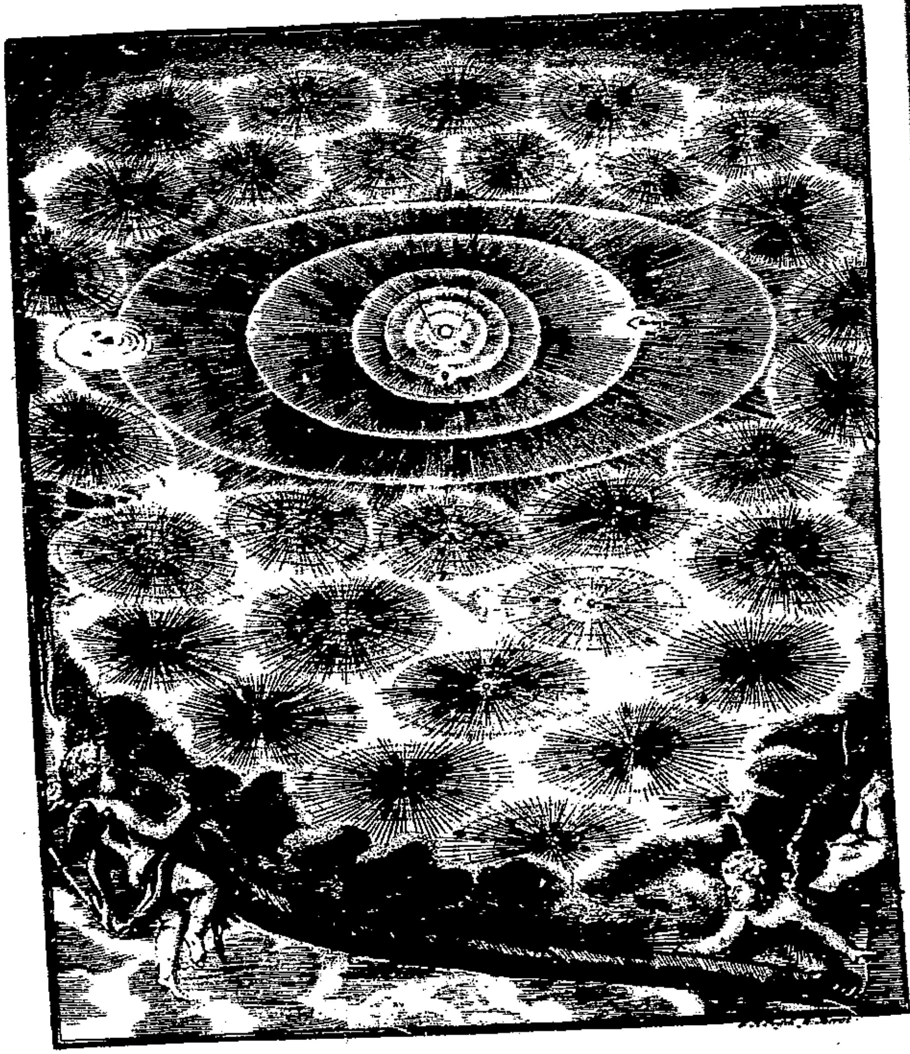

THEORIA MOTUUM PLANETARUM ET COMETARUM.

CONTINENS

METHODUM FACILEM EX ALIQUOT OBSERVATIONIBUS ORBITAS CUM PLANETARUM TUM COMETARUM DETERMINANDI.

UNA CUM CALCULO, QUO COMETÆ, QUI ANNIS 1680. ET 1681. ITEMQUE EJUS, QUI NUPER EST VISUS, MOTUS VERUS INVESTIGATUR.

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Berolini Sumtibus AMBROSII HAUDE.
Bibliop. Reg. & Acad. Scient. privilegiati.

De Motu planetarum & cometarum circa ſolem motorum.

I.

Si leges motus, quas planetæ primarii atque cometæ in curſu ſuo circa ſolem obſervant, conſulamus, non ſolum ellipſes, ſed etiam omnis generis ſectiones conicæ orbitas cometarum repræſentare poſſunt. Neque enim aliud diſcrimen inter planetas & cometas intercedit, niſi orbitæ figura, quæ ſi fuerit ellipſis non multum a circulo recedens, ſidus in ea motum planetæ nomen obtinuit, ſin autem orbita vehementer a circulo abhorreat, ſive ſit ellipſis admodum oblonga, ſive parabola, ſive adeo hyperbola, ejusmodi ſidera cometæ appellantur. Utrique autem, planetæ ſcilicet & cometæ, in curſu ſuo easdem ſequi leges motus merito ſtatuuntur, ſecundum quas tempora, quibus data ſpatia in orbitis ſuis abſolvunt, ſint in ratione compoſita arearum circum ſolem deſcriptarum directa, & reciproca ſubduplicata laterum rectorum, seu ut vocari alias solent, parametrorum orbitarum: Sol autem in altero foco cujusque orbitæ fixus æstimatur. Ex hac autem lege non solum planetarum cometarumve motus in orbibus sive ellipticis sive parabolicis sive hyperbolicis definiri, sed etiam hæ ipsæ orbitæ per observationes aliquot determinari possunt.

2. Ex isto etiam fonte derivatæ sunt tabulæ astronomicæ motuum planetarum, ope quarum ex dato tempore seu anomalia media inveniri solet anomalia vera seu coæquata, & vicissim; quæ methodus vero tantum ad orbitas ellipticas circulo vicinas est accommodata. Quare si eandem theoriam in sensu latissimo stabilire atque ad motus etiam cometarum, qui in hyperbolis incedant, transferre velimus, ab instituto hoc consueto, quo relatio inter anomaliam veram & mediam definiri solet, recedere, alioque modo cursum ad calculum revocare oportebit. Primum enim incommodum methodi receptæ in hoc consistit, quod anomaliæ ab aphelio computentur: aphelium autem solæ orbitæ ellipticæ admittunt, parabolicæ autem & hyperbolicæ eo penitus carent. Quamobrem huic incommodo medela afferetur, anomalias a perihelio, quod in omnis generis orbitas æque competit, computando. Deinde etiam anomalia media uti non licebit, quod a tempore periodico pendeat, orbitæ autem parabolicæ & hyperbolicæ tempore periodico destituantur. Loco anomaliæ mediæ igitur ipsum tempus a momento, quo sidus in perihelio versatur, computatum adhibebo; anomalia vera autem erit distantia sideris heliocentrica a perihelio. His igitur

igitur præmissis sequentia problemata evolvam, ex quibus natura motuum coelestium cum theoretice cognosci, tum etiam usus in praxi dilucide perspici poterit.

Problema I.

3. Data area, quam planeta vel cometa dato tempore circa solem descripsit, invenire latus rectum orbitæ.

Solutio.

Quoniam hic quantitates diversi generis, areæ scilicet & tempora inter se conferuntur, utrumque genus secundum certam constantemque mensuram exprimi debebit. Posita igitur distantia Solis a terra media $= 100000$, in ejusmodi unitatibus cunctas magnitudines exprimemus; atque adeo latus rectum orbitæ, quod quærimus, ex hac unitate definiri oportet. Aream autem circa solem descriptam per eandem mensuram in ejusmodi partibus quadratis, quarum 100000 semiaxem orbis magni constituunt, exhiberi pono. Sit igitur A area, quam planeta cometave circa solem descripsit, ad mensuram allumtam revocata; atque b denotet semissem lateris recti, seu applicatam ad axem transversum in foco normalem, in eadem mensura definiendam. Tempus denique posthac perpetuo exprimamus in diebus naturalibus temporis medii, & fractionibus diei decimalibus: sitque tempus propositum, quo area A circa solem confecta perhibetur, $= T$ diebus; hoc igitur modo quantitates A, b, & T, quarum in hoc problemate mentio fit, ad numeros absolutos reducentur. Cum igitur ex natura motus corporum coelestium circa solem motorum sit tempus

 T pro-

T proportionale areæ A divisæ per $\sqrt{b}$, fiet $\frac{A}{T\sqrt{b}}$ = numero cuidam constanti, qui sit $= m$; ita ut sit $T = \frac{A}{m\sqrt{b}}$ seu $\sqrt{b} = \frac{A}{mT}$, & $b = \frac{AA}{mmTT}$; dummodo ergo iste numerus m fuerit cognitus, problema erit resolutum, eo quod semilatus rectum b reperitur expressum in ejusmodi partibus, quarum 100000 distantiam solis a terra mediam conficiunt. Ad numerum itaque m definiendum casum jam cognitum evolvamus. Cum scilicet constet, terram circa solem in orbita sua circumferri tempore anni siderei seu $365^d, 6^h, 8', 30''$; si ponamus T = 365, 256, & A = areæ totius orbitæ terrestris & b = semilateri recto orbitæ terræ, dabit fractio $\frac{A}{T\sqrt{b}}$ verum valorem numeri m. Sit ergo semiaxis transversus orbitæ terræ $= c$, quem sumsimus = 100000, erit semiaxis conjugatus $= \sqrt{bc}$. Denotet 1 : π rationem diametri ad peripheriam, ita ut sit $\pi = 3,14159265$, erit area circuli radio c descripti $= \pi cc$, quæ erit ad aream orbitæ terræ ut c ad $\sqrt{bc}$, unde fit area orbitæ terrestris $A = \pi c\sqrt{bc}$, ideoque $m = \frac{A}{T\sqrt{b}} = \frac{\pi c\sqrt{c}}{T}$ seu numerus m dabitur per numeros cognitos $\pi = 3,14159265$; $c = 100000$, & T = 365, 256. Hinc per logarithmos valor numeri m reperietur:

$l\,\pi$	=	0,4971498727
$lc\sqrt{c}$	=	7,5000000000
		7,9971498727
subtr. lT	=	2,5625973588
erit $l\,m$	=	5,4345525139.
ideoque m	=	271989,735

Inven-

Invento ergo numero $m = 271989,735$, erit orbitæ cujuscunque circa solem descriptæ latus rectum $2b = \frac{2AA}{mmTT}$. Q. E. J.

Coroll. 1.

4. Si igitur cognita fuerit area, quam cometa dato tempore circa solem conficit, hinc latus rectum orbitæ, in qua movetur cometa, reperitur ita, ut ejus ratio ad distantiam Solis a terra mediam exhibeatur.

Coroll. 2.

5. Sin autem latus rectum orbitæ, quod sumimus $= 2b$ jam fuerit cognitum, tempus assignari poterit, quo data orbitæ area circa solem absolvitur. Sit enim area hæc descripta = A erit tempus $T = \frac{A}{m\sqrt{b}}$ dier: siquidem, quod ubique est tenendum, longitudines in ejusmodi partibus exprimantur, quarum 100000 semiaxem transversum orbis magni constituunt, & sit $m = 271989,735$.

Coroll. 3.

6. Vicissim ergo ex dato latere recto orbitæ $2b$ area assignari potest, quam planeta vel cometa dato tempore T circa solem describit: tempore enim T in diebus expresso erit area descripta $A = mT\sqrt{b}$.

Problema II.

7. Datis sectionis conicæ ABM distantia verticis a foco AS una cum latere recto, cujus semissis est BS, invenire rela- Fig. 1.

relationem, quæ inter diſtantiam cujusvis puncti M a foco S & anomaliam veram ſeu angulum ASM intercedit.

Solutio.

Sit diſtantia verticis a foco AS, ſeu pro orbitis planetarum cometarumve diſtantia perihelii A à ſole S nempe $AS = a$, & ſemiſſis lateris recti, nempe applicata $BS = b$. Dabit ergo recta AS producta axem ſectionis conicæ, ad quem ex puncto M demittatur perpendiculum MP; erit ex natura ſectionum conicarum diſtantia $MS = a + \frac{(b-a)}{a}.AP$. Cum autem ſit $AP = a + PS$; ſi ponatur diſtantia $MS = y$ & angulus ſeu anomalia vera $ASM = v$, erit poſito ſinu toto $= 1$, $\cos v = \frac{-PS}{MS} = \frac{-PS}{y}$, ideoque $PS = -y \cos v$, & $AP = a - y \cos v$. Hoc ergo valore ſubſtituto fiet $MS = y = a + \frac{(b-a)}{a}(a - y\cos v) = b - \frac{(b-a)y\cos v}{a}$. Hinc igitur prodit $\cos v = \frac{a(b-y)}{y(b-a)}$ & $y = \frac{ab}{a+(b-a)\cos v}$. Ex data ergo anomalia vera v reperitur diſtantia planetæ ſeu cometæ a ſole y; & viciſſim ex cognita hac diſtantia definitur anomalia vera. Q. E. J.

Coroll. 1.

8. Si anomalia vera ASM evaneſcit, ſitque $v = 0$, erit $\cos v = 1$. hocque caſu prodit diſtantia a ſole $y = a$; nempe puncto M in A incidente ſit $MS\ (y) = AS\ (a)$. Simili modo ſi anomalia vera $ASM = v$ fiat angulus rectus, erit

$\cos v = 0$, & fiet $y = b$; id quod perſpicuum eſt, quoniam punctum M in B incidit.

Coroll. 2.

9. Quod ſi ponamus $v = 180°$, exhibebit y diſtantiam aphelii a ſole; erit autem ob $\cos v = -1$, hæc diſtantia $= \frac{ab}{2a-b}$; ad quam ſi addatur diſtantia perihelii a ſole $= a$ prodibit axis transverſus orbitæ $= \frac{2aa}{2a-b}$; & ſemiaxis transverſus $= \frac{aa}{2a-b}$, unde diſtantia foci a centro orbitæ erit $= \frac{a(b-a)}{2a-b}$; & excentricitas $= \frac{b-a}{a}$.

Coroll. 3.

10. Si igitur fuerit $b = a$, ſectio conica erit circulus, centrum in S & radium $AS = a$ habens. Sin autem ſit $b > a$, curva erit ellipſis, quoad b fiat $= 2a$, quo caſu curva abit in parabolam ob axem transverſum $\frac{2aa}{2a-b}$ infinitum. At ſi fiat $b > 2a$, tum axis transverſus ſit negativus, quo indicatur, curvam eſſe hyperbolam.

Coroll. 4.

11. Si fuerit $b < a$, curva ſemper erit ellipſis, ſed punctum A erit vertex a foco S remotior, ideoque aphelium repræſentabit; punctum ergo orbitæ e diametro oppoſitum

erit

erit perihelium, cujus a foco S distantia erit $= \frac{ab}{2a-b}$, quæ minor est quam a si fuerit $b < a$.

Problema III.

Fig. 2. 12. Si dentur duæ cometæ planetæve a sole distantiæ FS & GS una cum angulo FSG intercepto; atque insuper cognitum sit latus rectum orbitæ; determinare ipsam orbitam AFG.

Solutio.

Sit semilatus rectum $= b$, ac ponatur distantia $FS = f$ $GS = g$, atque angulus $FSG = \Phi$; quæ sunt data: ex quæsitis autem sit distantia perihelii a sole $AS = a$, & anomalia seu angulus $ASF = v$, erit angulus $ASG = v + \Phi$. His positis problema præcedens duas nobis suppeditabit æquationes; $SF = f = \frac{ab}{a + (b-a)\cos v}$ & $SG = g = \frac{ab}{a + (b-a)\cos(v+\Phi)}$, ex quibus pro a duplex oritur valor $a = \frac{bf\cos v}{b - f + f\cos v} = \frac{bg\cos(v+\Phi)}{b - g + g\cos(v+\Phi)}$; qui inter se æquati dant $(b-g)f\cos v = (b-f)g\cos(v+\Phi) = (b-f)g(\cos v.\cos\Phi - \sin v.\sin\Phi)$, ideoque $\frac{(b-g)f}{(b-f)g} = \cos\Phi - \tan v.\sin\Phi$, ex qua æquatione determinabitur anomalia v per quantitates mere cognitas, scilicet $\tan v = \cot\Phi -$

(b–g)

$\frac{(b-g)f}{(b-f)g\sin\Phi}$ seu $\tan v = \frac{(b-f)g\cos\Phi - (b-g)f}{(b-f)g\sin\Phi}$. Invento autem angulo $ASF = v$, distantia perihelii a sole erit $AS = a = \frac{bf\cos v}{b - f + f\cos v}$. Ex datis vero $AS = a$ & semilatere recto $= b$ orbita AFG definitur per probl. præc. Q. E. J.

Coroll. 1.

13. Si igitur detur area FSG una cum tempore, quo planeta seu cometa spatium FG percurrit, per problema primum reperitur latus rectum, ac proinde ex hoc problemate definietur ipsa orbita AFG.

Coroll. 2.

14. Primo scilicet ex datis semilatere recto b & distantiis $FS = f$, $GS = g$ una cum angulo $FSG = \Phi$ cognoscetur positio axis AS ex angulo $ASF = v$, cum sit $\tan v = \cot\Phi - \frac{(b-g)f}{(b-f)g\sin\Phi}$; & cognito angulo v erit $AS = a = \frac{bf\cos v}{b - f + f\cos v}$.

Coroll. 3.

15. Ex natura autem sectionum conicarum patet, si fuerit A orbitæ perihelium, rectarum FS & GS illam, quæ sit brevior, propiorem esse perihelio A; Quare cum sæpenumero incertum sit, utra rectarum FS & GS designari debeat littera f, breviorem semper hac littera insigniri conveniet.

Coroll. 4.

16. Si tangens anomaliæ v prodeat negativa, indicio hec erit vel angulum ASF esse recto majorem (duobus tamen

 rectis

rectis minorem) vel esse negativum, atque perihelium A intra loca F & G cadere. Cum igitur positio perihelii A sit anceps, & in duo loca, e diametro opposita incidat, ea erit vera, quæ breviori distantiæ f erit propior, altera vero positio dabit aphelium.

Problema IV.

Fig. 3. 17. Data orbita planetæ seu cometæ AM, invenire tempus, quo anomalia vera quævis ASM absolvitur.

Solutio.

Sit distantia perihelii a sole $AS = a$, semilatus rectum $= b$; & anomalia vera proposita $ASM = v$, atque distantia $SM = y$, erit $y = \frac{ab}{a + (b-a)\cos v}$, seu $\cos v = \frac{a(b-y)}{y(b-a)}$. Sit insuper area $ASM = A$; erit per probl. 1. tempus quæsitum, quo planeta vel cometa spatium AM conficit $= \frac{A}{m\sqrt{b}}$ dierum. Quare ad tempus hoc definiendum aream ASM definiri oportet. Capiatur elementum Mm, & ducta Sm erit angulus $MSm = dv$, ideoque area trianguli $MSm = \frac{1}{2}yy\,dv$, quæ cum sit differentiale areæ $ASM = A$, erit $dA = \frac{1}{2}yy\,dv$, hincque $A = \frac{1}{2}\int yy\,dv$, & si loco y ejus valor in v substituatur, erit $A = \frac{1}{2}\int \frac{aabb\,dv}{(a + (b-a)\cos v)^2}$ quæ formula ut integrari queat, ponatur tang $\frac{1}{2}$ ASM $=$ tang $\frac{1}{2} v = t$, erit $dv = \frac{2\,dt}{1+tt}$; & $\cos v = \frac{1-tt}{1+tt}$, his substitutis fit $A =$ $\int aa$

$\int \frac{aabb\,dt(1+tt)}{(b+(2a-b)tt)^2}$. Sit $A = \frac{\alpha t}{b+(2a-b)tt} + \beta \int \frac{dt}{b+(2a-b)tt}$, atque comparatione instituta erit $\alpha + \beta = aab$, & $\beta - \alpha = \frac{aabb}{2a-b}$ hincque $\alpha = -\frac{aab(b-a)}{2a-b}$ & $\beta = \frac{a^3 b}{2a-b}$, ex quibus colligitur area $A = \frac{a^3 b}{(2a-b)} \int \frac{dt}{b+(2a-b)tt} - \frac{aab(b-a)t}{(2a-b)(b+(2a-b)tt)}$. Hujus integralis constituendi sunt quatuor casus, quorum primus est:

I. Si $b = a$, quo curva fit circulus; hic casus promtissime ex prima formula resolvitur, qui dat $A = \frac{1}{2}\int aa\,dv = \frac{1}{2} aav$; vel si hoc integrale per t expressum desideretur ob $v = 2$ A tang. t, fiet area $A = aa$ A tang. t.

II. Sit $b > a$ ita tamen ut sit $b < 2a$, quo casu orbita erit ellipsis, sitque ut vidimus $A = \frac{a^3 b}{2a-b} \int \frac{dt}{b+(2a-b)tt} - \frac{aab(b-a)t}{(2a-b)(b+(2a-b)tt)}$. Est vero $\int \frac{dt}{b+(2a-b)tt} = \frac{1}{\sqrt{b(2a-b)}}$ A tang $t\sqrt{\frac{2a-b}{b}} = \frac{1}{\sqrt{b(2a-b)}}$ A sin $\frac{t\sqrt{(2a-b)}}{\sqrt{(b+(2a-b)tt}}$, $= \frac{1}{\sqrt{b(2a-b)}}$ A cos $\frac{\sqrt{b}}{\sqrt{(b+(2a-b)tt)}}$

Hinc fit $\int \frac{dt}{b+(2a-b)tt} = \frac{1}{2\sqrt{b(2a-b)}}$ A sin $\frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt}$ sequentes ergo prodibunt expressiones aream A exhibentes

$A =$

$$A = \frac{a^3 \sqrt{b}}{(2a-b)\sqrt{(2a-b)}} \text{ A tang } t\sqrt{\frac{2a-b}{b}} - \frac{aab(b-a)t}{(2a-b)(b+(2a-b)tt)}$$

vel $A = \frac{a^3 \sqrt{b}}{2(2a-b)\sqrt{(2a-b)}} \text{ A fin } \frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt} - \frac{aa(b-a)\sqrt{b}}{2(2a-b)\sqrt{(2a-b)}} \cdot \frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt}$ feu $A = \frac{a^3 \sqrt{b}}{2(2a-b)\sqrt{(2a-b)}} \left(\text{A fin } \frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt} - \frac{(b-a)}{a} \cdot \frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt}\right)$

III. Sit $b = 2a$, erit orbita parabola, atque erit ex priori æquatione $A = \int aadt(1+tt) = aa(t+\frac{1}{3}t^3)$, qui eft unicus cafus, quo area algebraice poteft exhiberi.

