DE PROJECTIONE GEOGRAPHICA
SUPERFICIEM SPHAERICAE.

Auctore
L. EVLERO.

§. 1.

Cum in superiori dissertazione omnes plane modos possibilis expendiendum, quibus superficies sphaerica in plano repraesentari potest, vt singulae portiones minima per se inter lineas exhibantur: inde quidem flatim mapparum hydrographicarum Mercatoris constructio pariter atque Hemi-sphaerium polarium, le prodebat, quemadmodum autem ambra Hemisphaerium, superiorificet et inferius, vt quidem node contrui solet cum, meis formulis cohaerens, vix patebat, cum tamen ita repraesentatio cadem proprietate fit praedita. Hanc obtinem accuratius inquirere constitui, quomodo etiam ita repraesentandus modus cum formulis generalibus ibi datis esse dignus contenderat, ex isque luculenter derivari queat.

§. 2. Formulae aucti generales, quas pro huiusmodi constructionibus erudiram in locis enim speriam in Sphaera mittenda a polo fuerit $v$ eisque longitudo ad certo Meridiano fixe computata $r$, id punctum in plano per binas coordinas orthogonales $x$ et $y$ sit determinari debet, vt sit

$$ x = \Delta - \left( \frac{1}{2} \cot \varphi \left( \cot t + V - r \cos \varphi \right) \right)$$

$$ y = \Delta \left( \frac{1}{2} \cot \varphi \left( \cot t + V - r \cos \varphi \right) \right) - \Delta \left( \frac{1}{2} \cot \varphi \left( \cot t + V - r \cos \varphi \right) \right)$$

quae formulae quoque ita exhibere licet, vt sit

$$ x = \Delta \left( \frac{1}{2} \cot \varphi \left( \cot t + V - r \cos \varphi \right) \right)$$

$$ y = \Delta \left( \frac{1}{2} \cot \varphi \left( \cot t + V - r \cos \varphi \right) \right)$$

et cum sit

$$ \frac{x}{\cot \varphi \left( \cot t + V - r \cos \varphi \right)} = \tan \frac{1}{2} \varphi \left( \cot t + V - r \cos \varphi \right)$$

hae formulæ etiam ita exhiberi possunt:

$$ x = \Delta \left( \tan \frac{1}{2} \varphi \left( \cot t + V - r \cos \varphi \right) \right) + \Delta \left( \tan \frac{1}{2} \varphi \left( \cot t + V - r \cos \varphi \right) \right)$$

$$ y = \Delta \left( \tan \frac{1}{2} \varphi \left( \cot t + V - r \cos \varphi \right) \right) - \Delta \left( \tan \frac{1}{2} \varphi \left( \cot t + V - r \cos \varphi \right) \right)$$

R 3 ybi
vbi manifestum est, ex prioribus formulis, si character indefinitus functionis \( \Delta \) omitatur, priores formulæ praebere mappas hydrographicas, postremas vero constructionem Haemi-sphaerii finit borealis sint australis.

§ 3. Quo nunc facilius appareat, quomodo etiam reliqua proiectiones eidem principio innitae ex nostris formulis deduci quanta, rationem ipsis proiectionis, quae vulgo stereographica vocari solent hic accuratus cuolam. Hoc autem modo superneces sphærae in planum Sphaeram tangens ita proieci foliet, quæadmodum a spectatore in puncto contactue opposito constitueto secundum regulas Perspectuæ cerneretur.

Tab. I. Fig. 5. Referat igitur circulus \( \Delta MC \) Sphaeram, recta autem \( EF \) planum, quod Sphaeram in puncto \( C \) tangat; tum vero \( A \) fit punctum ipsi \( C \) oppositum in quo spectator sit constitutus. iam suum in Sphæra puncto quocunque \( M \), si per id ex \( A \) producatur recta \( AMS \), rectæ \( EF \) occurrens in puncto \( F \), erit \( S \) proiektum puncti \( M \). Hinc si radius sphærae ponatur \( = z \), vt sit diameter \( AC = 2z \), arcus vero \( CM \) flatatur \( = z \), erit angulus \( CAM = \frac{z}{2} \), vade fit intercalatum.

