LETTRE XXVII.

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GOLDBACH à EULER.

SOMMAIRE. Même sujet. Réponse à la lettre précédente.

(Petrop.) d. 24. Nov. 1739.

Gratissima mihi fuerunt quae heri scripsisti; mea solutio haec est: Sit

$$1+\frac{1}{2^n}+\frac{1}{3^n}+\frac{1}{4^n}+\text{etc.}=\alpha\pi^n$$

$$1+\frac{1}{2^{2n}}+\frac{1}{3^{2n}}+\frac{1}{4^{2n}}+\text{etc.}=\beta\pi^{2n}$$

$$\frac{1}{2^n}+\frac{1}{3^n}+\frac{1}{5^n}+\frac{1}{7^n}+\frac{1}{8^n}+\frac{1}{11^n}+\text{etc.}=M,$$

cujus denominatores, posita n 1, sunt producta primorum numero imparium

$$1+\frac{1}{4^n}+\frac{1}{6^n}+\frac{1}{9^n}+\frac{1}{10^n}+\text{etc.}=N,$$

cujus denominatores, posita $n=1$, sunt producta primorum numero parium.

erit $\alpha\pi^n+\frac{\beta\pi^n}{\alpha}=2M$, $\alpha\pi^n-\frac{\beta\pi^n}{\alpha}=2N$. Sed nescio, an methodus Tua valeat ad determinandam v. gr. rationem inter terminos affirmativos et negativos hujus seriei

$$\frac{1}{4^n}-\frac{1}{8^n}+\frac{1}{9^n}-\frac{1}{12^n}+\frac{1}{16^n}-\frac{1}{18^n}-\frac{1}{20^n}+\text{etc.}$$

cujus denominatores, posita $n=1$, sunt omnes potestates numerorum et omnia earum multipla; termini notati signo $+$ continent denominatores productos ex primis numero paribus, termini notati signo $-$, ex imparibus, quam rationem tamen eruere potero si operae pretium visum fuerit.

Sed multo magis Tibi, opinor, placebit quod heri inveni: Sit $1+\frac{1}{2^n}+\frac{1}{3^n}+\frac{1}{4^n}+\text{etc.}=\alpha\pi^n$, $\frac{1}{2^n}+\frac{1}{3^n}+\frac{1}{5^n}+\frac{1}{7^n}+\frac{1}{11^n}+\text{etc.}=P$ (cujus seriei denominatores continent omnes numeros primos) erit

$$\frac{1}{2^{2n}}+\frac{1}{3^{2n}}+\frac{1}{5^{2n}}+\frac{1}{7^{2n}}+\frac{1}{11^{2n}}+\text{etc.}=(P-1)^2+1-\frac{2}{\alpha\pi^n},$$

modo sit $n>1$. Vale et fave —

Goldbach.

Note marginale d'Euler.

$$A=1+\frac{1}{2^n}+\frac{1}{3^n}+\frac{1}{4^n}+\frac{1}{5^n}+\frac{1}{6^n}+\frac{1}{7^n}+\frac{1}{8^n}+\text{etc.}$$

$$\frac{1}{A}=1+\frac{\alpha}{2^n}+\frac{\beta}{3^n}+\frac{\gamma}{4^n}+\frac{\delta}{5^n}+\frac{\varepsilon}{6^n}+\frac{\zeta}{7^n}+\frac{\eta}{8^n}+\text{etc.}$$

$\frac{\mu}{p^n}$ terminus generalis.

Si p est numerus primus erit $-\frac{1}{p^n}$

is p prod. ex duobus numeris primis inaequalibus: $+\frac{1}{p^n}$

prod. ex duobus numeris primis aequalibus: $+\frac{0}{p^n}$

si p prod. ex tribus inaequalibus	abc erit	$-\frac{1}{p^n}$
	aab	$+\frac{0}{p^n}$
	aaa	$+\frac{0}{p^n}$
si p prod. ex quatuor inaequalibus	$abcd$	$+\frac{1}{p^n}$
	$aabc$	$+\frac{0}{p^n}$
	$aabb$	$+\frac{0}{p^n}$
	a^3b	$+\frac{0}{p^n}$

LETTRE XXVIII.

EULER à GOLDBACH.

SOMMAIRE. Suite des recherches précédentes.

d. 26 Novembr. 1739.

Considerans rationem, quae intercedit inter summam seriei $\frac{1}{2^{2n}}+\frac{1}{3^{2n}}+\frac{1}{5^{2n}}+\frac{1}{7^{2n}}+$ etc. et hanc expressionem

$$(P-1)^2+1-\frac{2}{a\pi^n},$$

deprehendi seriem aliquanto esse minorem ac fore

$$(P-1)^2+1-\frac{2}{a\pi^n}=\frac{1}{2^{2n}}+\frac{1}{3^{2n}}+\frac{1}{5^{2n}}+\frac{1}{7^{2n}}+\text{etc.}$$

$$\left.\begin{array}{l} +2\,.\,\text{summa factorum ex ternis} \\ -2\,.\,\text{summa factorum ex quaternis} \\ +2\,.\,\text{summa factorum ex quinis} \\ -\text{etc.} \end{array}\right\}\ \text{terminis inaequalibus}$$