IV. Sit $b > 2a$ erit orbita hyperbolica, fietque $A = \frac{aab(b-a)t}{(b-2a)(b-(b-2a)tt)} - \frac{a^3 b}{b-2a}\int\frac{dt}{b-(b-2a)tt}$. At hæc integratio a logarithmis pendet, fitque $\int\frac{dt}{b-(b-2a)tt} = \frac{1}{2\sqrt{b(b-2a)}}\, l\frac{\sqrt{b}+t\sqrt{(b-2a)}}{\sqrt{b}-t\sqrt{(b-2a)}}$. Ex his ergo reperitur area quæfita $A = \frac{aab(b-a)t}{(b-2a)(b-(b-2a)tt)} - \frac{a^3\sqrt{b}}{2(b-2a)\sqrt{(b-2a)}}\, l\frac{\sqrt{b}+t\sqrt{(b-2a)}}{\sqrt{b}-t\sqrt{(b-2a)}}$ five $A = \frac{a^3\sqrt{b}}{2(b-2a)\sqrt{(b-2a)}}\left(\frac{b-a}{a}\cdot\frac{2t\sqrt{b(b-2a)}}{b-(b-2a)tt} - l\frac{\sqrt{b}+t\sqrt{(b-2a)}}{\sqrt{b}-t\sqrt{(b-2a)}}\right)$

His igitur fingulis cafibus fi reperta fuerit area ASM$=$A, erit tempus, quo ea abfolvitur $= \frac{A}{m\sqrt{b}}$, quæ expreffio tempus in diebus exhibebit, fi fuerit $m = 271989,735$ & A & b ea menfura definiantur, quam indicavi. Q.E.J.

Coroll.

Corol. I.

18. Si ergo orbita fuerit circulus, erit tempus, quo angulus ASM $= v$ abfolvetur $= \frac{av\sqrt{a}}{2m}$ ob. $b = a$: quod quidem ex motu uniformi fponte liquet. Ceterum quia in circulo perihelium non datur, anomaliæ veræ nomen improprie adhibetur.

Coroll. 2.

19. Si orbita fuerit ellipfis, quo area A commodius exprimatur, quæratur angulus ω, ut fit tang. $\frac{1}{2}\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}$ tang. $\frac{1}{2}v = \frac{t\sqrt{(2a-b)}}{\sqrt{b}}$. Angulo autem hoc ω invento, erit area $A = \frac{a^3\sqrt{b}}{2(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\text{fin }\omega\right)$, ideoque tempus per AM $= \frac{a^3}{2m(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\text{ fin }\omega\right)$.

Coroll. 3.

20. Si orbita fuerit hyperbola, quæratur pariter angulus ω, ut fit tang $\frac{1}{2}\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ tang $\frac{1}{2}v$, feu $t = \frac{\sqrt{b}}{\sqrt{(b-2a)}}$ tang $\frac{1}{2}\omega$. Quo angulo ω invento erit area ASM $=$ A $= \frac{a^3\sqrt{b}}{2(b-2a)^{\frac{3}{2}}}\left(\frac{(b-a)}{a}\text{ tang }\omega - l\text{ tang }(45^\circ+\frac{1}{2}\omega)\right)$, hincque tempus per arcum AM erit $= \frac{a^3}{2m(b-2a)^{\frac{3}{2}}}\left(\frac{(b-a)}{a}\text{tang}\,\omega - l\text{ tang. }(45^\circ+\frac{1}{2}\omega)\right)$

Coroll.

Coroll. 4.

21. Cum autem hic logarithmus tangentis anguli $45^\circ + \frac{1}{2}\omega$ per integrationem sit inventus, perspicuum est, eum ex canone logarithmorum hyperbolicorum esse depromendum, posito sinu toto $= 1$. Hujusmodi vero canone deficiente, sumatur ex tabulis vulgaribus logarithmus tang $(45^\circ + \frac{1}{2}\omega)$, & subtracto 10 ab ejus characteristica, residuum, multiplicetur per numerum 2, 302585092994, atque productum erit log. hyperb. tang $(45^\circ + \frac{1}{2}\omega)$; ubi notari convenit, hujus numeri 2, 30258509 &c. logarithmum esse $=$ 0, 3622156886.

Scholion 1.

22. Eadem expressio logarithmica l tang$(45^\circ + \frac{1}{2}\omega)$ reperitur in hydrographia, si quæratur in mappa Nautica ad mentem Mercatoris construenda latitudo crescens respondens latitudini ω in superficie terræ. Quare ex tabulis nauticis, quibus latitudines crescentes (Gall. les latitudes croissantes) ad singulos gradus exhiberi solent, hi valores l tang $(45^\circ + \omega)$ sumi poterunt. Interim tamen sine his tabulis calculus satis expedite instituetur hoc modo. Sit id quod quæritur nempe l tang$(45^\circ + \frac{1}{2}\omega)$ $= x$. Tum in tabulis logarithmorum tangentium consuetis quæratur log. tang $(45^\circ + \frac{1}{2}\omega)$, a quo auferatur log. tang 45° seu 10, 00000, residuum ponatur $=$ R, eritque $x = 2$, 302585092994. R, ideoque sumendis logarithmis erit $lx =$ $lR +$ 0, 3622156886; unde ope tabulæ logarithmorum solitorum facile valor ipsius $x = l$ tang $(45^\circ + \frac{1}{2}\omega)$ reperitur. Sit exempli gratia angulus $\omega = 37^\circ, 22', 40''$. erit $\frac{1}{2}\omega =$ 18°,

$18^\circ, 41', 20''$ & $45^\circ + \frac{1}{2}\omega = 63^\circ, 41', 20''$. Tum ex tabulis erit.

l tang $(45^\circ + \frac{1}{2}\omega)$	$=$	10, 3058582068
subtr. l tang 45°	$=$	10, 0000000000
erit R	$=$	0, 3058582068
ideoque lR	$=$	9, 4855201380
addatur	$=$	0, 3622156886
erit lx	$=$	9, 8477358266
unde obtinetur x	$=$	0, 7042645474

Scholion. 2.

23. Circa tempus in ellipsi assignandum notari oportet arcum seu angulum ω non more solito in gradibus & minutis, sed in partibus radii, qui ponitur $= 1$, exprimi debere. Scilicet si ω esset 180°, tum pro ω substitui deberet longitudo semiperipheriæ 3, 1415926535. Hinc igitur valor cujusvis anguli ω in partibus decimalibus radii 1 exprimi poterit; convertatur arcus ω in minuta secunda, sitque $\omega = n''$, atque manifestum est, hanc institui debere proportionem ob $180^\circ = 648000''$, ut sit 648000 : 3, 1415926535 $= n$: valorem ω, eritque ideo

$$\text{valor } \omega = \frac{3,\ 1415926535}{648000}\, n. \quad \text{Cum}$$

ergo sit l 648000	$=$	5, 8115750059
subtr. l 3, 14159 &c.	$=$	0, 4971498727
a log. n subtrahatur	– – –	5, 3144251332

& residuum erit logarithmus ipsius ω in partibus decimalibus radii expressi. Vel ad logarithmum numeri n addatur constanter 4, 6855748668, eritque summa, postquam ejus characteristica fuerit denario minuta, logarithmus valoris quæsiti anguli ω.

 Proble-

Problema V.

Fig. 2. 24. Datis duabus a Sole distantiis FS & GS una cum angulo ad solem FSG, ac praeterea tempore, quo planeta vel cometa spatium FG absolvit, invenire latus rectum orbitae, hincque ipsam orbitam determinare, siquidem angulus FSG fuerit minimus.

Solutio.

Sit distantia FS $= f$, & GS $= g$, atque angulus FSG $= \Phi$ qui cum ponatur minimus, portio sectionis conicae FG non sensibiliter a linea recta discrepabit, erit ergo area FSG proxime triangulum rectilineum, hincque ejus area $= \frac{1}{2} fg \sin\Phi$. Ob curvaturam autem arcus FG haec area erit aliquanto major, ideoque ea propius definietur hac expressione $\frac{1}{2} fg\Phi$, quippe quae, si orbita fuerit circulus, adeo veram aream praebet. Accuratius tamen haec area ita assignabitur; sit FS $= y$, GS $= z$, & angulus ASF $= v$; tum autem ponatur area ASF $=$ F & area ASG $=$ G; erit ob ASG $= v + \Phi$; area $F = \frac{1}{2}\int yy\,dv$ & $G = \frac{1}{2}\int zz\,dv$. Sit distantia perihelii a sole AS $= a$, & semilatus rectum $= b$; erit $y = \frac{ab}{a + (b-a)\cos v}$ & $z = \frac{ab}{a + (b-a)\cos(v+\Phi)}$, hinc fit $dy = \frac{ab(b-a)\,dv \sin v}{(a+(b-a)\cos v)^2} = \frac{(b-a)yy\,dv \sin v}{ab}$, ideoque $\frac{dy}{dv} = \frac{(b-a)yy\sin v}{ab}$. Cum nunc ex y nascatur z, si loco v scribatur $v + \Phi$, erit $z = y + \frac{\Phi\,dy}{dv} + \frac{\Phi^2 ddy}{2dv^2} +$

$\Phi^3 d^3 y$

$\frac{\Phi^3 d^3 y}{6dv^3} +$ &c. posito dv constante. Cum autem simili modo ex area F nascatur area G, si loco v scribatur $v + \Phi$ erit $G = F + \frac{\Phi\,dF}{dv} + \frac{\Phi^2 ddF}{2dv^2} + \frac{\Phi^3 d^3 F}{6dv^3} +$ &c. ideoque area quaesita FSG $= G - F = \frac{\Phi\,dF}{dv} + \frac{\Phi^2 ddF}{2dv^2} + \frac{\Phi^3 d^3 F}{6dv^3} +$ &c. & ob $F = \frac{1}{2}\int yy\,dv$ erit $\frac{dF}{dv} = \frac{1}{2} yy$; hincque porro $\frac{ddF}{dv^2} = \frac{y\,dy}{dv}$; $\frac{d^3 F}{dv^3} = \frac{y\,ddy + dy^2}{dv^2}$; $\frac{d^4 F}{dv^4} = \frac{y d^3 y + 3dy\,ddy}{dv^3}$; $\frac{d^5 F}{dv^5} = \frac{y d^4 y + 4dy\,d^3 y + 3ddy^2}{dv^4}$, &c. Hinc ergo prodit $G - F = \frac{1}{2} yy\Phi + \frac{\Phi^2 y\,dy}{2dv} + \frac{\Phi^3 (y\,ddy + dy^2)}{6dv^2} + \frac{\Phi^4 (y d^3 y + 3dy\,ddy)}{24 dv^3} +$ &c. at est $\frac{1}{2} yz\Phi = \frac{1}{2} yy\Phi + \frac{\Phi\Phi\, y\,dy}{2dv} + \frac{\Phi^3 y\,ddy}{4dv^2} + \frac{\Phi^4 y\,d^3 y}{12dv^3} +$ &c. Quo ipsius $\frac{1}{2} yz\Phi$ valore subtracto remanebit $G - F = \frac{1}{2}\Phi yz + \frac{\Phi^3 (2dy^2 - y\,ddy)}{12dv^2} + \frac{\Phi^4 (3\,dy\,ddy - y\,d^3 y)}{24 dv^3} + \frac{\Phi^5 (6ddy^2 + 8dy\,d^3 y - 3y\,d^4 y)}{240 dv^4} +$ &c. Quare cum sit $\frac{dy}{dv} = \frac{(b-a)yy\cos v}{ab}$; erit $\frac{ddy}{dv^2} = \frac{(b-a)yy\cos v}{ab} +$

 $2(b-a)$

$\frac{2(b-a)\, y\, dy \sin v}{ab\, dv} = \frac{(b-a)\, yy \cos v}{ab} + \frac{2(b-a)^2 y^3 (\sin v)^2}{a^2 b^2}$;

$\frac{d^3 y}{dv^3} = \frac{-(b-a)\, yy \sin v}{ab} + \frac{6(b-a)^2 y^3 \sin v \cos v}{a^2 b^2} + \frac{6(b-a)^3 y^4 (\sin v)^3}{a^3 b^3}$; & $\frac{d^4 y}{dv^4} = -\frac{(b-a)\, yy \cos v}{ab} - \frac{8(b-a)^2 y^3 (\sin v)^2}{aabb} + \frac{6(b-a)^2 y^3 (\cos v)^2}{a^2 b^2} + \frac{36(b-a)^3 y^4 (\sin v)^2 \cos v}{a^3 b^3} + \frac{24(b-a)^4 y^4 (\sin v)^4}{a^4 b^4}$. &c. Ex his conficitur $\frac{2dy^2 - y\, ddy}{dv^2} = -\frac{(b-a)\, y^3 \cos v}{ab}$;

$\frac{3dy\, ddy - y\, d^3 y}{dv^3} = \frac{(b-a)\, y^3 \sin v}{ab} - \frac{3(b-a)^2 y^4 \sin v \cos v}{a^2 b^2}$;

$\frac{6ddy^2 + 8\, dy\, d^3 y - 3y\, d^4 y}{dv^4} = \frac{3(b-a) y^3 \cos v}{ab} + \frac{4(b-a)^2 y^4 (4(\sin v)^2 - 3(\cos v)^2)}{a^2 b^2} - \frac{36(b-a)^3 y^5 (\sin v)^2 \cos v}{a^3 b^3}$.

Erit ergo area $G - F = \frac{1}{2} \Phi y z - \frac{\Phi^3 (b-a) y^3 \cos v}{12ab} + \frac{\Phi^4 (b-a) y^3 \sin v}{24ab^2} - \frac{\Phi^4 (b-a)^2 y^4 \sin v \cos v}{8a^2 b^2} + \frac{\Phi^5 (b-a) y^3 \cos v}{80ab} + \frac{\Phi^5 (b-a)^2 y^4 (\sin v)^2}{15a^2 b^2} - \frac{\Phi^5 (b-a)^2 y^4 (\cos v)^2}{20a^2 b^2} - \frac{3\Phi^5 (b-a)^3 y^5 (\sin v)^2 \cos v}{20a^3 b^3}$ + &c. atque $z = y + \frac{\Phi (b-a) y^2 \sin v}{ab} +$

$\Phi^2 (b-a)$

$\frac{\Phi^2 (b-a)\, yy \cos v}{2ab} + \frac{\Phi^2 (b-a)^2 y^3 (\sin v)^2}{a^2 b^2}$ ... Cum igitur sit $\cos v = \frac{a(b-y)}{y(b-a)}$, & $\cos(v+\Phi) = \cos v \cos \Phi - \sin v.$ $\sin \Phi = \frac{a(b-z)}{z(b-a)}$, erit $\sin v = \frac{a(b-y)}{y(b-a)} \cdot \cot \Phi - \frac{a(b-z)}{z(b-a)}$ cosec. Φ. Substituatur tantum valor pro $\cos v$, erit $G - F = \frac{1}{2} \Phi y z - \frac{\Phi^3 yy(b-y)}{12b} + \frac{\Phi^4 (b-a)\, y^3 \sin v}{24ab} - \frac{\Phi^4 (b-a)\, y^3 (b-y) \sin v}{8abb}$ & loco $\sin v$ valore substituto $G - F = \frac{1}{2} \Phi y z - \frac{\Phi^3 yy(b-y)}{12b} + \frac{\Phi^4}{24bb} y^3 (3y - 2b) \left(\frac{b-y}{y} \cot \Phi - \frac{(b-z)}{z} \text{cosec. } \Phi\right)$. Quia autem nulla est ratio, cur y magis in sit quam z, hanc expressionem ita adornabimus, ut vero proxime sit $G - F = \frac{1}{2} \Phi y z - \frac{1}{12} \Phi^3 y z + \frac{\Phi^3 y z \sqrt{yz}}{12b}$ rejectis terminis sequentibus. Cum vero sit $\sin \Phi = \Phi - \frac{1}{6} \Phi^3 +$ &c. erit hæc area $FSG = \frac{1}{2} y z \sin \Phi + \frac{\Phi^3 y z \sqrt{yz}}{12b}$. Sit jam tempus, quo spatium FG absolvitur $= T$ dierum; erit $T = \frac{yz \sin \Phi}{2m\sqrt{b}} + \frac{\Phi^3 y z \sqrt{yz}}{12mb\sqrt{b}}$ existente $m = 271989, 735$. Quoniam $\sin \Phi$ non multum a Φ discrepat, ponatur $T = \frac{yz \sin \Phi}{2m\sqrt{b}} + \frac{yz\sqrt{yz}}{12mb\sqrt{b}} (\sin \Phi)^3$,

& for-

& formetur ad b inveniendum hæc æquatio $\frac{\sqrt{yz}}{\sqrt{b}}\sin\Phi = \frac{2mT}{\sqrt{yz}} + Q$, fietque $T = T + \frac{Q\sqrt{yz}}{2m} + \frac{2m^2T^3}{3yz\sqrt{yz}}$, hincque $Q = \frac{-4m^3T^3}{3y^2z^2}$. Ex his obtinetur $\frac{\sqrt{yz}}{\sqrt{b}}\sin\Phi = \frac{2mT}{\sqrt{yz}} - \frac{4m^3T^3}{3y^2z^2}$, & $\frac{\sin\Phi}{\sqrt{b}} = \frac{2mT}{yz} - \frac{4m^3T^3}{3y^2z^2\sqrt{yz}}$. Erit ergo $\frac{\sqrt{b}}{\sin\Phi} = \frac{yz}{2mT} + \frac{mT}{3\sqrt{yz}}$, ideoque semilatus rectum $b = \left(\frac{yyzz}{4m^2T^2} + \frac{1}{3}\sqrt{yz}\right)(\sin\Phi)^2$. Quo cognito orbita per probl. III definietur. Q. E. J.

Coroll. 1.

25. Si ergo cometæ duæ fuerint cognitæ a Sole distantiæ $FS = f$, & $GS = g$, una cum angulo $FSG = \Phi$ ac præterea observatum sit tempus per spatium F G, quod sit $= T$ dier. erit orbitæ semilatus rectum $b = \left(\frac{ffgg}{4m^2T^2} + \frac{1}{3}\sqrt{fg}\right)(\sin\Phi)^2$, calculus autem sæpe facilius instituetur quærendo $\sqrt{b} = \left(\frac{fg}{2mT} + \frac{mT}{3\sqrt{fg}}\right)\sin\Phi$.

Coroll. 2.

26. Hinc ex cognito tempore T area FSG propius definiri, & quantum superet triangulum rectilineum FSG ducta corda FG comprehensum, assignari potest. Excessus scilicet

scilicet erit segmentum FG inter arcum curvæ & cordam contentum, quod est $= \frac{fg\sqrt{fg}}{12b}(\sin\Phi)^3 = \frac{m^2T^2\sin\Phi}{3\sqrt{fg}}$

Coroll. 3.

27. Cognito hoc modo latere recto, cujus semissis $= b$, statim reperietur positio perihelii A, posito enim angulo $ASF = v$, erit $\tan v = \cot\Phi - \frac{(b-g)f}{(b-f)\sin\Phi}$; hincque porro definitur distantia perihelii a sole $AS = a = \frac{bf\cos v^2}{b-f+f\cos v}$.

Coroll. 4.

28. Inventis ergo anomalia vera ASF, & rectis a & b assignari poterit tempus, quod ad spatium AF absolvendum impenditur, hincque cum datum sit tempus, quo planeta vel cometa in loco F hæsit, definiri poterit temporis momentum, quo in perihelio est versatus.

Scholion 3.

29. Ad hoc ergo tempus definiendum inserviet methodus in probl. IV. exposita, cujus tres constituendi sunt casus, quorum primus locum habet, si curva AFG fuerit ellipsis seu $2a > b$. Hoc casu quæratur angulus ω, ita ut sit $\tan\frac{1}{2}\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}\tan\frac{1}{2}v$; hocque cognito erit tempus per spatium $AF = \frac{a^3}{2m(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\sin\omega\right)$ dierum. Alter casus adhiberi debet, si curva AFG fuerit hyperbola, cum ob

$b > 2a$ quæratur angulus ω, ut fit $\tang \frac{1}{2}\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ $\tang \frac{1}{2} v$. quo cognito erit tempus per AF $= \frac{a^3}{2m(b-2a)^{\frac{3}{2}}} \left(\frac{b-a}{a}\tang\omega - l\tang(45^\circ + \frac{1}{2}\omega)\right)$. Tertius cafus pro curva AFG parabola eft confiderandus, fi fit $b = 2a$, tum fumto $t = \tang\frac{1}{2}v$, erit tempus per AF $= \frac{a\sqrt{a}}{m\sqrt{2}}(t + \frac{1}{3}t^3)$ dierum. Quemadmodum autem calculus pro parabola eft facillimus, ita idem maxime impeditur, fi orbita tantum proxime ad parabolam accefferit, Dum enim $2a - b$ vel $b - 2a$ quantitas minima evadit, cum angulus ω fit nimis parvus, tum vero in temporis expreffione denominator tam exiguus, ut minimus error in angulo ω commiffus maximam aberrationem in tempore parere poffit. Hanc ob rem his cafibus, quibus orbita cometæ proxime ad parabolam accedit, conveniet tempus idonea approximatione exhibitum potius quam verum ufurpare.

Problema VI.

Fig. 3. 30. Si orbita Cometæ non multum a parabola discrepet five fit ellipfis five hyperbola, definire [illegible] ūs, quo data quævis. Anomalia vera ASM conficitur.

Solutio.

Sit ut hactenus diftantia AS $= a$, & femi-latus rectum $= b$; quoniam $2a$ & b non multum a fe invicem difcrepant, ponatur $2a - b = \delta$, erit δ quantitas minima, & affirmativa quidem, fi curva fuerit ellipfis, at negativa fi curva fit hyperbola. Jam fit anomalia vera propofita ASM $= v$, ac ponatur $\tang \frac{1}{2} v = t$, erit ut fupra (17) vidimus area ASM $= \frac{\int aabbdt(1+tt)}{(b+\delta tt)^2}$; quæ ftatuatur $= \frac{aabbz}{b+\delta tt}$, atque ob $b + \delta = 2a$ reperietur hic valor pro $z = \frac{t}{b} + \frac{2at^3}{3bb} - \frac{2a\delta t^5}{3.5b^3} + \frac{2a\delta^2t^7}{5.7b^4} - \frac{2a\delta^3t^9}{7.9b^5}$ &c. ergo area ASM $=$

$$\frac{aab}{b+\delta tt}\left(t + \frac{2at^3}{3b} - \frac{2a\delta t^5}{3.5b^2} + \frac{2a\delta^2t^7}{5.7b^3} - \frac{2a\delta^3t^9}{7.9b^4} + \&c.\right)$$

Vel cum fit $\frac{b}{b+\delta tt} = 1 - \frac{\delta tt}{b} + \frac{\delta^2t^4}{bb} - \frac{\delta^3t^6}{b^3} + \&c.$

$$\text{erit area ASM} = aa\left(t + \tfrac{1}{3}t^3 - \frac{4a\delta}{5bb}t^5 + \frac{6a\delta\delta}{7b^3}t^7 - \frac{8a\delta^3}{9b^4}t^9 \;\&c.\right.$$
$$\left. + \frac{\delta\delta}{bb}t^5 - \frac{\delta^3}{b^3}t^7 + \frac{\delta^4}{b^4}t^9\right]$$

Quare cum tempus fit $= \frac{\text{ar. ASM}}{m\sqrt{b}}$ dierum, erit tempus quo anomalia vera ASM $= v$ abfolvitur in diebus expreffum $=$

$$\frac{aa}{m\sqrt{b}}\left(t + \tfrac{1}{3}t^3 - \frac{4a\delta}{5bb}t^5 + \frac{6a\delta^2}{7b^3}t^7 - \frac{8a\delta^3}{9b^4}t^9 \;\&c.\right.$$
$$\left. + \frac{\delta\delta}{bb}t^5 - \frac{\delta^3}{b^3}t^7 + \frac{\delta^4}{b^4}t^9\right]$$

quæ expreffio, etfi in infinitum progreditur, tamen citiffime conver-

convergit, si $\delta = 2a - b$ fuerit quantitas vehementer parva, uti statuimus. Q. E. J.

