

## ADDITAMENTUM II.

*De motu projectorum in medio non resistente, per  
Methodum maximorum ac minimorum  
determinando.*

1. **Q**uoniam omnes naturæ effectus sequuntur quandam maxi-  
mimi minimive legem ; dubium est nullum , quin in  
lineis curvis , quas corpora projecta , si a viribus quibuscumque  
sollicitentur , describunt , quæpiam maximi minimive proprietas  
locum habeat. Quænam autem sit ista proprietas , ex princi-  
piis metaphysicis a priori definire non tam facile videtur : cum  
autem has ipsa curvas , ope Methodi directæ , determinare li-  
ceat ; hinc , debita adhibita attentione , id ipsum , quod in ipsis  
curvis est maximum vel minimum , concludi poterit. Spectari  
autem potissimum debet effectus a viribus sollicitantibus oriun-  
dus ; qui cum in motu corporis genito consistat , veritati con-  
sentaneum videtur hunc ipsum motum , seu potius aggrega-  
tum omnium motuum qui in corpore projecto insunt , mini-  
mum esse debere. Quæ conclusio etsi non satis confirmata vi-  
deatur , tamen , si eam cum veritate jam a priori nota consenti-  
re ostendero , tantum consequetur pondus , ut omnia dubia quæ  
circa eam suboriri queant penitus evanescant. Quin- etiam cum  
ejus veritas fuerit evicta , facilius erit in intimas Naturæ leges  
atque causas finales inquirere ; hocque assertum firmissimis ra-  
tionibus corroborare.

2. Sit massa corporis projecti  $= M$ , ejusque , dum spatio-  
lum  $= ds$  emetitur , celeritas debita altitudini  $= v$ ; erit quan-  
titas motus corporis in hoc loco  $= M \sqrt{v}$ ; quæ per ipsum  
spatiolum  $ds$  multiplicata , dabit  $M ds \sqrt{v}$  motum corporis  
collectivum per spatiolum  $ds$ . Jam dico lineam a corpore des-  
criptam

criptam ita fore comparatam, ut, inter omnes alias lineas iisdem terminis contentas, sit  $\int M ds \sqrt{v}$ , seu, ob  $M$  constans,  $\int ds \sqrt{v}$  minimum. Quod si autem curva quæsita tanquam esset data spectetur, ex viribus sollicitantibus celeritas  $\sqrt{v}$  per quantitates ad curvam pertinentes definiri, ideoque ipsa curva per Methodum maximorum ac minimorum determinari potest. Ceterum hæc expressio ex quantitate motus petita æque ad vires vivas traduci poterit; posito enim tempusculo, quo elementum  $ds$  percurritur,  $= dt$ ; quia est  $ds = dt \sqrt{v}$ , fiet  $\int ds \sqrt{v} = \int v dt$ ; ita ut, in curva a corpore projecto descripta, summa omnium viarum vivarum, quæ singulis temporis momentis corporis insunt, sit minima. Quamobrem neque ii qui vires per ipsas celeritates, neque illi qui per celeritatum quadrata æstimari oportere statuunt, hic quicquam quo offendantur reperient.

3. Primum igitur, si corpus a nullis prorsus viribus sollicitari ponamus, ejus quoque celeritas, ad quam hic solum attendo (directionem enim ipsa Methodus maximorum & minimorum complectetur), nullam patietur alterationem; eritque ideo  $v$  quantitas constans, puta  $= b$ . Hinc corpus a nullis viribus sollicitatum, si utcunque projiciatur, ejusmodi describet lineam, in qua sit  $\int ds \sqrt{b}$  vel  $\int ds = s$  minimum. Via ergo hæc, inter omnes iisdem terminis contentas, ipsa erit minima; atque adeo recta: prorsus uti prima Mechanicæ principia postulant. Hunc quidem casum non adeo hic affero, quo principium meum confirmari putem; quamcunque enim, loco celeritatis  $\sqrt{v}$ , aliam assumfissim functionem ipsius  $v$ , eadem prodiisset via recta; verum a casibus simplicissimis incipiendo facilius ipsa consensus ratio intelligi poterit.

4. Progredior ergo ad casum gravitatis uniformis, seu quo corpus projectum ubique, secundum directiones ad horizontem normales, deorsum sollicitetur a vi constante acceleratrice  $= g$ .

*Fig. 26.* Sit  $AM$  curva, quam corpus in hac hypothesi describit, sumatur recta verticalis  $AP$  pro axe, ac ponatur abscissa  $AP = x$ , applicata  $PM = y$ , & elementum curvæ  $Mm = ds$ ; erit ergo, ex natura sollicitationis,  $dv = g dx$ , &  $v = a + gx$ . Hinc curva

curva ita erit comparata, ut in ea sit  $\int ds \sqrt{a+gx}$  minimum. Ponatur  $dy = p dx$ , ut sit  $ds = dx \sqrt{1+pp}$ , atque minimum esse debet  $\int dx \sqrt{a+gx} (1+pp)$ ; quia expressio cum forma generali  $\int Z dx$  comparata dat  $Z = \sqrt{a+gx} (1+pp)$ ; quare, cum positum sit  $dZ = M dx + N dy + P dp$ , erit  $N = 0$  &  $P = \frac{p \sqrt{a+gx}}{\sqrt{1+pp}}$ . Quia ergo valor differentialis est  $N = \frac{dP}{dx}$ ; ob  $N = 0$ , fiet praesenti casu  $dP = 0$ , &  $P = \sqrt{C}$ . Habebitur ergo  $\sqrt{C} = \frac{p \sqrt{a+gx}}{\sqrt{1+pp}} = \frac{dy \sqrt{a+gx}}{ds}$ ; unde fit  $C dx^2 + C dy^2 = dy^2 (a+gx)$ , &  $dy = \frac{dx \sqrt{C}}{\sqrt{a-C+gx}}$ ; quæ integrata dat  $y = \frac{2}{g} \sqrt{C(a-C+gx)}$ .

5. Manifestum quidem est hanc æquationem esse pro Parabola. At ejus consensum cum veritate attentius considerasse juvabit. Primum ergo patet tangentem hujus curvæ esse horizontalem, seu  $dx = 0$ ; ubi est  $a - C + gx = 0$ . Cum igitur principium abscissarum A ab arbitrio nostro pendeat, sumatur id in hoc ipso loco, fietque  $C = a$ ; tum vero ipse axis per hoc punctum curvæ summum transeat, ita ut, posito  $x = 0$ , fiat simul  $y = 0$ . His consideratis, æquatio pro curva erit hæc  $y = 2 \sqrt{\frac{ax}{g}}$ ; quam non solum patet esse pro Parabola; sed etiam, cum celeritas in punto A sit  $\sqrt{a}$ , altitudo CA, ex qua corpus labendo ab eadem vi  $g$  sollicitatum eam ipsam acquirit celeritatem, qua in punto A movetur, erit  $= \frac{a}{g}$ ; hoc est, quartæ parametri parti æquatur; prorsus uti ex doctrina motus projectorum per Methodum directam intelligitur.