\[ CS = 2z, \tan z = \frac{a}{r} = \frac{a}{z}, \cos z = 2y = \frac{z}{y} \]

§ 4. Si ad \( AC \) ex \( M \) ducatur normalis \( MP \) erit \( MP = \sin z \), ac si circulus circa axem \( AC \) conuerit concipiantur, punctum \( M \) descriptum circulum plano tangenti parallellum, cuius radius \( = MP = \sin z \), qui ergo super plano idem circulo referatur, cuius radius \( = CS = \tan z \), ita vt radius istius circuli in sphærae ut habeat ad radius proiectionis, vt \( PM \) ad \( CS \), hoc est vt \( AP:AC \) vel \( PA \) \( M \) ad \( \Delta S \). Anguli autem in circulo sphærae radio \( CM \) descripto aequales erunt angulis in proiectione super plano.

§ 5. Nunc in sphæra concipiamus punctum \( m \), ipsi \( M \) proximum, cui in proiectione respondeat punctum \( s \), ita vt elemen-
elementum \( Mm \) exprimatur per spatium \( S \), et quaeramus
rationem inter hae duo elementa \( Mm \) et \( S \). Ac primo qui-
dem patet fore angulum \( ASU = 90^\circ \) et \( AS = AC \). At vero
anguli \( AMm \) mensura est emittis arcus \( AM \), vnde erit angulus
\( AMm = 90^\circ - \alpha \), ideoque aequalis angulo \( AC \); vnde sequitur tri-
angulum \( AMmO \) triangulo \( AS \), vnde erit \( MmS = AMAS \). Hoc
ergo ratio conexit cum ea, quam invenient inter circulam in sphaera radio \( PM \) descriptam et circulum in
radio \( CS \) descriptum; quamobrem haec ratio etiam
aequalis erit et, quia elementa similia in duobis his circulis in-
ter se tendent. Atque hinc manifestum est, si in sphaera portio
in parte parva circa elementum \( Mm \) descripta concepiatur, eius
projectionem ipsi fore similem, ita ut haec projectione eadem legi
et adstitiat, ex qua max fœrmulis generalibus eliceram.

§ 6. Referat vt ante circulam \( AGC \) sphaeram, cuius su-
perficies provincia sit in planum \( EF \); quod sphaeram in puncto
\( C \) tangat, ac fiat unum punctum \( G \). Terrae polum existere in
puncto \( G \). Vocemus arcum \( CG = g \) et per praecedentia iste po-
nis in plano exhibitur in puncto \( H \), vt sit \( GH = z \) tag. ;
glam vero consideremus punctum sphaeræ quodcunque in \( M \)
zeus distantia a polo sit \( GM = v \), angulus vero \( CGM = r \),
qui ergo denotabit longitudinem loci \( M \) in Meridiano \( GC \), at
que ad triangulum sphaericum comprehensum ducatur arcus \( CM \),
quo facto, si in projectione \( S \) sit punctum \( f \) responsum, qui
\( CS = z \) tag.; \( CM \), angulus vero \( ESC = \) angulus \( CG \). Ad
locum igitur huius puncti \( S \) designandum in triangulo sphaeric
c \( CGM \) quaerit oportet tam latus \( CM \) quam angulum \( CGM \).

§ 7. In triangulo antem \( sphaeric 
\( CG \) et \( GM \) cum angulo intercepto \( CGM \),
vnde per regulas Trigonometriae sphaericæ reperitur

\[
\text{cof.} \; CM = \text{cof.} \; g \cdot \text{cof.} \; v + \text{fin.} \; g \cdot \text{fin.} \; v \cdot \text{cof.} \; r
\]
unde cum fit
\[ C \cdot S = \begin{tag} \frac{2 \sin C \cdot M}{\sin C \cdot M} \end{tag} = 2 \sqrt{1 - \cos C \cdot M} \]

ex postrema formula statim habemus
\[ C \cdot S = 2 \sqrt{1 - \cos C \cdot M} \]

Praeterea vero reperitur \( G \cdot C \cdot M = \frac{\sin C \cdot M}{\cos C \cdot M} \)
quae ergo formula simul exprimit in projectione tangentem
anguli \( E \cdot C \cdot S \).