Coroll. 1.

31. Quia est $2a = b + \delta$, si in serie infinita hic valor ubique loco $2a$ substituatur, reperietur tempus, quo anomalia vera ASM $= v$, cujus semissis tangens ponitur $= t$, absolvitur $= \frac{aa}{m\sqrt{b}}\Big(t + \frac{1}{3}t^3 - \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 - \frac{4\delta^3}{9b^3}t^9$ &c.
$+ \frac{3\delta\delta}{5bb}t^5 - \frac{4\delta^3}{7b^3}t^7 + \frac{5\delta^4}{9b^4}t^9\Big]$

Coroll. 2.

32. Quoniam pro hyperbola fit δ numerus negativus, omnes termini prodibunt affirmativi, si enim pro hyperbola ponatur $b - 2a = \delta$ erit tempus per arcum AM $= \frac{aa}{m\sqrt{b}}$
$\Big(t + \frac{1}{3}t^3 + \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 + \frac{4\delta^3}{9b^3}t^9$ &c.
$+ \frac{3\delta\delta}{5bb}t^5 + \frac{4\delta^3}{7b^3}t^7 + \frac{5\delta^4}{9b^4}t^9\Big]$

Coroll. 3.

33. Series hæ maxime convergunt, quo minor fuerit angulus ASM; sin autem hic angulus v fiat tantus, ut ejus semissis multum superet semirectum, ideoque ejus tangens t unitatem longe superet; tum convergentia diminuetur. His ergo casibus expediet methodo directa uti. Quando quidem angulus ille ω ita sumtus, ut sit tang $\frac{1}{2}\omega = \frac{\sqrt{(2a - b)}}{\sqrt{b}}$

tang

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tang $\frac{1}{2}v$, vel tang $\frac{1}{2}\omega = \frac{\sqrt{(b - 2a)}}{\sqrt{b}}$ tang $\frac{1}{2}v$, prodit adhuc notabilis magnitudinis.

Problema VII.

Fig. 3

34. Si orbita cometæ non admodum a parabola discrepet, ex dato tempore, quod vel ante vel post appulsum ad perihelium elapsum sit, locum cometæ in Orbita, hoc est anomaliam veram ASM, una cum distantia cometæ a sole invenire.

Solutio.

Quoniam primo orbita cometæ est data, ponatur distantia perihelii a sole AS $= a$, semilatus rectum $= b$; & quoniam orbita non multum a parabola differre statuitur posito $2a - b = \delta$, erit δ quantitas valde parva. Deinde sit tempus, quod vel ante appulsum ad perihelium effluxerit, vel post, in diebus expressum $= T$; Anomalia vera autem, quæ quæritur, ASM sit $= v$, ac $t =$ tang $\frac{1}{2}v$. Jam ex præcedente problemate habebimus hanc æquationem

$$T = \frac{aa}{m\sqrt{b}}\Big(t + \frac{1}{3}t^3 - \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 - \frac{4\delta^3}{9b^3}t^9 \text{ \&c.} \\ + \frac{3\delta\delta}{5bb}t^5 - \frac{4\delta^3}{7b^3}t^7 + \frac{5\delta^4}{9b^4}t^9\Big]$$

ex qua valorem ipsius t erui oportebit. Ponamus $\frac{mT\sqrt{b}}{aa} = n$ brevitatis gratia, ut sit $n = t + \frac{1}{3}t^3$
-2δ

$$-\frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 - \frac{4\delta^3}{9b^3}t^9 \quad \&c.$$
$$+\frac{3\delta\delta}{5bb} \quad -\frac{4\delta^3}{7b^3} \quad +\frac{5\delta^4}{9b^4}$$

Sit primum orbita cometæ vera parabola, ideoque $\delta = o$, erit $n = t + \frac{1}{3}t^3$, atque $t^3 + 3t - 3n = o$; cujus æquationis cubicæ per regulam Cardani radix erit:

$$t = \sqrt[3]{(\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)})} - \sqrt[3]{(-\tfrac{3}{2} + \sqrt{(\tfrac{9}{4}nn + 1)})} \text{ vel}$$
$$t = \sqrt[3]{(\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)})} - \frac{1}{\sqrt[3]{(\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)})}} \text{ vel etiam}$$
$$t = \frac{1}{\sqrt[3]{(-\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)})}} - \sqrt[3]{(-\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)})}$$

Vel igitur ex his formulis, vel ex tabulis in hunc finem computatis valor ipsius t erui poterit, quo cognito dabitur anomalia vera ASM $= v$, per æquationem tang $\frac{1}{2}v = t$. Deinde autem habebitur distantia cometæ a sole SM $= y = \frac{ab}{a + (b-a)\cos v}$. Cum vero pro parabola sit $b = 2a$; erit $y = \frac{2a}{1 + \cos v} = \frac{a}{(\cos \frac{1}{2}v)^2}$. Sic itaque si orbita cometæ vera fuerit parabola, determinabitur ad datum tempus locus cometæ in orbita.

Sin autem orbita cometæ fuerit ellipsis maxime excentrica vel hyperbola non multum a parabola abhorrens, ita ut sit $\delta = 2a - b$ quantitas vehementer parva, posito $\frac{mT\sqrt{b}}{aa} = n$, hæc æquatio resolvi debebit:

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ita ut
posito
$n = t$

$$n = t + \tfrac{1}{3}t^3 - \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 - \frac{4\delta^3}{9b^3}t^9 \quad \&c.$$
$$+\frac{3\delta\delta}{5bb}t^5 \quad -\frac{4\delta^3}{7b^3} \quad +\frac{5\delta^4}{9b^4}$$

Sumatur primo tantum hæc æquatio $n = t + \frac{1}{3}t^3$, & per modum præcedentem quæratur valor ipsius t, sitque $t = \theta$, ita ut sit $n = \theta + \frac{1}{3}\theta^3$; eritque ob δ valde parvam quantitatem θ valor ipsius t vero proximus. Sit igitur verus valor $t = \theta + A\theta^3 + B\theta^5 + C\theta^7 + \&c.$ erit

$$t^3 = \theta^3 + 3A\theta^5 + 3B\theta^7 \;\&c.$$
$$\qquad\qquad\qquad + 3A^2\theta^7$$
$$t^5 = \theta^5 + 6A\theta^7, \&c. \quad \& \; t^7 = \theta^7.$$

quibus substitutis in æquatione orietur $n = \theta + \frac{1}{3}\theta^3 =$

$$\theta + A\theta^3 + B\theta^5 + C\theta^7 + \&c.$$
$$+ \tfrac{1}{3}\theta^3 + A\theta^5 + B\theta^7$$
$$- \frac{2\delta}{5b}\theta^5 + A^2\theta^7$$
$$+ \frac{3\delta\delta}{5bb}\theta^5 - \frac{2\delta}{b}A\theta^7 \quad \&c.$$
$$+ \frac{3\delta\delta}{bb}A\theta^7$$
$$+ \frac{3\delta^3}{7bb}\theta^7$$
$$- \frac{4\delta^3}{7b^3}\theta^7$$

Singulis ergo terminis more solito ad nihilum reductis erit

$A = o$;

$A = 0;\quad B = \frac{2\delta}{5b} - \frac{3\delta\delta}{5bb};\ C = -B - \frac{3\delta\delta}{7bb} + \frac{4\delta^3}{7b^3} =$
$-\frac{2\delta}{5b} + \frac{6\delta\delta}{35bb} + \frac{4\delta^3}{7b^3} = \frac{2\delta}{b}\left(\frac{\delta}{b}+1\right)\left(\frac{2\delta}{7b}-\frac{1}{5}\right)$

Cognito ergo ipſius t valore vero proximo θ, erit verus valor

$$t = \theta + \left(\frac{2\delta}{5b} - \frac{3\delta\delta}{5bb}\right)\theta^5 - \left(\frac{2\delta}{5b} - \frac{6\delta\delta}{35bb} - \frac{4\delta^3}{7b^3}\right)\theta^7:$$

Tum vero ob $t = \text{tang}\ \frac{1}{2}\ v$ cognoscetur anomalia vera $ASM = v$; hincque porro prodibit diſtantia cometæ a ſole $SM = y = \frac{ab}{a + (b-a)\,\text{coſ}\,v} = \frac{b}{1 + \frac{b-a}{a}\,\text{coſ}\,v}$. Q. E. J.

Coroll. 1.

35. Pro ellipſi ergo maxime oblonga δ ob quantitatem affirmativam, erit verus valor ipſius t major quam valor θ, qui ex hypotheſi orbitæ parabolicæ eſt erutus.

Coroll. 2.

36. Pro hyperbola autem ubi fit δ quantitas negativa verus valor ipſius t minor erit quam θ; utroque autem caſu ob tantas ipſius θ poteſtates, expreſſio pro t vehementer convergit, ſiquidem fit $\theta < 1$, quod fit ſi anomalia vera v fuerit angulo recto minor.

Scholium.

37. Quod ſi autem anomalia vera v multum excedat angulum rectum, ita ut δ vel t fiat numerus unitate multo major, hincque expreſſio illa pro t inventa divergat potius quam convergat, tum iſta methodo uti non conveniet, niſi forte δ fit quan-

quantit = tur min qua relа ſubſidiu anomali valor nem in vis caſu cometaı vera ı ſole 38. J. cognita, periheli affir- ſole SM), qui Sit = b; eı ;ativa quod ve n ca- diebus e enter quæritu era v ut fit tai tempus ılum hinc- æquatio con- Eulei δ fit quan-

quantitas tantopere exigua, ut per eam omnes termini efficiantur minimi. Oportebit igitur his caſibus expreſſionem veram, qua relatio inter tempus & anomaliam veram exhibetur, in ſubſidium vocare; atque ex ea modum, quo ad datum tempus anomalia vera aſſignari poſſit, eruere. Hanc itaque operationem in ſequentibus problematis fuſius ſum oſtenſurus, ut quovis caſu oblato theoria adhiberi, per eamque verus motus tam cometarum quam planetarum definiri queat.

Problema VIII.

38. Si orbita planetæ vel cometæ fuerit ellipſis quæcunque cognita, ad datum tempus, vel ante vel poſt appulſum ejus ad perihelium A, ejus anomaliam veram ASM, atque diſtantiam a ſole SM aſſignare.

Solutio.

Sit perihelii a ſole diſtantia $AS = a$, & ſemilatus rectum $= b$; erit $2a > b$, quia orbita ponitur ellipſis; Tempus autem, quod vel ante vel poſt appulſum ad perihelium effluxerit, fit in diebus expreſſum $= T$. Nunc ponatur anomalia vera, quæ quæritur $ASM = v$, ſitque alius angulus ω ita comparatus ut fit $\text{tang}\ \frac{1}{2}\ \omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}\ \text{tang}\ \frac{1}{2}\ v$, eritque ut vidimus (19), tempus $T = \frac{a^3}{2m\,(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\ \text{ſin}\,\omega\right)$ ex qua æquatione angulum ω erui oportet. Fiet ergo $\omega - \frac{(b-a)}{a}$ ſin ω =

$\sin \omega = \frac{2 . m (2 a - b) \frac{3}{2} T}{a^3}$. Convertatur hæc expreſſio $\frac{2 m (2 a - b) \frac{3}{2} T}{a^3}$ quæ eſt cognita, in arcum circuli cujus radius $= 1$, quod ita fiet: Sumatur expreſſionis $\frac{2 m (2 a - b) \frac{3}{2} T}{a^3}$ logarithmus, ad quem addatur conſtanter hic logarithmum 5, 3144251332; atque ſumma erit logarithmus anguli hujus quæſiti in minutis ſecundis expreſſi. Vel cum ſit logarithmus $m = 5{,}4345525139$, ad log. $\frac{(2 a - b) \frac{3}{2} T}{a^3}$ addatur conſtanter iſte logarithmus 11, 0500076428, & numerus logarithmo reſultanti reſpondens dabit angulum in minutis ſecundis expreſſum. Sit iſte angulus $= u$, qui erit ille ipſe, quem Aſtronomi anomaliam mediam vocare ſolent, qui ergo hoc pacto ex dato tempore T inveniri, vel ex tabulis aſtronomicis, ſi quæſtio de planeta quopiam inſtituatur, depromi poteſt. Priori autem modo ſi iſte angulus u in minutis ſecundis exprimatur erit $l\, u = 11{,}0500076428 + \frac{3}{2} l (2 a - b) + l T - 3 l a$, unde deficientibus tabulis aſtronomicis, anomalia media u expedite reperitur. Inventa erga jam ſit hæc anomalia media u, eritque $u = \omega - \frac{(b - a)}{a} \sin \omega$. Hæc æquatio commodiſſime aliquot tentaminibus reſolvetur. Scilicet cum ſit $\omega \gtrless u$, ſumatur pro arbitrio angulus pro ω, & computetur angulus $= \frac{(b - a)}{a}$ $\sin \omega$,

$\sin \omega$, quod fiet, ſi a $l \frac{b - a}{a} + l \sin \omega$ ex tabulis ſumto ſubtrahatur 4, 6855748668, logarithmus enim reſiduus dabit numerum minutorum ſecundorum ipſi $\frac{(b - a)}{a} \sin \omega$ æquivalentium; hoc igitur facto pro $\omega - \frac{(b - a)}{a} \sin \omega$ reperietur angulus vel major vel minor quam u; priori caſu valor pro ω aſſumtus erat nimis magnus, poſteriori vero nimis parvus; ſic igitur corrigendo hypotheſin factam mox angulus ω prope verus cognoſcetur. Sit valor jam prope verus pro ω inventus $= \varrho$; verus autem valor ſit $\omega = \varrho + z$, erit ſin $(\varrho + z) = \sin \varrho + z \cos \varrho$; ideoque $u = \varrho + z - \frac{(b - a)}{a} \sin \varrho - (b - u)\, z \cos \varrho$

& hinc naſcitur $z = \frac{u - \varrho + \frac{(b - a)}{a} \sin \varrho}{1 - \frac{(b - a)}{a} \cos \varrho}$, ubi in numeratore pars $\frac{b - a}{a} \sin \varrho$ pari modo in angulum converti debet, ut ſupra oſtendimus. Denominator autem erit merus numerus, & ob ſinum totum $= 1$, logarithmi coſϱ characteriſtica denario minui debet. Hoc ergo modo reperitur angulus z, qui ad angulum ϱ additus dabit verum angulum quæſitum ω. Sin autem angulus ϱ aſſumtus nimium a vero diſceſſerit, hoc modo multo propior valor pro ω orietur, qui tum loco ϱ ſubſtitutus vero proximum valorem pro ω ſuppeditabit. Quodſi autem

tem hac methodo repertus fuerit angulus ω, hinc facile invenietur anomalia vera ASM $= v$ ope hujus formulæ tang $\frac{1}{2} v$ $= \frac{\sqrt{b}}{\sqrt{(2a-b)}}$ tang $\frac{1}{2}\,\omega$. Tandem vero erit diſtantia planetæ ſeu cometæ a ſole SM $= \frac{ab}{a+(b-a)\,\text{coſ}\,v}$. Q. E. J.

Coroll. 1.

39. Coefficiens $\frac{b-a}{a}$, qui in hoc calculo occurrit, vocari ſolet orbitæ excentricitas; (9) hujus ergo logarithmus imprimis notari debet, quippe quo cognito totus calculus facillime inſtituetur.

Coroll. 2.

40. Ex data ergo orbitæ excentricitate $\frac{b-a}{a}$ hujus calculi ope ad anomaliam mediam quamvis propoſitam expedite anomalia vera reſpondens definietur: Hocque modo tabulæ æquationis motuum planetarum & cometarum, qui quidem in ellipſibus revolvuntur, ſupputabuntur.

Problema IX.

Fig. 3. 41. Si cometa in hyperbola circa ſolem moveatur, atque tempus, quo per perihelium A tranſit, fuerit notum, ad datum quodvis tempus locum cometæ in orbita ſeu anomaliam veram ASM, ejusque a ſole diſtantiam SM ſupputare.

Solutio.

inve- g ½ v plane- ocari impri- lime lcu- dite ulæ n in que da- ma- tio.

Solutio.

Sit iterum perihelii a ſole diſtantia AS $= a$, & ſemilatus rectum $= b$, eritque $b > 2a$. Tempus autem propoſitum a momento, quo Cometa in perihelio hæret, diſtet intervallo $=$ T dierum. Deinde ſit anomalia vera, quam inveniri oportet, ASM $= v$; ac concipiatur alius angulus ω, ut ſit tang $\frac{1}{2}\,\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ tang $\frac{1}{2}\,v$; eritque (20), T $= \frac{a^3}{2m(b-2a)^{\frac{3}{2}}}\left(\frac{b-a}{a}\text{ tang }\omega - l\text{ tang }(45^\circ + \tfrac{1}{2}\,\omega)\right)$. Statuatur $\frac{2m(b-2a)^{\frac{3}{2}}\,\text{T}}{a^3} = u$, erit u quantitas cognita, atque illi, quæ ante anomaliæ mediæ nomine occurrerat, analoga: hoc vero caſu expediet u in meris numeris expreſſam retinere, quam in angulum convertere. His monitis erit itaque $u = \frac{b-a}{a}$ tang $\omega - l$ tang $(45^\circ + \frac{1}{2}\,\omega)$ unde tentando pro ω ejusmodi valor eruatur qui a vero non multum diffideat. Sit ergo ϱ angulus non multum a vero anguli ω valore diſcrepans, ac ponatur $\omega = \varrho + z$, ob z angulum valde parvum erit tang. $\omega = \frac{\text{tang }\varrho + z}{1 - z\,\text{tang }\varrho}$ ſeu tang $\omega =$ tang $\varrho + z\,(1 + (\text{tang }\varrho)^2)$ $=$ tang $\varrho + \frac{z}{(\text{coſ }\varrho)^2}$. Ad l tang $(45^\circ + \frac{1}{2}\,\omega)$ inveniendum ponatur l tang $(45^\circ + \frac{1}{2}\varrho) =$ R, eritque l tang $(45^\circ + \frac{1}{2}\,\omega)$ valor ipſius R, qui prodit ſi loco ϱ ſcribatur $\varrho + z$,

 prodit

prodit autem is $= R + \frac{z\, dR}{d\varrho}$. Erit vero $dR = \frac{\frac{1}{2} d\varrho}{(\text{cof}(45^\circ + \frac{1}{2}\varrho))^2 \text{tang}(45^\circ + \frac{1}{2}\varrho)} = \frac{d\varrho}{\text{cof}\,\varrho}$; ideoque l tang $(45^\circ + \frac{1}{2}\omega) = l\,\text{tang}(45^\circ + \frac{1}{2}\varrho) + \frac{z}{\text{cof}\,\varrho}$. His fubftitutis erit

$$u = \frac{b-a}{a}\,\text{tang}\,\varrho + \frac{(b-a)z}{a(\text{cof}\,\varrho)^2} - l\,\text{tang}\,(45 + \tfrac{1}{2}\varrho) - \frac{z}{\text{cof}\,\varrho},$$

unde nafcitur $z = \dfrac{u + l\,\text{tang}\,(45^\circ + \frac{1}{2}\varrho) - \frac{(b-a)}{a}\,\text{tang}\,\varrho}{\frac{1}{(\text{cof}\,\varrho)^2}\left(\frac{b-a}{a} - \text{cof}\,\varrho\right)}$

feu $z = \dfrac{u + l\,\text{tang}\,(45 + \frac{1}{2}\varrho) - \frac{(b-a)}{a}\,\text{tang}\,\varrho}{\frac{(b-a)}{a} - \text{cof}\,\varrho}(\text{cof}\,\varrho)^2$. Hoc modo reperietur numerus pro z, dummodo ea regula ad l tang $(45 + \frac{1}{2}\rho)$ exprimendum obfervetur, quæ fupra (22) eft tradita. Hic autem numerus pro z inventus in angulum eft convertendus, modo, quem ante expofui. Hisque factis habebitur verus valor anguli $\omega = \varrho + z$, qui autem, fi adhuc dubium fuperfit, denuo pro ρ ipfe poni, ficque ulterius corrigi poterit. Invento autem angulo ω, ftatim reperietur tang $\frac{1}{2}v = \frac{\sqrt{b}}{\sqrt{(b - 2a)}}$ tang $\frac{1}{2}\omega$, atque adeo definietur anomalia vera quæfita ASM $= v$, ex qua porro elicitur diftantia cometæ a fole SM $= y = \frac{ab}{a + (b-a)\,\text{cof}\,v}$. Q. E. J.

Scholion.

Scholion.

42. Ad ufum hujus calculi, qui minus eft ufitatus in aftonomia, uberius declarandum, ponamus cometam in hyperbola æquilatera deferri, atque perihelii ejus a fole diftantiam effe = diftantiæ terræ a fole mediæ, nempe effe . . . . $a = 100000$
unde fiet $b = a(1 + \sqrt{2})$, feu $b = 241421,356$
& quæri debeat locus hujus cometæ centum diebus poft tranfitum ejus per perihelium; ita ut fit $T = 100$. Primo ergo quæri debet u ut fit $l\,u = l\,2m + l\,T + \frac{3}{2}\,l\,41421,356 - 3\,l\,100000$; unde fequens calculus nafcitur.

