DE

C A S I B V S

QVIBVS HANC FORMVLMAM

\[ x^4 + k \times x \times y \times y + y^4 \]

AD QUADRATVM REDVCRE LICET.

Auctore

L. E V L E R O.

Conuentui exhib. die 28. April. 1777.

§. 1.

Super hac formula primum obseruo, inde omnes caufus excludi debere, quibus numerorum \( x \) et \( y \) altervter effet \( = 0 \), quoniam tum haec formula \( \sponte \) euaderet quadratum, quicunque numeri loco \( k \) acciperentur. Secundo porro obseruo, si numeretur vel \( k = \) \( \ast \) vel \( k = \ast \), formulam iam \( \sponte \) esse quadratum, quicunque numeri pro \( x \) et \( y \) flatuerentur. Tertio vero obseruari convexit, omnia quadrata negativa loco \( k \) assumpta nulla difficultate laborare. Si enim ponatur \( k = -n \times n \), formula euadet \( x^4 - n \times n \times x \times y \times y + y^4 \), quae ergo in quadratum abit, sumerdo vel \( x = n \times y \), vel \( y = n \times x \), sicque pro littera \( k \) vitro fe offerunt caufus \( k = \pm \ast \) et \( k = -n \times n \); unde quaeftio in hoc verfa'ur, vt omnes reliqui valores pro \( k \) inuaefigentur, quibus redudio formulae profitaæ ad quadratum locum habere queat.

D 2

§. 2.
§. 2. Cum igitur postulentur omnes numeri integri pro $k$ accipiendi, quibus formula reductionem ad quadratum admittit, methodus Diophantea varios modos suppeditat id praefandi. Verum quacunque vtamur methodo, semper aliquod dubium relinquitur, an inde omnes plane valores idonei obtineantur; etiamsi facile fit innumerables valores idoneos exhibere, vt hoc modo omnes numeri inepti cognosci queant, cuiusmodi sunt $k = 1$, vel $k = 3$, vel $k = 4$, vel $k = 5$, vel $k = 6$, etc. pro quibus iam solide demonstratum est, reductionem ad quadratum nullo modo locum habere possit.

§. 3. Quod si enim quadrati, cui formula nostra æquari debet, radix statuatur $x \div \frac{2p}{q}x$, prodit $k = \frac{2p}{q} + \frac{2q}{q} \div \frac{q}{x} - \frac{2q}{x}$, qui valor vt fiat integer, primo patet pro $q$ lumi debere divisiorem ipsius $y$ $y$, id quod eo pluribus modis fieri potest, quo plures faiores numerus $y$ involuitor; unde iam patet istam methodum nimir esse vagam, quam vt omnes plane causis in genere exhiberi queant. Si igitur huic conditioni fatissecerimus, vt sit $y \div x = aq$, aequatio iuventa dabit $k \div x = \frac{2p}{q} - \frac{2q}{x} - aq$. Requiritur igitur porro vt formula $2p \div x = aq$, quae $2x = aq$, divisio per $q$ admittat, quod si fuerit esseum, et $Q$ sit valor huic fractioi aequalis, insuper, cum iam fit $k = \frac{2}{x} - aq$, effici debet, vt quantitas $Q - aq$ euadat divisibilis per $x$. Ex quo iam fatis intelligitur, hac methodo perfeclam enumerationem omnium valorem idoneorum ipsius $k$ sperari non possit.

§. 4. Idem defensus se exercit, quando radicem quadratam formulae propositae statuimus $x \div q \div x y \div y y$, tum enim faca evolutione reperitur $k \div xy$.
\[ h \times y = \frac{2p}{q} (x + y) + \frac{2p}{q} x y, \]
\[ h = \frac{2p}{q} \cdot \frac{x + 2y}{x} + 2 + \frac{2p}{q}, \]
quae forma etiam facile ad numeros integros reuocatur, hincque infiniti numeri idonei erui possunt, tamen pariter ingens reliquitur dubium, num hoc modo omnes plane valores idonei, nullo praetermissio, obtineri queant.