6. Sollicitetur, ut ante, corpus ubique verticaliter deorsum, at ipsa vis sollicitans non sit constans, sed pendeat utcunque ab altitudine CP. Scilicet posita abscissa CP =  $x$ , sit vis qua corpus in M deorsum nititur =  $X$  functioni cuicunque ipsius  $x$ . Si ergo vocetur applicata PM =  $y$ , elementum arcus Euleri de Max. & Min. R<sub>r</sub> M<sub>ap</sub>

$Mm = ds$ , &  $dy = pdx$ ; erit  $d v = Xdx$ , &  $v = A + \int Xdx$ ; unde minimum esse debet hæc expressio  $\int dx \sqrt{(A + \int Xdx)(1 + pp)}$ , ex qua pro curva descripta AM obtinebitur hæc æquatio . .

$$\sqrt{C} = \frac{p \sqrt{(A + \int Xdx)}}{\sqrt{(1 + pp)}} \quad \& \quad p = \frac{\sqrt{C}}{\sqrt{(A - C + \int Xdx)}} = \frac{dy}{dx};$$

seu  $y = \int \frac{dx \sqrt{C}}{\sqrt{(A - C + \int Xdx)}}$ . Tangens ergo curvæ erit horizontalis ubi  $\int Xdx = C - A$ . Hæc vero eadem æquatio træectoriæ corporis per Methodum directam reperitur.

Fig. 27. 7. Sollicitetur nunc corpus in M a duabus viribus, altera horizontali = Y secundum directionem MP, altera verticali = X secundum directionem MQ. Sit autem X functio quæcunque rectæ verticalis MQ = CP = x, & Y functio quæcunque applicatæ PM = y. Positis ergo ut ante  $dy = pdx$ , erit  $d v = -Xdx - Ydy$ , fietque  $v = A - \int Xdx - \int Ydy$ ; unde minimum esse debet hæc formula  $\int dx \sqrt{(1 + pp)(A - \int Xdx - \int Ydy)}$ . Differentietur  $\sqrt{(1 + pp)(A - \int Xdx - \int Ydy)}$ , atque prodibit . . . . .

$$\frac{-Xdx \sqrt{(1 + pp)}}{2\sqrt{(A - \int Xdx - \int Ydy)}} - \frac{Ydy \sqrt{(1 + pp)}}{2\sqrt{(A - \int Xdx - \int Ydy)}} + \frac{pd p \sqrt{(A - \int Xdx - \int Ydy)}}{\sqrt{(1 + pp)}}. \text{ Hinc posito}$$

$$N = \frac{-Y \sqrt{(1 + pp)}}{2\sqrt{(A - \int Xdx - \int Ydy)}}, \quad \& \quad P = \frac{p \sqrt{(A - \int Xdx - \int Ydy)}}{\sqrt{(1 + pp)}},$$

erit pro curva quæsita hæc æquatio o =  $N - \frac{dP}{dx}$ , seu  $Ndx = dP$ .

$$\text{Hinc ergo fit } \frac{-Ydx \sqrt{(1 + pp)}}{2\sqrt{(A - \int Xdx - \int Ydy)}} = \frac{dp \sqrt{(A - \int Xdx - \int Ydy)}}{(1 + pp) \sqrt{(1 + pp)}} - \frac{pXd x - pYdy}{2\sqrt{(1 + pp)(A - \int Xdx - \int Ydy)}}$$

$$\text{seu } \frac{dp \sqrt{(A - \int Xdx - \int Ydy)}}{(1 + pp) \sqrt{(1 + pp)}} = \frac{Xdy - Ydx}{2\sqrt{(1 + pp)(A - \int Xdx - \int Ydy)}},$$

ideoque  $\frac{2dp}{1 + pp} = \frac{Xdy - Ydx}{A - \int Xdx - \int Ydy}$ . Hanc æquationem veritati esse consentaneam patebit, si loco  $A - \int Xdx - \int Ydy$  ponatur

ponatur  $v$ , erit enim  $\frac{2vdp}{(1+pp)^{3/2}} = \frac{Xdy - Ydx}{\sqrt{1+pp}}$ . At est radius osculi  $r = \frac{(1+pp)^{3/2}dx}{dp}$ , quo introducto est  $\frac{2v}{r} = \frac{Ydx - Xdy}{ds}$ ; ubi est  $\frac{2v}{r}$  vis corporis centrifuga, &  $\frac{Ydx - Xdy}{ds}$  exprimit vim normalem ex viribus sollicitantibus ortam; quarum virium æqualitas utique in omni motu projectorum locum habet.

8. Aequatio autem inventa  $\frac{2dp}{1+pp} = \frac{Xdy - Ydx}{A - \int Xdx - \int Ydy}$  ita generaliter est integrabilis, si multiplicetur per  $\frac{p(A - \int Xdx - \int Ydy)}{1+pp}$ ; fiet enim  $\frac{2pd़p(A - \int Xdx - \int Ydy)}{(1+pp)^2} - \frac{pp Xdx + Ydy}{1+pp} = 0$ , quæ integrata dat  $\frac{p^2 \int Xdx + \int Ydy - A}{1+pp} = C$ , seu  $\int Ydy - p^2 \int Xdx = A + C + Cpp$ , unde  $p = \frac{\sqrt{(B + \int Ydy)}}{\sqrt{(C + \int Xdx)}}$ , posito  $B$  pro  $-A - C$ . Cum ergo sit  $p = \frac{dy}{dx}$ , erit  $\int \frac{dy}{\sqrt{(B + \int Ydy)}} = \int \frac{dx}{\sqrt{(C + \int Xdx)}}$ , aequatio pro curva quærita, in qua variabiles  $x$  &  $y$  sunt a se invicem separatae. Vel si constantes  $B$  &  $C$  in negativas convertantur, erit  $\int \frac{dy}{\sqrt{(B - \int Ydy)}} = \int \frac{dx}{\sqrt{(A - \int Xdx)}}$ . Ex quibus et si curvæ constructio facilis habetur, tamen aequationes algebraicæ, quoties quidem in ipsis continentur, non tam facile eruuntur. Sint  $X$  &  $Y$  functiones similes & quidem potestates ipsarum  $x$  &  $y$ , ita ut sit  $\int \frac{dy}{\sqrt{(b^n - y^n)}} = \int \frac{dx}{\sqrt{(a^n - x^n)}}$ , quæ aequatio; si  $n=1$ , præbet Parabolam; si  $n=2$ , Ellipsin centrum in  $C$  habentem: et si hoc casu utraque integratio quadraturam Circuli

requirit. Verisimile ergo videtur etiam aliis casibus, quibus neutra integratio succedit, curvas algebraicas satisfacere; quarum autem inveniendarum Methodus adhuc desideratur.