$l\,2m$	=	5, 7 3 5 5 8 2 5
$l\,T$	=	2, 0 0 0 0 0 0 0
$l\,41421,356$	=	4, 6 1 7 2 2 4 3
femiff.	=	2, 3 0 8 6 1 2 2
		1 4, 6 6 1 4 1 9 0
$3\,l\,100000$	=	1 5, 0 0 0 0 0 0 0
$l\,u$	=	9, 6 6 1 4 1 9 0
unde u	=	0, 4 5 8 5 8 4

Deinde eft $\frac{b-a}{a} = 1,41421356 = \sqrt{2}$

ideoque $l\,\frac{b-a}{a} =$ 0, 1 5 0 5 1 5 0

Statuatur primum tang $\omega = \frac{a}{b-a}\,u$; neglecto altero termino

erit ob $l\,u$	=	9, 6 6 1 4 1 9 0
fubtr. $l\,\frac{b-a}{a}$	=	0, 1 5 0 5 1 5 0
l tang ω	=	9, 5 1 0 9 0 4 0

ideoque

ideoque $\omega = 17°, 58'$,
& hinc fiet angulus $45° + \frac{1}{2}\omega = 53°, 59'$,
cujus tangens eſt $= 1,3755403$.

Patet ergo angulum $17°, 58'$ vehementer nimis eſſe parvum & ſi pro ω ſumatur $30°$, adhuc iſte valor nimis erit parvus, prodit enim pro u hic numerus 0, 26719, ſin autem ſtatuatur $\omega = 40°$, prodit pro u iſte numerus 0, 4237, quare cum ſit $u = 0,458584$, perſpicuum eſt hunc valorem non multum a veritate abludere, atque ex his duabus poſitionibus 30° & 40° conjecturam faciendo reperietur ω propemodum eſſe debere 42°, qui valor ergo pro ϱ accipiatur, ita ut ſit $\varrho = 42°$, & $\frac{1}{2}\varrho = 21°$ atque $45° + \frac{1}{2}\varrho = 66°$. Calculus itaque ſic inſtituatur

l tang 66° − 10	=	0, 3 5 1 4 1 6 9
hujus log.	=	9, 5 4 5 8 2 2 6
addatur per (22)	. .	0, 3 6 2 2 1 5 7.
Erit ergo		9, 9 0 8 0 3 8 3
l tang$(45° + \frac{1}{2}\varrho)$	=	0, 8 0 9 1 6 7
Porro eſt l tang ϱ	=	9, 9 5 4 4 3 7 4
add. $l \frac{b-a}{a}$	=	0, 1 5 0 5 1 5 0
$l \frac{b-a}{a}$ tang ϱ	=	1, 1 0 4 9 5 2 4
Ergo $\frac{b-a}{a}$ tang ϱ	=	1, 2 7 3 3 6 4

Quo circa ad u	=	0, 4 5 8 5 8 4
addatur l tang $(45° + \frac{1}{2}\varrho)$	=	0, 8 0 9 1 6 7
		1, 2 6 7 7 5 1
ſubtrahatur $\frac{b-a}{a}$ tang ϱ	=	1, 2 7 3 3 6 4
erit numerator	=	−0, 0 0 5 6 1 3
		$a = b$

3.
parvum
parvus,
tuatur ω
ſit $u =$
a veri-
° conje-
42°, qui
= 21°
ituatur
5 8 4
1 6 7
7 5 1
3 6 4
6 1 3
$a = b$

$\frac{b-a}{a}$	=	1, 4 1 4 2 1 3 5
cof ϱ	=	7 4 3 1 4 4 8
Denom.	=	0, 6 7 1 0 6 8 7
Jam eſt l cof ϱ	=	9, 8 7 1 0 7 3 5
dupl. $l(\text{cof}\,\varrho)^2$		9, 7 4 2 1 4 7 0
Add. l num.		7, 7 4 9 1 9 5 0
		7, 4 9 1 3 4 2 0
ſubtr. l den.	=	9, 8 2 6 7 6 7 0
$l - z$	=	7, 6 6 4 5 7 5 0
ſubtr.		4, 6 8 5 5 7 4 9
		2, 9 7 9 0 0 0 1

ergo $-z = 952, 79'' = 15', 53''$

Erit ergo $\omega = \varrho + z = 41°, 44', 7''$.

Invento ergo angulo $\omega = 41°, 44', 7''$, erit tang $\frac{1}{2}v = \frac{\sqrt{b}}{\sqrt{(b-2a)}}$ tang $\frac{1}{2}\omega$: at ob $b = a(1 + \sqrt{2})$ erit $\frac{b}{b-2a} = \frac{1+\sqrt{2}}{\sqrt{2}-1} = (1+\sqrt{2})^2$ ideoque $\sqrt{\frac{b}{b-2a}} = 1 + \sqrt{2} = 2,41421356$, & ob $\frac{1}{2}\omega = 20°, 52', 3\frac{1}{2}''$ calculus ita ſe habebit.

Ad. $l \sqrt{\frac{b}{b-a}}$	=	0, 3 8 2 7 7 5 6
add. l tang $\frac{1}{2}\omega$	=	9, 5 8 1 1 7 0 9
l tang $\frac{1}{2}v$	=	9, 9 6 3 9 4 6 5
Ergo $\frac{1}{2}v$	=	42°, 37', 28''
Ac propterea v	=	85°, 14', 56''

Atque ideo cometa iſte tempore 100 dierum poſt tranſitum per perihelium deſcripſit angulum ASM $= v = 85°$, 14',

$14', 56''$. Erit ergo hoc tempore ejus a sole distantia SM $=$

$$y = \frac{ab}{a+(b-a)\operatorname{cof} v} = \frac{b}{1+\frac{b-a}{a}\operatorname{cof} v}.$$

Quare ad $l \operatorname{cof} v$	$=$	8, 9 1 8 1 7 4 7
add. $l\,\frac{b-a}{a}$	$=$	0, 1 5 0 5 1 5 0
		9, 0 6 8 6 8 9 7
ergo $\frac{b-a}{a}\operatorname{cof} v$	$=$	0, 1 1 7 1 3 6
& denom.	$=$	1, 1 1 7 1 3 6
Jam a $l\,b$	$=$	5, 3 8 2 7 7 5 6
subtr. l denom.	$=$	0, 0 4 8 1 0 6 1
erit $l\,y$	$=$	5, 3 3 4 6 6 9 5
& SM $= y$	$=$	2 1 6 1 0 7.

Cum igitur distantia a Sole in perihelio esset 100000, erit post 100 dies cometæ a solo distantia $=$ 216107. Hocque exemplum ad calculum hujus methodi illustrandum sufficiet.

Problema X.

Fig. 4. 43. Datis duobus planetæ cometæve locis F & H a se invicem non multum remotis una cum tempore, quo spatium FH est confectum, ad quodvis tempus medium ejus locum verum G assignare.

Solutio.

Quo hoc problema commodius resolvere queamus, applicemus id ad motum terræ, quam motu medio in distantia a sole mediocri circulum percurrere concipiamus. Existente ergo sole in s sint f & b duo terræ loca, & tempus per fb sit $=$ T dierum.

dierum. ut hact $l\,\frac{2m\,T}{c\sqrt{c}}$ summæ & c in 5500c fsb in T in d tempo ut duc fsb, sg in 90°– $\frac{(\beta-\alpha)}{2T}$ hincq Est ve

dierum. Sit distantia media terræ a sole $fs = bs = c = 100000$ ut hactenus sumsimus, hincque reperietur angulus fsb, ad $l\,\frac{2m\,T}{c\sqrt{c}}$ addendo 5,3144251332, numerus enim respondens summæ exhibebit angulum fsb in minutis secundis, seu ob m & c in numeris data, ad l T addatur hic logarithmus 2,5500076427, & numerus summæ respondens dabit angulum fsb in minutis secundis expressum. Dispertiatur jam tempus T in duas partes α, & β ita ut sit $T = \alpha + \beta$ atque elapsa temporis parte α manifestum est terram futuram esse in g, ita ut ducta gs, sit ang. $fsg : gsb = \alpha : \beta$; erit ergo $fsg = \frac{\alpha}{T} fsb$, & $gsb = \frac{\beta}{T} fsb$. Ducatur corda fb, secans radium sg in o, & quæratur sagitta og hoc modo: ob ang. $sfo = 90° - \frac{1}{2} fsb$, & $sof = 90 + \frac{1}{2} fsb - \frac{\alpha}{T} fsb = 90 + \frac{(\beta-\alpha)}{2T} fsb$, erit $so : sf = \operatorname{cof}\frac{1}{2} fsb : \operatorname{cof}\frac{\beta-\alpha}{2T} fsb$, hincque $so = c\,\frac{\operatorname{cof}\frac{1}{2} fsb}{\operatorname{cof}\frac{\beta-\alpha}{2T} fsb}$; ac propterea sagitta $go =$

$$\frac{c\left(\operatorname{cof}\frac{\beta-\alpha}{2T} fsb - \operatorname{cof}\frac{1}{2} fsb\right)}{\operatorname{cof}\frac{\beta-\alpha}{2T} fsb} = \frac{2c \sin\frac{\beta}{2T} fsb . \sin\frac{\alpha}{2T} fsb}{\operatorname{cof}\frac{\beta-\alpha}{2T} fsb}$$

Est vero $\frac{1}{2T} fsb$ angulus constans, qui erit $= 29', 34$ 98''

$\frac{98''}{1000}$ seu $= 1774, 098''$, cujus numeri logarithmus est 3, 2489776471. Quodsi ergo hic angulus $\frac{1}{2T} f s b = 29', 34 \frac{''98}{1000}$ ponatur $= \tau$, & tempora α & β in diebus exprimantur, erit sagitta $g o = \frac{2c \sin \beta \tau . \sin \alpha \tau}{\cos(\beta - \alpha)\tau}$.
His præmissis observatus sit planeta seu cometa in F tempore autem $T = \alpha + \beta$ elapso in H, & quæratur ubi is sit futurus elapso tempore tantum α, postquam in F est versatus. Sit G locus quæsitus, & quoniam tempora in eadem orbita sunt ut areæ circa solem S descriptæ, erit area FSG: ar. GSH $= \alpha : \beta$. Ducatur corda FH, secans SG in O, & sequenti modo sagitta GO definietur. Quoniam curvatura orbitæ FGH proficiscitur a vi centripeta, ponamus vim centripetam qua cometa seu planeta spatium FGH peragrans ad solem S sollicitatum, esse constantem $= P$; quia enim loca F & H satis sibi sunt vicina, hæc hypothesis a veritate sensibiliter non recedit; Quo minus autem discrepet, ponamus P eam esse vim centripetam, quæ in loco medio G exercetur, ejusque directionem rectæ SG esse parallelam. Simili modo sit vis centripeta terram in orbita circulari retinens $= p$, quæ quidem per se erit constans, at ponamus pariter, terram ab ea dum spatium $f g b$ percurrit, constanter in directione $g s$ sollicitari. In F & f ducantur tangentes FMN, $f m n$, & ex H & b ipsis SG & $s g$ parallelæ HN & $b n$, erunt hæc intervalla HN & $b n$ effectus

nus est
f s b
in die-
in α τ
τ
mpore
futurus
Sit G
unt ut
= α: β.
do sa-
profi-
ometa
atum,
nt vi-
; Quo
ntripe-
onem
ripeta
er se
tium
In
is SG
: b n
ctus

effectus ab istis viribus centripetis producti: & quia hi effectus eodem tempore $= \alpha + \beta$ producuntur, erunt ii ipsis viribus proportionales, quare erit HN : $b n = P : p$. Erunt porro ex natura motus æquabilis quo tangentes FN & $f n$ describerentur, si nulla adesset vis sollicitans, spatia FM: MN $= \alpha : \beta$; & $f m : m n = \alpha : \beta$, unde ob triangula FOM & FHN similia erit quoque FO: OH $= \alpha : \beta$. Cum igitur & corda FH & ratio $\alpha : \beta$ detur ad locum medium G inveniendum secetur corda FH in O, ut sit FO: OH $= \alpha : \beta$, & radius ex S per O productus transibit per locum G quæsitum. Superest ergo ut sagitta GO definiatur, quod ex effectu virium ita præstabitur: Erit nempe HN: GM $=$ FN^2: FM^2 $= (\alpha + \beta)^2 : \alpha^2$; & pari modo $b n : g m = f n^2 : f m^2 = (\alpha + \beta)^2 : \alpha^2$, unde fit HN: GM $= b n : g m$ seu HN: $b n =$ GM: $g m$: est vero quoque HN: $b n =$ OM: $o m$; unde conducitur HN: $b n =$ OG: $o g = P : p$; ideoque erit OG $= \frac{P}{p} o g$. Cum igitur sit $o g = \frac{2c \sin \alpha \tau, \sin \beta \tau}{\cos(\beta - \alpha)\tau}$; erit sagitta quæsita OG $= \frac{2Pc \sin \alpha \tau . \sin \beta \tau}{p \cos(\beta - \alpha)\tau}$. Divisa ergo corda FH in ratione temporum $\alpha : \beta$ in O, ductaque SO si capiatur OG $= \frac{2Pc \sin \alpha \tau, \sin \beta \tau}{p \cos(\beta - \alpha)\tau}$ erit G locus planetæ seu cometæ quæsitus, siquidem loca extrema F & H non multum sint a se invicem remota. Cum autem vires centripetæ sint in ratione reciproca duplicata distantiarum a sole, erit

P, $p =$

$P : p = cc : SG^2$: vel potius, quia vim fumi oportet mediam inter extremas, erit ad veritatem accommodatius $P : p = 4cc : (SF + SH)^2$. Hac virium centripetarum ratione in computum ducta erit fagitta $OG = \frac{8c^3 \sin \alpha\tau . \sin \beta\tau}{(SF + SH)^2 \cos(\beta - \alpha)\tau}$.

Q. E. J.

Corol. 1.

44. Si ratio temporum $\alpha : \beta$, atque adeo ratio fegmentorum cordæ FO : OH non multum a ratione æqualitatis discrepet, tum fine errore loco $\frac{FS + SH}{2}$ fcribi poterit SG ita ut fit $OG = \frac{2c^3 \sin \alpha\tau . \sin \beta\tau}{SG^2 \cos(\beta - \alpha)\tau}$. Ac fi ratio $\alpha : \beta$ fuerit penitus ratio æqualitatis erit $OG = \frac{2c^3 \sin(\alpha\tau)^2}{SG^2} = \frac{c^3 fv . 2\alpha\tau}{SG^2}$.

Coroll. 2.

45. Si ergo viciffim detur locus G, in rectis SF & SH, quæ repræfentant loca heliocentrica, puncta F & H affignari poterunt, in quibus planeta feu cometa tempore α dier. antequam ad G, & tempore β dier. poftquam in G fuerat, verfatur. Nam ex puncto G, fumatur $GO = \frac{2c^3 \sin \alpha\tau . \sin \beta\tau}{SG^2 . \cos(\beta - \alpha)\tau}$ & per punctum O agatur recta FOH, ita ut fit FO : HO $= \alpha : \beta$, erunt F & H loca quæfita.

Coroll. 3.

47. Quo minora ergo fuerint temporum intervalla α & β & quo propius ad rationem æqualitatis accedant, eo minus

nus hæc determinatio a veritate aberrabit. Neque vero plerumque aberratio a veritate fit fenfibilis, nifi angulus FSH fit fatis magnus & circiter 10 vel 15 gradus fuperet, quæ latitudo fatis eft magna, ut hinc utiles methodi ad orbitas tam planetarum quam cometarum deriventur.

Problema XI.

47. Datis planetæ feu cometæ tribus locis geocentricis a fe invicem non nimis remotis, una cum ejus diftantia vera a terra tempore obfervationis mediæ; definire ejus planetæ feu cometæ orbitam veram, in qua circa folem movetur.

Solutio.

Tempore primæ obfervationis fit terra in f exiftente fole in S, fitque longitudo folis $= f$. In obfervatione fecunda fit terra in g, & longitudo folis $= g$; in obfervatione tertia fit terra in H, & longitudo folis $= h$. Tempus autem inter primam & fecundam obfervationem fit $= \alpha$; & tempus inter fecundam & tertiam obfervationem $= \beta$. Erit ergo angulus $f S g = g - f$; & angulus $g S h = h - g$: atque per theoriam folis dabuntur diftantiæ Sf, Sg, & Sh. Repræfentet fcilicet tabula planum eclipticæ, fitque in prima obfervatione longitudo cometæ (feu planetæ) obfervata $= F$, quam recta $f \xi$ repræfentet: in obfervatione fecunda fit cometæ longitudo obfervata $= G$, per rectam $g \eta$ repræfentata; & in obfervatione tertia fit longitudo cometæ $= H$, quam recta $h \theta$ exhibeat. Denique in obfervatione prima fit Fig. 5.

fit

latitudo cometæ $= \zeta$, in ſecunda $= \eta$, in tertia $= \theta$. Ex longitudinibus ergo obſervatis erunt anguli

$$S f \zeta = F - f, \quad S r \eta = G - f; \quad f m g = G - F$$
$$S g \eta = G - g; \quad S p \zeta = F - g; \quad g n b = H - G$$
$$S b \theta = H - b; \quad S s \theta = H - g;$$
$$S q y = G - b;$$

ducatur fx ipſi gm & $g\eta$ ipſi bn parallela, erit ob datos in triangulo Sfx omnes angulos cum latere Sf;

$$Sx = \frac{\text{ſin}(G-f)}{\text{ſin}(G-g)} Sf; \; fx = \frac{\text{ſin}(g-f)}{\text{ſin}(G-g)} Sf; \; \& \; gx = \frac{\text{ſin}(G-f)}{\text{ſin}(G-g)}$$

$Sf - Sg$. Deinde ob datos in triangulo fpx angulos cum latere Sx erit $fp = \frac{\text{ſin}(G-g)}{\text{ſin}(F-g)} fx = \frac{\text{ſin}(g-f)}{\text{ſin}(F-g)} Sf$; $px = \frac{\text{ſin}(G-F)}{\text{ſin}(F-g)} fx$, hinc eſt $px : fp = \text{ſin}(G-F) : \text{ſin}(G-g) = gx : fm$, unde obtinetur $fm = \frac{\text{ſin}(G-f)}{\text{ſin}(G-F)} Sf - \frac{\text{ſin}(G-g)}{\text{ſin}(G-F)} Sg$.

Tum erit $pm = \frac{\text{ſin}(g-f)}{\text{ſin}(F-g)} Sf + fm$. Ergo $\text{ſin}(G-g) : pm = \text{ſin}(F-g) : gm$ unde fit $gm = \frac{\text{ſin}(F-g)}{\text{ſin}(G-g)} fm + \frac{\text{ſin}(g-f)}{\text{ſin}(G-g)} Sf$ ſeu

$$gm = \frac{\text{ſin}(G-f)\,\text{ſin}(F-g) + \text{ſin}(g-f)\,\text{ſin}(G-F)}{\text{ſin}(G-g)\,\text{ſin}(G-F)} Sf - \frac{\text{ſin}(F-g)}{\text{ſin}(G-F)} Sg.$$

at eſt $\text{ſin}(G-f)\,\text{ſin}(F-g) + \text{ſin}(g-f)\,\text{ſin}(G-F) = \text{ſin}(F-f)\,\text{ſin}(G-g)$ ergo $gm = \frac{\text{ſin}(F-f)}{\text{ſin}(G-F)} Sf - \frac{\text{ſin}(F-g)}{\text{ſin}(G-F)} Sg$. Simili autem modo ratiocinando reperiuntur lineæ gn & bn eritque

Ex lon- -F G latos in (G−f) (G−g) um la- −F) −g) fx, x: fm, g) F) Sg. pm= Sf ſeu −g) F) Sg. (G−g) autem e m=

$$fm = \frac{\text{ſin}(G-f)}{\text{ſin}(G-F)} Sf - \frac{\text{ſin}(G-g)}{\text{ſin}(G-F)} Sg$$
$$gm = \frac{\text{ſin}(F-f)}{\text{ſin}(G-F)} Sf - \frac{\text{ſin}(F-g)}{\text{ſin}(G-F)} Sg$$
$$gn = \frac{\text{ſin}(H-g)}{\text{ſin}(H-G)} Sg - \frac{\text{ſin}(H-b)}{\text{ſin}(H-G)} Sb$$
$$bn = \frac{\text{ſin}(G-g)}{\text{ſin}(H-G)} Sg - \frac{\text{ſin}(G-b)}{\text{ſin}(H-G)} Sb.$$

Inventis jam punctis m & n, quibus ad ſequentem calculum erit opus, ſit G locus cometæ verus in ſecunda obſervatione, unde perpendicularis demittatur in eclipticam $G\eta$: ac ponatur ejus a terra diſtantia $= r$, erit ob latitudinem obſervatam $= \eta$, intervallum $g\eta = r \cos \eta$; & $G\eta = r \sin \eta$, dabitur ergo locus cometæ ad eclipticam relatus η, hincque lineæ $m\eta = g\eta - gm$; & $n\eta = g\eta - gn$. Ex η ducatur recta ad ſolem $S\eta$, atque demiſſo ex S in $g\eta$ perpendiculo SM, erit $SM = Sg \sin(G-g)$ & $gM = Sg \cos(G-g)$: hinc erit $M\eta = g\eta - gM$, & $\tan \eta SM = \frac{\eta M}{SM} = \cot S\eta M$; atque $S\eta = \frac{SM}{\sin S\eta M}$. Deinde reperietur latitudo heliocentrica, erit enim $\tan GS\eta = \frac{G\eta}{S\eta}$ & diſtantia a ſole $GS = \frac{G\eta}{\sin GS\eta} = \frac{S\eta}{\cos GS\eta}$. Sint nunc F & H loca cometæ in prima & tertia obſervatione, unde perpendicula in planum eclipticæ demiſſa cadere debent in rectas $f\zeta$ & $b\theta$, eritque

ducta

ducta corda FH, a radio SG in O secto, FO : HO $= \alpha : \beta$, & erit sagitta $GO = \frac{2c^3 \sin \alpha\tau . \sin \beta\tau}{SG^2 \cos(\beta-\alpha)\tau}$ (44). Demittatur quoque ex O perpendiculum Oo in planum eclipticæ, quod incidet in rectam $S\eta$; eritque ηo : GO $= S\eta$: SG; ideoque $\eta o =$ GO cos GSη. Cum igitur sit $\zeta o \theta$ corda projectionis orbitæ in ecliptica, ob $\theta o : \zeta o =$ HO : FO $= \beta : \alpha$, per punctum o duci debet recta $\zeta o \theta$ intra crura $m\zeta$ & $n\theta$, ita ut sit $\zeta o : \theta o = \alpha : \beta$, hoc est in ratione data. Ad hoc efficiendum in $S\eta$ producta capiatur oi ut sit $lo : oi = \alpha : \beta$, & per i ducatur ipsi $m\zeta$ parallela $i\theta$, cujus cum $n\theta$ intersectio θ dabit positionem cordæ $\zeta o \theta$ quæsitam. Sequenti autem modo per calculum puncta ζ & θ determinabuntur. Sit ang. $S\eta M = \mu$, qui cum detur erit in triangulo $m\eta l$, sin $(G-F+\mu) : m\eta = \sin(G-F) : \eta l = \sin\mu : ml$ erit ergo

$$\eta l = \frac{\sin(G-F)}{\sin(G-F+\mu)} m\eta \;\&\; ml = \frac{\sin\mu}{\sin(G-F+\mu)} m\eta$$ hinc

ob $lo = \eta l - \eta o$ dabitur lo: eritque $oi = \frac{\beta}{\alpha} lo$ & $li = \frac{(\alpha+\beta)}{\alpha} lo = \lambda\theta$, ducta $\lambda\theta$ parallela ipsi li. Nunc consideretur triangulum $\theta k \lambda$, eritque $\sin(H-F) : \lambda\theta = \sin(G-F+\mu) : k\theta = \sin(\mu - H + G) : k\lambda$; unde fit

$$k\theta = \frac{\sin(\mu+G-F)}{\sin(H-F)} \lambda\theta; \;\&\; k\lambda = \frac{\sin(\mu-H+G)}{\sin(H-F)} \lambda\theta;$$ sicque cognoscun-

cognoscentur puncta θ & λ; ob $km = \frac{\sin(H-G)}{\sin(H-F)} mn$ & $nk = \frac{\sin(G-F)}{\sin(H-F)} mn$; eritque $l\lambda = ml + km - k\lambda$.