§ 8. Nunc in projectione ex puncto \( S \) ad rectam fixam \( E \cdot F \), quippe in quam cadit polus \( H \), ducamus perpendicularum \( S \cdot X \), ac vocemus coordinatas \( C \cdot X = x \) et \( X \cdot S = y \); et cum fit
\[ C \cdot S = \frac{2 \sin C \cdot M}{\cos C \cdot M} \]

\( x = \frac{2 \sin C \cdot M}{1 + \cos C \cdot M} \) erit
\[ y = \frac{2 \sin C \cdot M}{1 + \cos C \cdot M} \]

unde patet fore
\[ x = \frac{\sin C \cdot M}{\cos C \cdot M} \]

Praeterea vero ex iam inventis erit
\[ x \cdot x + y \cdot y = C \cdot S^2 = \frac{1 - \cos C \cdot M}{1 + \cos C \cdot M} \]

ex quibus duabus aequationibus ambas coordinatas \( x \) et \( y \) seorsim definitre licebit.

§ 9. Facilius autem carum valores directe frequenti modo reperire licebit. Cum fit
\[ \begin{align*}
C \cdot M &= \sin G \cdot C \cdot M : \sin \cdot t \\
C \cdot M &= \sin G \cdot C \cdot M = \sin \cdot t \sin \cdot v,
\end{align*} \]

quo valore introducto fieri
\[ \begin{align*}
\text{tag} \cdot G \cdot C \cdot M &= \frac{\sin G \cdot C \cdot M}{\cos G \cdot C \cdot M} \\
\text{et} \quad y &= \frac{2 \sin C \cdot M}{1 + \cos C \cdot M}. \quad \text{Quia igitur inuenimus} \\
\text{cof.} \cdot G \cdot C \cdot M &= \cos G \cdot C \cdot M + \sin G \cdot C \cdot M \cdot \text{cof.} \cdot t
\end{align*} \]

ex quibus valoribus statim colligimus
\[ x = \frac{2 \left( \sin G \cdot C \cdot M \cdot \cos G \cdot C \cdot M + \sin G \cdot C \cdot M \cdot \cos t \right)}{1 + \cos G \cdot C \cdot M} \]

§ 10.
§. 10. Quod si ergo hic ponamus $v = 0$, locus poli $H$ in projectione prodir pro debet; tum autem reperietur

$$x = \frac{\sin g}{\cos g} = 2 \tan \frac{g}{2} = C H.$$  

At vero $v = 0$. Hinc igitur etiam locum alterius poli in projectione assignare poterimus, ponendo $v = 180^\circ$; tum autem reperietur $x = \frac{\sin g}{\cos g}$ et $y = 0$. Vnde si ex altera parte alter polus statutur in $K$, erit interium

$$x = \frac{\sin g}{\cos g} = 2 \cot \frac{g}{2},$$  

Tum vero si capiamus $CE = CF = 2$, erit $EF$ diametrum circilium, quod referatur Hemisphaerium totum circa centrum $C$ descriptum, cuius ergo diametrum erit $EF = 4$, hoc est duplo major quam diametrum $H$.  

§. 11. Vt nunc in hac projectione Acquatum designemus, ponamus $v = 90^\circ$, atque $x$ et $y$ sint coordinatae Aquatoris in projectione; tum autem erit

$$x = \frac{\sin g}{\cos g} = \cot \frac{g}{2},$$  

et $y = \frac{\tan g}{\cos g}$.  

Sura autem jam vidimus esse $x = x + y = \frac{1 - \sin g}{1 + \sin g}$, hic concipimus fore

$$\frac{2x}{1 - \sin g} = \left(\frac{1 - \sin g}{1 + \sin g}\right)^2,$$  

vnde fit $\cot g = \frac{2x}{1 - \sin g}$.  