79. Urgeatur corpus M perpetuo versus punctum fixum secundum directionem MC, vi quæ sit ut functio quæcunque distantiae MC. Positis ut ante  $CP = x$ ,  $PM = y$ , &  $dy = p dx$ ; sit  $CM = \sqrt{(x^2 + y^2)} = t$ , atque sit T ea functio ipsius  $t$ , quæ exprimit vim centripetam. Resolvatur hæc vis in laterales secundum MQ & MP, erit vis trahens secundum

$$MQ = \frac{Tx}{t}; \text{ & vis secundum } MP = \frac{Ty}{t}; \text{ ex quibus oritur}$$

$$\text{acceleratio } dv = -\frac{Tx dx}{t} - \frac{Ty dy}{t} = -T dt, \text{ ob } x dx +$$

$$y dy = t dt; \text{ unde fit } v = A - \int T dt. \text{ Quamobrem minimum}$$

esse debet hæc expressio  $\int dx \sqrt{(1 + pp)} (A - \int T dt)$ .

Jam, secundum Regulæ præceptum, differentietur quantitas,

$$\sqrt{(1 + pp)} (A - \int T dt), \text{ prodabitque}$$

$$-\frac{T d t \sqrt{(1 + pp)}}{2 \sqrt{(A - \int T dt)}} + \frac{p dp \sqrt{(A - \int T dt)}}{\sqrt{(1 + pp)}}.$$

$$\text{Ob } dt = \frac{x dx + y dy}{t}, \text{ erit ergo } N = \frac{-Ty \sqrt{1 + pp}}{2 t \sqrt{(A - \int T dt)}}$$

$$\text{& } P = \frac{p \sqrt{(A - \int T dt)}}{\sqrt{(1 + pp)}}, \text{ ex quibus efficitur æquatio pro curva } N dx = dP, \text{ quæ præbet,}$$

$$-\frac{Ty dx \sqrt{(1 + pp)}}{2 t \sqrt{(A - \int T dt)}} = \frac{dp \sqrt{(A - \int T dt)}}{(1 + pp) \sqrt{(1 + pp)}} - \frac{p T dt}{2 \sqrt{(1 + pp)} (A - \int T dt)},$$

hæcque reducta abibit in istam,

$$\frac{T(x dy - y dx)}{2 t (A - \int T dt)} = \frac{dp}{1 + pp}.$$

10. Quamvis hæc æquatio quatuor contineat litteras diversas, tamen debita dexteritate integrari potest. Cum enim sit  $y dy + x dx = t dt = py dx + x dx$ , erit  $dx = \frac{t dt}{x + py}$  &  $dy = \frac{p t dt}{x + py}$ , qui valores in æquatione substituti dabant

$$\frac{(px - y) T dt}{2(x + py)(A - \int T dt)} = \frac{dp}{1 + pp}, \text{ seu } \frac{T dt}{2(A - \int T dt)} = \frac{dp(px - y)}{(1 + pp)(px - y)}.$$

Ha-

Harum expressionum utraque per logarithmos est integrabilis, est enim  $\int \frac{T dt}{2(A - \int T dt)} = -\frac{1}{2} \ln(A - \int T dt)$ , &  $\int \frac{dp(x+py)}{(1+pp)(px-y)} \text{ resolvitur in } \int \frac{x dp}{px-y} - \int \frac{pd p}{1+pp} = \int \frac{px-y}{\sqrt{1+pp}}; \text{ ita ut sit } \frac{C}{\sqrt{A-\int T dt}} = \frac{px-y}{\sqrt{1+pp}}; \text{ qua ex-}$   
quatione declaratur, celeritatem corporis in  $M$ , quæ est  $= \sqrt{A - \int T dt}$ , esse reciproce ut perpendicular ex  $C$  in tangentem demissum; quæ est proprietas palmaria horum motuum.

11. Hoc vero idem Problema commodius resolvi potest ipsam rectam  $CM$  pro altera variabili assumendo. Verum Methodus supra tradita non postulat, ut ambæ variabiles sint coordinatae orthogonales, dummodo sint ejusmodi binæ quantitates quibus determinatis simul curvæ punctum determinetur. Hanc ob causam, non liceret distantiam  $CM$  cum perpendicular ex  $C$  in tangentem demisso pro binis illis variabilibus accipere; quoniam etiamsi detur & distantia a centro & perpendicularum in tangentem, hinc tamen locus puncti curvæ non definitur. Nil hil autem impedit, quo minus distantia  $CM$ , & arcus circuli  $BP$  centro  $C$  descripti, in locum duarum variabilium substituantur; quia dato arcu  $BP$ , & distantia  $CM$  curvæ punctum  $M$  que determinatur ac per coordinatas orthogonales. Hac ergo annotatione usus Methodi multo latius extenditur, quam alioquin videri queat.

12. Sit igitur distantia corporis a centro  $MC = x$ , & vis quæ corpus ad centrum  $C$  sollicitatur sit  $= X$  functioni cuicunque ipsius  $x$ . Centro  $C$ , radio pro lubitu assumpto  $BC = c$ , describatur circulus, cuius arcus  $BP$  teneat locum alterius variabilis  $y$ , ita ut sit  $Pp = dy = pdx$ . Ex vi autem sollicitante est  $dv = -Xdx$ , unde  $v = A - \int X dx$ . Centro  $C$ , radio  $CM = x$ , describatur arcus  $Mn$ , erit  $Mn = dx$ ; &  $CP :: Pp = CM : Mn$ , unde fit  $Mn = \frac{px dx}{c}$ , & elementum spa-

Fig. 28.

rii  $Mm = dx \sqrt{1 + \frac{p^2 x^2}{c^2}}$ . Quamobrem minimum esse

debet hæc formula  $\int dx \sqrt{(A - \int X dx)(1 + \frac{ppxx}{cc})}$ , ex  
 qua oritur valor differentialis  $\frac{I}{dx} d. \frac{pxx\sqrt{(A - \int X dx)}}{c\sqrt{(cc + ppxx)}}$ , qui,  
 per Regulam, nihilo æqualis positus, præbabit hanc æquationem:  
 $\sqrt{C} = \frac{pxx\sqrt{(A - \int X dx)}}{c\sqrt{(cc + ppxx)}}$ , seu  $Cc^{\frac{1}{4}} + Cccppxx =$   
 $(A - \int X dx)ppx^{\frac{1}{4}}$ , ex qua fit  
 $P = \frac{cc\sqrt{C}}{\sqrt{(A - \int X dx)x^{\frac{1}{4}} - Cccxx}} = \frac{cc\sqrt{C}}{x\sqrt{(A - \int X dx)xx - Ccc}}$   
 seu  $dy = \frac{ccdx\sqrt{C}}{x\sqrt{(A - \int X dx)xx - Ccc}}$ , quæ eadem æqua-  
 tio etiam per Methodum directam invenitur.