§. 5. Nuper autem, cum haec perpendijsem, incidit in methodum profus singularem, quae primo intuitu adeo naturae quaestionis aduerfari videtur. Confidero enim valores litterae \( \kappa \) quae effec formula irrationalis, in binomio \( P + \sqrt{Q} \) contenta, ita ut sit \( h = P + \sqrt{Q} \). Euidens enim est, postquam in genere omnem valores pro \( P \) et \( Q \) fuerint inuenti, id insuper essenti debere, ut \( Q \) reddatur numerus quadratus; hoc autem valore substituto formula propofita abibit pariter in tale binomium, cuius pars rationalis erit \( x^4 + P x x y y + y^4 \), irrationalis vero \( x x y y \sqrt{Q} \), quod igiitur quadratum essenti debet. Confiat autem hoc fieri non posse, nisi quadratrum partis rationalis, ablato quadrato partis irrationalis, fiat quadratum; hinc autem peruenitur ad sequentem formam:

\[
x^8 + 2P x^5 y y + 2 x^4 y^4 + 2P x x y y + y^8
\]
\[ + P P
\]
\[ - Q \]

§. 6. Cum iam haec forma in genere debeat esse quadratum, quicunque numeri pro \( x \) et \( y \) accipientur, manifestum est eius radicem aliam formam habere non posse, nisi vel \( x^8 + P x x y y + y^4 \) vel \( x^8 + P x x y y - y^4 \). At vero prior hic locum habere nequit; perduceret enim ad \( Q = 0 \);
vnde adhibeamus alteram formam, cuius quadratum est
\[ x^8 + 2px^6y^2 - 2px^4y^4 - 2px^2y^6 + y^8, \]
+ PP

cui si formula inventa aequalis statuatur, peruenitur ad
hanc aequationem:
\[ 4x^4y^4 - Qx^4y^4 + 4pxxy^6 = 0, \]
quae per \( xxy^4 \) duifsa praebet:
\[ 4pyy - Qxx + 4xx = 0, \]
cui vt satisiat, statuatur \( P = fxx, \) bincque fponte se prodict
\( Q = fy + x, \) vbi id commodi fumus affectui, vt nullae amplius fractiones sint abigendae.

§ 7. Quoniam igitur inuenimus \( P = fxx \) et \( Q = 4fyy + 4, \) binomium pro numero \( k \) accipiendum fiet \( k = fxx + 2\sqrt{(fyy + x)}, \) nihilque iam amplius supereft, nifi vt formula \( fyy + x \) reddatur quadratum, quae cum sit ipsa formula Pelliana, unideris et, hoc infinitis modis praebant poftf, dum pro \( f \) pro lubitu omnes numeros positivos assumere licet, exceptis solis numeris quadratis; quamobrem hoc transferri poterunt, quae circa hanc formulam iam olim sum commentatus, vbi pro singulis valoribus \( f \) vsque ad 100 valores requisitos ipius \( y \) in tabula sum complexus:
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§ 8. Quin etiam, si loco $k$ istum valorem substituamus, deprehendemus, formulam nostram propositam revoca fieri quadratum. Prodit enim

$$x^4 - f x^4 y y^4 + y^4 + 2 x x y y' (f y y' + 1),$$

quae manifesto est quadratum huius formae:

$$y y' + x x y' (f y y' + 1)$$

quemadmodum periculum facienti mox patebit. Ex quo intelligimus, etiam pro omnibus valoribus idoneis litterae $k$ statim radicem quadratam ipsius formuae propositae assignari possit. Ita si fuerit $f = 2$ et $y = 2$, hinc fit $k = 2 x x + 6$, ex quo valore formulae euidit

$$9 x^4 - 24 x x + 16 = (3 x x - 4)^2.$$

§ 9. Contemplamur iam accuratius formulam pro $k$ inuentam $k = f x x + 2 y (f y y + 1)$, vbi per se manifestum est, membra posterioris radicale tam positivum quam negativum accipit possit, ita vt sit $k = f x x + 2 y (f y y + 1)$; quare si primo sumamus $x = 1$ et $y = 1$, erit $k = f + 2 y (f + 1)$. Imaginat haec formula reddatur rationalis, ponatur $f + 1 = n n$, ideoque $f = n n - 1$, eritque,