Qui valor substitutus in acutatione pro $x$ praebet

$$4 \sin g - (x + y) \cot g = -4 \cot g.$$  

Vnde fit $x = x + y = \frac{(x \sin g + \cot g)}{\cot g}$, vnde colligimus hunc acuationem: $y + (2 \tan g - x)^2 = \frac{2x}{\cot g}$. Hinc patet, Acquatum in projectione fore circulm-radio $= \frac{2x}{\cot g}$ descriptum.  

Ad centrum autem huius circuli inueniendum capiatur interium $CI = 2 \tan g$, vt fiat $IX = 2 \tan g - x$, et cum fieri debeat $XS + 1X = \frac{x}{\cot g}$, patet fore $IS = \frac{x}{\cot g}$ hoc est quantitatio constanti. Erit ergo hoc ipsum punctum $I$ centrum circuli Acquatum referentis, existente $CI = 2 \tan g$. Quare ex tab. I. Fig. 6.
C erigatur perpendiculum $\mathbf{CD}=\mathbf{2}$, et cum hinc fiat recta $\mathbf{ID}=\frac{z}{\mathbf{cof. g}}$, patet Aequatorem descriptum iri, si ex centro $\mathbf{I}$ et radio $\mathbf{ID}$ circulus delineetur.

§. 12. Definiamus nunc quoque omnes circulos Aequatori parallelos in nostra proiectione, atque vt calculos taedio-fos cuitemus statuamus breuitatis gratia $a=2\sin. g\ \mathbf{cof. a}$, $b=2\mathbf{cof. g}\ \mathbf{sin. a}$, $e=x+2\ \mathbf{cof. g}\ \mathbf{cof. a}$ et $d=\mathbf{sin. g}\ \mathbf{sin. a}$ et $e=4-4\ \mathbf{cof. g}\ \mathbf{cof. a}$, vbi $\alpha$ scripsimus loco $\psi$, vt distantia Paralleli a polo sit $=\alpha$, quo facto nostrae aequationes ita habebunt: $x=\frac{e-b\mathbf{cof. t}}{e-d\mathbf{cof. t}}$ et $x+y^2=\frac{e-a\mathbf{cof. t}}{e-d\mathbf{cof. t}}$, ex quorum priore colligitur $\mathbf{cof. t}=\frac{e}{b-d\alpha}$, qui $\psi$ in altera substitutus praebet $x+y^2=\frac{e}{b-d\alpha}$. Restitutis vero valouribus assumptis erit $x+y^2=\frac{e}{b-d\alpha}$, quae aequatio reducita ad hanc formam

$$y^2=\left(\frac{x}{\mathbf{cof. g}+\mathbf{cof. a}}-x\right)^2=\frac{z\mathbf{sin. a}}{(\mathbf{cof. g}+\mathbf{cof. a})^2}$$

monstrat, proiectionem Paralleli propositi esse circulum, cuius radius $=\frac{2\mathbf{sin. a}}{\mathbf{cof. g}+\mathbf{cof. a}}$, centrum autem in ipso axe $\mathbf{EF}$ esse fictum puta in puncto $\mathbf{L}$ sit $\psi$ $\mathbf{L}=\frac{2\mathbf{sin. g}}{\mathbf{cof. g}+\mathbf{cof. a}}$.

§. 13. Innefitigemus nunc etiam proiectionem omnium

Tab. I. Meridianorum; ac primo quidem cum sumto $\psi=0$ ipsa recta

Fig. 6. H K referet Meridianum principalem, a quo reliquis computus, ponamus declinationem Meridiani quaestiti ab hoc principali esse $=\beta$, vt $\psi=t=\beta$, et aequationes nostrae erunt

$$x=\frac{1}{\mathbf{cof. g}+\mathbf{cof. a}}\left(\mathbf{sin. g}\ \mathbf{cof. a}-\mathbf{cof. b}\ \mathbf{cof. g}\ \mathbf{fin. a}\right)$$
$$y=\frac{1}{\mathbf{cof. g}+\mathbf{cof. a}}\left(\mathbf{fin. b}\ \mathbf{fin. a}\right)$$