13. Ex his igitur casibus perfectissimus consensus principii hic  
 stabiliti cum veritate eluet: utrum autem iste consensus in casi-  
 bus imagis complicatis locum quoque sit habiturus, dubium su-  
 peresse potest. Quamobrem quam late pateat istud prin-  
 cipium diligentius erit investigandum, quo plus ipsi non  
 tribuatur quam ejus natura permittit. Ad hoc explicandum,  
 omnis motus projectorum in duo genera distribui debet; quo-  
 rum altero celeritas corporis, quam in quavis loco habet, a so-  
 lo situ pendet; ita ut, si ad eundem situm revertatur, eandem  
 quoque sit recuperatum celeritatem; quod evenit, si corpus  
 vel ad unum vel ad plura centra fixa trahatur viribus, quæ sint ut  
 functiones quæcunque distantiarum ab his centris. Ad alterum  
 genus refero eos projectorum motus, quibus celeritas corporis  
 per solum locum in quo hæret non determinatur; id quod usu  
 venit, vel si centra illa ad quæ corpus sollicitatur fuerint mobi-  
 lia, vel si motus fiat in medio resistente. Hac facta divisio-  
 ne; notandum est, quoties motus corporis ad prius genus per-  
 tineat, hoc est, si corpus non solum ad unum sed ad quocunque  
 centra fixa sollicitetur viribus quibuscunque, toties in motu hoc  
 summam omnium motuum elementarium fore minimam.

14. Hoc ipsum autem postulat indoles Propositionis: dum  
 enim, inter datos terminos, ea quæritur curva, in qua sit  $\int ds \sqrt{v}$   
 minimum; eo ipso assumitur, celeritatem corporis in utroque  
 termino

termino eandem esse, quæcunque curva corporis viam constitutæ. Quocunque autem fuerint centra virium fixa, celeritas corporis in quovis loco  $M$ , exprimitur functione determinata ambarum variabilium  $C P = x$ , &  $PM = y$ . Sit igitur  $v$  functio quæcunque ipsarum  $x$  &  $y$ , ita ut sit  $dv = Tdx + Vdy$ ; atque videamus, an principium nostrum veram exhibiturum sit projectoriam corporis. Cum autem sit  $dv = Tdx + Vdy$ ; corpus perinde movebitur, ac si sollicitetur in  $M$  a duabus viribus, altera  $T$  in directione abscissæ  $x$  parallela, altera vero  $V$  in directione parallela applicatis  $y$ , ex quibus oritur vis tangentialis  $= \frac{Tdx + Vdy}{ds}$ , & vis normalis  $= \frac{Vdx + Tdy}{ds}$ . Debet autem,

ex natura motus liberi, esse  $\frac{2v}{r} = \frac{Vdx + Tdy}{ds} = \frac{V + Tp}{\sqrt{1+pp}}$ ;

ad quam æquationem si Methodus maximorum ac minimorum deducat, erit utique principium nostrum veritati conforme.

15. Cum igitur, per hoc principium, debeat esse  $\int dx \sqrt{v(1+pp)}$  minimum, differentietur quantitas  $\sqrt{v(1+pp)}$ , atque, ob  $dv = Tdx + Vdy$ , orientur:

$\frac{Tdx \sqrt{1+pp}}{2\sqrt{v}} + \frac{Vdy \sqrt{1+pp}}{2\sqrt{v}} + \frac{pdःp \sqrt{v}}{\sqrt{1+pp}}$ , ex quo obtinetur pro curva quæsita sequens æquatio, secundum præcepta tradita,

$$\frac{Vdx\sqrt{1+pp}}{2\sqrt{v}} = d \cdot \frac{p\sqrt{v}}{\sqrt{1+pp}} = \frac{dp\sqrt{v}}{(1+pp)^{3/2}} + \frac{p(Tdx + Vdy)}{2\sqrt{v}(1+pp)}$$

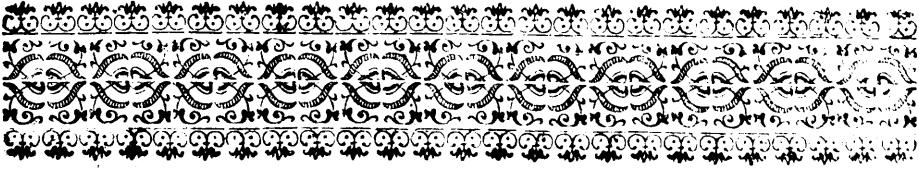
$$\text{seu } \frac{dp\sqrt{v}}{(1+pp)^{3/2}} = \frac{Tpdx - Vdx}{2\sqrt{v}(1+pp)}. \text{ At est radius osculi in}$$

$$M = \frac{(1+pp)dx\sqrt{1+pp}}{dp}; \text{ qui si ponatur } = r, \text{ erit}$$

$$\frac{2v}{r} = \frac{Tp - V}{\sqrt{1+pp}}; \text{ omnino uti per Methodum directam inventur. Dummodo ergo vires sollicitantes ita fuerint comparatae, ut ea reduci queant ad duas vires } T \& V, \text{ secundum directiones coordinatis } x \& y \text{ parallelas sollicitantes, quæ sint ut functiones quæcunque harum variabilium } x \& y, \text{ tum semper in curva.}$$

curva descripta erit motus corporis per omnia elementa collectus minimus.

16. Tam late ergo hoc principium patet, ut solus motus a resistentia medii perturbatus excipiens videatur; cuius quidem exceptionis ratio facile perspicitur, propterea quod hoc casu corpus per varias vias ad eundem locum perveniens non eandem acquirit celeritatem. Quamobrem, sublata omni resistentia in motu corporum projectorum, perpetuo haec constans proprietas locum habebit, ut summa omnium motuum elementarium sit minima. Neque vero haec proprietas in motu unius corporis tantum cernetur, sed etiam in motu plurium corporum conjunctim; quae quomodounque in se invicem agant, tamen semper summa omnium motuum est minima. Quod, cum hujusmodi motus difficulter ad calculum revocentur, facilius ex primis principiis intelligitur, quam ex consensu calculi secundum utramque Methodum instituti. Quoniam enim corpora, ob inertiam, omni status mutationi reluctantur; viribus sollicitantibus tam parum obtemperabunt, quam fieri potest, siquidem sint libera; ex quo efficitur, ut, in motu genito, effectus a viribus ortus minor esse debeat, quam si ullo alio modo corpus vel corpora fuissent promota. Cujus ratiocinii vis, etiamsi nondum satis perspiciatur; tamen, quia cum veritate congruit, non dubito quin, ope principiorum senioris Metaphysicæ, ad majorem evidenter evehi queat; quod negotium aliis, qui Metaphysicam profitentur, relinquo,