$$k = n n + 2 n - 1 = (n + 1)^2 - 2.$$

Sicque pro $k$ iam habentur omnes numeri quadrati binario minuti, unde usque ad centum pro $k$ fumi poterunt sequentes valores:

$$2, 7, 14, 23, 34, 47, 62, 79, 98.$$

§ 10. Maneat $y = 1$, at $x$ relinquatur indefinitum, sumtoque $f = n n - 1$ prohbit ista formula:

$$k = (n n - 1) x x + 2 n,$$

quae
quae iam infinitam multitudinem valorum idoneorum pro \( k \) suppeditat. Vbi imprimis notasse inuabit, nihil impedire, quominus pro \( x \) fractiones accipientur, dummodo valor ipsius \( k \) prodeat numerus integer, quandoquidem sola ratio inter \( x \) et \( y \) est spedita; unde si prodierit \( x = \frac{p}{q} \), quoniam sumimus \( y = r \), sumi debet \( x : y = p : q \).

§. 11. Percurramus igitur causas simpliciores numeri \( n \); ac si eueniat \( vt \ n n - 1 \) habeat saeulum quadratum, puta \( n n - 1 = m a a \), statui poterit \( x = \frac{z}{a} \), etque hinc \( k = m z z + 2 n \), tum vero erit \( x : y = z : a \), hincque nata est sequens tabula:

\[
\begin{array}{c|c|c|c}
\hline
\text{Novæ Adæ Acad. Imp. Scient. T. X.} & E & n \\
\hline
\end{array}
\]
§. 12. Haec enim adhuc pro $f$ folos numeros integros admisisimus; verum eam si dixi admissi possunt, dummodo pro $h$ numeri integri resistent. Quod $h$ enim in genere fiatamus $x = 2v$, fiet $h = 4f vv + \sqrt{\(4f^2v^2 - 4\)}$, ubi eundem ess sufficiere dummodo $4f$ fuerit numerus integer. Ponatur ergo $4f = g$, eritque $h = g vv + \sqrt{\(g^2 v^2 - 4\)}$, ubi quia pro $x$ summamus numerum parem, intelligitur hic pro...
pro \( y \) tantum numeros impares accipi debeo, quia aliquo in causis praecedentibus reuerteremur.

\$\$ 13. Iam in hac formula flatuamus \( y = x \), vt fit \( k = g\sqrt{v} \) \( v \) \( \pm \sqrt{(g + 4)} \), et nunc \( v \) \( g + 4 \) equaet quadratum, primo omnium sumi poterit \( g = 3 \), unde ortur \( k = -3 \) \( v \) \( \pm 1 \); vbi cum fit \( x = 2 \) \( v \), erit \( x : y = 2 \) \( v : 1 \), ex qua formula meri numeri negativi pro \( k \) resultant, qui vs-que ad centum erunt:

\[-2, -4, -1, -13, -26, -28, -47, -49, -74, 76, 76,\]

ad quos infuper, vt initio innuimus, quadrata negativa accedunt, scilicet:

\[-1, -4, -9, -16, -25, -36, -49, -64,\]

\[-81, -100.\]

\$\$ 14. Pro reliquis valoribus ipsius \( g \) flatuamus \( g + 4 = nn \), fietque \( k = (n\,n - 4)\,v\,v \pm n \). Hic ergo, vt supra, si equat \( n\,n - 4 = ma \), et loxo \( a \, v \) scribatur \( x \), erit \( k = m \, x \, x \pm n \); vbi cum fit \( v = x \), erit \( x = \frac{a\,x}{2} \), ideoque \( x = \frac{2\,x}{a} \), hincque ratio inter \( x \) et \( y \) erit \( x : y = 2 \, z : a.\)

\$\$ 15. In hac autem formula sufficit pro \( n \) numero tantum impares sumisse, quandoquidem ex paribus praecedentibus formulae redirent. Hoc notato sequentes formulae speciales pro \( k \) obtinentur:

\[
\begin{align*}
E & = \frac{x}{y} \\
\end{align*}
\]
§. 16. Ex his iam formulis haud difficulter omnes valores numeri \( h \), vsque ad 100, computari possunt, qui cum sponte distinguantur in positius et negativos, vtrosque foelifm in tabulis subiuntis referamus, et cuilibet valori adiungamus rationes inter \( x \) et \( y \), unde hi numeri producuntur.

