contineatur intra limites 2 & 0.

44. Cum

44. Cum igitur ex æquatione $(1+tt)\,ddz = m(m+1)\,z\,dt^2$ fit $z = fT + gV$, habebitur, loco T & V valores inventos subſtituendo, verus valor pro z in t expreſſus, unde porro reperietur $y = \frac{adz}{zdt}$ & $x = \frac{atdz}{zdt}$; ſicque curva quæſita conſtruetur. Quod autem m unitate minus eſſe debeat, ſolutioni non admodum nocet, quoniam, uti in æquatione *Riccatiana* uſu venit, ex caſibus his, quibus $m < 1$ reliqui omnes deduci poſſunt.

Problematis Conſtructio generalis mechanica.

45. Etſi Problemati propoſito abunde equidem ſatisfeciſſe mihi videor; tamen, quia poſterior claſſis curvarum tranſcendentium ex calculo vix ac ne vix quidem cognoſci poteſt, coronidis loco hic adjiciam conſtructionem generalem omnium curvarum, Problema reſolventium, quæ, etſi eſt mechanica, a motu quippe tractorio petita, tamen ad delineationem curvarum quæſitarum ingens ſubſidium afferet.

TAB. II Fig. 1 & 5. 46. Conſtruatur primum curva acb ſuper axe cp hac lege, ut, poſito angulo $ADM = \varphi$, capiatur abſciſſa $cp = -2\,c\,l$ tag $\frac{1}{2}\varphi$, & applicata $pm = -a\,l$ ſin φ. Ubi notandum eſt, logarithmos tangentis anguli $\frac{1}{2}\varphi$, & ſinus anguli φ, ſumi debere hyperbolicos. Unde, ſi vulgares tabulas adhibere velimus, a logarithmis in tabula extantibus ſubtrahi debet logarithmus ſinus totius, atque logarithmus reſiduus multiplicari per numerum conſtantem 2, 302585093. Qui numerus ſi vocetur $= n$, erit tabulas vulgares uſurpando $cp = 2nc$ (*log.* ſin. tot. − *log.* tag $\frac{1}{2}\varphi$) & $pm = na$ (*log.* ſin. tot. − *log.* ſin. φ).

47. Poſtquam curva acb hoc modo erit deſcripta, ſumatur filum longitudinis $CD = a$, ejusque alter terminus m ſecundum ductum curvæ acb ita promoveatur, ut alter terminus n deſcribat curvam tractoriam $fneg$, quod infinitis modis fieri poteſt; ab arbitrio enim noſtro pendet, ubi filo poſitionem ad curvam normalem tribuere velimus. Quin etiam tractoria ad convexam curvæ acb partem deſcribi poteſt.

48. His factis, pro angulo $ADM = \varphi$ pro lubitu aſſumto capiatur curvæ acb punctum m ex hoc angulo φ natum, ex quo ducatur tractoriæ tangens mn, eaque producta in q, biſecetur angulus pmq, ſitque ſemiſſis $= \psi$. Tum erit recta $DM = c$ tag ψ; ſeu fiat ut ſinus totus ad tangentem anguli $pmr = \psi$, ita recta conſtans c ad rectam DM. Pro quovis ergo angulo ADM hoc modo invenitur linea DM, ſicque curvæ quæſitæ AMB ſingula puncta M aſſignabuntur.

* * *

NOVA ACTA ERUDITORUM,

publicata Lipſiæ

Calendis Junii Anno MDCCXLIV.

Pars II.

HISTOIRE GENERALE DE PORTUGAL, etc.

id eſt,

HISTORIA PORTUGALLIÆ GENERALIS, concinnata ſtudio & opera Domini DE LA CLEDE.

Tomi VIII.

Pariſiis, apud Theodorum Craſſum, 1735, 12.

Alph. 7. plag. 19.

Magnum opus aggreſſus eſt Autor, complexus continuis *octo Voluminibus* omnem Portugalliæ hiſtoriam. *Tomo primo* expoſuit originem, mores, & bella Luſitanorum priſcorum, ſtatum eorum ſub Romano imperio Gothisque, & invaſione, a Mauris tolerata. *Tomo ſecundo* ediſſeruit originem regni, in Luſitania fundati, vitasque Regum primorum usque ad *Ferdinandum*. *Tomus tertius* complectitur interregnum aliquot annorum, vitas *Joannis I*, *Eduardi*, *Alphonſi V*, & initia regni *Joannis II*. *Quartus* continet reliqua *Joannis II*, vitam *Emanuelis*, ac *Joannis III* acta priora. *Quintus* exhibet reliqua *Joannis* ejusdem, vitasque Regum, *Sebaſtiani*, Cardinalis *Henrici*, nec non *Philippi II*, qui Luſitaniam Hiſpaniæ adjecit. *Sextus* continet res in India geſtas, dum regno præfuerunt *Sebaſtianus*, *Henricus*, *Philippus II*, & *Philippus*