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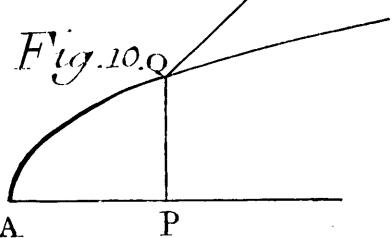
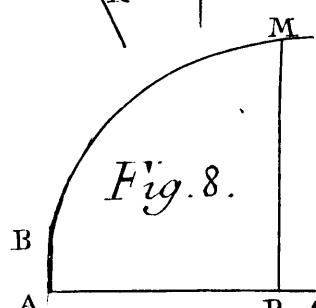
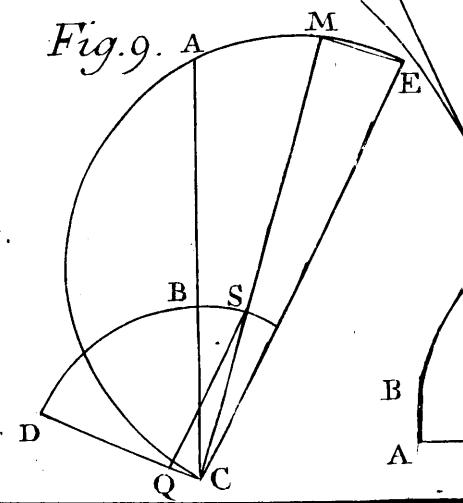
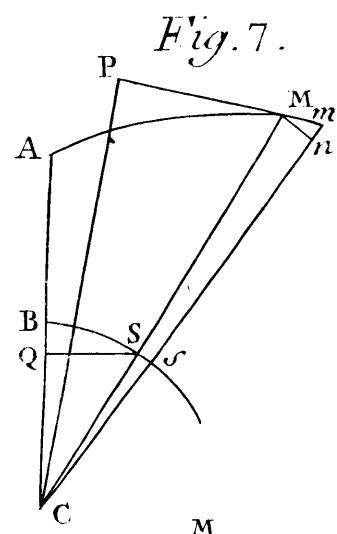
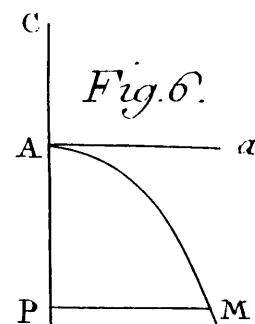
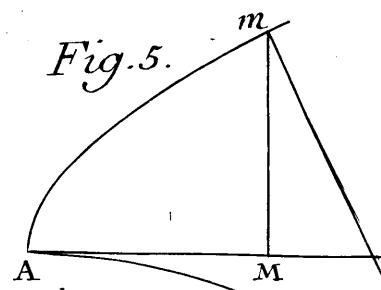
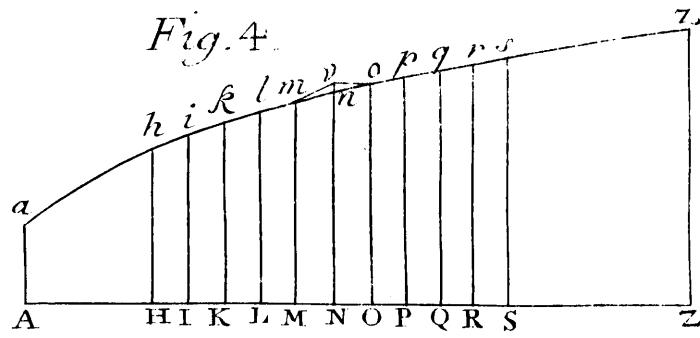
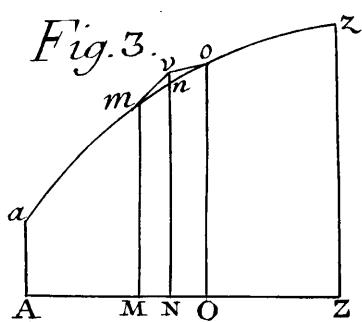
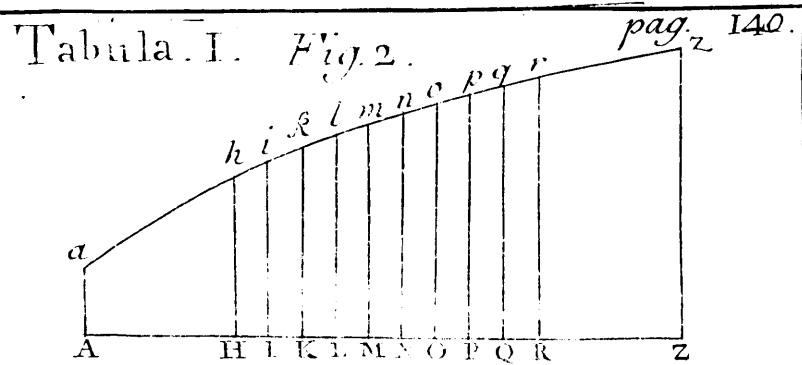
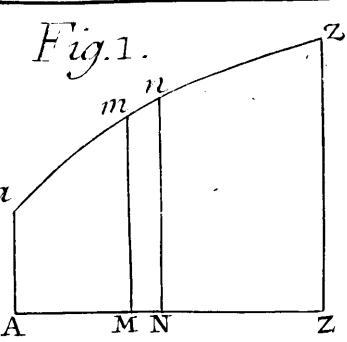
*Monitum ad Bibliopegam.*

Tabulæ omnes Figurarum ad calcem compingantur, vel duæ priores post paginam 244 inferantur, posteriores tres ad calcem ponantur.

*Avis au Relieur.*

Il placera les cinq Planches de Figures à la fin du Livre, ou bien, il mettra les deux premières à la page 244, & les trois dernières à la fin.







Tabula II.

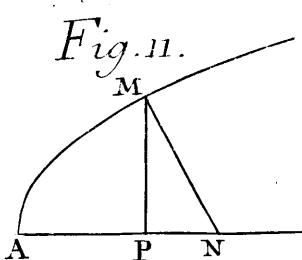


Fig. 14.

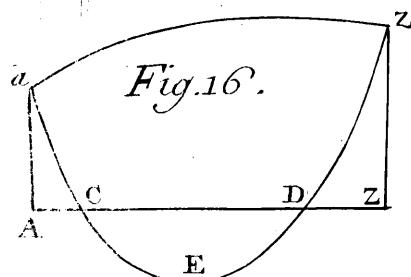
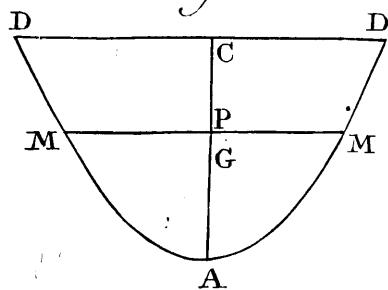


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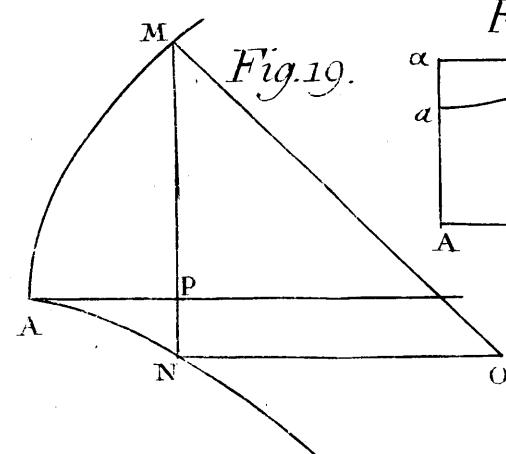


