LETTRE XX.

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EULER à GOLDBACH.

SOMMAIRE. Problème de la géométrie des courbes.

(Plié en forme de lettre, mais sans suscription, signature et date).

Problema. (Fig. 4.) Si ex curva AMB curva Amb ita formetur, ut recta MAm per punctum fixum A ducta perpetuo capiatur ejusdem longitudinis; invenire casus, quibus hae duae curvae prodeunt inter se similes et aequales, ad axes AB, Ab inter se normales relatae.

Solutio. Posita longitudine constante $Mm = Dd = AB = 2a$, sit $AP = x$, $PM = y$, atque sumta nova variabili z, sit Q talis functio ipsius z, quae posita z negativa, abeat in sui ipsius negativam, cujusmodi sunt mz, $mz^3 + nz$, etc. Sequenti modo per z coordinatae x et y determinabuntur:

$$x = \frac{(a+z)\sqrt{(aa+zz+2Q)}}{\sqrt{2(aa+zz)}};\quad y = \frac{(a+z)\sqrt{(aa+zz-2Q)}}{\sqrt{2(aa+zz)}}.$$

Eliminandis ergo z et Q, infinitae prodibunt aequationes inter x et y, ac proinde innumerabiles curvae AMB problemati satisfacientes $Q.\ E.\ I.$

Corollarium 1. Erit ergo $\sqrt{(xx+yy)} = a + z$. Atque $x : y = \sqrt{(aa+zz+2Q)} : \sqrt{(aa+zz-2Q)}$.

Corollarium 2. Sumta $AC = \frac{a}{\sqrt{2}}$, fiet $CD = \frac{a}{\sqrt{2}}$ atque $AD = a$, punctoque D in altera curva sui homologum d respondebit in generatione.

Exemplum. Sit $Q = naz$, erit

$$xx : yy = aa + 2naz + zz : aa - 2naz + zz,$$

seu

$$xx\begin{pmatrix}2aa + xx + yy - 2a\sqrt{(xx+yy)}\\ +2naa \qquad -2na\sqrt{(xx+yy)}\end{pmatrix} =$$

$$yy\begin{pmatrix}2aa + xx + yy - 2a\sqrt{(xx+yy)}\\ -2naa \qquad +2na\sqrt{(xx+yy)}\end{pmatrix},$$

$$2a\left((n+1)xx + (n-1)yy\right)\sqrt{(xx+yy)} =$$

$$2aa\left((n+1)xx + (n-1)yy\right) + x^4 - y^4,$$

unde sequens oritur aequatio pro curva satisfaciente

$$x^8 - 2x^4y^4 + y^8 - 4na^2\left((n+1)x^2 + (n-1)y^2\right)(xx+yy)^2 + 4a^4\left((n+1)x^2 + (n-1)y^2\right)^2 = 0,$$

quae jam innumerabiles praebet curvas quaesitas.