Deinde fiat $\beta : \alpha = l\lambda : l\zeta$, eritque $l\zeta = \frac{\alpha}{\beta} l\lambda$, ideoque & punctum ζ est repertum. Quia ergo in triangulo $\zeta k \vartheta$ dantur latera $k\zeta$ & $k\theta$ cum angulo $\zeta k \theta = H - F$, reperietur latus $\zeta\theta$ cum angulo $k\zeta\theta$, unde erit angulus $S o \zeta = 180° - k\zeta\theta - \mu - G + F$; ergo & latus $\zeta\theta$ & ejus positio erit cognita. Tum vero ex latitudinibus observatis erit $F\zeta = f\zeta$ tang ζ, & $H\theta = b\vartheta$ tang ϑ; ex quo & cordæ FH positio innotescit. Producatur corda HF, donec in plano eclipticæ cum $\vartheta\zeta$ in N occurrat, eritque recta SN intersectio orbitæ cometæ & eclipticæ, ideoque linea nodorum; & in N quidem erit nodus ascendens, si latitudines observatæ fuerint boreales, & loca cometæ ita sint disposita, ut figura repræsentat. Ad punctum N inveniendum, fiat $H\vartheta - F\zeta : \zeta\vartheta = H\vartheta : \vartheta N$, unde fit $\vartheta N = \frac{\zeta\vartheta}{H\vartheta - F\zeta} . H\vartheta$; & $\frac{H\vartheta - F\zeta}{\zeta\vartheta}$ dabit anguli HNϑ tangentem. Deinde in triangulo $S o N$ ob data latera So & No cum angulo $S o N = S o \zeta$, reperientur anguli NSo, SNo, cum latere SN. Hinc primum si angulus $NS\eta + \eta S g$ a longitudine terræ in observatione media, quæ est $= 6' + g$, subtrahatur, remanebit longitudo heliocentrica nodi N. Tum ex o in SN demittatur perpendiculum oP, ductaque recta OP angulus OPo monstrabit inclinationem

clinationem orbitæ cometæ ad eclipticam, Erit vero $oP = No . \text{ſin } SNo$; & $Oo = No . \text{tang } HN\vartheta$, unde erit $\text{tang } OPo = \frac{\text{tang } HN\vartheta}{\text{ſin } SNo} = \frac{H\vartheta - F\zeta}{\zeta\vartheta . \text{ſin } SNo}$. Deinde erit $\text{coſ } SNH = \frac{NP}{NO} = \frac{NP}{No} \cdot \frac{No}{NO} = \text{coſ } SNo . \text{ coſ } HN\vartheta$. Denique invenitur $NF = \frac{F\zeta}{\text{ſin } HN\vartheta}$; & $HN = \frac{H\vartheta}{\text{ſin } HN\vartheta}$, atque hinc in plano orbitæ cometæ SNFH dabuntur latus S N; angulus SNH, & latera NF & NH, unde reperientur anguli NSF & NSH cum lateribus SF & SH; ideoque dabuntur duæ cometæ a ſole diſtantiæ FS & HS, una cum angulo FSH & tempore $= \alpha + \beta$ dier. quo cometa ex F in H pervenit, ex quibus orbita cometæ invenietur per probl. V. Q. E J.

Coroll. 1.

48. Quo igitur hæc orbitæ cometæ vel planetæ determinatio magis ſit exacta; neceſſe eſt ut obſervationes non nimis à ſe invicem diſtent, atque ut ratio inter tempora $\alpha : \beta$ proxime ſit ratio æqualitatis. Tum vero hoc maxime requiritur, ut obſervationes ſumma cura ſint inſtitutæ atque ut diſtantia cometæ a terra in obſervatione media a veritate quam minimum aberret.

Coroll. 2.

48. Quando longitudo cometæ in obſervatione media $g\eta$ in ipſam longitudinem ſolis gS incidit, tum angulus $S\eta g$ ſit vel

fit v
dioc
ræ f
eſt f
dum
præ
moc
Not
T
T
Obſ
)
H
S f
S g
S b

ſit vel nullus vel 180°, hocque adeo caſu calculus non mediocriter contrahitur.

Scholion 1.

50. Operatio hæc vel per accuratam deſcriptionem figuræ ſuper charta ſatis magna inſtitui poteſt, vel, quod magis eſt ſuadendum, per calculum trigonometricum; quemadmodum ergo hunc calculum quam breviſſime inſtitui oporteat prætermiſſo omni ratiocinio, hic ob oculos ponamus, quo commodius eo ad orbitas cometarum inveſtigandas uti liceat. Notentur igitur primum quæ ſunt per obſervationes data:

Tempus elapſum ab Obſervatione I ad II $= \alpha$ dieb.
Tempus elapſum ab Obſervatione II ad III $= \beta$ dieb.

Obſervat.	Long. Solis	Diſt. ☉ a ♁	Long. Com.	Lat. Com.
I	f	Sf	F	ζ
II	g	Sg	G	η
III	b	Sb	H	θ

Hinc definiantur anguli

$$fmg = \zeta m \eta = G - F$$
$$gnb = \eta n \theta = H - G$$
$$fkb = \zeta k \theta = H - F$$

itemque

$Sf\zeta = F - f$; $Sr\eta = G - f$; $Ss\theta = H - g$
$Sg\eta = G - g$; $Sp\zeta = F - g$; $Sq\eta = G - b$
$Sb\theta = H - b$; Ex his reperitur

$$fm = \frac{\text{ſin } Sr\eta}{\text{ſin } fmg} Sf - \frac{\text{ſin } Sg\eta}{\text{ſin } fmg} Sg$$
$$gm = \frac{\text{ſin } Sf\zeta}{\text{ſin } fmg} Sf - \frac{\text{ſin } Sp\zeta}{\text{ſin } fmg} Sg$$

$gn =$

$$g\,n = \frac{\text{fin } S\,r\,\theta}{\text{fin } g\,n\,b}\, S\,g - \frac{\text{fin } S\,b\,\theta}{\text{fin } g\,n\,b}\, S\,b$$

$$b\,n = \frac{\text{fin } S\,g\,\eta}{\text{fin } g\,n\,b}\, S\,g - \frac{\text{fin } S\,g\,\eta}{\text{fin } g\,n\,b}\, S\,b$$

unde $mn = gm - gn$; & $mk = \frac{\text{fin}\, g\,n\,b}{\text{fin}\, f\,k\,b}\, mm$; $nk = \frac{\text{fin}\, f\,m\,g}{\text{fin}\, f\,k\,b}\, m\,n$

Sit in obſervatione II diſtantia cometæ a terra $= r$ erit $G\eta = r$ fin η; $g\eta = r$ cof η; $m\eta = g\eta - gm$; $n\eta = g\eta - gn$. Porro eſt tang $\frac{1}{2}(g\eta S - gS\eta) = \frac{gS - g\eta}{gS + g\eta}$ cot $\frac{1}{2} S g \eta$: unde erit $g\eta S = \frac{1}{2}(g\eta S - gS\eta) + 90 - \frac{1}{2} Sg\eta$; & $gS\eta = 90 - \frac{1}{2} Sg\eta - \frac{1}{2}(g\eta S - gS\eta)$

atque $S\,\eta = \frac{\text{fin } S\,g\,\eta}{\text{fin } g\,\eta\,S}\, S\,g$.

Deinde eſt tang $GS\eta = \frac{G\,\eta}{S\,\eta}$; & $SG = \frac{G\,\eta}{\text{fin } GS\eta}$. Ponatur angulus $1774^{rs}\,3^{\circ}\,3''$ $= \tau$, ut ſit $l\tau = 3,2489776$ & quærantur anguli $\alpha\tau$ & $\beta\tau$; ſitque $c = 100000$; erit $GO = \frac{2c^3 \text{ fin } \alpha\tau.\ \text{fin } \beta\tau}{SG^2 \text{ cof } (\beta-\alpha)\tau}$; & $\eta o = GO$ cof $GS\eta$; $So = S\eta - o\eta$

Deinde eſt ang. $\zeta l o = g\eta S + \zeta m \eta$; & $l\eta = \frac{\text{fin } \zeta\, m\,\eta}{\text{fin } \zeta\, l\, o}\, m\,\eta$

& $ml = \frac{\text{fin}\, g\,\eta\,S}{\text{fin}\, \zeta l o}\, m\eta$: indeque $lo = l\eta - \eta o$, ex hiſque $\lambda\vartheta = \frac{\gamma + \delta}{\alpha}\, lo$; $k\theta = \frac{\text{fin}\, \zeta l o}{\text{fin}\, \zeta k \vartheta}\, \lambda\vartheta$; & (ob $k\vartheta\lambda = \zeta l o - \zeta k \vartheta$)

$k\lambda =$

$k\lambda = \frac{\text{fin}\, k\vartheta\lambda}{\text{fin}\, \zeta k \vartheta}\, \lambda\vartheta$, unde $l\lambda = ml + km - k\lambda$, $l\zeta = \frac{\alpha}{\epsilon}\, l\lambda$

& $k\zeta = km + ml + l\zeta$, atque tang $\frac{1}{2}(k\vartheta\zeta - k\zeta\vartheta) = \frac{k\zeta - k\vartheta}{k\zeta + k\vartheta}$ cot $\frac{1}{2}\zeta k\vartheta$, & $k\vartheta\zeta = 90 - \frac{1}{2}\zeta k\vartheta + \frac{1}{2}(k\vartheta\zeta - k\zeta\vartheta)$; $k\zeta\vartheta = 90 - \frac{1}{2}\zeta k\vartheta - \frac{1}{2}(k\vartheta\zeta - k\zeta\vartheta)$ atque $\vartheta\zeta = \frac{\text{fin}\, \zeta k \vartheta}{\text{fin}\, k\zeta\vartheta}\, k\vartheta$; unde $So\zeta = 190^{\circ}\, l\zeta\vartheta - \zeta l o$.

His inventis erit $f\zeta = fm + ml + l\zeta$: $b\vartheta = bn + nb + k\vartheta$; hinque $F\zeta = l\zeta$ tang ζ & $H\vartheta = b\vartheta$ tang ϑ, ac porro tang $HN\vartheta = \frac{H\vartheta - F\zeta}{\zeta\vartheta}$ & $\vartheta N = \frac{H\vartheta}{\text{tang } HN\vartheta}$; ob $\vartheta o = \frac{\epsilon}{\alpha + \beta}\zeta\vartheta$, erit $No = N\theta - \theta o$. Deinceps eſt tang $\frac{1}{2}(SNo - oSN) = \frac{oS - oN}{oS + oN}$ cot $\frac{1}{2} So\zeta$ & $SNo = 90 - \frac{1}{2} So\zeta + \frac{1}{2}(SNo - oSN)$; $oSN = 90 - \frac{1}{2} So\zeta - \frac{1}{2}(SNo - oSN)$ & $SN = \frac{\text{fin } So\zeta}{\text{fin } oSN}\, No$, unde longitudo heliocentrica nodi $N = 6^{s} + g - gS\eta - oSN$, & inclinationis orbitæ ad Eclipticam tang $= \frac{\text{tang } HN\theta}{\text{fin } SNo}$. Deinde eſt cof $SNH =$ cof SNo. cof $HN\vartheta$ & $FN = \frac{F\zeta}{\text{fin } HN\theta}$; $HN = \frac{H\theta}{\text{fin } HN\theta}$. Denique fiet tang $\frac{1}{2}(NFS - NSF) = \frac{SN - NF}{SN + NF}$ cot $\frac{1}{2}$ SNH; $NFS = 90 - \frac{1}{2} SNH + \frac{1}{2}(NFS - NSF)$ $NSF = 90 - \frac{1}{2} SNH - \frac{1}{2}(NFS - NSF)$ & $SF = \frac{\text{fin } SNH}{\text{fin } NFS}\, SN$; &

tang

tang ½ (NHS−NSH) $= \frac{SN - NH}{SN + NH}$ cot ½ SNH; unde NHS $=$ 90 − ½ SNH + ½ (NHS − NSH) & NSH $=$ 90 − ½ SNH − ½ (NHS−NSH), & SH $= \frac{\text{fin SNH}}{\text{fin NHS}}$ SN atque FSH $=$ NSH − NSF.

Scholion. 2.

51. Inventis duabus diſtantiis FS & HS cum angulo FSH & tempore inter hæc loca elapſo $= \alpha + \beta$ dier. orbita ſequenti calculo definietur. Sit

diſtantia FS $= y$, tempus $\alpha + \beta = T$
diſtantia HS $= z$, & ang FSH $= \Phi$

ob $m = 271989, 735$ & $lm = 5, 4345525139$ erit orbitæ latus rectum $b = \left(\frac{yyzz}{4m^2T^2} + \frac{1}{2}\sqrt{yz}\right)(\text{fin}\,\Phi)^2$. Sit diſtantia perihelii a ſole $= a$; & anomalia vera loci F ſit $= v$, erit tang $v = \cot\Phi - \frac{(b-z)y}{(b-y)z\,\text{fin}\,\Phi}$, & $a = \frac{by\cos v}{b - y + y\cos v}$. Sumatur tang $\frac{1}{2}\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}$ tang $\frac{1}{2}v$, ſi quidem fuerit $2a > b$ & orbita ellipſis: eritque tempus, quo cometa a perihelio ad locum F pervenit, in diebus expreſſum $= \frac{a^3}{2m(2a-b)^{\frac{3}{2}}}$ $\left(\omega - \frac{(b-a)}{a}\,\text{fin}\,\omega\right)$, ſin autem fuerit $b > 2a$ & curva hyperbola, ſumatur tang $\frac{1}{2}\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ tang $\frac{1}{2}v$, & erit tempus quo

NHS = NH − ½ − NSF. angulo er. orbi rit orbi. Sit di F ſit = cos v y cos v m fuerit a peri (2a−b)³⁄₂ hyperbo tempus quo

quo cometa a perihelio ad F pervenit in diebus $= \frac{a^3}{2m(b-a)^{\frac{3}{2}}}$ $\left(\frac{b-a}{a}\,\text{tang}\,\omega - l\,\text{tang}\,(45^\circ + \frac{1}{2}\omega)\right)$. At ſi orbita cometæ fuerit vel parabola vel ad eam proxime accedat, ponatur tang $\frac{1}{2}v = t$; & $\delta = 2a - b$ & $n = \frac{\delta}{b} = \frac{2a-b}{b}$, eritque tempus, quo cometa a perihelio ad F pervenit, in diebus expreſſum $=$

$$\frac{aa}{m\sqrt{b}}\left(t + \tfrac{1}{3}t^3 - \tfrac{2}{3}n\,t^3 + \tfrac{3}{5}nn\,t^5 - \tfrac{4}{7}n^3\,t^7 + \&c. \right.$$
$$\left. + \tfrac{1}{5}nn\,t^5 - \tfrac{4}{7}n^3\,t^7 + \tfrac{5}{9}n^4\,t^9 \right)$$

Cum igitur tempus conſtet, quo cometa in F eſt verſatus, ſimul temporis momentum habebitur, quo cometa per perihelium tranſiit. Denique ſi ab anomalia vera $= v$ loci F ſubtrahatur angulus FSN, habebitur diſtantia nodi N a perihelio. Ceterum hoc problema parum utilitatis habere videatur, cum nunquam parallaxis cometæ tam exacte definiri queat, quam ad hoc inſtitutum opus eſt; at vero ſummus uſus hujus reſolutionis perſpicietur in problemate ſequenti.

Problema XII.

52. Ex aliquot obſervationibus cometæ ejus orbitam veram determinare.

Solutio.

Ex obſervationibus cometæ, quibus vel ejus diſtantiæ a ſtellis fixis, vel altitudines meridianæ ſunt menſuratæ, elicientur

tur ejus longitudines ac latitudines geocentricæ, & ad unamquamque obſervationem notetur temporis momentum, quo eſt facta, ſecundum tempus medium. Ex his cometæ locis in ordinem diſpoſitis eligantur tria non nimis a ſe invicem remota, ita ut media ratione temporis fere medium inter jaceat inter extremas. Tum pro lubitu in obſervatione media fingatur cometæ diſtantia a terra, ac per problema præcedens ex his tribus locis & diſtantia ficta determinetur orbita cometæ, quæ erit vera, ſi diſtantia illa ficta cum vera conveniat, contra autem falſa. Sumatur ergo obſervatio quædam quarta a tribus illis electis longiſſime remota, atque ex orbita eruta ad ejus tempus computetur locus cometæ geocentricus, qui ſi cum obſervato congruat, docebit orbitam erutam eſſe veram, ſin minus congruat, eſſe falſam. Fingantur ergo ſimul duæ tresve diſtantiæ, quas cometa in obſervatione media habuerit, inter ſe diverſæ, & ex qualibet definiatur orbita cometæ, & diſpiciatur quantum quæque ab obſervatione quarta diſcrepet. Atque hinc etiamſi nulla orbita hoc modo inventa ſit vera, tamen facile intelligetur, quænam ad veritatem propius accedat; atque adeo, ſi diſſenſus non fuerit admodum magnus, per interpolationem vera cometæ orbita erui poterit. Sin autem diſſenſus ſit adhuc nimis magnus, tum ſaltem diſtantia cometæ in obſervatione media propius cognoſcetur; quare fingendis aliquot

novis

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it orbi-
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bſerva-
aliquot
novis

novis diſtantiis a veritate minus abhorrentibus, ſi ex iis pari modo orbitæ definiantur, & cum obſervatione quarta conferantur, per interpolationem ſatis exacte vera orbita cometæ concludi poterit. Q. E. J.

Coroll. 1.

53. Quoniam prima fictio ideo tantum fit, ut orbita cometæ veræ tantum propinqua obtineatur, non opus eſt, ut calculus omni cura inſtituatur, ſed ſufficiet per conſtructionem geometricam ex diſtantiis fictis orbitam eruere.

Coroll. 2.

54. Eodem modo poſſent etiam planetarum orbitæ investigari, atque ex quatuor obſervationibus definiri. At quoniam in planetis per continuas obſervationes tempora periodica & lineæ nodorum facile determinantur, his cognitis certior aperitur via ad eorum orbitas definiendas, quam potius ſequi convenit.

Scholium.

55. Hujus methodi utilitas aptiſſime per exemplum docebitur, ſimulque patebit, quemadmodum commodiſſime calculum ſatis prolixum inſtitui oporteat. Hunc in finem cometam, qui circa finem A. 1680 & initium ſequentis anni

 apparuit

apparuit, potissimum investigari conveniet, quippe quo vix ullus alius majori cura atque opportunitate non solum est observatus, sed etiam ejus orbita per calculum determinata. Cum igitur hujus cometæ & observationes per quadrimestre fere temporis spatium habeantur accuratissimæ, & ipsa ejus orbita Virorum Celeberrimorum Neutoni & Halleji studiis sit determinata, etiamsi actum agere videar, tamen hoc exemplo mea methodus non solum maxime illustrabitur, sed etiam ex consensu fortissime confirmabitur. Observationes autem hujus cometæ in Principiis Math. Philosophiæ Neutonianæ recensentur, unde eas hic depromam.

Cometæ

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mestre
sa ejus
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, sed
tiones
Neuto-
ıetæ

Cometæ, qui circa finem anni 1680. & initium anni 1681. apparuit, loca observata ad tempus medium stili veteris & meridianum Londinensem reducta.

	d. h. m.	Longitudo	Latitudo	
A.1680.M.Nov.	16,17,10	6ˢ,8°,0′	0°,44,′	austr.
	17,17,10	6,12,52	1, 0	
	18,21,44	6,18,40	1, 18	
	19,17,10	6,22,48	1, 30	
	20,17,—	6,27,52	1, 45	
	21,17,—	7, 2,56	1, 58	
	23,17,15	7,12,58	2, 20	
	24,17,30	7,17,53	2, 29	
M.Dec.	12 4,46,0″	9, 6,31,21″	8,26,0″	bor.
	21, 6,36,59	10, 5, 7,38	21,45,30	
	24, 6,17,52	10,18,49,10	25,23,24	
	26, 5,20,44	10,28,24, 6	27, 0,57	
	29, 8, 3, 2	11,13,11,45	28,10, 5	
	30, 8,10,26	11,17,39, 5	28,11,12	
A.1681.M.Jan.	5, 6, 1,38	0, 8,49,10	26,15,26	
	9, 7, 0,53	0,18,43,18	24,12,42	
	10, 6, 6,10	0,20,40,57	23,44, 0	
	13, 7, 8,55	0,25,59,34	22,17,36	
	25, 7,58,42	1, 9,35,48	17,56,54	
	30, 8,21,53	1,13,19,36	16,40,57	
M.Febr.	2, 6,34,51	1,15,13,48	16, 2, 2	
	5, 7, 4,41	1,16,59,52	15,27,23	
	25, 8,41,—	1,26,18,17	12,46,54	
	27, 8,26,—	1,27, 4,24	12,36,12	
M.Mart.	1,11,10,—	1,27,53, 6	12,24,52	
	2, 8,10,—	1,28,12,27	12,20, 0	
	5,11,39,—	1,29,20,51	12, 3,30	
	9, 8,38,—	2, 0,43, 3	11,45,53	

Investi-

Investigatio Orbitæ hujus Cometæ.