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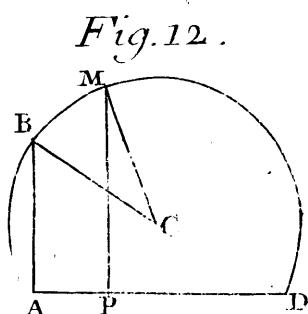


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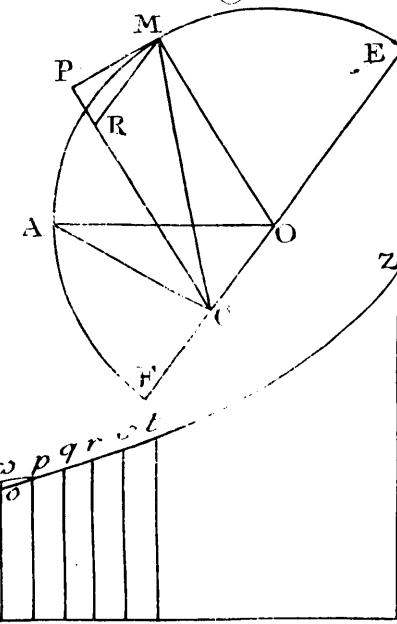


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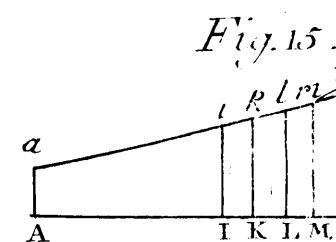


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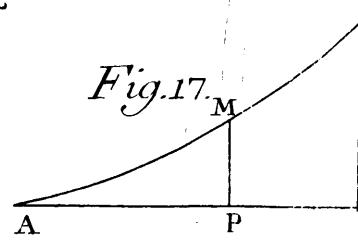


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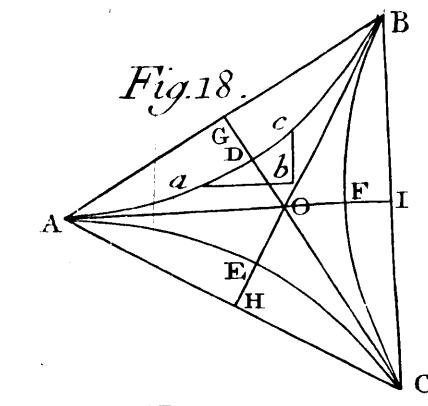


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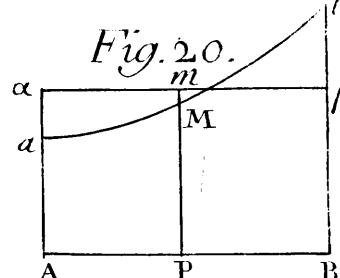


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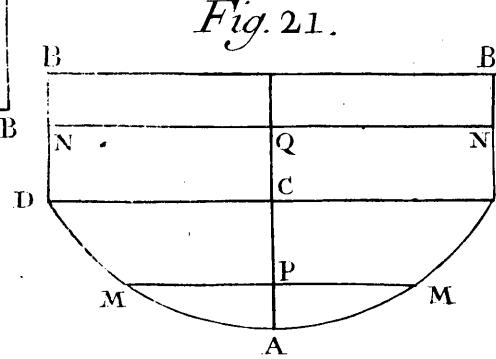
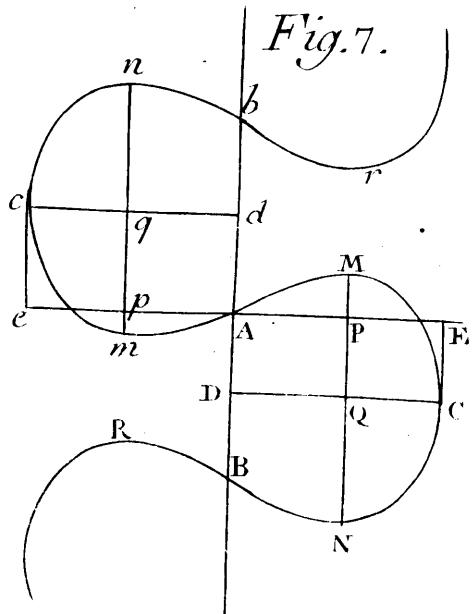
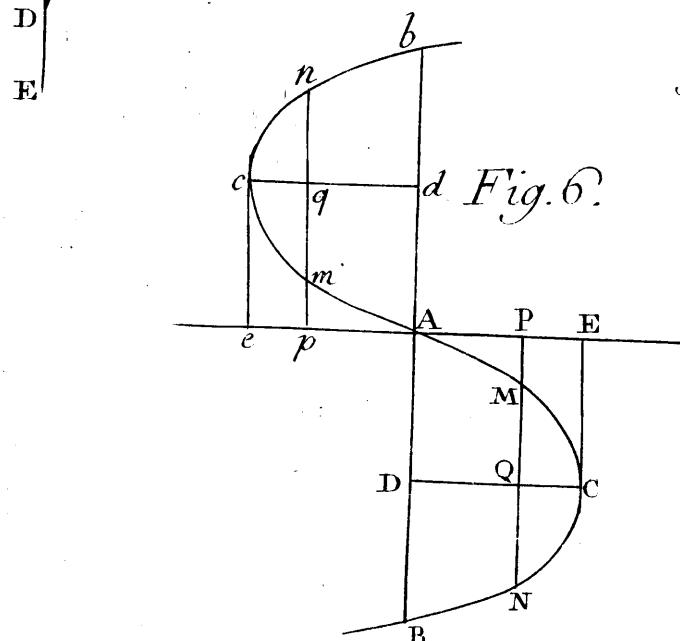
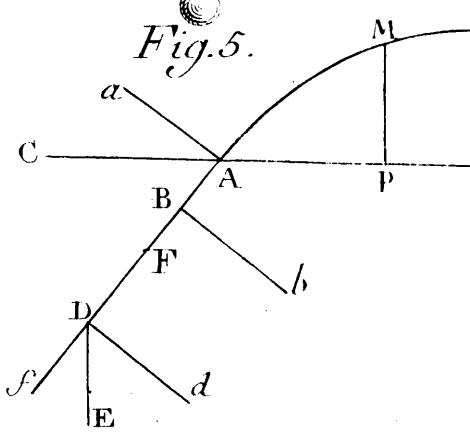
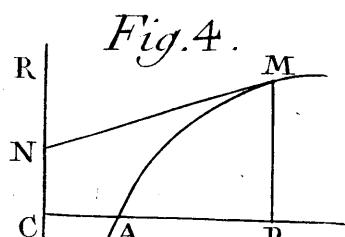
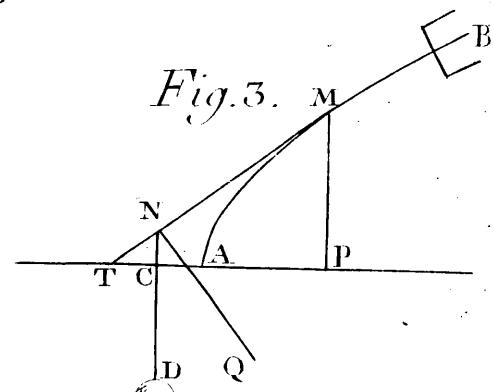
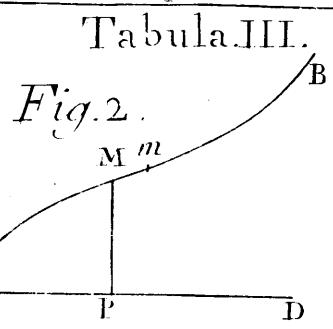
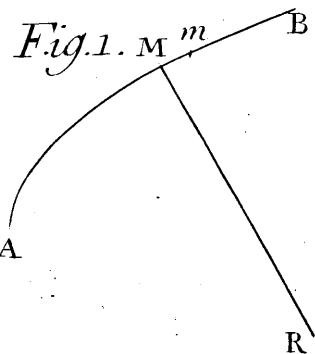