et

$$x+y^2=\left(\frac{1}{\mathbf{cof. g}+\mathbf{cof. a}}\right)^2$$

ex quibus aequationibus quantitatem $\psi$ eliminare oportet. Hunc in finem consideremus formulam

$$y=\frac{\mathbf{fin. b}\ \mathbf{fin. a}}{\mathbf{cof. g}\ \mathbf{fin. a}+\mathbf{cof. g}\ \mathbf{fin. a}}$$
$$x=\mathbf{fin. b}\ \mathbf{tang. a}-\mathbf{cof. g}\ \mathbf{tang. a}$$

ex qua colligitur $\psi=\frac{\mathbf{fin. b}\ \mathbf{tang. a}}{\mathbf{cof. g}\ \mathbf{tang. a}}$.
§. 14. Quo nunc facileius hoc valore in reliquis aequationibus vt queamus, formamus hanc aequationem:

\[ 4 - xx - yy = \frac{\text{cof} g \text{cof} v + \text{cof} \beta \text{fin} g \text{fin} v}{x + \text{cof} g \text{cof} v + \text{cof} \beta \text{fin} g \text{fin} v} \]

quae per \( y \) diuita praebet

\[ \frac{4 - xx - yy}{y} = \frac{\text{cof} g + 4 \text{cof} \beta \text{fin} g \text{tang} g}{\text{fin} \beta \text{fin} g} \]

in qua loco tang. \( y \) valorem ante inuentum scribamus, vnde fiet

\[ \frac{4 - xx - yy}{y} = \frac{4 \text{cof} \beta + 4 \text{fin} \beta \text{cof} g}{\text{fin} \beta \text{fin} g} \]

ex qua deducimus

\[ xx + yy = 4 - \frac{4 \text{cof} \beta + 4 \text{fin} \beta \text{cof} g}{\text{fin} \beta \text{fin} g} \]

quae aequatio itidem est pro circulo, vnde tuto concludere possumus, omnes circulos maximos in spheara ductos etiam per arcus circulares exprimi, vel adeo per lineas rectas.

§. 15. Quo nunc tam centrum quam radium cuiusque Meridiani pro nostra proiezione assignemus, aequationem inventam in hanc formam transfundamus:

\[ \left( \frac{\text{cof} g}{\text{fin} \beta \text{fin} g} + x \right)^2 + \left( \frac{\text{cof} \beta}{\text{fin} \beta \text{fin} g} + y \right)^2 = \frac{\text{fin} \beta \text{fin} g}{\text{fin} \beta \text{fin} g}. \]

Sint igitur puncta H et K poli in proiezione, ita vt fit

\[ \text{CH} = 2 \text{tang} \gamma g = \frac{\text{fin} \beta \text{fin} g}{\text{cof} \beta \text{cof} g} \quad \text{et} \quad \text{CK} = 2 \cot \gamma g = \frac{\text{fin} \beta \text{fin} g}{\text{cof} \beta \text{cof} g} \]

ideoque totum intervallum \( HK = \frac{\text{fin} \beta \text{fin} g}{\text{cof} \beta \text{cof} g} \) eiusque semissis \( \frac{\text{fin} \beta \text{fin} g}{\text{cof} \beta \text{cof} g} \)

quod medium in punctum O incidat, erit \( CO = \frac{\text{fin} \beta \text{fin} g}{\text{cof} \beta \text{cof} g} \); hinc sumto \( CX = x \) erit \( OX = \frac{\text{cof} \beta \text{fin} g}{\text{fin} \beta \text{fin} g} + x \). Ex O erigatur perpendiculum \( \frac{\text{cof} \beta \text{fin} g}{\text{fin} \beta \text{fin} g} \); furtamine \( XL \) ipsi O N aequali erit \( LS = \frac{\text{fin} \beta \text{fin} g}{\text{fin} \beta \text{fin} g} + y \), quocirca esse oportet \( OX = (\text{fin} \beta \text{fin} g) \)

\[ + LS = OY. \]

ideoque \( NS = \frac{\text{fin} \beta \text{fin} g}{\text{fin} \beta \text{fin} g} \). Vnde patet, punctum N esse centrum Meridiani descripti, radium vero \( \frac{\text{fin} \beta \text{fin} g}{\text{fin} \beta \text{fin} g} \), qui radius praecipue aequalis erit rectae N H, quod egregie cum natura rei conuenit; quandoquidem omnes Meridiani etiam in proiezione per polos N et H transire debent.
Comparatio huius projectionis cum formulis generalibus.