Eligantur tres sequentes observationes Mense Januario A. 1681. factæ

Tempus	Long. Solis	Dist.Sol.a Ter.	Long.Comet.	Lat. Cometæ
5d,6h,1',38''	9s,26°,22',18''	9 8 3 6 3, 5	0s,8°,49',10''	26°,15',26''
9, 7 ,0,53	10, 0 ,29 , 2	9 8 4 0 7, 0	0 ,18,43 ,18	24 ,12 ,42
13,7 ,8,55	10, 4 ,33 ,20	9 8 4 5 8, 8	0 ,25,59 ,34	22 ,17 , 36

Hinc erit

$\alpha = 4^d, 0^h, 59', 15'' = 4, 0411; \; l\alpha = 0, 6064996$

$\beta = 4^d, 0^h, 8', 2'' = 4, 0055; \; l\beta = 0, 6026567$

$\alpha + \beta = 8, 0466; \; \& \; l(\alpha + \beta) = 0, 9056124$

$f = 9^s, 26°, 22', 18''; \; F = 0^s, 8°, 49', 10''; \; \zeta = 26°, 15', 26''$

$g = 10, 0, 29, 2; \; G = 0, 18, 43, 18; \; \eta = 24, 12, 42$

$h = 10, 4, 33, 20; \; H = 0, 25, 59, 34; \; \theta = 22, 17, 36$

$fmg = \zeta m \eta = G - F = 9°, 54', 8''$

$gnh = \eta n \theta = H - G = 7°, 16', 16''$

$fkh = \zeta k \theta = H - F = 17, 10, 24$

$Sf\zeta = F - f = 72, 26, 52$

$Sg\eta = G - g = 78, 14, 16$

$Sh\theta = H - h =$ [illegible]$, 26, 14$

$Sr\eta = G - f = 82, 21, 0$

$Sp\zeta = F - g = 68, 20, 8$

$Ss\theta = H - g = 85, 30, 32$

$Sq\eta = G - h = 74, 9, 58$

$Sf = 98363, 5; \; lSf = 4,9928340$

$Sg = 98407, 0; \; lSg = 4,9930260$

$Sh = 98458, 8; \; lSh = 4,9932545$

Jam sequentes calculi instituantur.

lSf

$lSf = 4, 9928340$

subt. $l \sin fmg = 9, 2354458$

$5, 7573882$

$l \sin Sr\eta = 9, 9961174$

$l \sin Sf\zeta = 9, 9792946$

$l \frac{\sin Sr\eta}{\sin fmg} Sf = 5, 7535056$

$l \frac{\sin Sf\zeta}{\sin fmg} Sf = 5, 7366828$

$lSg = 4, 9930260$

$l \sin fmg = 9, 2354458$

$5, 7575802$

$l \sin Sg\eta = 9, 9907836$

$l \sin Sp\zeta = 9, 9681848$

$l \frac{\sin Sg\eta}{\sin fmg} Sg = 5, 7483638$

$l \frac{\sin Sp\zeta}{\sin fmg} Sg = 5, 7257650$

$\frac{\sin Sr\eta}{\sin fmg} Sf = 566898, 9$

$\frac{\sin Sg\eta}{\sin fmg} Sg = 560226, 8$

$fm = 6672, 1$

$\frac{\sin Sf\zeta}{\sin fmg} Sf = 545359, 4$

$\frac{\sin Sp\zeta}{\sin fmg} Sg = 531820, 4$

$gm = 13539, 0$

$gn = 5875, 0$

$mn = 7664, 0$

$lSg = 4, 9930260$

$l \sin gnh = 9, 1023116$

$5, 8907144$

$l \sin Ss\theta = 9, 9986644$

$l \sin Sg\eta = 9, 9907836$

$l \frac{\sin Ss\theta}{\sin gnh} Sg = 5, 8893788$

$l \frac{\sin Sg\eta}{\sin gnh} Sg = 5, 8814980$

$lSh = 4, 9932545$

$l \sin gnh = 9, 1023116$

$5, 8909429$

$l \sin Sh\theta = 9, 9951318$

$l \sin Sq\eta = 9, 9832008$

$l \frac{\sin Sh\theta}{\sin gnh} Sh = 5, 8860747$

$l \frac{\sin Sq\eta}{\sin gnh} Sh = 5, 8741437$

$\frac{\sin Ss\theta}{\sin gnh} Sg = 775137, 7$

$\frac{\sin Sh\theta}{\sin gnh} Sh = 769262, 7$

$gn = 5875, 0$

$\frac{\sin Sg\eta}{\sin gnh} Sg = 761198, 5$

$\frac{\sin Sq\eta}{\sin gnh} Sh = 748417, 0$

$hn = 12781, 5$

$lmn = 3, 8844555$

$l \sin fkh = 9, 4702096$

$4, 4142459$

mk

mk =	3285, 168	l fin gnb =	9, 1023116
nk =	4463, 666	l fin fmg =	9, 2354458
		$l\,mk$ =	3, 5165575
		$l\,nk$ =	3, 6496917

Ut jam labori in ſcrutando valore diſtantiæ $gG = r$ vero proximo parcamus, conſulamus Theoriam Neutoni, quæ pro hoc tempore exhibet diſtantiam cometæ a ſole = 110970, unde elicitur diſtantia cometæ a terra = 72747. Hanc obrem pro r aſſumam hos duos valores 72700 & 72800.

$S g\eta = 78°, 14', 16''$
$\frac{1}{2} S g\eta = 39, 7', 8$
Compl. $= 50, 52', 52''$

Sit ergo r =	72700	72800
$l r$ =	4, 8615344	4, 8621314
add l fin η =	9, 6128990	9, 6128990
add l coſ η =	9, 9600122	9, 9600122
$l G\eta$ =	4, 4744334	4, 4750304
$l g\eta$ =	4, 8215466	4, 8221436
$g\eta$ =	66305, 05	66396, 26
gm =	13539, 0	13539, 0
gn =	5875, 0	5875, 0
$m\eta$ =	52766, 05	52857, 26
$n\eta$ =	60430, 05	60521, 26
Sg =	98407, 0	98407, 0
$g\eta$ =	66305, 05	66396, 26
$Sg + g\eta$ =	164712, 05	164803, 26
$Sg - g\eta$ =	32101, 95	32010, 74
$l(Sg - g\eta)$ =	4, 5065314	4, 5052957
$l(Sg + g\eta)$ =	5, 2167254	5, 2169658
	9, 2898060	9, 2883299
l tang $(90 - \frac{1}{2} Sg\eta)$ =	10, 0897890	10, 0897890
l tang $\frac{1}{2}(g\eta S - gS\eta)$ =	9, 3795950	9, 3781189

$\frac{1}{2}(g\eta S - gS\eta)$

$\frac{1}{2}(g\eta S - gS\eta)$ =	13°, 28', 38''	13°, 25', 59''
$90 - \frac{1}{2} Sg\eta$ =	50, 52, 52	50, 52, 52'
$g\eta S$ =	64, 21 30	64, 18, 51
$gS\eta$ =	37, 24, 14	37, 26, 53
Ad $l\,Sg$ =	4, 9930260	4, 9930260
add l fin $Sg\eta$ =	9, 9907836	9, 9907836
	14, 9838096	14, 9838096
ſubtr. l fin $g\eta S$ =	9, 9549744	9, 9548136
$l\,S\eta$ =	5, 0288352	5, 0289960
a $l\,G\eta$ =	4, 4744334	4, 4750304
l tang $GS\eta$ =	9, 4455982	9, 4460344
$GS\eta$ =	15°, 35', 20''	15°, 36', 14''
A $l\,G\eta$ =	4, 4744334	4, 4750304
ſubtr. l fin $GS\eta$ =	9, 4293210	9, 4297284
$l\,SG$ =	5, 0451124	5, 0453020
$l\,\tau$ =	3, 2489776	
$l\,\alpha$ =	0, 6064996	
$l\,\beta$ =	0, 6026567	
$l\,\alpha\tau$ =	3, 8554772	
$l\,\beta\tau$ =	3, 8516343	
$\alpha\tau$ =	7169'',3 =	1°, 59', 29''
$\beta\tau$ =	7106'',1 =	1°, 58', 26''
$(\alpha - \beta)\tau$ =	63, 2 =	1', 3''
l fin $\alpha\tau$ =	8, 5409422	
l fin $\beta\tau$ =	8, 5371103	
$l\,2c^3$ =	15, 3010300	
	32, 3790825	
ſubt. l coſ$(\alpha - \beta)\tau$ =	10, 0000000	
	12, 3790825	12, 3790825
ſubtr 2 $l\,SG$ =	10, 0902248	10, 0906040

lGO =	2, 2888577	2, 2884785
add. l cos G S η =	9, 9837231	9, 9836924
l η o =	2, 2725808	2, 2721709
η o =	187,32	187,14
ab S η =	106864,9	106904,5
S o =	106677,6	106717,4
Ang. g η S =	64°,24′,30″	64°,18′,51″
ζ m η =	9, 54, 8	9, 54, 8
ζ l o =	74, 15, 38	74, 12, 59″
l m η =	4, 7223546	4, 7231046
subtr. l sin ζ l o =	9, 9834031	9, 9833086
	4, 7389515	4, 7397960
add { l sin ζ m η =	9, 2354458	9, 2354458
{ l sin g η S =	9, 9549744	9, 9548136
l l η =	3, 9743973	3, 9752418
l m l =	4, 6939259	4, 6946096
Ergo l η =	9427, 52	9445, 87
subtr. η o =	187, 32	187, 14
l o =	9240, 20	9258, 73
l l o =	3, 9656814	3, 9665515
add $l\,\frac{(\alpha+\beta)}{\alpha}$ =	0, 2991128	0, 2991128
erit l λ θ =	4, 2647942	4, 2656643
Ang. ζ l o =	74°,15′,38′	74°,12′,59″
subtr. ζ k θ =	17°,10, 24	17, 10, 24
k θ λ =	57 ,5 ,14	57 ,2 ,35
l λ θ =	4, 2647942	4, 2656643
subtr. l sin ζ k θ =	9, 4702096	9, 4702096
	4, 7945846	4, 7954547
add { l sin ζ l o =	9, 9834031	9, 9833086
{ l sin k θ λ =	9, 9240200	9, 9238032
		l k θ =

l k θ =	4, 7779877	4, 7787633
l k λ =	4, 7186046	4, 7192579
m l =	49422, 63	49500, 50
k m =	3285, 17	3285, 17
k l =	52707, 80	52785, 67
subtr. k λ =	52312, 40	52391, 15
l λ =	395, 40	394, 52
l l λ =	2, 5970367	2, 5960690
add. $l\,\frac{\alpha}{\beta}$ =	0, 0038429	0, 0038429
l. l ζ =	2, 6008796	2, 5999119
½ ζ k θ =	8°,35′,12″	8°,35′,12″
90 − ½ ζ k θ =	81°,24′,48″	81°,24′,48″
k ζ =	3285, 17	3285, 17
m l =	49422, 63	49500, 50
l ζ =	398, 91	398, 02
k ζ =	53106, 71	53183, 69
k θ =	59977, 42	60084, 63
k θ + k ζ =	113084, 13	113268, 32
k θ − k ζ =	6870, 71	6900, 94
l (k θ − k ζ) =	3, 8370016	3, 8389082
l (k θ + k ζ) =	5, 0534016	5, 0541084
	8, 7836000	8, 7847998
l tang (90 − ½ ζ k θ) =	10, 8210294	10, 8210294
	9, 6046294	9, 6058292
½ (k ζ θ − k θ ζ) =	21°,55′,7″	21°,58′,24″
½ (k ζ θ + k θ ζ) =	81, 24, 48	81, 24, 48
k ζ θ =	103, 19,55	103, 23,12
k θ ζ =	59, 29,41	59, 26,24
k ζ θ + ζ l o =	177, 35,33	177, 36,11
S o ζ =	2°, 24,27	2, 23,49

A l sin

A l fin $\zeta k \theta$ =	9, 4702096	9, 4702096
fubtr. l fin $k \zeta \theta$ =	9, 9881354	9, 9880368
	9, 4820742	9, 4821728
add $l\, k\theta$ =	4, 7779877	4, 7787633
$l\, \theta\zeta$ =	4, 2600619	4, 2609361
fm =	6672, 1	6672, 1
$m\, l$ =	49422, 6	49500, 5
$l\, \zeta$ =	398, 9	398, 0
erit $f\zeta$ =	56493, 6	56570, 6
$b\, n$ =	12781, 5	12781, 5
$n\, k$ =	4463, 6	4463, 6
$k\, \vartheta$ =	59977, 4	60084, 6
erit $b\, \vartheta$ =	77222, 6	77329, 8
ad $l\, f\zeta$ =	4, 7519993	4, 7525908
add $l$tang ζ =	9, 6931129	9, 6931129
lFζ =	4, 4451122	4, 4457037
$l\, b\vartheta$ =	4, 8877445	4, 8883470
add. l tang ϑ =	9, 6127775	9, 6127775
lHϑ =	4, 5005220	4, 5011245
Ergo Hϑ =	31660, 8	31704, 8
Fζ =	27868, 4	27906, 4
Hϑ−Fζ =	3792, 4	3798, 4
l (Hϑ − Fζ) =	3, 5789141	3, 5796007
fubtr. $l\, \zeta\vartheta$ =	4, 2600619	4, 2609361
$l$tang HNϑ =	9, 3188522	9, 3186646
a l Hϑ =	4, 5005220	4, 5011245
l Nϑ =	5, 1816698	5, 1824599
Ang. HNϑ =	11°,46′,15″	11°,45′,57″
Nϑ =	151939, 2	152215, 9

$l\zeta\vartheta$ =

$l\, \zeta\theta$ =	4, 2600619	4, 2609361
fubtr. $l \frac{\alpha+\beta}{\beta}$ =	0, 3029557	0, 3029557
$l\, \theta o$ =	3, 9571062	3, 9579804
θo =	9059, 5	9077, 8
N o =	142879, 7	143138, 1
$\frac{1}{2}$ S$o\zeta$ =	1°,12′,13$\frac{1}{2}$″	1°,11′,54$\frac{1}{2}$″
90−$\frac{1}{2}$S$o\zeta$ =	88°,47′,46$\frac{1}{2}$″	88°,48′, 5$\frac{1}{2}$″
So =	106677, 6	106717, 4
No =	142879, 7	143138, 1
No+So =	249557, 3	249855, 5
No − So =	36202, 1	36420, 7
A l(No − So) =	4, 5587338	4, 5613482
fubtr. l(No+So) =	5, 3971703	5, 3976889
	9, 1615635	9, 1636583
add l tang (90−$\frac{1}{2}$S$o\zeta$) =	11, 6775226	11, 6794316
l tang $\frac{1}{2}$ (oSN − SNo) =	10, 8390861	10, 8430899
$\frac{1}{2}$ (oSN − SNo) =	81°,45′,29″$\frac{1}{2}$	81°,49′,58″
$\frac{1}{2}$ (oSN + SNo) =	88, 47, 46 $\frac{1}{2}$	88, 48, 5$\frac{1}{2}$
oSN =	170, 33, 16	170, 38, 3$\frac{1}{2}$
SNo =	7, 2, 17	6, 58, 7$\frac{1}{2}$
ab $l$fin S$o\zeta$ =	8, 6233157	8, 6214086
fubtr. l fin oSN =	9, 2151360	9, 2114815
	9, 4081797	9, 4099271
add. lNo =	5, 1549705	5, 1557552
lSN =	4, 5631502	4, 5656823
Ang. gSη =	37°,24′,14″	37°,26′,53″
add oSN =	5^s,20, 33, 16	5^s,20, 38, 3
fubtr.	6^s,27, 57, 30	6^s,28, 4, 56″
a 6^s+g =	16, 0, 29, 2	16, 0, 29 2

Long. Nodi Ascend.	9ˢ,2°,31',32''	9ˢ,2°,24', 6''
l tang HNθ =	9, 3188523	9, 3186645
subtr. l sin SN♁ =	9, 0882372	9, 0839610
l tang. Inclinat.	10, 2306151	10, 2347035
Inclin. Orbitæ ad Eclipt. =	59°,32',38''	59°,46',45''
l cos SN♁ =	9, 9967152	9, 9967798
add. l cos HNθ =	9, 9907700	9, 9907779
l cos SNH =	9, 9874852	9, 9875577
Ergo SNH =	13°,41',19''	13°,38',59''
a { l F♀ =	4, 4451122	4, 4457037
l Hθ =	4, 5005220	4, 5011245
subtr. l sin HNθ =	9, 3096223	9, 3094424
l FN =	5, 1354899	5, 1362613
l HN =	5, 1908997	5, 1916821
½ SNH =	6°, 50',39½''	6°, 49',29½''
90 − ½ SNH =	83°, 9',20½''	83°,10, 30½''
SN =	36572, 12	36785, 97
NF =	136612, 3	136855, 2
NF + SN =	173184, 4	173641, 2
NF − SN =	100040, 2	100069, 2
a l(NF − SN) =	5, 0001745	5, 0003004
subt. (lNF + SN) =	5, 2385087	5, 2396528
	9, 7616658	9, 7606476
add l tang (90 − ½ SNH) =	10, 9207206	10, 9219679
l tang ½ (NSF − NFS) =	10, 6823864	10, 6826155
½ (NSF − NFS) =	78°,15',42¼''	78°,16',4¾''
½ (NSF + NFS) =	83, 9, 20½	83, 10, 30½
NSF =	161, 25, 3	161, 26, 35
NFS =	4, 53, 38	4, 54, 26⅙
A l sin SNH =	9, 3740724	9, 3728853
subtr l sin NFS =	8, 9310033	8, 9321860

add.

add
l tang

	0, 4430691	0, 4406993
add. lSN =	4, 5631502	4, 5656823
lSF =	5, 0062193	5, 0063816
NH =	155202, 8	155482, 7
SN =	36572, 12	36785, 97
NH + SN =	191774, 9	192268, 7
NH − SN =	118630, 7	118696, 7
A l(NH − SN) =	5, 0741971	5, 0744387
subt. l(NH + SN) =	5, 2827918	5, 2839086
	9, 7914053	9, 7905301
add l tang (90 − ½ SNH) =	10, 9207206	10, 9219679
l tang ½ (NSH − NHS) =	10, 7121259	10, 7124980
½ (NSH − NHS) =	79°,1',9⅛''	79°,1',42½
½ (NSH + NHS) =	83, 9, 20½	83,10, 30½
NSH =	162, 10,29⅔	162, 12,13
NHS =	4, 8,11⅓	4, 8,48
A l sin SNH =	9, 3740724	9, 3728853
subtr. l sin NHS =	8, 8581311	8, 8591973
	0, 5159413	0, 5136880
Add lSN =	4, 5631502	4, 5656823
lSH =	5, 0790915	5, 0793703
NSH =	162°,10',29⅔''	162°,12',13''
NSF =	161, 25, 3	161, 26, 35
FSH =	0°,45',26⅔''	0°,45',38''
FS = y =	101442, 3	101480, 3
HS = z =	119975, 2	120052, 2
lT =	0, 9056124	0, 9056124
φ =	0°,45',26⅔''	0°, 45',38''
l sin φ =	8, 1211936	8, 1229950
l cos φ =	9, 9999635	9, 9999638

l cot φ =

$l \cot \Phi =$	11, 8787699	11, 8769688
$l y =$	5, 0062193	5, 0063816
$l z =$	5, 0790915	5, 0793703
$l yz =$	10, 0853108	10, 0857519
$l yyzz =$	20, 1706216	20, 1715038
$l 4 =$	0, 6020600	
$l m^2 =$	10, 8691050	
$l T^2 =$	1, 8112248	
$l 4m^2T^2 =$	13, 2823898	13, 2823898
$l \frac{yyzz}{4m^2T^2} =$	6, 8882318	6, 8891140
add. $2 l \sin \Phi =$	16, 2423872	16, 2459900
	3, 1306190	3, 1351040
Pars prior $=$	1350, 887	1364, 910
$l \sqrt{yz} =$	5, 0426554	5, 0428759
fubt. $l 3 =$	0, 4771213	0, 4771213
	4, 5655341	4, 5657546
add $2 l \Phi =$	16, 2423872	16, 2459900
	0, 8079213	0, 8117446
pars poft. $=$	6, 426	6, 482
Ergo $b =$	1357, 313	1371, 392
$y =$	101442, 3	101480, 3
$z =$	119975, 2	120052, 2
$y-b =$	100085, 0	100108, 9
$z-b =$	118617, 9	118680, 8
A $l(z-b) =$	5, 0741502	5, 0743804
fubtr. $l(y-b) =$	5, 0003690	5, 0004728
	0, 0737812	0, 0739076
Add $l \frac{y}{z} =$	9, 9271278	9, 9270113

fubtr.

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	10, 0009090	10, 0009189
fubt. $l \sin \Phi =$	8, 1211936	8, 1229950
l poft. fubtr.	1, 8797154	1, 8779239
Paff fubtrah. $=$	75, 80807	75, 49600
$a \cot \Phi =$	75, 64322	75, 33014
$-$ tang $v =$	0, 16485	0, 16586
Ergo $- v =$	9°,21′,40″	9°,25′, 4″
Anom. vera loci F feu $v =$	170°,38′,20″	170°,34′,56″
fubtr. FSN $=$	161°,25′, 3″	161°,26′,35″
Dift. Nodi ☊ a perihelio	9°,13′,17″	9°, 8′,21″
Ad. $l y =$	5, 0062193	5, 0063816
add. $l - \cos v =$	9, 9941775	9, 9941065
$l - y \cos v =$	5, 0003968	5, 0004881
add. $l b =$	3, 1326800	3, 1371615
l Num $=$	8, 1330768	8, 1376496
$- y \cos v =$	100091, 41	100112, 45
add. $y - b =$	100085, 0	100108, 9
Denom.	200176, 4	200221, 3
l Denom. $=$	5, 3014129	5, 3015103
$l a =$	2, 8316639	2, 8361393
Diftantia perih: a Sole $a =$	678, 678	685, 708
Ergo $2a =$	1357, 356	1371, 416
& $b =$	1357, 313	1371, 392
$\delta = 2a - b =$	0, 043	0, 024

Eft ergo orbita cometæ utroque cafu Ellipfis maxime oblonga & parabolæ proxima;

$a l \delta =$	(−2), 6334685	(−2), 3802112	
fubtr. $l b =$	3, 1326800	3, 1371615	
$n = l n =$	5, 5007885	5, 2430497	Charact. 10. minuatur.