Fig. 21.



## Tabula III.

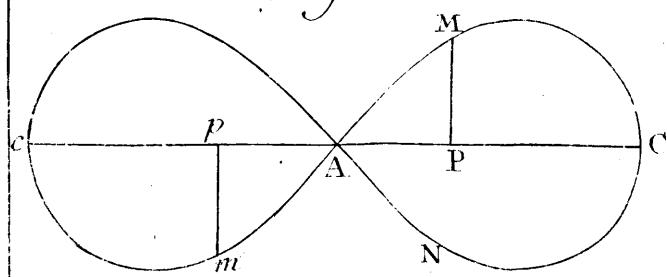
Additamentum.





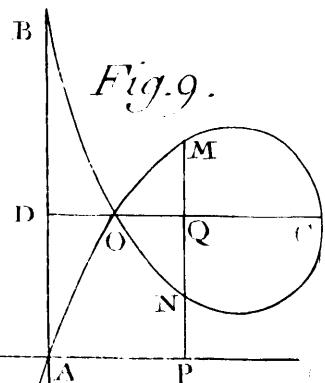
Tabula IV.

*Fig. 8.*

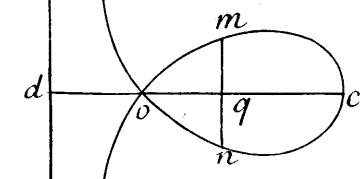
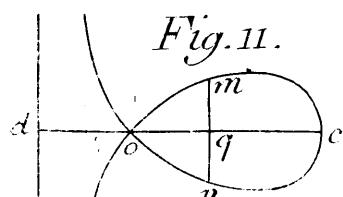


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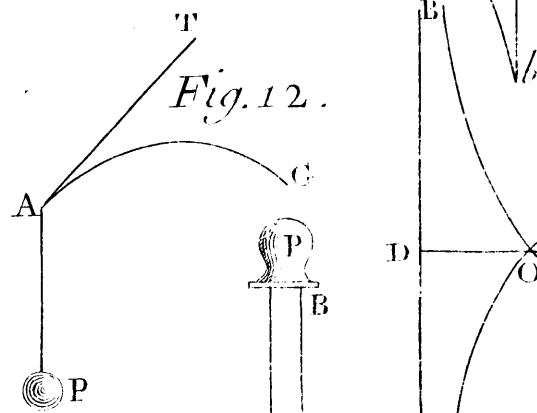
*Fig. 9.*



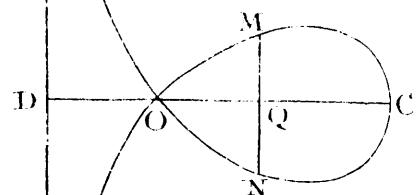
*Fig. 11.*



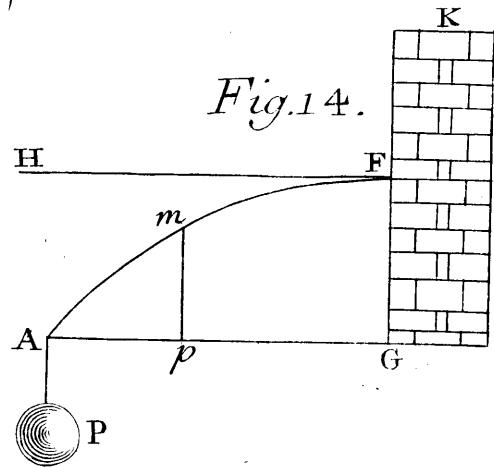
*Fig. 12.*



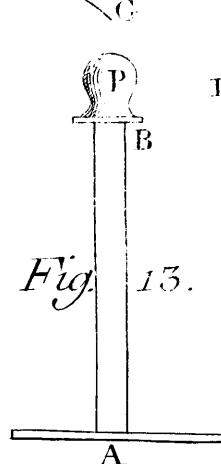
*Fig. 10.*



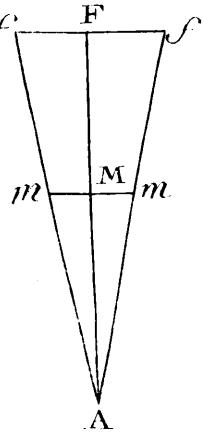
*Fig. 14.*



*Fig. 13.*



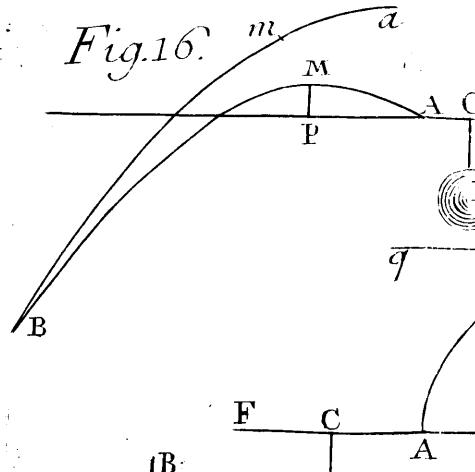
*Fig. 15.*





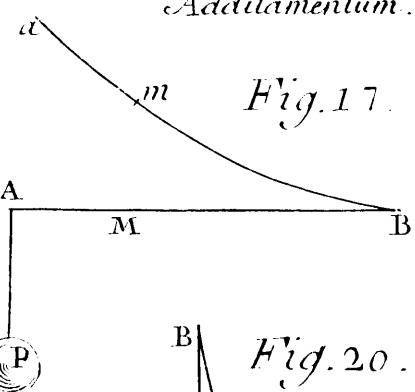
Tabula V.

*Fig. 16.*



Additamentum.