§. 16. Hic igitur quaeritur, cuius modi forma functioni \( \Delta \) tribui debet, vt projectione modo descripta inde securatur. Ac primo patet, potestates prae laiores in ea occurrere non posse, quia alioquin multa angulorum \( t \) et \( v \) ingredentur; deinde vero haec functione debet esse fractioni, quoniam formae pro \( x \) et \( y \) inuentae sunt fractiones. Hanc ob caussam functioni \( \Delta : z \) talem formam generalem tribuamus \( \frac{a + b}{c + d} \) : at vero pro \( z \) sumamus formam postremam supra expositam, quae erat \( z = \tan(\frac{1}{2}v)(\text{cof. } t + \sqrt{1 - \text{fin. } t}) \)

ita vt nostra functione enatat:

\[
a + b \tan(\frac{1}{2}v)(\text{cof. } t + \sqrt{1 - \text{fin. } t})
\]

\[
c + d \tan(\frac{1}{2}v)(\text{cof. } t + \sqrt{1 - \text{fin. } t})
\]

quae, loco \( \tan(\frac{1}{2}v) \) scribendo \( \frac{\text{fin. } v}{1 + \text{cof. } v} \) induet hanc formam:

\[
a(1 + \text{cof. } v) + b \text{fin. } v(\text{cof. } t + \sqrt{1 - \text{fin. } t})
\]

\[
c(1 + \text{cof. } v) + d \text{fin. } v(\text{cof. } t + \sqrt{1 - \text{fin. } t})
\]

§. 17. Pro calculi commodo loco huius formae vtamur hac concinniore: \( \frac{a + b}{c + d} \) ut fit.

\[
P = a(1 + \text{cof. } v) + b \text{fin. } v \text{cof. } t ; Q = b \text{fin. } v \text{fin. } t
\]

\[
R = c(1 + \text{cof. } v) + d \text{fin. } v \text{cof. } t ; S = d \text{fin. } v \text{fin. } t
\]

Hinc autem coordinatae \( x \) et \( y \) ita prodibunt determinatae:

\[
x = \frac{P + Q \sqrt{1}}{R + S \sqrt{1}} + \frac{P - Q \sqrt{1}}{R - S \sqrt{1}}
\]

\[
y = \frac{P + Q \sqrt{1}}{R + S \sqrt{1}} - \frac{P - Q \sqrt{1}}{R - S \sqrt{1}}
\]

unde colligimus.

\[
x = \frac{2PR + 2QS}{R^2 + S^2} \text{ et } y = \frac{2QR - 2PS}{R^2 + S^2}
\]

§. 18. Quod si iam loco \( P, Q, R, S \) valores assumtos refticuamus, pro denominator communis reperiemus.

\[
R + S \equiv c(1 + \text{cof. } v)^2 + 2c \text{d}(1 + \text{cof. } v) \text{fin. } v \text{cof. } t + 2 \text{dfin. } v
\]

\[
= (1 + \text{cof. } v)(c(1 + \text{cof. } v) + 2 \text{dfin. } v \text{cof. } t + d \text{d}(1 - \text{cof. } v).)
\]

Tum
Tum vero pro numeratore ipsius $x$ fiet

$$PR = QS - (x + \text{cof. } v)(ac(x + \text{cof. } v) + (bc + ad)\text{ fin. } v \text{ cof. } s + b d (x - \text{cof. } v)).$$

denique pro numeratore ipsius $y$

$$QR - PS = (x + \text{cof. } v)(bc - ad)\text{ fin. } v \text{ fin. } t$$

sicque pro coordinatis nanciscimus has expressiones:

$$x = \frac{a c (x + \text{cof. } v) + z (bc - ad)\text{ fin. } v \text{ cof. } t + z c d \text{ fin. } v \text{ cof. } t + d a (1 - \text{cof. } v)}{a (bc - ad)\text{ fin. } v \text{ fin. } t}$$

$$y = \frac{z c d \text{ fin. } v \text{ cof. } t + d a (1 - \text{cof. } v)}{a (bc - ad)\text{ fin. } v \text{ fin. } t}.$$