$\frac{1}{2} v =$

$\frac{1}{2}v$ =	85°,19′,10″	85°,17′,28″
$l\,t$ =	1, 0868576	1, 0842248
n =	0, 0000316	0, 0000175
$l\,t^3$ =	3, 2605728	3, 2526744
$l\,t^5$ =	5, 4342880	5, 4211240
$l\,t^7$ =	7, 6080032	7, 5895736
$l\,n\,t^5$ =	0, 9350765	0, 6641737
$l\,n\,n\,t^7$ =	(−2), 6095802	(−2), 0756730
t =	12, 21399	12, 14017
$\frac{1}{3}t^3$ =	607, 36750	596, 4215
$t+\frac{1}{3}t^3$ =	619, 5815	608, 5616
subtr.	3, 4445	1, 8460
add.	0, 0174	0, 0051
$t+\frac{1}{3}t^3-\frac{2}{5}nt^5+\frac{3}{7}n^2t^7$ − &c. =	616, 1544	606, 7207
hujus log.	2, 7896895	2, 7829888
$2\,l\,a$ =	5, 6633278	5, 6722786
subtr. $l\sqrt{b}$ =	1, 5663400	1, 5685808
	4, 0969878	4, 1036978
subtr. $l\,m$ =	5, 4345525	5, 4345525
	8, 6624353	8, 6691473
add.	2, 7896895	2, 7829888
	1, 4521248	1, 4521361
A Perih. ad F dies. =	28, 3221	28, 3228
seu		
At est tempus in F =	28^d,7^h,43′,50″	28^d,7^h,44′,50″
A 1680 Dec.	36^d,6, 1, 38	39^d,9 1, 38
Cometa per perihelium transiit A. 1680 Decembr.	7^d,22^h,17′,48″	7^d,22^h,16′,48″
Distantia Perih. a Sole =	678, 678	685, 708
semilatus rectum =	1357, 313	1371, 392
Dist. Nodi ☊ a Perihelio =	9°,13′,17″	9°, 8′,21″
Longitudo Nodi ☊ =	9^s,2°,31′,32″	9^s,2°,24′, 6″
Inclinatio Orbitæ ad Eclipt	59°,32′,38″	59°,46′,45″

Compa-

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A. 16
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Dist. S
A
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Comparentur jam hæ binæ determinationes cum observatione quadam quarta; quem in finem eligatur observatio A. 1680 die 12 Decembris facta, quæ ita se habebat.

A. 1680. Dec. 12^d, 4^h, 46′, 0″
Longitudo Cometæ; 9^s, 6°, 31′, 21″
Latitudo Cometæ bor. 8°, 26′, 0″
Longitudo Solis 9^s, 1°, 51′, 23″
Dist. Solis a terra = 98275, ejus log. = 4,9924431

Temp. propositum A. 1680 Mens. Dec.	12^d,4^h,46′,0″	12^d,4^h,46′,0″
subtr. temp. Perihelii	7, 22,17,48	7, 22,16,48″
Erit tempus T =	4 , 6,28,12	4 , 6,29,12
& in diebus est T =	4, 26958	4, 27027
lT =	0, 6303852	0, 6304553
add $l\,m$ =	5, 4345525	5, 4345525
	6, 0649377	6, 0650078
subtr. $l\frac{aa}{\sqrt{b}}$ =	4, 0969878	4, 1036978
Vid. Probl. VII. $l\,n$ =	1, 9679499	1, 9613100
& n =	92, 88592	91, 47660
$\frac{1}{2}n$ =	46, 44296	45, 73830
Ergo $\frac{3}{2}n$ =	139, 32888	137, 21490
$l\,\frac{3}{2}n$ =	2, 1440412	2, 1374012
$2\,l\,\frac{3}{2}n$ =	4, 2880824	4, 2748024
$\frac{9}{4}nn$ =	19413, 55	18827, 93
ad 1	19414, 55	18828, 93
$l(\frac{9}{4}nn+1)$ =	4, 2881271	4, 2748255
$l\sqrt{(\frac{9}{4}nn+1)}$ =	2, 1440636	2, 1374128
$\sqrt{(\frac{9}{4}nn+1)}$ =	139, 3361	137, 2185
add. $\frac{3}{2}n$ =	139, 3289	137, 2149

$l(\frac{3}{2}n+\sqrt{}$

	278, 6650	274, 4334
$l(\frac{3}{4}n + \sqrt{(\frac{9}{16}nn + 1)}) =$	2, 4450823	2, 4384369
l partis majoris =	0, 8150274	0, 8128123
l partis minoris =	9, 1849725	9, 1871876
Pars major =	6, 531718	6, 498489
Pars minor =	0, 153099	0, 153882
Eritque θ =	6, 378619	6, 344607
$l\,\theta$ =	0, 8047267	0, 8024047
$l\frac{2\delta}{b}$ =	(−5), 8018185	(−5), 5440797
$l\frac{2\delta}{b}\theta$ =	(−4), 6065452	(−4), 3464844
$2\,l\,\theta$ =	1, 6094534	1, 6048094
$l\frac{2\delta}{b}\theta^3$ =	(−2), 2159986	(−3), 9512938
	0, 4771213	0, 4771213
$l\frac{2\delta}{3b}\theta^3$ =	(−3), 7388773	(−3), 4741725
A tang θ =	81°,5′,24″	81°,2′,35″
feu	291924″	291755″
huj. log. =	5, 4652698	5, 4650183
add.	4, 6855749	4, 6855749
	0, 1508447	0, 1505932
$l\frac{2\delta}{b}$ =	(−5), 8018185	(−5), 5440797
$l\frac{2\delta}{b}$ A tang θ =	(−5), 9526632	(−5), 6946729
θ =	6, 378619	6, 344607
fubtr. $\frac{2\delta}{b}\theta$ =	0, 000404	0, 000222

add. 2δ

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	6, 378215	6, 344385
add. $\frac{2\delta}{3b}\theta^3$ =	0, 005481	0, 002980
$\frac{2\delta}{b}$ A tang θ =	0, 000089	0, 000049
t = tang $\frac{1}{2}v$ =	6, 383785	6, 347414
l tang $\frac{1}{2}v$ =	10, 8050782	10, 8025969
$\frac{1}{2}v$ =	81°, 5′,50″	81°,2′,49″
& v =	162, 11, 40″	162, 5, 38
b =	1357, 313	1371, 392
a =	678, 678	685, 708
$b-a$ =	678, 635	685, 684
$l(b-a)$ =	2, 8316362	2, 8361240
$l\,a$ =	2, 8316639	2, 8361393
$l\frac{b-a}{a}$ =	9, 9999723	9, 9999847
l − cof v =	9, 9786825	9, 9784370
$l\frac{b-a}{a}$ =	9, 9999723	9, 9999847
	9, 9786548	9, 9784217
$-\frac{b-a}{a}$ cof v =	0, 9520392	0, 9515284
Denom. =	0, 0479607	0, 0484715
$l\,b$ = l num. =	3, 1326800	3, 1371615
l Den. =	8, 6808855	8, 6854865
$l\,y$ =	4, 4517945	4, 4516750
Ab v =	162°,11′,40″	162°,5′,38″
fubtr. dift. Nodi a Perih. =	9, 13, 17	9, 8, 21
Dift. Cometæ a Nodo =	152, 58, 23	152, 57,17

In triangulo ergo fphærico ad B rectangulo CNP repræfentet N nodum afcend. NP eclipticam & NC orbitam cometæ, dantur. Fig. 6.

 NC =

sin CP = sin NC. sin N.
tang NP = cos N. tang NC.

Fig. 7.

NC =	152°,58′,23″	152°,57′,17″
ang. CNP =	59, 32, 38	59, 46, 45
l sin NC =	9, 6574473	9, 6577197
l sin N =	9, 9355161	9, 9365599
l sin CP =	9, 5929634	9, 5942796
Latitudo heliocentr CP =	23°,3′,39″	23°,8′,6″
l cos N =	9, 7049036	9, 7018564
l − tang NC =	9, 7076705	9, 7080138
l − tang NP =	9, 4125741	9, 4098702
Ergo NP =	165°,30′,10″	165°,35′,20″
seu NP =	5ˢ,15°,30′,10″	5ˢ,15°,35′,20″
Add. longitudo Nodi =	9, 2, 31,32	9, 2, 24, 6
Longitudo heliocentrica	2, 18, 1,42	2, 17,59,26
Longitudo terræ =	3, 1,51,23	3, 1,51,23
Ergo ang. *c*ST =	13, 49,41	13°,51,57
CS*c* =	23, 3,39	23, 8, 6
ly = *l*CS =	4, 4517945	4, 4516750
add. *l* sin CS*c* =	9, 5929634	9, 5942796
l cos CS*c* =	9, 9638300	9, 9635903
*l*C*c* =	4, 0447579	4, 0459546
*l*S*c* =	4, 4156245	4, 4152653
add. *l* sin *c*ST =	9, 3784143	9, 3795762
l cos *c*ST =	9, 9872269	9, 9871565
l c P =	3, 7940388	3, 7948415
*l*SP =	4, 4028514	4, 4024218
SP =	25284, 32	25259, 32
ab ST =	98275	98275
Erit TP =	72990, 68	73015, 68
A *l c*P =	3, 7940388	3, 7948415
subtr. *l*TP =	4, 8632675	4, 8634162

l tang.

l tang. ST*c* =	8, 9307713	8, 9314253
ST*c* =	4°,52′,24″	4°,52′,51″
Addatur long. Solis	9ˢ,1,51,23	9ˢ,1,51,23
Long. Com. Geocentrica	9ˢ,6°,43′,47″	9ˢ,6°,44′,14″
A *l*TP =	4, 8632675	4, 8634162
subtr. *l* cos ST*c* =	9, 9984272	9, 9984223
*l c*T =	4, 8648403	4, 8649939
*a l*C*c* =	4, 0447579	4, 0459546
l tang CT*c* =	9, 1799176	9, 1809607
Latitudo Geocentrica	8°,36′,18″	8°,37′,32″

Accedit ergo prior hypothesis qua sumsimus $r = 72700$ propius ad veritatem, ex quo concludimus valorem verum ipsius r multo minorem esse debere assumto. Atque ex latitudinibus concluditur iste valor ipsius r diminui debere 835, ob longitudines autem diminui deberet 2763. Medium ergo sumendo foret valor ipsius $r = 72700 - 1800 = 70900$ Hac itaque positione orbita cometæ multo magis a parabola ad figuram ellipticam reducetur, qua cognita hujus cometæ tempus periodicum definiri poterit. Quæ investigatio cum operæ pretium sit, tribuamus iterum ipsi r duos valores intra quos verus contineatur.

Sit ergo r =	70000	72000
erit *l r* =	4, 8450980	4, 8573325
add. *l* sin η =	9, 6128990	9, 6128990
l cos η =	9, 9600122	9, 9600122
*l*Gη =	4, 4579970	4, 4702315
*l*3η =	4, 8051102	4, 8173447

gη =

gη =	63842, 5	65666, 6
gm =	13539, 0	13539, 0
gn =	5875, 0	5875, 0
mη =	50303, 5	52127, 6
nη =	57967, 5	59791, 6
Sg =	98407, 0	98407, 0
gη =	63842, 5	65666, 6
Sg + gη =	162249, 5	164073, 6
Sg − gη =	34564, 5	32740, 4
l (Sg − gη) =	4, 5386302	4, 5150840
l (Sg + gη) =	5, 2101833	5, 2150387
	9, 3284469	9, 3000453
l tang (90 − ½ Sgη) =	10, 0897890	10, 0897890
l tang ½ (gηS − gSη) =	9, 4182359	9, 380[illegible]43
[illegible] ½ (gηS − gSη) =	14°, 40′, 46″	1[illegible], 12″
½ (gηS + gSη) =	50, 52, 52	5[illegible] 52, [illegible]2
gηS =	65, 33, 38	64, 40, 4
gSη =	36, 12, 6	37, 5, 40
A l Sg fin Sgη =	14, 9838096	14, 9838096
fubtr. l fin gηS =	9, 9592327	9, 9560926
l Sη =	5, 0245769	5, 0277170
a l Gη =	4, 4579970	4, 4702315
l tang GSη =	9, 4334201	9, 4425145
GSη =	15°, 10′, 41″	15°, 29′, 2″
A l Gη =	4, 4579970	4, 4702315
fubtr. l fin GSη =	9, 4180021	9, 4264582
l SG =	5, 0399949	5, 0437733
	12, 3790825	12, 3790825
fubtr. 2 l SG =	10, 0799898	10, 0875466
l GO =	2, 2990927	2, 2915359
add l cof GSη =	9, 9845798	9, 9839443

l ηo =

l ηo =	2, 2836725	2, 2754802
Ergo ηo =	192, 164	188, 573
ab Sη =	105822, 22	106590, 12
So =	105630, 06	106401, 55
Ang. gηS =	65°, 33′, 38″	64°, 40′, 4″
ζmη =	9, 54, 8	9, 54, 8
ζlo =	75, 27, 46	74, 34, 12
A l mη =	4, 7015982	4, 7170677
fubtr. l fin ζlo =	9, 9858686	9, 9840573
	4, 7157296	4, 7330104
add { l fin ζmη =	9, 2354458	9, 2354458
add { l fin gηS =	9, 9592327	9, 9560926
l lη =	3, 9511754	3, 9684562
l ml =	4, 6749623	4, 6891030
Ergo lη =	8936, 67	9299, 43
fubtr. ηo =	192, 16	188, 57
lo =	8744, 51	9110, 86
l lo =	3, 9417355	3, 9595593
add. $l\,\frac{\alpha+\beta}{\alpha}$ =	0, 2991128	0, 2991128
l λθ =	4, 2408483	4, 2586721
Ab ang. ζlo =	75°, 27′, 46″	74°, 34′, 12″
fubtr. ζkθ =	17, 10, 24	17, 10, 24
kθλ =	58, 17, 22	57, 23, 48
A l λθ =	4, 2408483	4, 2586721
fubtr. l fin ζkθ =	9, 4702096	9, 4702096
	4, 7706387	4, 7884625
add. { l fin ζlo =	9, 9858686	9, 9840573
add. { l fin kθλ =	9, 9297837	9, 9255293

l kθ =

$l k\theta =$	4, 7565073	4, 7725198
$l k\lambda =$	4, 7004224	4, 7139918
$m l =$	47311, 02	48876, 81
add. $km =$	3285, 17	3285, 17
$k l =$	50596, 19	52161, 98
ſubtr. $k\lambda =$	50167, 50	51759, 70
$l\lambda =$	428, 69	402, 28
$l l\lambda =$	2, 6321433	2, 6045284
add. $l \frac{\alpha}{\epsilon} =$	0, 0038429	0, 0038429
$l l\zeta =$	2, 6359862	2, 6083713
$k l =$	50596, 19	52161, 98
$l\zeta =$	432, 50	405, 85
$k\zeta =$	51028, 69	52567, 83
$k\theta =$	57082, 07	59227, 02
$k\theta + k\zeta =$	108110, 76	111794, 85
$k\theta - k\zeta =$	6053, 38	6659, 19
$l (k\theta - k\zeta) =$	3, 7819979	3, 8234215
$l (k\theta + k\zeta) =$	5, 0338690	5, 0484219
	8, 7481289	8, 7749996
l tang $(90 - \frac{1}{2}\zeta k\theta) =$	10, 8210294	10, 8210294
	9, 5691583	9, 5960290
$\frac{1}{2} (k\zeta\theta - k\theta\zeta) =$	20°,20′,44″	21°,31′,42″
$\frac{1}{2} (k\zeta\theta + k\theta\zeta) =$	81, 24 48	81, 24, 48
$k\zeta\theta =$	101, 45, 32	102, 56, 30
$k\theta\zeta =$	61, 4, 4	59, 53, 6
$\zeta l o =$	75°,27, 46	74, 34, 12
$k\zeta\theta + \zeta l o =$	177, 13, 18	177, 30, 42
$S o \zeta =$	2, 46, 42	2, 29, 18
A l ſin $\zeta k\theta =$	9, 4702096	9, 4702096
ſubtr. l ſin $k\zeta\theta =$	9, 9997889	9, 9888258

add. $l k\theta$

	9, 4794207	9, 4813838
add. $l k\theta =$	4, 7565073	4, 7725198
$l \vartheta\zeta =$	4, 2359280	4, 2539036
$f m =$	6672, 1	6672, 1
$m l =$	47311, 0	48876, 8
$l\zeta =$	432, 5	405, 8
Erit $f\zeta =$	54415, 6	55954, 8
$b k =$	17245, 1	17245, 1
$k\theta =$	57082, 1	59227, 0
erit $b\theta =$	74327, 2	76472, 1
$l f\zeta =$	4, 7357234	4, 7478373
l tang $\zeta =$	9, 6931129	9, 6931129
$l F\zeta =$	4, 4288363	4, 4409502
$l b\theta =$	4, 8711478	4, 8835030
l tang $\theta =$	9, 6127775	9, 6127775
$l H\theta =$	4, 4839253	4, 4962805
Ergo $H\theta =$	30473, 7	31353, 1
$F\zeta =$	26843, 3	27602, 6
$H\theta - F\zeta =$	3630, 4	3750, 5
$l (H\theta - F\zeta) =$	3, 5599545	3, 5740892
$l \zeta\theta =$	4, 2359280	4, 2539036
l tang $HN\theta =$	9, 3240265	9, 3201856
a $l H\theta =$	4, 4839253	4, 4962805
$l N\theta =$	5, 1598988	5, 1760949
Ang. $HN\theta =$	11°,54′,28″	11°,48′,21″
$N\theta =$	144510, 3	150001, 3
A $l \zeta\vartheta =$	4, 2359280	4, 2539036
ſubtr. $l \frac{\alpha + \epsilon}{\epsilon} =$	0, 3029557	0, 3029557
$l \vartheta o =$	3, 9329723	3, 9509479
$\theta o =$	8569, 83	8931, 98

$N o =$

N☉ =	135940, 5	141069, 3
½ S☉ς =	°,23′,21″	1°,14′,39″
90 − ½ S☉ς =	88, 36, 39	88, 45, 21
S☉ =	105630, 06	106401, 55
N☉ =	135940, 5	141069, 3
N☉ + S☉ =	241570, 5	247470, 8
N☉ − S☉ =	30310, 5	34667, 8
l (N☉ − S☉) =	4, 4815930	4, 5399263
l (N☉ + S☉) =	5, 3830439	5, 3935239
	9, 0985491	9, 1464024
l tang (90 − ½ S☉ς) =	11, 6152831	11, 6631758
l tang ½ (☉SN − SN☉) =	10, 7138322	10, 8095782
½ (☉SN − SN☉) =	79°, 3′,40″	81°,11′,15″
½ (☉SN + SN☉) =	88, 36, 39	88, 45, 21
☉SN =	167, 40, 19	169, 56, 36
SN☉ =	9, 32, 59	7, 34, 6
ab l sin S☉ς =	8, 6854914	8, 6376495
subtr. l sin ☉SN =	9, 3294159	9, 2420989
	9, 3560755	9, 3955506
l N☉ =	5, 1333489	5, 1494324
l SN =	4, 4894244	4, 5449830
Ang. gS♈ =	36, 12, 6	37, 5, 40
seu	1ˢ,6°, 12′,6″	1ˢ,7°, 5′,40″
☉SN =	5, 17,40,19	5, 19,56,36
	6, 23,52,25	6, 27, 2,16
a 6ˢ + g =	16, 0, 29, 2	16, 0, 29, 2
Long. Nodi ascend.	9, 6, 36,37	9, 3, 26,46
l tang HN♈ =	9, 3240265	9, 3201856
subtr. l sin SN☉ =	9, 2198555	9, 1196138
	10, 1041710	10, 2005718

Inclinatio

In

Inclinatio Orb. ad Eclipt.	51°,48′,24″	57°,47′,2″
l cos SN☉ =	9, 9939394	9, 9962000
add. l cos HN♈ =	9, 9905525	9, 9907146
l cos SNH =	9, 9844919	9, 9869146
SNH =	15°,13′,15″	13°,59′,40″
a { l Fς =	4, 4288363	4, 4409502
l H♈ =	4, 4839253	4, 4962805
subtr. l sin HN♈ =	9, 3145790	9, 3109002
l FN =	5, 1142573	5, 1300500
l HN =	5, 1693463	5, 1853803
½ SNH =	7°,36′,37½″	6°,59′,50″
90 − ½ SNH =	82, 23, 22½	83, 0, 10″
SN =	30862, 02	35073, 82
NF =	130094, 03	134911, 87
NF + SN =	160956, 05	169985, 7
NF − SN =	99232, 01	99838, 05
l (NF − SN) =	4, 9966517	4, 9992961
l (NF + SN) =	5, 2067073	5, 2304124
	9, 7899444	9, 7688837
l tang (90 − ½ SNH) =	10, 8741495	10, 9110303
l tang ½ (NSF − NFS) =	10, 6640939	10, 6799140
½ (NSF − NFS) =	77°,46′,18½″	78°,11′,48″
½ (NSF + NFS) =	82, 23, 22½	83, 0, 10
NSF =	160, 9, 41	161, 11, 58
NFS =	4, 37, 4	4, 48, 22
l sin SNH =	9, 4191955	9, 3835062
subt. l sin NFS =	8, 9058402	8, 9231624
	0, 5133553	0, 4603438
l SN =	4, 4894244	4, 5449830
l SF =	5, 0027797	5, 0053268