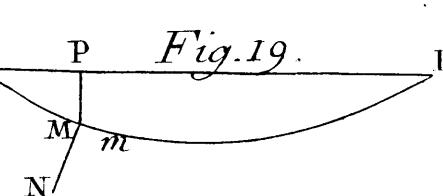
*Fig. 17.*



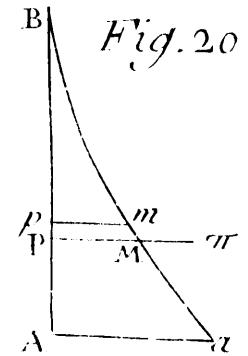
*Fig. 21.*



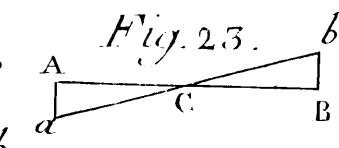
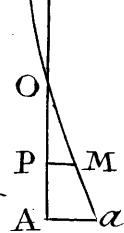
*Fig. 19.*



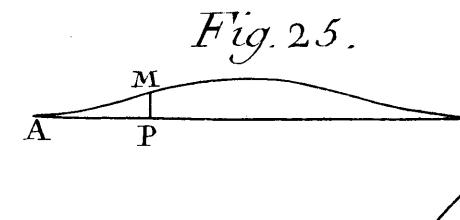
*Fig. 20.*



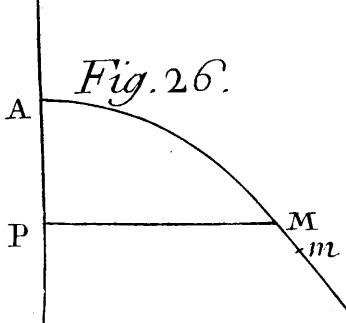
*Fig. 22.*



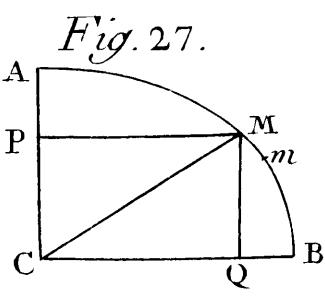
*Fig. 25.*



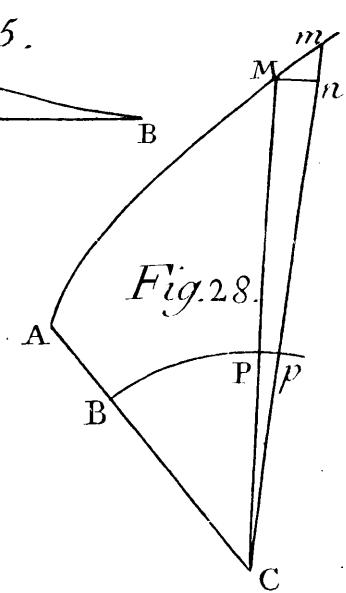
*Fig. 26.*



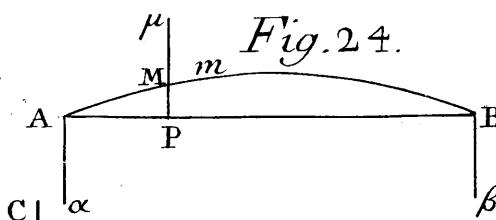
*Fig. 27.*



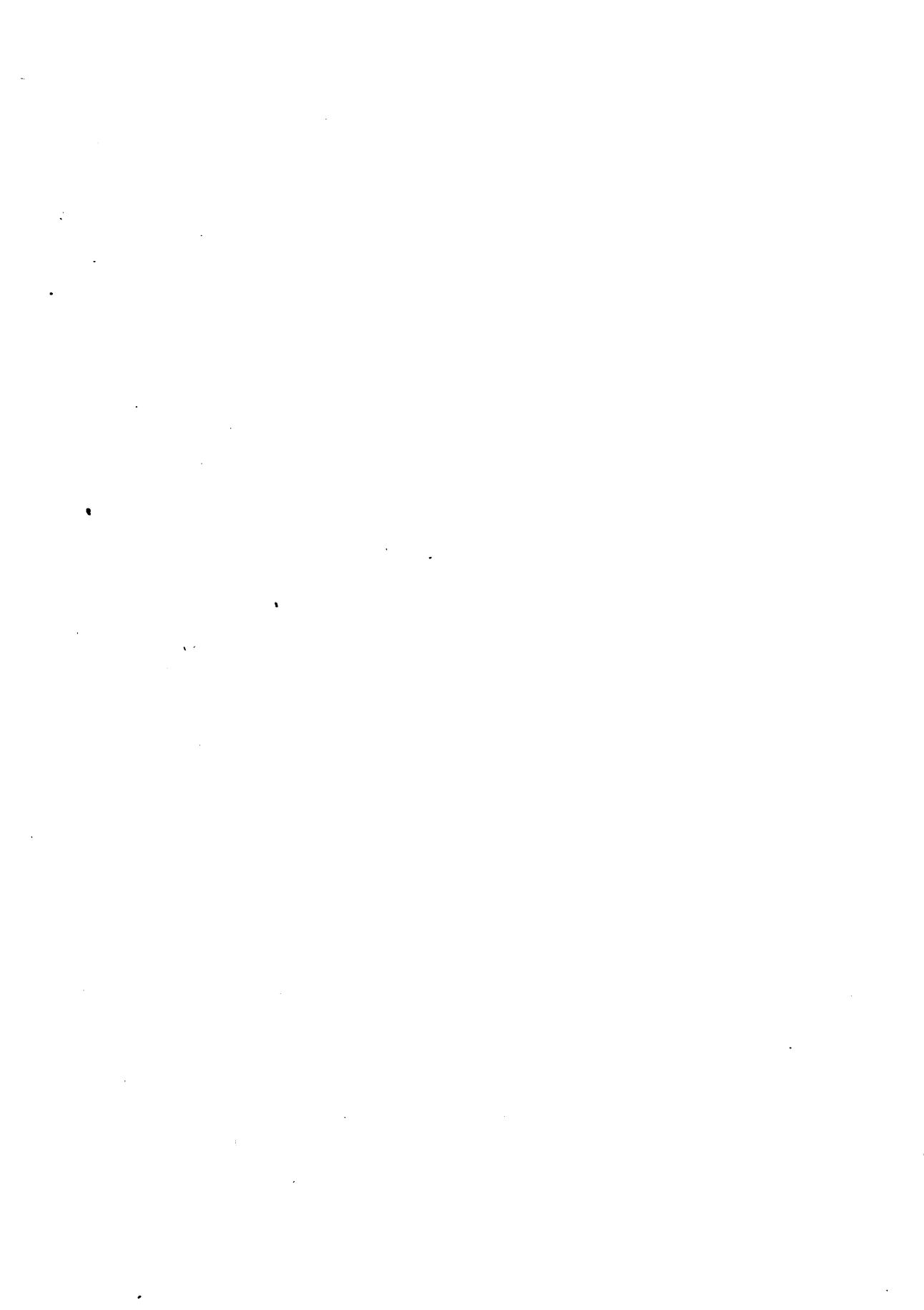
*Fig. 28.*



*Fig. 24.*









7.21  
26.2