§. 19. Quod si iam habet formulas cum iis comparemus, quas supra inuenimus, quae erant

$$x = \frac{2 \text{ fin. } g \text{ cof. } v + \text{ cof. } g \text{ fin. } v \text{ cof. } g}{x + \text{ cof. } g \text{ cof. } v + \text{ fin. } g \text{ fin. } v \text{ cof. } t}$$

et

$$y = \frac{2 \text{ fin. } t \text{ fin. } v}{1 + \text{ cof. } g \text{ cof. } v + \text{ fin. } g \text{ fin. } v \text{ cof. } t}.$$

Egregium iam consistere comprehendimus: at facile erit constanter $a, b, c, d$ ita assumere, vt consequens fiat perfectus. Primo igitur vt de nominatore ad identitatem perductum, requiritur, vt sit $cc - dd = x$, $cc - dd = \text{cof. } g$ et $2 cd = \text{fin. } g$.

Ex duas prioribus fit

$$c = \frac{1 + \text{ cof. } g}{2} = \text{cof. } g$$

et

$$d = \frac{1 - \text{ cof. } g}{2} = \text{fin. } g.$$

Vnde sit $c = \text{cof. } g$ et $d = \text{fin. } g$, quibus valoribus iam tertiae conditioni satisfit; hie enim

$$2 c d = 2 \text{ fin. } g \text{ cof. } g = \text{fin. } g.$$

Pro numeratore ipsius $x$ perfectus consequens postulat, vt fiat

$$ac + bd = c, ac - bd = \text{fin. } g, bc + ad = - \text{cof. } g$$

ubi si loco $c$ et $d$ valores modo inuentos scribamus, hie

$$a \text{ cof. } g + b \text{ fin. } g = c, a \text{ cof. } g - b \text{ fin. } g = \text{fin. } g,$$

$$b \text{ cof. } g + a \text{ fin. } g = - \text{cof. } g.$$

Ex bis prioribus fit

$$a = \frac{\text{fin. } g}{2 \text{ cof. } g} = \text{fin. } g,$$

porro

$$b = \frac{2 \text{ fin. } g}{2 \text{ fin. } g} = - \text{cof. } g.$$

Hisque valoribus etiam tertiae conditioni sponte satisfit. Tantum

$S$ 3

igitur.
igitur supereft, vt etiam videamus, an ifti valores cum numeratore ipsius y conueniant, quo requiritur, vt fit $bc - ad = 1$; et vero $bc = -\cot \frac{1}{g}$ et $ad = \sin \frac{1}{g}$ vnde fit $bc - ad = -1$.

Probe autem notandum est, ambas coordinatas tam positiva quam negativa sumi posse, ita ut hic percuta identitas agnonci debeat.

§. 20. His valoribus inuentis manifestum est, formulas nostras generales perducturas suffice ad hanc profectionem stereographicam, si pro functione $\Delta : z$ assumussemus statim hanc formam:

$$\frac{\tan \frac{1}{g} - z}{\tan \frac{1}{g} + z \sin \frac{1}{g}} = \frac{\sin \frac{1}{g} - z}{\cos \frac{1}{g} + z \sin \frac{1}{g}}$$

Ceterum hic observari conueniet, iustum casum ad vius practicos, quos in Geographia poftulamus maxime esse accommodatum, quandoquidem veram figuram regionum terrestrum non admodum detorquet. Imprimis autem notari meretur, quod in hac profectione non solum omnes Meridiani et Paralleli circulii vel adeo lineis rectis exhibeantur, sed etiam omnes circuli maximini in Sphaera descripsi etiam per arcus circulares vel adeo lineas rectas exprimantur, dum e contrario aliae Hypotheses, quae pro functione $\Delta$ fingi posse, his commodis penitus essent caritarea.