NH =

NH =	147688, 39	153242, 89
SN =	30862, 02	35073, 82
NH+SN =	178550, 41	188316, 71
NH − SN =	116826, 37	118169, 07
2 l (NH − SN) =	5, 0675410	5, 0725038
l (NH+SN) =	5, 2517608	5, 2748889
	9, 8157802	9, 7976149
l tang (90−½SNH) =	10, 8741495	10, 9110303
	10, 6899297	10, 7086452
½ (NSH − NHS) =	78°,27′,30¾″	78°,55′,59¼
½ (NSH+NHS) =	82, 23, 22½	83, 0, 10
NSH =	160, 50, 53⅛	161, 56, 9 ¼
NSF =	160, 9, 41	161, 11, 58
FSH =	0, 41, 12⅛	0, 44, 11¼
NHS =	3, 55, 52	4, 4, 11
A l fin SNH =	9, 4191955	9, 3835062
fubtr. l fin NHS =	8, 8360518	8, 8510771
	0, 5831437	0, 5324291
l SN =	4, 4894244	4, 5449830
l SH =	5, 0725681	5, 0774121
FS = y =	100642, 1	101234, 1
HS = z =	118186, 6	119512, 2
Φ =	0°, 41′,12⅛	0, 44′,11¼″
l fin Φ =	8, 0786123	8, 1090132
l cof Φ =	9, 9999688	9, 9999641
l cot Φ =	11, 9213565	11, 8909509
l y =	5, 0027797	5, 0053268
l z =	5, 0725681	5, 0774121
l yz =	10, 0753478	10, 0827389

l ηηzz =

l yyzz =	20, 1506956	20, 1654778
fubtr.	13, 2823898	13, 2823898
	6, 8683058	6, 8830880
add 2 l fin Φ =	16, 1572246	16, 2180264
	3, 0255304	3, 1011144
Pars prior.	1060, 548	1262, 160
l V yz =	5, 0376739	5, 0413694
	0, 4771213	0, 4771213
	4, 5605526	4, 5642481
2 l fin Φ =	16, 1572246	16, 2180264
	0, 7177772	0, 7822745
Pars pofter. =	5, 221	6, 057
Ergo b =	1065, 769	1268, 217
y =	100642, 1	101234, 1
z =	118186, 6	119512, 2
y − b =	99576, 4	99965, 9
z − b =	117120, 9	118244, 0
A l (z−b) =	5, 0686344	5, 0727790
fubtr. l (y−b) =	4, 9981564	4, 9998519
	0, 0704780	0, 0729271
add l $\frac{y}{z}$ =	9, 9302116	9, 9279147
	10, 0006896	10, 0008418
fubtr. l fin Φ =	8, 0786123	8, 1090132
	1, 9220773	1, 8918286
Pars fubtr. =	83, 57518	77, 95224
a cot. Φ =	83, 43658	77, 79487
− tang v =	0, 13860	0, 15737
Anom. vera loci F = v =	172°, 6′,32″	171°, 3′,24″
fubtr. FSN =	160, 9, 41	161°,11, 58

Dift. Nodi

Diſt. Nodi ☊ a Perihel.	11, 56, 51	9, 51, 26
Ad ly =	5, 0027797	5, 0053268
add $l-\text{coſ}\,v$ =	9, 9958679	9, 9946878
$l-y$ coſ v =	4, 9986476	5, 0000146
add lb =	3, 0276634	3, 1031935
l Num.	8, 0263110	8, 1032081
$-y$ coſ v =	99689, 10	100003, 36
$y-b$ =	99576, 4	99965, 9
Den.	199265, 5	199969, 2
l Denom.	5, 2994320	5, 3009631
la =	2, 7268790	2, 8022450
Diſt. Perih. a Sole = a =	533, 1864	634, 2274
Ergo $2a$ =	1066, 3728	1268, 4548
b =	1065, 769	1268, 217
$\delta = 2a - b$ =	0, 603	0, 237
$l\delta$ =	9, 7803173	9, 3747483
lb =	3, 0276634	3, 1031935
Char. 10 minuenda. ln =	6, 7526539	6, 2715548
$\frac{1}{2}v$ =	86°, 3′, 16″	85°, 31′, 42″
lt =	1, 1613272	1, 1067702
lt^3 =	3, 4839816	3, 3203106
lt^5 =	5, 8066360	5, 5338510
lt^7 =	8, 1292904	7, 7473914
lnn =	13, 5053078	12, 5431096
lnt^5 =	2, 5592899	1, 8054058
ln^2t^5 =	9, 3119438	8, 0769606
ln^2t^7 =	1, 6345982	0, 2905010
ln^3t^7 =	8, 3872521	6, 5620558
ln^3t^9 =	9, 5485793	7, 6688260

$t =$

t =	14, 4986	12, 7870
$\frac{1}{3}t^3$ =	1015, 9220	696, 9303
$\frac{3}{5}n^2t^5$ =	0, 1231	71
$\frac{3}{7}n^2t^7$ =	18, 4766	8366
	1049, 0203	710, 5610
ſubtr. $\frac{2}{5}nt^5$ =	144, 9940	25, 5544
	904, 0263	685, 0066
ſubtr. $\frac{4}{7}n^3t^7$ =	139	2
$\frac{1}{9}n^3t^9$ =	1572	20
$t+\frac{1}{3}t^3+$&c.	903, 8552	685, 0044
$2\,la$ =	5, 4537580	5, 6044900
$l\sqrt{b}$ =	1, 5138317	1, 5515967
	3, 9399263	4, 0528933
lm =	5, 4345525	5, 4345525
	8, 5053738	8, 6183408
add. $l(t+\frac{1}{3}t^3+$&c.$)$ =	2, 9560989	2, 8356934
	1, 4614727	1, 4540342
A Perih. ad F dies	28, 9383	28, 4468
ſeu conſueto more	28ᵈ, 22ʰ, 31′, 9″	28ᵈ, 10ʰ, 43′, 23″
Com. in F, A. 1680. M. Dec.	36, 6, 1′, 38	36, 6, 1, 38
Cometa per Perihelium tranſiit A. 1680. M. Dec. =	7ᵈ, 7ʰ, 30′, 29″	7ᵈ, 19ʰ, 18′, 15″
Diſt. Perihelii a Sole a =	533, 1864	634, 2274
ſemilatus rectum b =	1065, 769	1268, 217
Diſt. Nodi ☊ a Perihelio =	11°, 56′, 51″	9°, 51′, 26″
Long. Nodi ☊ helioc.	9ˢ, 6°, 36′, 37″	9ˢ, 3°, 26′, 46″
Inclinatio ad Eclipticam	51°, 48′, 24″	57°, 47′, 2″

Computetur nunc pari modo locus cometæ geocentricus.

ad A. 1680. M. Dec.	12ᵈ, 4ʰ, 46′, 0″	12ᵈ, 4ʰ, 46′, 0″
Tempus Perih. ſubtr.	7, 7, 30, 29	7, 18, 19, 15

T =

$T =$	4, 21,15,30	4, 9,27,45
in diebus autem est $T =$	4, 8857	4, 3943
Ad Probl. VIII. $l\,(2a-b) =$	9, 7803173	9, 3747482
semiss.	9, 8901586	9, 6873742
$\frac{1}{2}\,l\,(2a-b) =$	9, 6704759	9, 0621225
$l\,T =$	0, 6889268	0, 6428897
add.	11, 0500076	11, 0500076
	11, 4094103	10, 7550198
subtr. $3\,l\,a =$	8, 1806370	8, 4067350
$l\,u =$	3, 2287733	2, 3482848
Ergo $u =$	1693,″4	223,″0
seu Anomalia media $u =$	0°,28′,13″	0°,3′,43″
$b-a =$	532, 583	633, 990
$l\,(b-a) =$	2, 7263872	2, 8020824
$l\,a =$	2, 7268790	2, 8022450
$l\,\frac{b-a}{a} =$	9, 9995082	9, 9998374
fit $\rho =$	21°, 30′	10°, 30′
l sin $\rho =$	9, 5640754	9, 2606330
add.	5, 3139333	5, 3142625
$l\,\frac{(b-a)}{a}$ sin $\rho =$	4, 8780087	4, 5748955
$\frac{b-a}{a}$ sin $\rho =$	75510,″7	37574,″7
add. $u =$	1693, 4	223, 0
	77204, 1	37797, 7
$u + \frac{b-a}{a}$ sin $\rho =$	21°, 26′,44″	10°, 29′,57″
subtr. $\rho =$	21°, 30′, 0″	10°, 30′, 0″

num. = −

num. = −	− 3′, 16″	− 3″
l cos $\rho =$	9, 9686779	9, 9926661
$l\,\frac{(b-a)}{a} =$	9, 9995082	9, 9998374
	9, 9681861	9, 9925035
$1 - \frac{(b-a)}{a} =$	0, 070634	0, 017112
erit $z =$	− 46′,15″	− 2′, 55″
Erit $\omega =$	20°, 43′,5″	10°,27′, 5″
$\frac{1}{2}\,\omega =$	10°, 21′,32½	5°, 13′,32½″
l tang $\frac{1}{2}\,\omega =$	9, 2619649	8, 9612284
subtr. $l\sqrt{\frac{2a-b}{b}} =$	8, 3763269	8, 1357774
l tang $\frac{1}{2}\,v =$	10, 8856380	10, 8254510
Ergo $\frac{1}{2}\,v =$	82°, 35′,9½″	81°,29′,56¾″
Et anomalia vera $v =$	165, 10, 19	162,59′,53
$l-$cos $v =$	9, 9852909	9, 9805920
$l\,\frac{b-a}{a} =$	9, 9995082	9, 9998374
	9, 9847991	9, 9804294
$-\frac{(b-a)}{a}$ cos $v =$	0, 965604	0, 955937
Den. =	0, 034395	0, 044062
$l\,b =$	3, 0276634	3, 1031935
lDen. =	8, 5364953	8, 6440642
$l\,y =$	4, 4911681	4, 4591293
Ab $v =$	165°,10′,10″	162°,59′,53″
Dist. ☊ a Perih.	11, 56,51	9, 51, 26
NC = Dist. Com. a ☊ =	153°, 13′,28″	153°, 8′,27″
CNP =	51°, 48′,24″	57, 47, 2

Fig. 6.

l sin NC =

l fin NC =	9, 6536916	9, 6549454
l fin N =	9, 8953998	5, 9273925
l fin CP =	9, 5490914	9, 5823379
l cof N =	9, 7911844	9, 7268202
l—tang NC =	9, 7029480	9, 7045221
l—tang NP =	9, 4941324	9, 4313423
Lat. helioc. CP =	20°,44′,12″	22°,28′,21″
Ergo NP =	162, 40, 22	164, 53, 28
feu NP =	5ˢ,12°, 40,22	5ˢ, 14,53,28
Add. long. ☊ =	9, 6, 36,37	9, 3,26,46
Longitudo heliocentr.	2, 19,16,59	2ˢ, 18,20,14
Longitudo terræ	3, 1,51,23	3, 1,51,23
Ang. *c*ST =	12°,34,24″	13°,31′, 6″
CS*c* =	20, 44, 12	22, 28, 21
ly=*l*CS =	4, 4911681	4, 4591293
add. *l* fin CS*c* =	9, 5490914	9, 5823379
l cof CS*c* =	9, 9709127	9, 9657017
*l*C*c* =	4, 0402595	4, 0414672
*l*S*c* =	4, 4620808	4, 4248310
add. { *l* fin *c*ST =	9, 3378364	9, 3687698
add. { *l* cof *c*ST =	9, 9894579	9, 9877967
*l*cP =	3, 7999172	4, 7936208
*l*SP =	4, 4515387	4, 4126277
SP =	28283, 9	25859, 9
ab ST =	98275,	98275, 0
TP =	69991	72415,
*a l*cP =	3, 7999172	3, 7936208
fubtr. *l*TP =	4, 8450422	4, 8598285
l tang ST*c* =	8, 9548750	8, 9337923
ST*c* =	5°,9′,1″	4°,54′,27″
Long. Solis	9ˢ,1, 51, 23	9, 1, 51, 23

Long.

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Long. Com. Geocentr.	9, 7, 0, 24	9, 6, 45,50
a *l*TP =	4, 8450422	4, 8598285
fubtr. *l*. cof ST*c* =	9, 9982431	9, 9984053
*l c*T =	4, 8467991	4, 8614232
a *l*C*c* =	4, 0402595	4, 0414672
l tang CT*c* =	9, 1934604	9, 1800440
Latitudo Geocentr. =	8°, 52′,24″	8°, 36′,27″

Hinc tam ex longitudinibus quam latitudinibus obfervatis liquet verum valorem ipfius *r* majorem effe quam 72000. Ex præcedente autem hypothefi collegimus, *r* minorem effe quam 72700. Sunt autem omnes quatuor tam longitudines quam latitudines obfervatis majores, ex quo fequitur, valores intra 72000 & 72700 contentos prabituros effe minores & longitudines & latitudines, ficque veritati magis confentaneas; ita ut intra hos limites minima longitudo & latitudo contineatur. Cum autem valores 72000 & 72700 pro *r* fubftituti æquales fere præbeant longitudines ac latitudines, minimum fere refpondebit valori medio 72350; quem tanquam verum ipfius *r* valorem affumamus; cum eum propius definire vix liceat.

Comparemus ergo has duas hypothefes inter fe.

Hypothefis *r* =	72000	7 2 7 0 0
Cometa per perihelium tranfiit A. 1680 M Dec.	7ᵈ, 19ʰ, 18′, 15″	7ᵈ, 22ʰ, 17′, 48″
Dift. Perih. a fole —	634, 227	678, 678
Semi latus rectum — —	1268, 217	1357, 313
Dift. nodi ☊ a Perih. —	9°, 51′, 26	9°, 13′,17″
Long. nodi ☊ helioc. —	9ˢ, 3°, 26, 46	9ˢ, 2°, 31′, 32″
Inclin. Orbitæ ad Eclipt.	57°, 47′, 2″	59°, 32′, 38″

Sumendo

Sumendo ergo inter hæc systemata medium, prodibit
Cometæ hujus sequens theoria quasi vera

I. Cometa per perihelium transiit A. 1680 M. Decembri	–	$7^d, 20^h, 48', 0''$
II. Distantia Perihelii a Sole	=	656, 4525
III. Semi-latus rectum	=	1312, 7650
IV. Distantia nodi ☊ a Perih.	=	$9^\circ, 32', 21''$
V. Longitudo helioc. nodi ☊	=	$9^s, 2^\circ, 59', 9'$
VI. Inclinatio Orbitæ ad Eclipt.	=	$58^\circ, 39', 50''$

Non multum discrepant hæ orbitæ Cometæ determinationes ab iis, quas Neutonus & Hallejus invenerunt, quarum ille constructione geometrica, hic vero calculo est usus, ita tamen ut locum nodorum, & inclinationem & tempus, quo per perihelium transiit, eadem retinuerit, quæ Neutonus per constructionem invenisset: uterque autem orbitam cometæ perfectam esse parabolam assumsit. Quare cum constructio geometrica in hoc negotio admodum sit lubrica, atque Hallejus præcipua capita hinc nata retinuerit, mirum non est, quendam dissensum inter hanc orbitæ determinationem & Neutonianam deprehendi, neque enim esset mirandum, si iste dissensus major prodiisset. Statuit autem Neutonus l. hunc Cometam in perihelio esse versatum A. 1680. M. Dec. $8^d, 0^h, 4'$; II. distantiam perihelii a sole = 607, 5; III. semi-latus rectum = 1215. IV. Distantiam nodi a perihelio = $9^\circ, 20'$: V. Long. nodi ☊ = $9^s, 1^\circ, 53'$ & VI. inclinationem orbitæ ad Eclipt. $61^\circ, 20', 20''$. Orbita autem hujus cometæ sic proxime inventa poterit secundum

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5. IV. Di-
odi ☊ =
, 20', 20''.
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dum methodum, quam in superiore dissertatione de Cometa A. 1742. tradidi, ulterius corrigi, atque cum accuratissimæ observationes per satis longum temporis spatium suppetant, fere ad eum perfectionis gradum perduci, ut axis transversus ejus orbitæ definiri, atque adeo ejus tempus revolutionis assignari queat. Quem laborem aliis relinquens interim ex hac cometæ orbita inventa, tanquam esset vera, tempus periodicum investigabo. Cum igitur sit

Distantia perihelii a sole a	=	656, 4525
erit $2a$	=	1312, 9050
subtrahatur b	=	1312, 7650
erit $2a - b$	=	0, 1400
At est la	=	2, 8172032
unde laa	=	5, 6344064
subtr. $l(2a-b)$	=	9, 1461280
l semi-axis transversi	=	6, 4882784
addatur semissis	=	3, 2441392
		9, 7324176
subtrahatur $lc\sqrt{c}$	=	7, 5000000
		2, 2324176

Unde erit tempus periodicum = 170, 77 annorum.

Reverteretur itaque iste cometa ad perihelium post annos centenos & septuaginta; & quamvis exiguum discrimen in valore $2a-b$ vehementer hoc tempus immutare possit, tamen non adeo a veritate aberrabit. Neque enim contra hanc reversionem quicquam valet objectio, quod idem cometa annis 170 antea non sit observatus, neque in Tabula Halleji unquam ante ullus sit observatus, qui cum hoc consentiret. Primum enim

statuš

ſtatus cometæ apparens plurimum a loco terræ in orbita ſua pendet, quæ ſi verſetur in iisdem ſignis, quibus cometa eſt proximus, cometa maxime fulgens conſpicietur, & caudæ longitudo apparens non ex ejus vera quantitate, ſed potiſſimum e ſitu relativo reſpectu noſtri æſtimatur. Utraque opportunitas locum habuit A. 1680, quo tempore non ſolum cometa ſatis
Fig. 8. prope ad locum terræ acceſſit, ſed etiam caudæ directio C *c* ita ratione terræ T erat diſpoſita, ut ejus magnitudo apparens ſeu angulus CT*c*, fuerit vehementer magnus; quanquam & hic cometa ob ſummam in perihelio vicinitatem cum ſole præ reliquis multo majori cauda inſtructus eſt putandus. Ex his autem intelligitur, ſi idem hic cometa vel in poſterum redeat, vel ante jam aliquoties ad perihelium acceſſerit, niſi terra eo tempore in iisdem fere orbitæ ſuæ locis ſit verſata, longe alia ſpecie hunc eundem cometam conſpici debuiſſe, ut ex terra viſus nullo modo pro eodem haberi potuerit. Tum vero ad quod maxime eſt attendendum, hic inſignis cometa ſæpius ad ſolem reverti poterit, incolis terræ prorſus inconſpicuus. In hoc enim Cometa idem, quod de cometa anni 1742 notavi, uſu venit, quod diſparuerit, fere antequam ejus diſtantia a terra diſtantiam ſolis ſuperaſſet. Quare cum terra eo tempore, quo iſte cometa ante redierit, in orbita ſua ita ſita eſſe poſſit, ut is perpetuo longius quam ſol a terra fuerit remotus, nequidem obſervari potuit; unde eo minus eſt mirandum, hunc cometam ante nunquam eſſe animadverſum, quantum quidem ex obſervationibus ſufficienti ſtudio inſtitutis colligere licet. Confir-

matur

:bita ſua eſt pro- æ longi- imum e rtunitas eta ſatis ctio C *c* pparens uam & ole præ Ex his redeat, erra eo ge alia x terra vero ad pius ad s. In avi, uſu diſtan- uo iſte is per- em ob- metam obſer- Confir. matur

matur itaque maxime illa conjectura, quam ante feci, nullum cometam nobis eſſe conſpicuum, niſi terræ ſit propior quam ſol, ſicque ſine dubio complures cometæ ad ſolem in orbitis ſuis ſæpius revertuntur, aſpectum noſtrum prorſus fugientes. Et hanc ob rem numerus cometarum ſyſtema ſolare exornantium multo erit major, quam ſuſpicamur; utrum autem quandoque cometas, qui ad aliarum ſtellarum fixarum ſyſtemata pertineant, ſpectare nobis liceat; hoc nullo modo affirmari poterit, cum eorum diſtantia plus quam millies ſuperare deberet eam, in qua cometæ noſtri ſyſtematis ſe demum ſpectandos exhibeant.

Quoniam ceterum cometæ ſunt corpora non admodum ingentia, & terram in perigæo celerrime præterire ſolent, effectus eorum ratione attractionis in perturbatione motus terræ non minimus eſſe nequit: niſi forte proxime ad terram accedant; quod quidem fieri non poteſt, niſi dum in alterutro nodo verſantes per ipſam terræ ſemitam tranſeant, ibique terram offendant; unde non ſolum ob attractionem mutuam, ſed etiam a mutuo conflictu effectus maxime funeſti reſultare deberent. Quod autem in majoribus diſtantiis motum terræ vix perturbare valeant, ratio eſt, quia vis ſolis, qua terra in orbita ſua retinetur, incomparabiliter eſt major, quam illa vis, quæ unquam a cometa proficiſci poſſet; ita ut vis cometæ a vi ſolis longiſſime ſuperetur ac penitus abſorbeatur. Hoc vero tantum eſt tenendum de viribus cometarum, quarum directiones in planum eclipticæ incidunt; aliter enim comparata eſt ratio earum viri-

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um, quarum directio ad planum eclipticæ est normalis. Cum enim hæ vires a vi solis non afficiantur, nihil impedit, quominus effectum suum producant; qui dum terram de plano eclipticæ vel boream vel austrum versus retrahere nituntur, actu orbitam terræ in aliud planum deducent; unde cum axis terræ non afficiatur, obliquitas eclipticæ atque positio punctorum æquinoctialium & solstitialium immutabitur, qui effectus eo magis erit sensibilis, quo propius cometa ad terram accesserit, simulque quo major eo tempore fuerit ejus latitudo. Hinc non dubitarim mutabilitatem obliquitatis eclipticæ, quæ nunc quidem extra dubium posita videtur, pariter ac variationem, quam in stellarum fixarum longitudine & latitudine deprehendunt, soli cometarum actioni tanquam veræ causæ adscribere; facile autem erit in posterum hanc conjecturam per observationes vel confirmare vel refellere, cum si hypothesis hæc esset vera, istæ mutationes subito post apparitionem cujusque cometæ contingere deberent. Quod quidem ad cometam anni 1742 attinet, ejus actione obliquitas eclipticæ aliquantum augeri debuisset, quamobrem Astronomi sunt rogandi, ut quam primum fieri licet, obliquitatem eclipticæ nunc quidem omni studio investigent, ut intelligi possit, utrum ea major reperiatur, quam ante hujus cometæ apparitionem est inventa, an secus. Generatim vero cometarum, qui in perigæo satis prope ad terram accedunt, simulque notabilem latitudinem habent, effectus ita erit comparatus; ut sequens tabella declarat:

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Si cometæ Latitudo fuerit borealis &

Locus solis sit in ♈. Puncta æquinoctialia non afficientur, obliquitas eclipticæ autem augebitur.

Locus solis sit in ♋. Puncta æquinoctialia promovebuntur, obliquitas eclipticæ non mutabitur.

Locus solis sit in ♎. Puncta æquinoctialia non afficientur, obliquitas eclipticæ diminuetur.

Locus solis in ♑. Puncta æquinoctialia regredientur, obliquitas eclipticæ non afficietur.

Sin cometæ latitudo fuerit australis, effectus isti erunt contrarii. Probe autem ista æquinoctiorum variatio distingui debet a præcessione æquinoctiorum satis perspecta, quæ non a mutabilitate eclipticæ, sed a mutatione ipsius axis terræ proficiscitur, qui a cometis non variatur. Hinc autem recepta Astronomorum regula, qua statuunt, puncta æquinoctialia quotannis 50″ regredi, non parum perturbabitur. Quod autem ab antiquissimis temporibus obliquitas eclipticæ satis sensibiliter sit minuta, concludendum est, vel plures cometas cum latitudine boreali, dum sol in signis borealibus versabatur, vel cum latitudine australi, dum sol in signis australibus versabatur, ad terram accessisse; horumque effectum prævaluisse.

Inve-