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Tabula
Tabula prior
exhibens omnes valores positivos ipfius \( h \)
centenario minores.

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| 47 | 10 3 | 94 | 1
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| 49 | 10 3 | 96 | 1
| 52 | 10 3 | 98 | 1
| 55 | 10 3 | 100 | 1.

E 3

Ta-
Tabula posterior
exhibens omnes valores negativos ipsius \( k \) centenario minores.

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</table>

§. 17. Quemadmodum in nostris formulis pro \( k \) inventis, quae sunt

\[ k = (n n - r) xx + 2 n \] et \( k = (n n + r) vv + n \),

loco \( x \) et \( y \) numeros fractiones admissimus, ita etiam pro \( n \) fractiones admissi poterunt, dummodo ita fuerint comparatae, ut inde valores integri pro \( k \) reperiantur, quo obseruato investigationem harum formularum multo facilius instituere poterit. Sumto enim \( y = r \), ut formula \( x^4 + k xx + r \) quadratum effici debeat, quaeque eum fuerit radix, eam semper fub...
fut hac formula: \( f x x + i \) comprehendere licet. Hinc autem statim prodict \( h = (f f - 1) x x + 2 f \), quae erat formula nostra prior, in qua si statuatur \( x = 2 \nu \) et \( 2 f = g \), prodict altera formula \( h = (g g - 4) \nu \nu + g \), cui respondet ratio \( \frac{x}{y} = \frac{2^2}{1} \).

§. 18. Quod si iam loco g fractiones introducere velimus, facile patet statui debere \( g = \frac{a}{b} \), ac praeterea \( \nu = b z \): hoc enim modo probit \( h = \frac{(x a - 4 b^2)}{b b} x x + \frac{a}{b b} \), cui respondet ratio \( \frac{a}{b}, \) atque hic pro \( a \) et \( b \) eiusmodi numeros accipit, vt pro \( h \) prodeant numeri integri. Requiritur ergo vt numerus \( a a x x + a \), hoc est \( a a x x = 1 \) diuisionem per \( b b \) admittat, tum enim erit

\[ h = a \frac{a a x x + 1}{b b} = 4 b b x x; \]

hocque adeo in genere praefarsi potest, ponendo \( a = b^4 + i \), erit enim

\[ h = \frac{b^4 - 1 b x x + b^4 + 1}{b b} \].

Hic iam ponatur \( x = \frac{1}{b^4 + 1} \), vt habeatur \( h = \frac{b^4 + b}{b b} \), vbi ergo \( t t + 1 \) per \( b b \) divisibile reddi debet, vnde prodict \( h = \frac{1 t + 1}{b b} + b b \). Quomodo unequal autem haec formula evaluatur, omnes numeri in ea contenti iam in formulis superioribus contineri videntur.

§. 19. Hinc igitur patet, in Analysis adhuc defiderari methodum certam, cujus ope omnes valores ipsius \( k \) assignari atque adeo quosque libuerit continuari quantaest. Quin etiam ex formula distra fortasse eiusmodi numeri erui posse videntur, qui in formulis integris supra exhibi-
hibitis non contineantur; veluti se mihi obtulit iste numerus \( k = 131 \), quem primo intuítu ex formulís supra datas derivári posse non videbatur, cum tamen in formulá \( (n n - 4) z z + n \) contineatur, si postó \( z = 6 \) pro \( n \) vel fractio \( \frac{11}{2} \) vel \( \frac{25}{9} \) súmatur. Postea vero deprehendi hunc ipsum numerum ex formulá \( k = 21 z z + 110 \) oriri; num autem hoc semper eueniat, etiamnunc dubitare licet, unde perfecít solutio etiamnunc plane vires Analythicos superare videtur. Quæstio igitur ista maximi momenti sequenti modo proponi potest:

Inuenire methodum, cuius ope omnes numeri integri assignari queant, qui ex formulá \( (n n - 4) z z + n \) resultare possint, si loco literarum \( n \) et \( z \) non solum numeri integri sed etiam fracti accipientur.

Huius autem quæstionis enódatio certe insignia incrementa in Analyfin Diophanteam effet illatura.